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TWISTOR-LIKE APPROACH IN THE GREEN — SCHWARZ D = 10 SUPERSTRING THEORY

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The Lagrangian and Hamiltonian mechanics of a recently proposed twistor-like Lorentz harmonic formulation of the $D = 10$, $N = IIB$ Green — Schwarz superstring are discussed. The equations of motion are derived and the classical equivalence of this formulation to the standard one is proved. Presented is the complete set of the covariant and irreducible first-class constraints generating the gauge symmetries of the theory, including κ-symmetry. The algebra of all gauge symmetries and symplectic structure characterizing the set of second-class constraints are derived. Thus, basis for the covariant BRST-BFV quantization of $D = 10$ superstring in the twistor-like approach is built.

Изучаются лагранжева и гамильтонова механики $D = 10$, $N = IIB$ суперструны Грина — Шварца в твисторном подходе. Твисторные переменные реализуются в форме спинорных лоренцевых гармоник. Выводятся уравнения движения суперструны и доказывается классическая эквивалентность твисторной и стандартной формулировок теории. Строится полный набор неприводимых, ковариантных связей первого рода, генерирующих калибровочные симметрии действия суперструны, включая κ-симметрию, и приводится их алгебра. Представляется симплектическая структура алгебры ковариантных связей второго рода. Обсуждается процедура ковариантного БРСТ-ЮФВ десятимерной суперструны.

1. INTRODUCTION

Superstrings in $D = 10$ [1,2,3] are discussed as the possible basis for building the selfconsistent quantum theory of gravity and the Unified theory of all the interactions. However, its covariant quantization is hampered by the problem of κ-symmetry covariant description because this fermionic symmetry [4] is infinitely reducible in the standard superstring formulation [1,2]. Unfortunately, the existing modern schemes [5—7] of covariant quantization have been developed only for the systems with the finite level of the constraint reducibility. (Remember, that such a problem appears already in the superparticle theory [8,3]).

The progress in solving the problem of covariant quantization is necessary for the correct choice of the superstring ground state, among the infinite number of solutions for $D = 10$ superstring compactification. As a result the infinitely many different effective 4-dimensional theories have appeared instead of the unique 10-dimensional one [9].

One way to solve the problem of covariant superstring quantization is to use the fact that the reducibility level of the symmetries is not invariant under possible reformulations of the theory [5—7]. In other words, two classically equivalent theories may have different level of reducibility of their symmetries. Thus some formulation of superstring theory, which includes auxiliary variables and is classically equivalent to the standard formulation [1—3], may have either finite level of the reducibility of κ-symmetry or even irreducible κ-symmetry*.

This way has been opened in the pioneer works of Nissimov, Pacheva and Solomon [14—16]. They have extended the phase space of $D = 10$, $N = 2$ Green — Schwarz superstring by adding the vector $SO(1,9)/[SO(1,1) \otimes SO(8)]$ harmonic variables $(u_m^{[\pm 2]}, u_m^{(i)})$ (see [13]) with two light-like vectors $u_m^{[\pm 2]}$ being replaced by the bilinear combinations of the $D = 10$ bosonic spinors $v^{\alpha\pm}$: $u_m^{[\pm 2]} = v^{\alpha\pm}\sigma_{m\alpha\beta}v^{\beta\pm}$.

The characteristic feature of the approach [14—17] is the formulation of the action functional in the Hamiltonian formalism with using the Lagrange multipliers method. The «harmonic» variables $(v^{\alpha\pm}, u_m^{(i)})$ and the momentum degrees of freedom canonically conjugated to them are involved into the action principle through the constraints which are chosen in such a way, that the additional variables are pure gauge ones. Thus equivalence of the «harmonic» superstring formulation [14—16] with the standard Green — Schwarz one is reached.

*Another way consists in attempts to extend the quantization scheme, developed by Batalin and Vilkovisky [6], to the case of systems with infinitely reducible symmetries (see [9—12] and Refs. therein). Such extensions use an infinitely reducible gauge-fixing conditions and produce free type effective actions including infinite number of fields for superparticles and superstrings.

However, the straightforward extension of the BV prescription [6] for the systems with infinitely reducible constraints leads to the well-known troubles [10,11]. So, the cohomologies of the superparticle BRST operator, calculated in this way differ from the state spectrum of the Brink — Schwarz superparticle obtained from the quantization in the light-cone gauge (see [10,11]). To achieve the correct BRST cohomology (i.e., state spectrum) it is necessary to modify not only BV-quantization prescription, but also the initial superparticle or superstring formulation. However, after this step, the second way is reduced to a variant of the first one.

The use of these variables permits Nissimov, Pacheva and Solomon to solve the problem of the covariant decomposition of the Grassmannian constraints into the irreducible first and second class ones. The second class fermionic constraints were transformed into the first class constraints using the introduced auxiliary fermionic variables, and the covariant quantization of the Brink — Schwarz superparticle and Green — Schwarz superstring theories was carried out [14—16] (see also [17]).

In parallel, the twistor approach [58] to superparticle and superstring theories has been developed [24—40, 42,43,45,46]. It is closely related with the approach of Nissimov, Pacheva and Solomon, in particular, both approaches use bosonic spinor variables as the auxiliary ones. However, the twistor approach puts forward the new concept explaining the nature of the set of auxiliary bosonic spinor variables necessary for the covariant decomposition of the Grassmannian constraints of superparticle or superstring theories. This concept proposed in Refs. [28—30] interprets the chosen bosonic spinor variables as the «superpartners» of the target superspace Grassmannian coordinate field $\theta^{\alpha I}$ with respect to worldsheet supersymmetry.

Such a treatment of the bosonic spinor variables reduces the arbitrariness in their choice and, in particular, fixes their number to be equal to $N(D-2)$, where N is the number of the target space supersymmetries and D is the dimension of target space-time.

On the base of the twistor approach the «mysterious» κ-symmetry is presented as the nonlinearly realized worldsheet supersymmetry when all auxiliary fields are excluded using their equations of motion [28—30].

In the frame of the superfield realization of the twistor approach the infinitely reducible κ-symmetry with algebra being closed only on mass shell is replaced by the local world-sheet supersymmetry transformations [28,29,30], which are irreducible and have the algebra closed off mass shell. So, the twistor approach seems to be a relevant base for the covariant superstring quantization, alternative to that developed in Refs. 14,15,16,17.

The doubly supersymmetric superfield action functionals have been proposed for the superparticle and heterotic superstring in $D = 3, 4, 6, 10$ [28—32], [34—40] as well as for $D = 3$, $N = 2$ Green — Schwarz superstring [59]. There are some problems in the construction of such superfield action functionals for $N = 2$ Green — Schwarz superstrings in $D = 4, 6, 10$ [59]), and, up to now, this problems is open. Nevertheless, the component twistor formulation [19,23,46] exists for these cases. These formulations are related to the discussed superfield ones rewritten in terms of components, when all the auxiliary variables, except for the bosonic spinor ones, are removed from the action using algebraic motion equations. Therefore, in twistor-like component superstring formulations [19,23,46] the world-sheet supersymmetry is realized nonlinearly, i.e., is represented as a κ-symmetry, and its algebra is closed on the mass shell only. However, the κ-symmetry remains irreducible in this formulation, and the number of the auxiliary bosonic spinor variables (twistors) is conserved to be the same as that in superfield ones. Hence, the formulations [19,23,46] still give the possibilties to investigate the machinery of the twistor approach in solving the problems related to the task of the covariant superstring quantization.

$D = 10$, $N = IIB$, supersrting formulation [23,46], being invariant under the (nonlinearly realized) extended local $n = (8, 8)$ world-sheet supersymmetry, includes in its configurational space two sets of auxiliary Majorana — Weyl bosonic spinor fields (twistor components) $v^{+}_{\alpha A}(\tau, \sigma)$ and $v^{-}_{\alpha \dot{A}}(\tau, \sigma)$ ($\alpha = 1,\ldots,16$; $A = 1,\ldots,8$; $\dot{A} = 1,\ldots,8$) taking their values in 8-dimensional s- and c-spinor representations of the «transverse» $SO(8)$ group. These twistor components are the superpartners of the Majorana — Weyl Grassmannian spinors $\theta^{\alpha 1}(\tau, \sigma)$ and $\theta^{\alpha 2}(\tau, \sigma)$ under the discussed world-sheet supersymmetry transformations.

Comprising the considered component twistor-like formulation with the one proposed by Nissimov, Pacheva and Solomon [14,15,16,17] we conclude that they differ not only in the form of the action functionals, but also in the sets of the auxiliary bosonic spinor fields. More exactly one may say that the additional twistor variables in the set [20,21,23] can be obtained by taking the square root of the transverse vector harmonic variables $u^{(i)}_{m}$ belonging to the NPS set [14—17]. In other worlds, the harmonic fields [14—17] are composite objects constructed from the twistor variables [20,21,23]. The importance of the latter difference for the problem of the covariant superstring quantization may be shown only by further investigations of the classical and quantum dynamics of supestring in the twistor approach. Now we note only that the square root extraction operation leads to nontrivial consequences in many cases [51].

Note that the auxiliary spinor variables similar to the discussed twistor variables have been previously used by Wiegmann [57] for the description of $N = 1$, $D = 10$ heterotic and Neveu — Schwarz — Ramond fermionic string in the covariant light-cone gauge. The paper [57] is closely related to the Lund — Regge geometric approach [60] and, especially, to its gauge interpretation [61], where the 2-dimensional $SO(1,1)$ and $SO(D-2)$ gauge fields and the Cartan embedding forms used in [57] have been introduced. However, Wiegmann does not consider the problem of building the covariant Hamiltonian formalism for the original heterotic string phase space extended by the addition of the twistor variables in an arbitrary gauge. Instead of it the author of [57] excludes original physical variables of the heterotic string

$\theta^{\alpha}(\tau, \sigma)$, i.e., the Grassmannian target space spinor coordinates, by means of functional integration. On the other hand, the original phase space of the heterotic string reduced in such a way is extended by the addition of the effective gauge fields [61] generated by the differential forms of embedding. As a result the Hamiltonian structures of the twistor [22,23,46] and effective [57] actions differ in principle.

Taking into account all the above-mentioned reasons we regard the investigation of the Lagrangian and Hamiltonian structures of the $D = 10$, $N = IIB$ Green — Schwarz superstring in the component twistor-like formulation [22,23,46] as a problem to be paid attention to. This is just the problem suggested for studying in the present paper.

Here we follow the line of papers [18—23, 46] and realize the twistor variables for $D = 10$, $N = IIB$ Green — Schwarz superstring as the pure spinor Lorentz harmonics which parametrize the $SO(1,9)/[SO(1,1) \otimes SO(8)]$ coset. These harmonics are obtained by taking the square root of the basic vectors of the moving Cartan repere attached to the superstring world-sheet. Newman and Penrose were the first to consider this interpretation of the twistor's components for $D = 4$ [44].

In papers [18,19,62,42,43] the Newman — Penrose dyades were used for the description of the massless superparticles, null superstrings and null supermembranes. In particular, in these papers shown was the principle role of the component twistor formulation for the action of null super-p-branes (i.e. massless superparticles for $p = 0$, null superstring for $p = 1$, null super-p-brane for $p = 2$) in 4-dimensional space-time for the solution of the problem of the covariant contsraint splitting and their conversion [7] into the Abelian first class constraints. As a result of the component twistor approach the problem of the covariant BRST-BFV quantization of null super-p-branes in $D = 4$ was solved [42,43].

In the case $D = 4$ the Newman — Penrose dyades are used for building the vector fields $u^{(n)}(\tau, \sigma^M)$ of the Cartan moving repere (an isotropic tetrade [44]) attached to the world hypersheet of (null) super-p-brane. And the twistor-like null super-p-brane action is the first order form functional constructed using the composed vector from these moving frame set.

This observation leads to the generalization of the Lorentz-harmonic approach [18,19,42,43] to the description of superstring and other extended supersymmetric objects (for example, supermembrane) in higher dimensions D [22,23,45,46]. The proposed generalization implies the necessity of the consideration of the D-dimensional spinor harmonics as generalized «dyades». Therefore, if the first order form action with auxiliary vector variables is known, the problem of the twistor-harmonic description of the superstring (and super-p-branes) imbedded into the D-dimensional space-time is reduced to constructing the realization of the Cartan repere (moving frame system) $u_m^{(n)}(\tau, \sigma^M) \equiv u_m^{(n)}(\xi^\mu)$ in terms of spinor $2^{[D/2]} \times 2^{[D/2]}$ harmonic matrix

$$v_\alpha^a \in \text{Spin}\,(1, D-1), \qquad \alpha = 1,\ldots,2^\nu; \qquad a = 1,\ldots,2^\nu \tag{1.1}$$

with $\nu = [D/2]$ or $(D-2)/2$ for Majorana — Weyl spinors in $D = 10$ (mod8) [18—23], [42,43,45,46].

But such task can be solved easily. The orthonormal repere

$$u_m^{(n)} u^{m(l)} = \eta^{(n)(l)} = \text{diag}\,(1, -1,\ldots,-1) \tag{1.2}$$

belongs to the $SO(1, D-1)$ group. The double covering of this group is Spin $(1, D-1)$. Thus the connection of the repere $u_m^{(n)}$, with harmonic variable matrix v_α^a is defined by means of the «square root» type universal relation

$$u_m^{(n)} \equiv 2^{-\nu} v_\alpha^a \,(C\Gamma_m)^{\alpha\beta}\, v_\beta^b\, (\Gamma^{(n)} C^{-1})_{ab}\,. \tag{1.3}$$

As the result of Eq.(1.1), the relation (1.3) may be rewritten in the following forms

$$u_m^{(n)}\, (\Gamma^m C^{-1})_{\alpha\beta} = v_\alpha^a\, (\Gamma^{(n)} C^{-1})_{ab}\, v_\beta^b\,, \tag{1.4a}$$

$$u_m^{(n)}\, (C\Gamma_{(n)})^{ab} = v_\alpha^a\, (C\Gamma_m)^{\alpha\beta}\, v_\beta^b\,. \tag{1.4b}$$

This is possible because, in general case, the following identities

$$\text{Sp}\,(v^T C\Gamma_{m_1\ldots m_k}\, v\, \Gamma^{(n)} C^{-1}) = 0, \qquad \text{(when } k > 1), \tag{1.5a}$$

$$\text{Sp}\,(v^T C\Gamma_m v\, \Gamma^{(n_1)\ldots(n_k)} C^{-1}) = 0, \qquad \text{(when } k > 1) \tag{1.5b}$$

are satisfied for the matrix $v_\alpha^a \in$ Spin $(1, D-1)$ (1.1)

The relations (1.1)—(1.3) are the basis of the twistor-like Lorentz harmonic approach to super-p-brane theories.

The discussed approach has been named harmonic one, because the condition (1.1) is not realized by expressing the matrix v_α^a as an exponential function of the Spin (1.9) Lie algebra generators; it is realized by the requirement, that v_α^a matrix should satisfy a set of the so-called harmonicity conditions

$$\Xi_M(v) = 0. \tag{1.6}$$

These conditions provide the satisfaction of all the relations (1.5a), (1.5b), as well as of the relations (1.3), by definition. And the use of them is more convenient, than the use of the straightforward exponential parametrization (this fact was evident already in the case of the compact space $SU(2)/U(1)$ [47]).

For the case of $D = 10$ superstring, the matrix v_α^a has one $SO(1,9)$ Majorana — Weyl spinor index $\alpha = 1,\ldots,16$ and one 16-dimenisonal index a of the right product of the $SO(1,1)$ and $SO(8)$ group. The latter may be decomposed into the two $SO(1,1) \in SO(8)$ invariant subsets of indices $a = (\overset{+}{A}, \overset{-}{\dot{A}})$. Here $A = 1,\ldots,8$ and $\dot{A} = 1,\ldots,8$ are the indices of (s) and (c) spinor representations of $SO(8)$ and $\pm$ symbols denote the Weyl weight under the transformations from $SO(1,1)$ group (which is identified with the Lorentz group of the world-sheet in the formulation [22,23,46]). Correspondingly, the 16×16 harmonic matrices v_α^a are decomposed into the two 16×8 blocks [20,21]

$$v_\alpha^a = (v_{\alpha A}^+, v_{\alpha \dot{A}}^-), \tag{1.7}$$

which transform covariantly under the left $SO(1,9)$ and right $SO(1,1) \otimes SO(8)$ transformations.

The corresponding $SO(1,9)_L \otimes [SO(1,1) \otimes SO(8)]_R$ invariant splitting of the composed Cartan repere (1.3) has the following form

$$u_m^{(n)} \equiv (u_m^{(0)}, u_m^{(1)}, \ldots, u_m^{(9)}) \equiv (u_m^{(I)}, u_m^{(i)}), \tag{1.8a}$$

$$u_m^{(I)} = (u_m^{(0)}, u_m^{(9)}) = \left(\frac{1}{2}(u_m^{[+2]} + u_m^{[-2]}, \frac{1}{2}(u_m^{[+2]} - u_m^{[-2]})\right), \tag{1.8b}$$

$$u_m^{(i)} = (u_m^{(1)}, \ldots, u_m^{(8)}), \tag{1.8c}$$

where the vectors $u_m^{[\pm 2]}$, $u_m^{(i)}$ are defined by the relations [20,21]

$$u_m^{[+2]} = \frac{1}{8}(v_A^+ \tilde{\sigma}_m v_A^+) \equiv \frac{1}{8} v_{\alpha A}^+ \tilde{\sigma}_m^{\alpha\beta} v_{\beta A}^+, \tag{1.9a}$$

$$u_m^{[-2]} = \frac{1}{8}(v_{\dot{A}}^- \sigma_m v_{\dot{A}}^-), \tag{1.9b}$$

$$u_m^{(i)} = \frac{1}{8}(v_A^+ \tilde{\sigma}_m v_{\dot{A}}^-)\gamma_{A\dot{A}}^i. \tag{1.9c}$$

The contracted $SO(1,9)$ spinor indices are omitted in (1.9b), (1.9c) and in the following formulas.

The harmonicity conditions (1.6) have the following form in the discussed case [20—23,46]

$$\Xi_{m_1 \ldots m_4} = u^{m(n)} \eta_{(n)(l)} \Xi^{(l)}_{m_1 \ldots m_4 m} = 0, \tag{1.10a}$$

$$\Xi_0 \equiv u^{[-2]}_m u^{m[+2]} - 2 = 0, \tag{1.10b}$$

where the expression

$$\Xi^{(n)}_{m_1 \ldots m_5} \equiv \mathrm{Sp}\,(v^T \tilde{\sigma}_{m_1 \ldots m_5} v \sigma^{(n)}) \equiv v^a_\alpha\, (\tilde{\sigma}_{m_1 \ldots m_5})^{\alpha\beta} v^b_\beta\, (\sigma^{(n)})_{ab} = 0 \tag{1.11}$$

vanishes as the consequence of Eqs. (1.10a) [46]. The last expression of the type (1.5a) vanishes identically because of the antisymmetric property of the matrix $(\tilde{\sigma}_{m_1 \ldots m_3})^{\alpha\beta}$ under the spinor index permutations.

The repere orthogonality conditions (1.2) are satisfied as the common consequence of the expressions (1.9), the conditions (1.10a) and the famous identity

$$\tilde{\sigma}^{\alpha\{\beta}_m \tilde{\sigma}^{\gamma\delta\}m} \equiv \frac{1}{3}\,(\tilde{\sigma}^{\alpha\beta}_m \tilde{\sigma}^{\gamma\delta m} + \text{cyclic permutations}\ (\alpha, \beta, \gamma)) = 0. \tag{1.12}$$

The normalization conditions for the composed repere (1.2), (1.9) are satisfied due to the harmonicity conditions (1.10a), (1.10b) and due to the identity (1.12).

Thus, the orthonormal repere in $D = 10$ space-time was constructed in terms of generalized dyades. So, after the construction of the first-order form superstring action using the auxiliary vector variables $n^{[\pm 2]}_m$, which belong to the moving frame system (repere), the twistor-like form of the superstring action can be achieved by the simple replacement of the $n^{[\pm 2]}_m$ by the composed vectors $u^{[\pm 2]}_m$ (1.9).

In such a way the action for $D = 10$, $N = IIB$ superstring was constructed [22,23].

Here we continue the program outlined in Refs. 22,23.

Lagrangian and Hamiltonian mechanics of the twistor-like Lorentz harmonic formulation of the superstring are constructed. The equations of motion are derived. The constraints decomposition onto the covariant and irreducible first and second class ones is carried out. We compute the algebra of the gauge symmetries of the theory in the Hamiltonian formalism and present the symplectic structure characterizing the set of the second class constraints. Thus we get all necessary information for the conversion of the second class constraints into Abelian first class ones (see [7]), forthcoming construction of the classical BRST charge and covariant quantization, which are the subjects of future works.

The paper is organized as follows.

To make clear the forthcoming description of superstring in twistor-like formulation we consider the bosonic string formulation with auxiliary vector variables in detail. This is done in Section 2, where the derivation of the motion equation and the construction of the Hamiltonian formalism for systems with harmonic variables are discussed using this simple example. For the reader convenience, the description is closed in this section.

In Section 3 we describe the twistor-like Lorentz harmonic superstring formulation [22,23] and discuss its equivalence to the standard one [1,3]. Here we derive all the equations of motion for the discussed superstring formulation.

Section 4 is devoted to the construction of the Hamiltonian formalism.

The primary constraints are derived and the so-called covariant momentum densities for the harmonic variable are introduced in Subsection 4.1. It is demonstrated that these momentum variables generate the current algebra of the $SO(1,9)$ on the Poisson brackets.

In Subsection 4.2 the Dirac prescription of the checking the constraint conservation during evolution is carried out, the covariant and irreducible first class constraints are derived.

In Section 5 the first class constraints are redefined. This redefinition leads to the simplification of the algebra generated by them on the Poisson brackets. Such algebra is presented in Subsection 5.1. The symplectic structure of the second class constraint system is derived in Subsection 5.3. The relation between the well-known Virasoro constraints and the reparametrization symmetry generators of the twistor-like formulation [22,23] is discussed in Subsection 5.3.

Our notations for the Majorana — Weyl spinor indices in $D = 10$ coincide with ones from Refs. 14,15 except for another choice of metric signature (see Eq. (1.2)).

2. BOSONIC STRING IN THE CARTAN MOVING FRAME FORMULATION

2.1. Action Principle and Equations of Motion. To make more clear the forthcoming description of superstring in twistor-like Lorentz-harmonic approach we consider the bosonic string formulation with the following action functional

$$S \equiv \int d^2\xi\, L(\xi) = \int d^2\xi\, e(\xi)\,(-(\alpha')^{-1/2} e^{\mu}_{f}\partial_{\mu}x^{m} n^{(f)}_{m} + c). \qquad (2.1)$$

This formulation uses two D-dimensional vector fields $n_m^{\{f\}} = (n_m^{(0)}, n_m^{(D-1)})$ belonging to the Cartan repere (moving frame system) $n_m^{(l)} = (n_m^{\{f\}}, n_m^{(i)})$* attached to the string world-sheet and defined by the orthonormality conditions

$$\Xi^{(n)(l)} \equiv n_m^{(n)} n^{m(l)} - \eta^{(n)(l)} = 0, \tag{2.2}$$

(where the Minkowski metric tensor $\eta^{(n)(l)} = \mathrm{diag}\,(+, -, \dots, -)$). Other additional set of the used auxiliary fields is the world-sheet zweinbein $e_\mu^f(\xi)$ $(\mu = (\tau, \sigma);\ f = 0, 1)$;

$$e_g^\mu e_\mu^f = \delta_g^f, \qquad e_\mu^f e_f^\nu = \delta_\mu^\nu, \qquad e \equiv \det\,(e_\mu^f). \tag{2.3}$$

The two-dimensional Lorentz group $SO(1,1)$ acts on the flat indices f, g of the zweinbein $e_\mu^f(\xi)$ as well as on the 2-valued index $\{f\}$ numbering the vectors from the set $n_m^{\{f\}}(\xi)$**. The basis of a two-dimensional vector space may be always chosen to be composed from two light-like vectors with the definite and opposite weights under the $SO(1,1)$ group. Thus it is convenient to work in terms of the light-like zweinbein components

$$e_\mu^f = \left(\frac{1}{2}\,(e_\mu^{|+2|} + e_\mu^{|-2|}),\ \frac{1}{2}\,(e_\mu^{|+2|} - e_\mu^{|-2|})\right),$$

$$e_f^\mu = \left(\frac{1}{2}\,(e^{\mu|-2|} + e^{\mu|+2|}),\ \frac{1}{2}\,(e^{\mu|-2|} - e^{\mu|+2|})\right),$$

$$e_\mu^{|+2|} e^{\mu|+2|} = 0 = e_\mu^{|-2|} e^{\mu|-2|},$$

$$e_\mu^{|+2|} e^{\mu|-2|} = 2 = e_\mu^{|-2|} e^{\mu|+2|},$$

$$\varepsilon^{\mu\nu} = \frac{1}{2}\, e\, (e^{\mu|+2|} e^{\nu|-2|} - e^{\mu|-2|} e^{\nu|+2|}), \tag{2.4}$$

$$(\varepsilon^{01} = -\,\varepsilon_{01} = 1),$$

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* The Latin letter l in the round brackets $n_m^{(l)}$ denotes the number of a vector from the moving frame set $n_m^{(0)}, n_m^{(1)}, \dots, n_m^{(D-1)}$. It is convenient to separate all the repere vectors $n_m^{(l)}$ into two sets $n_m^{(l)} = (m_m^{\{f\}}, n_m^{(i)})$, where $i = 1, \dots, D-2$ and $f = 0, 1$ (so $n_m^{\{0\}} = n_m^{(0)}$ and $n_m^{\{1\}} = n_m^{(D-1)}$).

**Thus the $SO(1,1)$ subgroup of the target space Lorentz group $SO(1,D\text{-}1)$ (acting on the repere variables matrix $n_m^{(l)}$ from the right) is identified with the Lorentz group of the string world-sheet in the discussed formulation (comprise with [22,23]).

$$g^{\mu\nu}=\frac{1}{2}(e^{\mu[+2]}e^{\nu[-2]}+e^{\mu[-2]}e^{\nu[+2]}),$$

$$\sqrt{-g}\equiv e, \qquad \delta^{\nu}_{\mu}=\frac{1}{2}(e^{[+2]}_{\mu}e^{\nu[-2]}+e^{[-2]}_{\mu}e^{\nu[+2]}), \tag{2.4}$$

$$\varepsilon_{\mu\nu}\, e^{\mu[-2]}e^{\nu[+2]}=2/e,$$

and light-like vectors

$$n^{(f)}_{m}=\left(\frac{1}{2}(n^{[+2]}_{m}+n^{[-2]}_{m}),\frac{1}{2}(n^{[+2]}_{m}-n^{[-2]}_{m})\right),$$

$$n^{[\pm 2]}_{m}=n^{(0)}_{m}\pm n^{(1)}_{m}=n^{(0)}_{m}\pm n^{(D-1)}_{m},$$

$$n^{[+2]}_{m}n^{m[+2]}=0=n^{[-2]}_{m}n^{m[-2]}, \qquad n^{[+2]}_{m}n^{m[-2]}=2 \tag{2.5}$$

(comprise with Eqs.(1.8b))*.

The variation of the action (2.1) with respect to the inverse zweinbeins e^{μ}_{f} gives the following relation

$$e^{f}_{\mu}(\xi)=\partial_{\mu}x^{m}n^{(f)}_{m}/c(\alpha')^{1/2}, \tag{2.6}$$

which is the simple expression for the souldary form $e^{f}(d\xi,\xi)=d\xi^{\mu}e^{f}_{\mu}(\xi)$ of the world-sheet, induced by embedding of the world-sheet into the D-dimensional Minkowski space-time. Taking into account Eq.(2.6), we may exclude the auxiliary zweinbein field from the action (2.1)

$$-(\alpha')^{1/2}e^{\mu}_{f}\partial_{\mu}x^{m}n^{(f)}_{m}=-2ce=-2\det(\partial_{\mu}x^{m}n^{(f)}_{m}), \tag{2.7}$$

$$S_{V-Z}=-(c\alpha')^{-1}\int d^{2}\xi\,\det(\partial_{\mu}x^{m}n^{(f)}_{m}). \tag{2.8}$$

The resulting action (2.8) coincides with one from Ref. 41, where the auxiliary vector fields from Cartan moving repere had been introduced for the first time for building string and superstring actions.

Thus the action (2.1) is the first order form representation for the «antisymmetric» action from Ref. 41.

Now let's discuss the relation of the discussed string formulation (2.1) with the standard Dirac — Nambu — Goto and Polyakov ones.

*In such form the coincidence of the repere variables $n^{(I)}_{m}$ with the vector harmonics from Ref.13 is evident. However, the repere variables were used for the first time for the string and superstring description in Ref.41.

It should be proved below, that the variation of the action (2.1) with respect to the auxiliary vector fields $n_m^{(f)}(\xi)$ leads to the following nontrivial equation

$$\partial_\mu x^m n_m^{(i)} = 0, \tag{2.9}$$

which means that the vectors $n_m^{(i)}$ are orthogonal to the string world-sheet. Eqs. (2.6), (2.9) and the completeness of the moving frame system

$$n_m^{(n)} \eta_{(n)(l)} n_p^{(l)} = \eta_{mp}$$

give possibility to express $\partial_\mu x^m(\xi)$ through $n_m^{(f)}(\xi)$

$$\partial_\mu x^m(\xi) = c(\alpha')^{1/2} e_\mu^g \eta_{gf} n_m^{(f)}, \tag{2.10a}$$

and vice versa

$$n_m^{(f)} = c^{-1}(\alpha')^{-1/2} \eta^{fg} e_g^\mu \partial_\mu x_m. \tag{2.11}$$

Taking into account Eqs. (2.7) and (2.11), as well as the definitions

$$e_f^\mu \eta^{fg} e_g^\nu \equiv g^{\mu\nu}, \qquad e = \sqrt{-g},$$

we may rewrite the action S (2.1) in the following form

$$S_p = -(2c\alpha')^{-1} \int d^2\xi \sqrt{-g}\, g^{\mu\nu} \partial_\mu x^m \partial_\nu x_m,$$

which is the known string action introduced in Ref. 48. From the other hand Eq. (2.10) leads to the following expression for the induced metric

$$g_{\mu\nu} = \partial_\mu x^m(\xi)\, \partial_\nu x_m(\xi)/c^2\alpha', \tag{2.10b}$$

which results in the relation

$$e = \sqrt{-g} = \det{}^{1/2}(\partial_\mu x^m(\xi)\, \partial_\nu x_m(\xi))/c^2\alpha'. \tag{2.10c}$$

The substitution of Eqs. (2.10a), (2.10c) into the functional (2.1) leads to the Dirac — Nambu — Goto action

$$S_{D-N-G} = -(c\alpha')^{-1} \int d^2\xi\, (\det(\partial_\mu x^m(\xi)\, \partial_\nu x_m(\xi)))^{1/2}.$$

At last, the variation of the action S (2.1) with respect to $x^m(\xi)$ gives the equation

$$\partial_\mu(e\, e_f^\mu n_m^{(f)}) = 0, \tag{2.12}$$

which may be rewritten in the standard form (see Ref. 48)

$$\partial_\mu(\sqrt{-g}\, g^{\mu\nu} \partial_\nu x^m) = 0, \tag{2.13}$$

using Eq. (2.11).

However, the derivation of Eq. (2.9), which is crucial for the conclusion presented above, is not so simple task. First of all note that the variational problem with respect to $n_m^{(l)}$ fields is the problem with conditional extreme due to the necessity to take into account the orthonormality conditions (2.2). It may be reformulated into the variational prblem with absolute extreme if we extend the action (2.1) by means of adding the conditions $\Xi^{(n)(l)}$ (2.2) with the corresponding Lagrange multipliers (see [41]). Another way for yielding the right motion equations is to restrict the class of admissible variations $\delta n_m^{(l)}$ by the variations which conserve the orthonormality conditions (2.2). The use of this method does not require the introduction of the Lagrange multipliers and seems to be simpler for the solution of the variational problems characterized by the sophisticated structure of constraints. The same method will be used below for studying $D = 10$ superstring dynamics in the twistor-like formulation [22,23].

2.2. Admissible Variations for Repere Variables. Let's discuss arbitrary set of D independent vector variables $n_m^{(l)}$ in D-dimensional space. The condition of the independence has the form $\det(n_m^{(l)}) \neq 0$. Thus the set of the variables $n_m^{(l)}$ discussed as $D \times D$ matrix belongs to the $GL(D, R)$ group. An arbitrary variation with respect to $n_m^{(l)}$

$$\delta = \delta n_m^{(l)} \partial / \partial n_m^{(l)} \tag{2.14a}$$

may be rewritten in the form

$$\delta = (n^{-1}\delta n)_{(k)}^{(l)} \, (n_m^{(k)} \partial / \partial n_m^{(l)}). \tag{2.14b}$$

In Eq. (2.14b) $(n^{-1}\delta n)_{(k)}^{(l)} \equiv (n^{-1})_{(k)}^{m} \, \partial n_m^{(l)}$ is the Cartan differential form, which is invariant under the left $GL(D, R)$ transformations. The differential operators $n_m^{(l)} \partial / \partial n_{m(k)}$, appeared in (2.14b), may be discussed as the covariant derivatives (see [47]) for the $GL(D, R)$ group.

Let's restrict the right $GL(D, R)$ transformations (acting on the numbers (l) of the vectors $n_m^{(l)}$) to be only from the Lorentz group $SO(1, D-1)$. Then the invariant metric tensor

$$\eta^{(n)(l)} = \mathrm{diag}\,(+, -, \ldots, -)$$

appears and we achieve the possibility of lovering and of rising the indices in the brackets. After this step we may transform Eq. (2.14b) into the form

$$\delta = (n^{-1}\delta n)_{(k)(l)}(n_m^{(k)}\partial/\partial n_{m(l)}) \tag{2.14c}$$

and to decompose the $GL(D, R)$ covariant derivatives $n^{(l)}\partial/\partial n_{(k)}$ onto the symmetric and antisymmetric parts

$$(n\partial/\partial n)^{(k)(l)} = \frac{1}{2}(\Delta^{(l)(k)} + K^{(l)(k)}),$$

$$\Delta^{(l)(k)} = n_m^{(k)}\partial/\partial n_{m(l)} - n_m^{(l)}\partial/\partial n_{m(k)}, \tag{2.15a}$$

$$K^{(l)(k)} = n_m^{(l)}\partial/\partial n_{m(k)} + n_m^{(k)}\partial/\partial n_{m(l)}. \tag{2.15b}$$

The corresponding decomposition of the Cartan differential form is defined by the relations

$$(n^{-1}\delta n)_{(k)(l)} = \widetilde{\Omega}_{(k)(l)}(\delta) + S_{(k)(l)}(\delta),$$

$$\widetilde{\Omega}_{(k)(l)}(\delta) = (n^{-1}\delta n)_{[(k)(l)]} \equiv \frac{1}{2}((n^{-1}\delta n)_{(k)(l)} - (n^{-1}\delta n)_{(l)(k)}), \tag{2.16a}$$

$$S_{(k)(l)}(\delta) = (n^{-1}\delta n)_{\{(k)(l)\}} \equiv \frac{1}{2}((n^{-1}\delta n)_{(k)(l)} + (n^{-1}\delta n)_{(l)(k)}). \tag{2.16b}$$

Taking into account Eqs.(2.15) and (2.16), the expression (2.14) for arbitrary variation may be presented in the form

$$\delta = \frac{1}{2}\widetilde{\Omega}_{(k)(l)}(\delta)\,\Delta^{(l)(k)} + \frac{1}{2}S_{(k)(l)}(\delta)\,K^{(l)(k)}. \tag{2.17}$$

It is easy to show that $\Delta^{(l)(k)}$ and $K^{(l)(k)}$ operators generate the $gl(D, R)$ Lie algebra

$$[\Delta_{(l_1)(l_2)}, \Delta_{(k_1)(k_2)}] = -2\eta_{(l_1)[(k_1)}\Delta_{(k_2)](l_2)} + 2\eta_{(l_2)[(k_1)}\Delta_{(k_2)](l_1)}, \tag{2.18a}$$

$$[\Delta_{(l_1)(l_2)}, K_{(k_1)(k_2)}] = 2\eta_{(l_1)\{(k_1)}K_{(k_2)\}(l_2)} - 2\eta_{(l_2)\{(k_1)}K_{(k_2)\}(l_1)}, \tag{2.18b}$$

$$[K_{(l_1)(l_2)}, K_{(k_1)(k_2)}] = 2\eta_{(l_1)\{(k_1)}\Delta_{(k_2)\}(l_2)} - 2\eta_{(l_2)\{(k_1)}\Delta_{(k_2)\}(l_1)}, \tag{2.18c}$$

with the subalgebra (2.18a) of the Lorentz group produced by $D(D-1)/2$ $\Delta_{(l)(k)}$ operators. The operators $K_{(l)(k)}$ are related with the factor space

$$GL(D, R)/SO(1, D-1)$$

and the number of them coincides with the number of the orthonormality conditions $\Xi^{(l)(k)}$.

Now it is evident that the admissible variation may include $\Delta_{(l)(k)}$ operators only. This statement is true due to the fact that the Lorentz rotations are the only transformations which conserve the orthonormality of repere. Let's, however, arrive this statement in a more formal way. This helps us to understand a more complicated case of spinor moving frame variables (i.e., Lorentz harmonics [20—23]).

The action of Δ and K operators on the variables $n_m^{(l)}$ may be easily determined (see Eqs.(2.15))

$$\Delta_{(l_1)(l_2)} n_{(l)m} = 2\eta_{(l)[(l_1)} n_{(l_2)]m}, \tag{2.19a}$$

$$K_{(l_1)(l_2)} n_{(l)m} = 2\eta_{(l)\{(l_1)} n_{(l_2)\}m}. \tag{2.19b}$$

So we have

$$\Delta_{(l_1)(l_2)} \Xi_{(k_1)(k_2)} = 2\eta_{(k_1)[(l_1)} \Xi_{(l_2)](k_2)} + 2\eta_{(k_2)[(l_1)} \Xi_{(l_2)](k_1)}, \tag{2.20a}$$

$$K_{(l_1)(l_2)} \Xi_{(k_1)(k_2)} = 4\eta_{(k_1)\{(l_1)} \eta_{(l_2)\}(k_2)} +$$

$$+ 2\eta_{(k_1)\{(l_1)} \Xi_{(l_2)\}(k_2)} + 2\eta_{(k_2)\{(l_1)} \Xi_{(l_2)\}(k_1)}. \tag{2.20b}$$

Eqs.(2.20) justify the statement that $\Delta_{(l)(k)}$ operators conserve the orthonormality conditions (2.2)

$$\Delta_{(l_1)(l_2)} \Xi_{(k_1)(k_2)} \Big|_{\Xi=0} = 0. \tag{2.21a}$$

At the same time

$$K_{(l_1)(l_2)} \Xi_{(k_1)(k_2)} \Big|_{\Xi=0} = 4\eta_{(k_1)\{(l_1)} \eta_{(l_2)\}(k_2)}. \tag{2.21b}$$

Hence the operators $K_{(l)(k)}$ destroy the repere orthonormality. Moreover, the differential form (2.16b), related to the operator $K_{(l_1)(l_2)}$ (see (2.17)), is reduced to the complete differential of the orthonormality condition $\Xi_{(l_1)(l_2)}$ on the surface (2.2)

$$S_{(l_1)(l_2)} \Big|_{\Xi=0} = \frac{1}{2} d\Xi_{(l_1)(l_2)}. \tag{2.21c}$$

So the following variations

$$\delta \Big|_{\Xi=0} = \frac{1}{2} \Omega^{(l)(k)}(\delta)\, \Delta_{(k)(l)} \tag{2.14d}$$

are admissible (i.e., conserve the repere orthonormality conditions (2.2)).

In Eq.(2.14d) the covariant $SO(1, D-1)$ derivative has the form (2.15a) and the expressions for the Cartan forms (2.16a) may be reduced to the following ones

$$\Omega^{(k)(l)}(\delta) = \tilde{\Omega}^{(k)(l)}(\delta)\Big|_{\Xi=0} = n^{(k)}_m \delta n^{m(l)} = -n^{(l)}_m \delta n^{m(k)} \tag{2.22}$$

on the surface defined by the orthonormality conditions (2.2).

It is interesting to note, that Eq.(2.14a) may be discussed as the definition of the covariant derivatives $\Delta_{(l)(k)}$. So, $\Delta_{(l)(k)}$ may be understood as the derivatives with respect to the Cartan forms $\Omega^{(k)(l)}(\partial)$.

If the repere variables become the fields living on the world-sheet

$$n^{(l)}_m = n^{(l)}_m(\xi^\mu),$$

then the variational analogs $\tilde{\Delta}(\xi)$ of the operators Δ should be used

$$\tilde{\Delta}^{(k)(l)}(\xi) \equiv n^{(l)}_m(\xi)\, \delta/\delta n_{m(k)}(\xi) - n^{(k)}_m(\xi)\, \delta/\delta n_{m(l)}(\xi), \tag{2.23}$$

and we should use the following form of the admissible variation

$$\delta\Big|_{\Xi(\xi)=0} = \frac{1}{2}\int d^2\xi\, \Omega^{(l)(k)}(\delta)\, \tilde{\Delta}_{(k)(l)}(\xi) \tag{2.24}$$

instead of one defined by Eq.(2.14d).

Now we are ready to discuss the derivation of Eq.(2.9).

Taking into account Eq.(2.24), it is easy to see, that the variation of the action (2.1) with respect to the repere fields $n^{(f)}_m$ is defined by the relation

$$\delta S \equiv \int d^2\xi\, \delta L(\xi) = \int d^2\xi\, e(\xi)\,(-(\alpha')^{-1/2} e^\mu_f \partial_\mu x^m\, \delta n^{(f)}_m) =$$

$$= -(\alpha')^{-1/2}\int d^2\xi\, e\, e^\mu_f \partial_\mu x^m \Omega^{(l)(k)}(\delta)\, (\Delta_{(k)(l)} n^{(f)}_m)\,(\xi). \tag{2.25}$$

We stress, that the simple covariant derivative (2.15a) is used in the last part of Eq.(2.25). This is the result of application of the variational derivative (2.23) included in the previous part of this equation.

Hence, we may conclude that:

i) The right equations of motion for the repere fields have the forms of the variations of the action (2.1) with respect to the Cartan forms (2.22)

$$\delta S/\delta\Omega^{(l)(k)}(\delta) = 0. \tag{2.26}$$

(These equations take into account the orthonormality conditions (2.2) automatically);

ii) These equations may be presented in terms of the Lagnrangian density and ordinary covariant derivatives as follows

$$\Delta^{(k)(l)}L(\xi) \equiv (n_m^{(l)}(\xi)\,\partial/\partial n_{m(k)}(\xi) - n_m^{(k)}(\xi)\,\partial/\partial n_{m(l)}(\xi))\,L(\xi) = 0. \quad (2.27)$$

This statement is true for cases, which are similar to the discussed one, where there are no time derivatives of repere fields in the action;

iii) The equations of motion are defined by the result of the action of the ordinary covariant derivatives (2.15a) on the fields $n_m^{(l)}$.

So the equations of motion for the $n_m^{(l)}$ fields have the form

$$e\, e_f^{\mu}\partial_{\mu}x^m\Delta_{(k)(l)}n_m^{\{f\}} = 0. \quad (2.28)$$

We may specify them as follows, using the Eq. (2.19a),

$$e\, e_f^{\mu}\partial_{\mu}x^m n_{m[(k)}\delta_{(l)]}^{\{f\}} = 0. \quad (2.29)$$

Thus it is evident, that motion equations for the fields $n_m^{(l)}$ give nontrivial consequences only for the cases $(k) = \{f\}$ or $(l) = \{f\}$. The equations (2.29) are satisfied identically when $(k) \neq \{f\}$ and $(l) \neq \{f\}$. This is the consequence of the gauge $SO(8)$ symmetry of the discussed action (2.1). The operators $\Delta^{(i)(j)}$ generate these transformations.

Eq. (2.29) is reduced to the following relation

$$e\, e_{[f}^{\mu}\partial_{\mu}x^m n_{m\{g\}]} = 0, \quad (2.30)$$

when both the indices (k) and (l) belong to the $\{f\}$-set. Eq. (2.30) is satisfied identically if Eq. (2.6) is taken into account. This fact corresponds to the $SO(1,1)$ gauge symmetry of the discussed action (2.1).

Hence the unique nontrivial consequence of Eq. (2.28) corresponds to the varying of the action with respect to the Cartan form $\Omega^{\{f\}(i)}$ describing the variations from the coset $SO(1, D-1)/[SO(1,1)\times SO(D-2)]$. It has the form of the relation

$$e\, e_f^{\mu}\partial_{\mu}x^m n_m^{(i)} = 0 \quad (2.31)$$

and is equivalent to Eq. (2.9).

Hence, the equations of motion for the discussed bosonic string formulation (2.1) are derived using the variational principle based on the concept of admissible variation (2.22), (2.24) of the repere fields. It is a simple task to derive the same equations of motion using arbitrary variations and the extended action functional completed by the products of the orthonormality conditions (2.2) $\Xi^{(n)(l)}$ on the Lagrange multipliers (see [41] for this approach applied to the second order form action).

However, for the case of twistor-like Lorentz harmonic formulation of superstring (see Section 3) the described form of variational principle simplifies the calculation significantly.

2.3. Hamiltonian Formalism and Covariant Momentum Densities. Now let's discuss the Hamiltonian formalism for the bosonic string formulation (2.1). The first order form of the action principle results in the following fact. All the expressions for momentum density variables

$$P_M(\xi) = -\partial L/\partial(\partial_\tau X^M) \equiv (P_m(\xi), P^m_{(l)}(\xi)), \tag{2.32}$$

canonically conjugated to the configurational space coordinates of the theory

$$X^M(\xi) \equiv (x^m(\xi), n^{(l)}_m(\xi)), \tag{2.33}$$

result in some constraints. For the discussed formulation of the bosonic string these primary constraints have the form

$$P_m - (\alpha')^{-1/2} e\, e^\tau_f n^{(f)}_m \approx 0, \tag{2.34a}$$

$$P^m_{(l)} \approx 0. \tag{2.34b}$$

However, the repere orthonormality conditions (2.2) should be discussed as the additional primary constraints

$$\Xi^{(n)(l)} \equiv n^{(m)}_m n^{m(l)} - \eta^{(n)(l)} \approx 0 \tag{2.35}$$

if the canonical momentum densities $P^m_{(l)}(\xi)$ for the repere variables $n^{(l)}_m(\xi)$ are used. Such extension of the set of constraints makes the Hamiltonian mechanics more complicated in the discussed case (see [41]). But the corresponding complication for the case of twistor-like formulation of $D = 10$ superstring [22,23] becomes drastic. Indeed, in the formulation [22,23] the complicated harmonicity conditions (1.10), (1.11) appear instead of the orthonormality conditions (2.2).

Henceforth, it is significant to work out the method, which allows one to exclude the conditions similar to (2.2) from the set of constraints and to discuss them as the strong relations. Such a method was used in fact in Refs. 13—17,20,21 and was grounded shortly in Ref.13 for the superparticle case (see also Refs.49,18). Here we justify this method in detail for the case of bosonic string formulation (2.1). Such justification makes more clear the forthcoming discussion for the case of twistor-like superstring formulation.

Let's return to the primary constraints (2.34). The first of them (2.34a) may be decomposed into the two relations, using the orthonormality conditions (2.2),

$$P^{(f)}_0 \equiv n^{m(f)} P_m \approx (\alpha')^{-1/2} e\, e^{f\tau} \equiv (\alpha')^{-1/2} e\, \eta^{fg} e^\tau_g, \tag{2.36a}$$

$$P_0^{(i)} \equiv n^{m(i)} P_m \approx 0. \tag{2.36b}$$

Eqs. (2.36) mean that the repere variables may be discussed as the matrix of the Lorentz transformations, which connect an arbitrary coordinate frame with the fixed one, where the string momentum density $P_0^{(l)}$ has only two nonvanishing components (which coincide with the τ-components of the zweinbein density $e\, e_f^{\mu}$)

$$P_0^{(l)} \equiv (P_0^{(f)}, P_0^{(i)}) = ((\alpha')^{-1/2} e\, e^{f\tau}, 0) = n^{m(l)} P_m. \tag{2.37}$$

Similar interpretation of the Cartan-Penrose representation rewritten in terms of $D = 4$ Lorentz harmonic matrix was given in Ref. 18.

Let's extend such interpretation to the case of harmonic sector and form the $SO(1, D-1)_L$ invariant momentum densities

$$P_{(l)(k)} \equiv n_{m(k)} P^m_{(l)} = -\, n_{m(k)} \partial L / \partial\, (\partial_\tau n_m^{(l)}). \tag{2.38}$$

After the division of $P_{(k)(l)}$ into the symmetric

$$\Sigma_{(l)(k)} \equiv n_{m(k)} P^m_{(l)} + n_{m(l)} P^m_{(k)} \tag{2.39}$$

and antisymmetric

$$\Pi_{(l)(k)} \equiv n_{m(k)} P^m_{(l)} - n_{m(l)} P^m_{(k)} \tag{2.40}$$

parts, we get $D(D+1)/2$ symmetric and $D(D-1)/2$ antisymmetric constraints equivalent to (2.34b)

$$\Sigma_{(k)(l)} \approx 0, \tag{2.41a}$$

$$\Pi_{(k)(l)} \approx 0. \tag{2.41b}$$

The Poisson brackets are defined by the relations

$$[P_M(\tau, \sigma), X^N(\tau, \sigma')]_P \equiv -\,[X^N(\tau, \sigma'), P_M(\tau, \sigma)]_P =$$
$$= \delta_M^N\, \delta\, (\sigma - \sigma'), \tag{2.42a}$$

or

$$[F, G]_P \equiv \int d\sigma\, (\delta F/\delta P_M(\sigma)\, \delta G/\delta X^M(\sigma) - \delta F/\delta X^M(\sigma)\, \delta G/\delta P_M(\sigma)) \equiv$$
$$\equiv \int d\sigma\, (\delta F/\delta P_m(\sigma)\, \delta G/\delta x^m(\sigma) - \delta F/\delta x^m(\sigma)\, \delta G/\delta P_m(\sigma)) +$$
$$+ \int d\sigma\, (\delta F/\delta P^m_{(l)}(\sigma)\, \delta G/\delta n_m^{(l)}(\sigma) - \delta F/\delta n_m^{(l)}(\sigma)\, \delta G/\delta P^m_{(l)}(\sigma)), \tag{2.42b}$$

where $F \equiv F\,[X^M(\sigma), P_M(\sigma)]$ and $G \equiv G\,[X^M(\sigma), P_M(\sigma)]$ are arbitrary functionals defined on the phase space of the system.

It may be justified that the variables $\Sigma_{(k)(l)}(\xi)$ and $\Pi_{(k)(l)}(\xi)$ realize a vector representation of the $gl\,(D, R)$ current algebra on the Poisson brackets (2.42)

$$[\Pi_{(l_1)(l_2)}(\sigma), \Pi_{(k_1)(k_2)}(\sigma')\,]_P =$$
$$= 2\,(\eta_{(l_1)[(k_1)}\Pi_{(k_2)](l_2)} - \eta_{(l_2)[(k_1)}\Pi_{(k_2)](l_1)})\,\delta\,(\sigma - \sigma'), \tag{2.43a}$$

$$[\Pi_{(l_1)(l_2)}(\sigma), \Sigma_{(k_1)(k_2)}(\sigma')\,]_P =$$
$$= -\,2\,(\eta_{(l_1)\{(k_1)}\Sigma_{(k_2)\}(l_2)} - \eta_{(l_2)\{(k_1)}\Sigma_{(k_2)\}(l_1)})\,\delta\,(\sigma - \sigma'), \tag{2.43b}$$

$$[\Sigma_{(l_1)(l_2)}(\sigma), \Sigma_{(k_1)(k_2)}(\sigma')\,]_P =$$
$$= -\,2\,(\eta_{(l_1)\{(k_1)}\Pi_{(k_2)\}(l_2)} + \eta_{(l_2)\{(k_1)}\Delta_{(k_2)\}(l_1)})\,\delta\,(\sigma - \sigma'). \tag{2.43c}$$

The constraints $\Pi_{(k)(l)}$ (2.40) form the representation of the $SO(1, D-1)$ current algebra and, consequently, do not change the constraints (2.35) in the week sense

$$[\Pi_{(l_1)(l_2)}(\sigma), n_{(l)m}(\sigma')\,]_P = -\,2\eta_{(l)[(l_1)}n_{(l_2)]m}\,\delta\,(\sigma - \sigma'), \tag{2.44}$$

$$[\Pi_{(l_1)(l_2)}(\sigma), \Xi_{(k_1)(k_2)}(\sigma')\,]_P =$$
$$= -\,2\,(\eta_{(k_1)[(l_1)}\Xi_{(l_2)](k_2)} + \eta_{(k_2)[(l_1)}\Xi_{(l_2)](k_1)})\,\delta\,(\sigma - \sigma'). \tag{2.45}$$

So it is natural to consider $\Pi_{(k)(l)}$ as the (covariant) momentum variables for the degrees connected with Lorentz subgroup $SO(1, D-1)$ of the $GL(D, R)$ group (i.e., to the orthonormal repere).

Contrary to $\Pi_{(k)(l)}$, the symmetric constraints $\Sigma_{(k)(l)}$ don't conserve the orthonormality conditions (2.35). Indeed,

$$[\Sigma_{(l_1)(l_2)}(\sigma), \Xi_{(k_1)(k_2)}(\sigma')\,]_P = 4\eta_{(k_1)\{(l_1)}\eta_{(l_2)\}(k_2)}\,\delta\,(\sigma - \sigma') +$$
$$+\,2\,(\eta_{(k_1)\{(l_1)}\Xi_{(l_2)\}(k_2)} + \eta_{(k_2)\{(l_1)}\Xi_{(l_2)\}(k_1)})\,\delta\,(\sigma - \sigma'), \tag{2.46}$$

or, in the week sense,

$$[\Sigma_{(l_1)(l_2)}(\sigma), \Xi_{(k_1)(k_2)}(\sigma')\,]_P \approx 4\eta_{(k_1)\{(l_1)}\eta_{(l_2)\}(k_2)}\,\delta\,(\sigma - \sigma'). \tag{2.47}$$

So it is natural to consider the combinations of the phase variables $P^m_{(l)}$ and $n^{(l)}_m$, presented by $\Sigma_{(k)(l)}$ and $\Xi^{(k)(l)}$, as a new canonically conjugated variables describing $D(D+1)/2$ degrees of freedom. Due to their vanishing in the week sense, the phase variables $\Xi_{(k)(l)}$ and $\Sigma_{(k)(l)}$ may be excluded from the string dynamics by the transition from Poisson brackets to the Dirac one (see [50])

$$[F, G]_D \equiv [F, G]_P + \frac{1}{4}\int d\sigma\, [F, \Xi^{(k)(l)}(\sigma)]_P\, [\Sigma_{(k)(l)}(\sigma), G]_P - \\ - \frac{1}{4}\int d\sigma\, [F, \Sigma_{(k)(l)}(\sigma)]_P\, [\Xi^{(k)(l)}(\sigma), G]_P. \qquad (2.48)$$

The momentum variables, remaining after the exclusion of $\Sigma_{(k)(l)}$, are the covariant momentum densities $\Pi_{(k)(l)}$. So it is important to express the Dirac brackets (2.48) in terms of $\Pi_{(k)(l)}$. With that end in view let us discuss the change of variables from $P^m_{(l)}$ momentum densities to $\Pi_{(k)(l)}$ and $\Sigma_{(k)(l)}$ ones. It is based on the evident relation

$$P^{(l)m} = (n^{-1})^m_{(r)}\, n^{(r)}_{m'}\, P^{m'(l)} = (n^{-1})^m_{(r)}\, P^{(r)(l)} = \\ = \frac{1}{2}(n^{-1})^m_{(r)}\, \Pi^{(r)(l)} + \frac{1}{2}(n^{-1})^m_{(r)}\, \Sigma^{(r)(l)}. \qquad (2.49)$$

Using (2.49) we find that

$$\delta/\delta\Pi^{(r)(l)}(\sigma) = \int d\sigma' \delta P^{(s)m}(\sigma')/\delta\Pi^{(r)(l)}(\sigma)\, \delta/\delta P^{(s)m}(\sigma') = \\ = \frac{1}{2}(n^{-1})_{m(r)}\, \delta/\delta P^{(l)}_m - \frac{1}{2}(n^{-1})_{m(l)}\, \delta/\delta P^{(r)}_m, \qquad (2.50)$$

$$\delta/\delta\Sigma^{(r)(l)}(\sigma) = \int d\sigma' \delta P^{(s)}_m(\sigma')/\delta\Sigma^{(r)(l)}(\sigma)\, \delta/\delta P^{(s)}_m(\sigma') = \\ = \frac{1}{2}(n^{-1})_{m(r)}\, \delta/\delta P^{(l)}_m + \frac{1}{2}(n^{-1})_{m(l)}\, \delta/\delta P^{(r)}_m \qquad (2.51)$$

and consequently

$$\delta/\delta P^{(l)m} = n^{(r)}_m\, (\delta/\delta\Pi^{(r)(l)} + \delta/\delta\Sigma^{(r)(l)}). \qquad (2.52)$$

Using the representation (2.52), the change of the momentum densities may be done in the Poisson and Dirac brackets. So Eq. (2.48) may be presented in the form

$$[F, G]_D = \int d\sigma\, (\delta F/\delta P_m(\sigma)\, \delta G/\delta x^m(\sigma) - \delta F/\delta x^m(\sigma)\, \delta G/\delta P_m(\sigma)) + \\ + \int d\sigma\, (\delta F/\delta\Pi^{(r)(l)}(\sigma)\, \tilde{\Delta}^{(r)(l)}(\sigma) G - \tilde{\Delta}^{(r)(l)}(\sigma) F\, \delta G/\delta\Pi^{(r)(l)}(\sigma)) + \\ + \int d\sigma\, (\delta F/\delta\Sigma^{(r)(l)}(\sigma)\, \tilde{K}^{(r)(l)}(\sigma) G - \tilde{K}^{(r)(l)}(\sigma) F\, \delta G/\delta\Sigma^{(r)(l)}(\sigma)) + \\ + \frac{1}{4}\int d\sigma\, [F, \Xi^{(k)(l)}(\sigma)]_P\, [\Sigma_{(k)(l)}(\sigma), G]_P -$$

$$-\frac{1}{4}\int d\sigma\,[F,\Sigma_{(k)(l)}(\sigma)\,]_P\,[\Xi^{(k)(l)}(\sigma),G\,]_P, \tag{2.53}$$

where the variational covariant derivatives $\tilde{\Delta}$ are defined by Eq. (2.23).

Let us discuss some functionals $\tilde{F}$, $\tilde{G}$ which are independent of the $\Sigma_{(k)(l)}(\sigma)$ variables

$$\tilde{F}\equiv\tilde{F}\,[x^m(\sigma),P_m(\sigma),n_m^{(l)}(\sigma),\Pi_{(k)(l)}(\sigma)\,],$$

$$\tilde{G}\equiv\tilde{G}\,[x^m(\sigma),P_m(\sigma),n_m^{(l)}(\sigma),\Pi_{(k)(l)}(\sigma)\,]. \tag{2.54}$$

The Dirac brackets (2.53) coincide with (the «covariant» version of) the Poisson ones on the class of such functions

$$[\tilde{F},\tilde{G}\,]_D=\int d\sigma\,(\delta\tilde{F}/\delta P_m(\sigma)\,\delta\tilde{G}/\delta x^m(\sigma)-$$

$$-\,\delta\tilde{F}/\delta x^m(\sigma)\,\delta\tilde{G}/\delta P_m(\sigma))+\int d\sigma\,(\delta\tilde{F}/\delta\Pi^{(r)(l)}(\sigma)\,\tilde{\Delta}^{(r)(l)}(\sigma)\tilde{G}-$$

$$-\,\tilde{\Delta}^{(r)(l)}(\sigma)\tilde{F}\,\delta\tilde{G}/\delta\Pi^{(r)(l)}(\sigma))=[\tilde{F},\tilde{G}\,]_P\,. \tag{2.55}$$

So, the ordinary Poisson brackets (2.55), together with the strong relations (2.2), may be used for the function with the properties (2.54).

Therefore we are free from the necessity of the inclusion of the orthonormality conditions (2.2) into the list of the Hamiltonian constraints if the phase space includes only the covariant momentum densities $\Pi_{(k)(l)}(\sigma)$ (2.40) for the repere variables. Such momentum densities are characterized by the property

$$[\Pi_{(k)(l)}(\sigma),\Xi^{(k)(l)}(\sigma')\,]_P\Big|_{\Xi=0}=0. \tag{2.56}$$

The similar prescription should be used below for the investigation of the Hamiltonian mechanics for the twistor-like superstring formulation [22,23]. It gives the possibility of taking into account the complicated harmonicity conditions (1.10), (1.11) as the «strong» relations and exclude them from the list of Hamiltonian constraints.

To clarify the nature of the covariant momentum densities $\Pi_{(k)(l)}(\sigma)$ let's prove that they may be defined as the derivatives of the Lagrangian density with respect to the τ-components of the Cartan differential form (2.22)

$$\Pi_{(l)(k)}(\sigma)=-\,\partial L/\partial\Omega_\tau^{(k)(l)}(\sigma). \tag{2.57}$$

The components $\Omega^{(k)(l)}_{\mu} = (\Omega^{(k)(l)}_{\tau}, \Omega^{(k)(l)}_{\sigma}) \equiv \Omega^{(k)(l)}(\partial_{\mu})$ of the Cartan differential form $\Omega^{(k)(l)}(d)$ (2.22) with respect to the holonomic basis $d\xi^{\mu} = (d\tau, d\sigma)$ are defined by the relation

$$\Omega^{(k)(l)}(d) = n^{(k)}_{m} \delta n^{m(l)} = d\xi^{\mu}\, \Omega^{(k)(l)}_{\mu} = d\tau\, \Omega^{(k)(l)}_{\tau} + d\sigma\, \Omega^{(k)(l)}_{\sigma}. \quad (2.58)$$

Indeed, using the completeness of the set of differential forms $\tilde{\Omega}^{(k)(l)}(\delta)$ and $S^{(k)(l)}(\delta)$ (2.17), we may decompose the derivative with respect to $\partial_{\tau} n^{(l)}_{m}$ as follows

$$\partial / \partial\, (\partial_{\tau} n^{(l)}_{m}) = \frac{1}{2}\, \partial \tilde{\Omega}^{(k)(l)}_{\tau} / \partial\, (\partial_{\tau} n^{(l)}_{m})\; \partial / \partial \tilde{\Omega}^{(k)(l)}_{\tau} + $$
$$ + \frac{1}{2}\, \partial S^{(k)(l)}_{\tau} / \partial\, (\partial_{\tau} n^{(l)}_{m})\; \partial / \partial S^{(k)(l)}_{\tau}. \quad (2.59)$$

Multiplying Eq.(2.59) by $n^{(s)}_{m}$ and taking the antisymmetric part of the resulting expression, we get the relation

$$\partial / \partial \tilde{\Omega}^{(s)(l)}_{\tau} = n_{m(s)}\, \partial / \partial\, (\partial_{\tau} n^{(l)}_{m}) - n_{m(l)} \partial / \partial\, (\partial_{\tau} n^{(s)}_{m}). \quad (2.60)$$

Henceforth, the expression (2.57) coincides with (2.40) (see also (2.35)), and we conclude that the covariant momentum density characterizing the property (2.56) is defined by the following expression

$$\Pi_{(l)(k)} = -\, \partial L / \partial \Omega^{(k)(l)}_{\tau}(\sigma) = n_{m(l)} P^{m}_{(k)} - n_{m(k)} P^{m}_{(l)}. \quad (2.61)$$

Finally, we should note, that the covariant momentum density (2.61) is the «classical analog» for the variational covariant derivative (2.23). This statement means, that the Poisson bracket of $\Pi_{(k)(l)}$ with any admissible functional, defined on the configurational space (2.33), coincides with the action of the variational covariant derivative (2.23) on the same functional

$$[\Pi^{(l)(k)}(\sigma), \tilde{F}\, [x^{m}, n^{(r)}_{m}]\,]_{P} = \tilde{\Delta}^{(l)(k)}(\sigma)\, \tilde{F}\, [x^{m}, n^{(r)}_{m}\,]. \quad (2.62)$$

The discussed properties (2.56), (2.61), (2.62) of the covariant momentum density $\Pi_{(k)(l)}$ should help us to find the corresponding variable for the twistor-like superstring formulation [22,23] and, thus, to simplify the investigation of its Hamiltonian mechanics (see Section 4).

3. $D = 10$ SUPERSTRING IN TWISTOR-LIKE LORENTZ HARMONIC FORMULATION

3.1. Action Functional. The twistor-like action functional for the $D = 10$, $N = IIB$ superstring has the form [22,23]

$$S = S_1 + S_{W-Z}, \tag{3.1}$$

$$S_1 = \int d^2\xi \, e(\xi) \left(-(\alpha')^{-1/2} e_f^{\mu} \omega_\mu^m n_m^{(f)} + c\right) =$$
$$= \int d\tau d\sigma \, e \left[-(\alpha')^{-1/2} \left(e^{\mu[+2]} u_m^{[-2]} + e^{\mu[-2]} u_m^{[+2]}\right) \omega_\mu^m + c\right] \equiv$$
$$\equiv \int d\tau d\sigma \, e \left(c + \frac{1}{16} (\alpha')^{-1/2} e^{\mu[+2]} \omega_\mu^m (v_{\dot{A}}^- \tilde{\sigma}_m v_{\dot{A}}^-) + \right.$$
$$\left. + \frac{1}{16} (\alpha')^{-1/2} e^{\mu[-2]} \omega_\mu^m (v_A^+ \tilde{\sigma}_m v_A^+)\right), \tag{3.1a}$$

$$S_{W-Z} \equiv -(c\alpha')^{-1} \int d\tau d\sigma \, \varepsilon^{\mu\nu} \left[i \omega_\mu^m \left(\partial_\nu \theta^1 \sigma_m \theta^1 - \partial_\nu \theta^2 \sigma_m \theta^2\right) + \right.$$
$$\left. + \partial_\mu \theta^1 \sigma^m \theta^1 \, \partial_\nu \theta^2 \sigma_m \theta^2 \right]. \tag{3.1b}$$

Here

$$\omega_\mu^m = \partial_\mu x^m - i \left(\partial_\mu \theta^1 \sigma^m \theta^1 + \partial_\mu \theta^2 \sigma^m \theta^2\right) \equiv$$
$$\equiv \partial_\mu x^m - i \left(\partial_\mu \theta^{\alpha 1} \sigma_{\alpha\beta}^m \theta^{\beta 1} + \partial_\mu \theta^{\alpha 2} \sigma_{\alpha\beta}^m \theta^{\beta 2}\right) \tag{3.2}$$

are the coefficients of the pullback of the $D = 10$, $N = 2B$ supersymmetric Cartan form [51] on the world-sheet

$$\omega^m = dx^m - i \left(d\theta^1 \sigma^m \theta^1 + d\theta^2 \sigma^m \theta^2\right) = d\xi^\mu \omega_\mu^m,$$

$x^m (m = 0, 1, \ldots, 9)$ are the ordinary (flat) space-time coordinates and $\theta^{\alpha I} = (\theta^{\alpha 1}, \theta^{\alpha 2})$ $(\alpha = 1, \ldots, 16)$ are the fermionic (Grassmannian) coordinates of the $D = 10$, $N = 2B$ superspace which have the properties of Majorana-Weyl spinors with respect to $SO(1,9)$ group. $\sigma_{\alpha\beta}^m$ are the symmetric 16×16 Pauli matrices for $D = 10$ space-time (see [14,15] for the notations). The conventions about the world-sheet zweinbeins $e_\mu^{[\pm 2]}$, $e^{\mu[\pm 2]}$ are collected in Eqs.(2.4).

The action (3.1) differs from the trivial supersymmetrization $(\partial_\mu x^m \to \omega_\mu^m)$ of the repere (moving frame) bosonic string formulation (2.1) by

i) adding the Wess — Zumino term (3.1b) and by

ii) replacement of the fundamental moving frame vectors $n_m^{(l)}$ (2.5) by the compound ones $u_m^{(l)}$ (1.8), (1.9) composed from the bosonic spinor variables (1.1)

$$n_m^{(l)} \rightarrow u_m^{(l)} \equiv \frac{1}{16} \mathrm{Sp}\,(v^T \tilde{\sigma}_m v \sigma^{(l)}) \equiv \frac{1}{16} v_\alpha^a \tilde{\sigma}_m^{\alpha\beta} v_\beta^b \sigma_{ab}^{(l)}, \tag{3.3}$$

$$v_\alpha^a = (v_{\alpha A}^+, v_{\alpha \dot{A}}^-) \in \mathrm{Spin}\,(1,9). \tag{3.4}$$

The orthonormality conditions (1.2) for the composed repere (3.3) are the straightforward consequences of the relation (3.4). To arrive the decomposition (1.8), (1.9) of the set of composed moving frame vectors (3.3) the following σ-matrix representation should be used

$$\begin{aligned}
\sigma_{ab}^0 &= \mathrm{diag}\,(\delta_{AB}, \delta_{\dot{A}\dot{B}}) = \tilde{\sigma}^{0ab}, \\
\sigma_{ab}^9 &= \mathrm{diag}\,(\delta_{AB}, -\delta_{\dot{A}\dot{B}}) = -\tilde{\sigma}^{9ab}, \\
\sigma_{ab}^{(i)} &= \begin{bmatrix} 0 & \gamma_{A\dot{B}}^i \\ \tilde{\gamma}_{\dot{A}B}^i & 0 \end{bmatrix} = -\tilde{\sigma}^{(i)ab}, \\
\sigma_{ab}^{[+2]} &\equiv (\sigma^0 + \sigma^9)_{ab} = \mathrm{diag}\,(2\delta_{AB}, 0) = (\tilde{\sigma}^0 - \tilde{\sigma}^9)^{ab} = \tilde{\sigma}^{[-2]ab}, \\
\sigma_{ab}^{[-2]} &\equiv (\sigma^0 - \sigma^9)_{ab} = \mathrm{diag}\,(0, 2\delta_{\dot{A}\dot{B}}) = (\tilde{\sigma}^0 + \tilde{\sigma}^9)^{ab} = \tilde{\sigma}^{[+2]ab}.
\end{aligned} \tag{3.5}$$

In Eqs. (3.5) $\gamma_{A\dot{B}}^i$ are the σ-matrices for $SO(8)$ group (see [3]), $\tilde{\gamma}_{\dot{A}B}^i \equiv \gamma_{B\dot{A}}^i$.

The presence of the Wess — Zumino term (3.1b) in the action (3.1) leads to the invariance of this action under the κ-symmetry transformations which explicit form was presented in Refs. 23,46. There are also evident reparametrization symmetry and the gauge symmetry under the right product of $SO(8)$ and $SO(1,1)$ groups.

The $SO(8)$ gauge symmetry transformations result in the arbitrary rotations of the eight spacelike composed vectors $u_m^{(i)}$ (see Eq. (1.8)) among themselves. And the $SO(1,1)$ ones result in the pseudorotations of the vectors $u_m^{(f)}$. To achieve the invariance of the action functional (3.1), they should be identified with the world-sheet Lorentz group transformations acting on the «flat» indices of the zweinbeins e_μ^f (see Eq. (2.4)).

The relations (3.4) together with the gauge symmetry under the right product of $SO(1,1)$ and $SO(8)$ groups permit us to identify the space of harmonic variables $\{(v_{\alpha A}^-, v_{\alpha \dot{A}}^+)\}$ with the coset space $SO(1,9)/[SO(1,1) \otimes SO(8)]$ [22,23,46]. We stress that the so-called «boost» symmetry is absent in the discussed superstring formulation (3.1) in distinction with the formulations of the $D = 10$ Green — Schwarz heterotic superstring presented in [38,39]. The cause of such distinction shall be discussed below.

3.2. Harmonic Variables, Composed Moving Frame Vectors and Admissible Variations. The relation (3.4) is realized by the requirement that variables $v^a_\alpha = = (v^-_{\alpha A}, v^+_{\alpha \dot{A}})$ must satisfy the harmonicity conditions (1.10) [20,21,46]

$$\Xi_{m_1 \ldots m_4} = u^{m(n)} \eta_{(n)(l)} \Xi^{(l)}_{m_1 \ldots m_4 m} \equiv$$

$$\equiv u^{m(n)} \eta_{(n)(l)} \mathrm{Sp}\,(v\, \tilde{\sigma}_{m_1 \ldots m_4 m} v \sigma^{(n)})^T = 0, \tag{3.6a}$$

$$\Xi_0 \equiv u^{[-2]}_m u^{m[+2]} - 2 \equiv \frac{1}{8} (v^-_A \tilde{\sigma}_m v^-_A)\ \frac{1}{8} (v^+_{\dot{A}} \tilde{\sigma}_m v^+_{\dot{A}}) - 2 = 0 \tag{3.6b}$$

(We stress that the equality

$$\Xi^{(n)}_{m_1 \ldots m_5} \equiv \mathrm{Sp}\,(v\, \tilde{\sigma}_{m_1 \ldots m_k} v \sigma^{(n)})^T = 0$$

[20,21] results from Eqs.(3.6a) [46]).

It is easy to see that Eqs. (3.6) kill the $210 + 1 = 211$ degrees of freedom and reduce the numbers of independent variables included in v^a_α to $45 = 256 - 211 = \dim SO(1,9)$. The equivalence of the restrictions (3.6) to the relation (3.4) was discussed in Ref. [46] in detail.

It is necessary to introduce the inverse harmonic matrix

$$(v^{-1})^\alpha_a \equiv (v^{-1})^\alpha_a \equiv (v^{-\alpha}_A, v^{+\alpha}_{\dot{A}})^T. \tag{3.7}$$

In contradistinction to the case of $D = 4$ [18,42,43,19], its elements cannot be expressed through the harmonic variables $v^+_{\alpha \dot{A}}$, $v^-_{\alpha A}$ in a simple and covariant way. This is explained by the impossibility to trasnform the subscript $D = 10$ Majorana — Weyl spinor index into the superscript one, since they describe the representations with different chiralities. Therefore it is convenient to discuss 256 variables $v^{-\alpha}_A$, $v^{+\alpha}_{\dot{A}}$ as the independent harmonics and to complete the set of harmonicity conditions by the 256 relations of the mutual invertness of the matrices $(v^{-1})^\alpha_a$ and v^a_α

$$(v^{-1})^\alpha_a v^b_\alpha = \delta^b_a:$$

$$\Xi^{[0]}_{AB} \equiv v^{-\alpha}_A v^+_{\alpha B} - \delta_{AB} = 0, \qquad \Xi^{[-2]}_{A\dot{B}} \equiv v^{-\alpha}_A v_{\alpha \dot{B}} = 0,$$

$$\Xi^{[+2]}_{\dot{A}\dot{B}} \equiv v^{+\alpha}_{\dot{A}} v^{+}_{\alpha \dot{B}} = 0, \qquad \Xi^{[0]}_{\dot{A}\dot{B}} \equiv v^{+\alpha}_{\dot{A}} v^{-}_{\alpha \dot{B}} - \delta_{\dot{A}\dot{B}} = 0, \tag{3.8}$$

(256 − 256 = 0, consequently additional degrees of freedom are not included in the theory).

Note that the distinction in the $SO(1,1)$ weights ± for the same $SO(8)$ ((*s*) or (*c*) spinor) index structure shall help us to distinguish the harmonics $v^{-}_{\alpha A}, v^{+}_{\alpha \dot{A}}$ (3.4) from the (3.7) ones $v^{-\alpha}_{A}$, $v^{+\alpha}_{\dot{A}}$ in the expressions, where the $SO(1,9)$ spinor indices are contracted and omitted (see, for example, Eqs.(3.11)).

It is easy to prove that the composed repere vectors $u^{(n)}_{m}$ (3.3) may be expressed through the inverse harmonic matrix (3.7) as well as through the ordinary one (3.4) (see Eqs.(1.9))

$$u^{(l)}_{m} \equiv \frac{1}{16}\,\mathrm{Sp}\,(v\tilde{\sigma}_{m} v^{T} \sigma^{(l)}) = \frac{1}{16}\,\mathrm{Sp}\,(v^{-1}\sigma_{m}(v^{-1})^{T}\tilde{\sigma}^{(l)}). \tag{3.9}$$

In terms of the harmonic variables $v^{-}_{\alpha A}$, $v^{+}_{\alpha \dot{A}}$ and $v^{-\alpha}_{A}$, $v^{+\alpha}_{\dot{A}}$ Eqs.(3.9) may be specified as follows (see Eqs.(1.8), (1.9) and the σ-matrix representation (3.5))

$$u^{(l)}_{m} = (u^{(l)}_{m}, u^{(i)}_{m}) \equiv \left(\frac{1}{2}(u^{[+2]}_{m} + u^{[-2]}_{m}), u^{(i)}_{m}, \frac{1}{2}(u^{[+2]}_{m} - u^{[-2]}_{m})\right), \tag{3.10}$$

$$u^{[+2]}_{m} = \frac{1}{8}(v^{+}_{\dot{A}}\tilde{\sigma}_{m} v^{+}_{\dot{A}}) = \frac{1}{8}(v^{+}_{\dot{A}}\sigma_{m} v^{+}_{\dot{A}}) \equiv \frac{1}{8} v^{+\alpha}_{\dot{A}} \sigma_{m\alpha\beta} v^{+\beta}_{\dot{A}}, \tag{3.11a}$$

$$u^{[-2]}_{m} = \frac{1}{8}(v^{-}_{A}\tilde{\sigma}_{m} v^{-}_{A}) = \frac{1}{8}(v^{-}_{A}\sigma_{m} v^{-}_{A}) \equiv \frac{1}{8} v^{-\alpha}_{A} \sigma_{m\alpha\beta} v^{-\beta}_{A}, \tag{3.11b}$$

$$u^{(i)}_{m} = \frac{1}{8}(v^{+}_{\dot{A}}\tilde{\sigma}_{m} v^{-}_{A})\gamma^{i}_{A\dot{A}} = -\frac{1}{8}(v^{-}_{A}\sigma_{m} v^{+}_{\dot{A}})\gamma^{i}_{A\dot{A}}. \tag{3.11c}$$

The orthonormality conditions (1.2) may be specified as follows

$$u^{(n)}_{m} u^{m(k)} = \eta^{(n)(k)} = \mathrm{diag}\,(1, -1, \ldots, -1): \tag{3.12}$$

$$u^{[+2]}_{m} u^{m[+2]} = 0, \qquad u^{[-2]}_{m} u^{m[-2]} = 0, \tag{3.12a,b}$$

$$u^{[\pm 2]}_{m} u^{m(i)} = 0, \tag{3.12c}$$

$$u^{[+2]}_{m} u^{m[-2]} = 2, \qquad u^{(i)}_{m} u^{m(j)} = -\delta^{(i)(j)}. \tag{3.12d,e}$$

To justify them explicitly the identity (1.12) and the consequences (1.4) of the harmonicity conditions (3.6), (3.8) should be used (see [46] for details). For the discussed $D = 10$ superstring case the relations (1.4) may be specified as follows

$$u^{(l)}_{m}\sigma^{m}_{\alpha\beta} = v^{a}_{\alpha}\sigma^{(l)}_{ab} v^{b}_{\beta}: \tag{3.13}$$

$$u^{[+2]}_m \sigma^m_{\alpha\beta} = 2\, v^+_{\alpha A} v^+_{\beta A}, \tag{3.13a}$$

$$u^{[-2]}_m \sigma^m_{\alpha\beta} = 2\, v^-_{\alpha \dot{A}} v^-_{\beta \dot{A}}, \tag{3.13b}$$

$$u^{(i)}_m \sigma^m_{\alpha\beta} = (v^+_{\alpha A} v^-_{\beta \dot{A}} + v^+_{\beta A} v^-_{\alpha \dot{A}})\, \gamma^i_{A\dot{A}}, \tag{3.13c}$$

$$u^{(l)}_m \tilde{\sigma}^{m\alpha\beta} = (v^{-1})^{\alpha}_{a}\, \tilde{\sigma}^{(l)ab} (v^{-1})^{\beta}_{b} : \tag{3.14}$$

$$u^{[+2]}_m \tilde{\sigma}^{m\alpha\beta} = 2\, v^{+\alpha}_{\dot{A}} v^{+\beta}_{\dot{A}}, \tag{3.14a}$$

$$u^{[-2]}_m \tilde{\sigma}^{m\alpha\beta} = 2\, v^{-\alpha}_{A} v^{-\beta}_{A}, \tag{3.14b}$$

$$u^{(i)}_m \tilde{\sigma}^{m\alpha\beta} = -\, (v^{-\alpha}_{A} v^{+\beta}_{\dot{A}} + v^{-\beta}_{A} v^{+\alpha}_{\dot{A}})\, \gamma^i_{A\dot{A}}, \tag{3.14c}$$

$$u^{(l)}_m \tilde{\sigma}^{ab}_{(l)} = v^a_\alpha \tilde{\sigma}^{\alpha\beta}_m v^b_\beta : \tag{3.15}$$

$$u^{[+2]}_m \delta_{AB} = (v^+_A \tilde{\sigma}_m v^+_B), \tag{3.15a}$$

$$u^{[-2]}_m \delta_{\dot{A}\dot{B}} = (v^-_{\dot{A}} \tilde{\sigma}_m v^-_{\dot{B}}), \tag{3.15b}$$

$$u^{(i)}_m \gamma^i_{A\dot{B}} = (v^+_A \tilde{\sigma}_m v^-_{\dot{B}}), \tag{3.15c}$$

$$u^{(l)}_m \sigma_{(l)ab} = (v^{-1})^{\alpha}_{a} \sigma_{m\alpha\beta} (v^{-1})^{\beta}_{b} : \tag{3.16}$$

$$u^{[-2]}_m \delta_{AB} = (v^-_A \sigma_m v^-_B), \tag{3.16a}$$

$$u^{[+2]}_m \delta_{\dot{A}\dot{B}} = (v^+_{\dot{A}} \sigma_m v^+_{\dot{B}}), \tag{3.16b}$$

$$u^{(i)}_m \gamma^i_{A\dot{B}} = -\, (v^-_A \sigma_m v^+_{\dot{B}}). \tag{3.16c}$$

For the forthcoming derivation of the equations of motion let's discuss the concept of an admissible variation for the case of spinor harmonic variables. This is the variation which doesn't destruct the harmonicity conditions (3.6), (3.8) (or, equivalently, the relation (3.4)). Such variation was discussed in detail for the case of the fundamental repere variables in Section (2) (see Eqs.(2.14a)—(2.14d)). Thus we may omit some evident steps in the discussion of the spinor harmonic case.

An arbitrary variation of the variables v^a_α and $(v^{-1})^{\alpha}_{a} \equiv v^{\alpha}_{a}$

$$\delta = \delta v^a_\alpha \frac{\partial}{\partial v^a_\alpha} + \delta v^\alpha_a \frac{\partial}{\partial v^\alpha_a}, \tag{3.17}$$

may be written in the form

$$\delta = (v^{-1}\delta v)_a^b \left(v_\alpha^a \frac{\partial}{\partial v_\alpha^b} - v_b^\alpha \frac{\partial}{\partial v_a^\alpha} \right), \tag{3.18}$$

where the conditions (3.8) were used explicitly. To specify Eq. (3.18) let's use Eq. (3.11), the consequences (3.13)—(3.16) of the harmonicity conditions and the known identities (see, for example, Refs. 14,15).

$$S_{\alpha\beta} \equiv S_{\{\alpha\beta\}} = \frac{1}{16} \sigma^m_{\alpha\beta} \, \mathrm{Sp}(\tilde{\sigma}_m S) + \frac{1}{5!16} (\sigma^{m_1 \ldots m_5})_{\alpha\beta} \, \mathrm{Sp}(\tilde{\sigma}_{m_1 \ldots m_5} S), \tag{3.19a}$$

$$A_{\alpha\beta} \equiv A_{[\alpha\beta]} = -\frac{1}{3!\,16} (\sigma^{m_1 m_2 m_3})_{\alpha\beta} \, \mathrm{Sp}\,(\tilde{\sigma}_{m_1 m_2 m_3} A), \tag{3.19b}$$

$$F_\alpha^\beta = \frac{1}{16} \delta_\alpha^\beta \, \mathrm{Sp}(F) - \frac{1}{32} (\sigma^{m_1 m_2})_\alpha^\beta \, \mathrm{Sp}(\sigma_{m_1 m_2} F) + \\ + \frac{1}{4!\,16} (\sigma^{m_1 \ldots m_4})_{\alpha\beta} \, \mathrm{Sp}(\sigma_{m_1 \ldots m_4} F). \tag{3.19c}$$

Indeed, varying Eq. (3.15)

$$\delta u_m^{(l)} \tilde{\sigma}_{(l)}^{ab} = 2(\delta v^a \tilde{\sigma}_m v^b) = (v^{-1}\delta v)_c^a \, \tilde{\sigma}_{(n)}^{cb} u_m^{(n)} + (v^{-1}\delta v)_c^b \, \tilde{\sigma}_{(n)}^{ca} u_m^{(n)} \tag{3.20}$$

and contracting the result (3.20) with the 10×16 matrix $(u^{m(k)} \sigma_{(k)bd})$ we obtain

$$u^{m(k)} \delta u_m^{(l)} \, (\sigma_{(k)} \tilde{\sigma}_{(l)})_d^a = 10\,(v^{-1}\delta v)_d^a + (\tilde{\sigma}_{(k)} v^{-1} \delta v \, \sigma^{(k)}). \tag{3.21}$$

It is easy to see that the left-hand side of Eq. (3.21) may be presented in the form $u^{m(k)} \delta u_m^{(l)} \, (\sigma_{(k)(l)})$. This results from the vanishing of the expression $u^{m(k)} \delta u_m^{(l)} \, \eta_{(k)(l)} = \frac{1}{2} \delta \Xi^{(k)(l)} \, \eta_{(k)(l)} = 0$ which is the consequence of the orthonormality conditions (3.12). The righ-hand side of Eq. (3.21) may be transformed using the identities (3.19c) for $v^{-1}\delta v$ and the relations

$$\tilde{\sigma}^{(k)} \sigma_{m_1 \ldots m_{2r}} \sigma_{(k)} = (10 - 4r)\, \tilde{\sigma}_{m_1 \ldots m_{2r}} = (-1)^r (10 - 4r)\, \sigma_{m_1 \ldots m_{2r}}. \tag{3.22}$$

Thus we derive from Eq. (3.21)

$$u^{m(k)} \delta u_m^{(l)} \, (\sigma_{(k)(l)}) = 10\,(v^{-1}\delta v) + \\ + \frac{1}{4} (\sigma^{m_1 m_2}) \, \mathrm{Sp}\,(\sigma_{m_1 m_2} v^{-1} \delta v) + \frac{1}{64} (\sigma^{m_1 \ldots m_4}) \, \mathrm{Sp}\,(\sigma_{m_1 \ldots m_4} v^{-1} \delta v). \tag{3.23}$$

Contracting Eq. (3.23) with the matrices $I, \sigma_{(k)(l)}, \sigma_{m_1 \ldots m_4}$, we produce the following relations

$$\mathrm{Sp}\,(v^{-1}\delta v) = 0 \Leftrightarrow v_A^- \delta v_A^+ = -v_A^+ \delta v_A^-, \tag{3.24}$$

$$-\frac{1}{8}\,\mathrm{Sp}\,(v^{-1}\delta v\,\sigma^{(k)(l)}) = u^{m(k)}\delta u^{(l)}_{m} = \Omega^{(k)(l)}(\delta), \tag{3.25}$$

$$\mathrm{Sp}\,(v^{-1}\delta v\,\sigma_{m_1\ldots m_4}) = 0, \tag{3.26}$$

which are straightforward consequences of the harmonicity conditions (3.6), (3.8). Taking into account Eqs.(3.24)—(3.26), it is easy to derive from the identity (3.19c) the following expression for $v^{-1}\delta v$

$$(v^{-1}\delta v) = \frac{1}{4}\,\Omega^{(k)(l)}(\delta)\,(\sigma_{(k)(l)}). \tag{3.27}$$

Thus the admissible variation which conserves the harmonicity conditions (3.6), (3.8) has the form

$$\delta = \frac{1}{2}\,\Omega^{(k)(l)}(\delta)\,\Delta_{(l)(k)}, \tag{3.28}$$

which coincides with Eq.(2.14d). However, in Eq.(3.28) the $SO(1,9)$ Cartan forms $\Omega^{(k)(l)}(\delta)$ are defined by Eq.(3.25) in terms of spinor harmonic variables (3.4), (3.7), and covariant derivatives $\Delta_{(k)(l)}$ are defined as follows

$$\Delta^{(l)(k)} \equiv \frac{1}{2}\,(\sigma^{(k)(l)})^{b}_{a}\left(v^{a}_{\alpha}\,\frac{\partial}{\partial v^{b}_{\alpha}} - v^{\alpha}_{b}\,\frac{\partial}{\partial v^{\alpha}_{a}}\right). \tag{3.29}$$

Taking into account the definition of the composed repere vectors (3.3) (or Eq.(3.39)) we may obtain the action of the covariant derivatives $\Delta_{(l_1)(l_2)}$ on the

$$\Delta_{(l_1)(l_2)}u_{(l)m} = 2\eta_{(l)[(l_1)}u_{(l_2)]m}, \tag{3.30}$$

Eq.(3.30) coincides with Eq.(2.19a). It may be also justified that $\Delta_{(l_1)(l_2)}$ operators generate the Lorentz group algebra (2.18a).

For the forthcoming discussion it is useful to specify Eqs.(3.28), (3.25), (3.29) as follows (the contracted spinor indices are omitted)

$$\delta = \frac{1}{2}\,\Omega^{(k)(l)}(\delta)\Delta_{(l)(k)} = \Omega^{(0)}(\delta)\Delta^{(0)} +$$

$$+\,\Omega^{[\mp 2]j}(\delta)\Delta^{[\pm 2]j} - \frac{1}{2}\,\Omega^{(i)(j)}(\delta)\Delta^{(i)(j)}, \tag{3.31}$$

$$\Omega^{(k)(l)}(\delta) = (-\,2\,\Omega^{(0)}(\delta),\ \Omega^{[\mp 2](j)}(\delta),\ \Omega^{(i)(j)}(\delta)): \tag{3.32}$$

$$\Omega^{(0)}(\delta) \equiv -\frac{1}{2}\,\Omega^{[+2][-2]}(\delta) = \frac{1}{2}\,u^{m[-2]}\delta u^{[+2]}_{m} =$$

$$= -\frac{1}{2}\,u^{m[+2]}\delta u^{[-2]}_{m} = \frac{1}{4}\,(v^{-}_{A}\delta v^{+}_{A} - v^{+}_{A}\delta v^{-}_{A}) = \frac{1}{2}\,v^{-}_{A}\delta v^{+}_{A}, \tag{3.32a}$$

$$\Omega^{[+2](i)} = u^{[+2]}_{m}\delta u^{m(i)} = \frac{1}{4}\,v^{+}_{A}\tilde{\gamma}^{i}_{AA}\delta v^{+}_{A} \equiv \frac{1}{4}\,v^{+\alpha}_{A}\,\tilde{\gamma}^{i}_{AA}\delta v^{+}_{\alpha A}, \tag{3.32b}$$

$$\Omega^{[-2](i)} = u_m^{[-2]}\delta u^{m(i)} = \frac{1}{4} v_A^- \gamma^i_{A\dot{A}} \delta v_{\dot{A}}^-, \tag{3.32c}$$

$$\Omega^{(i)(j)} = u^{m(i)} du_m^{(j)} = -\frac{1}{8}(v_A^- \gamma^{ij}_{AB} \delta v_B^+ + v_{\dot{A}}^+ \tilde{\gamma}^{ij}_{\dot{A}\dot{B}} \delta v_{\alpha\dot{B}}^-), \tag{3.32d}$$

$$\Delta^{(l)}(k) = (-2\Delta^{(0)}, \Delta^{[\mp 2](j)}, \Delta^{(i)(j)}) : \tag{3.33}$$

$$\Delta^{(0)} \equiv -\frac{1}{2}\Delta^{[+2][-2]} = v_A^+ \partial/\partial v_A^+ - v_A^- \partial/\partial v_A^- - v_{\dot{A}}^- \partial/\partial v_{\dot{A}}^- + v_{\dot{A}}^+ \partial/\partial v_{\dot{A}}^+, \tag{3.34a}$$

$$\Delta^{[+2](i)} = v_A^+ \gamma^i_{A\dot{A}} \partial/\partial v_{\dot{A}}^- - v_{\dot{A}}^+ \tilde{\gamma}^i_{\dot{A}A} \partial/\partial v_A^-, \tag{3.34b}$$

$$\Delta^{[-2](i)} = v_{\dot{A}}^- \tilde{\gamma}^i_{\dot{A}A} \partial/\partial v_A^+ - v_A^- \gamma^i_{A\dot{A}} \partial/\partial v_{\dot{A}}^+, \tag{3.34c}$$

$$\Delta^{(i)(j)} = \frac{1}{2}(v_A^+ \gamma^{ij}_{AB}\, \partial/\partial v_B^+ + v_{\dot{A}}^- \tilde{\gamma}^{ij}_{\dot{A}\dot{B}}\, \partial/\partial v_{\dot{B}}^- +$$

$$+ v_A^- \gamma^{ij}_{AB}\, \partial/\partial v_B^- + v_{\dot{A}}^+ \tilde{\gamma}^{ij}_{\dot{A}\dot{B}}\, \partial/\partial v_{\dot{B}}^+), \tag{3.34d}$$

$$\Delta_{(l_1)(l_2)} u_{(l)m} = -2\eta_{(l)[(l_1)} u_{(l_2)]m}:$$

$$\Delta^{(0)} u_m^{[\pm 2]} = \pm 2 u_m^{[\pm 2]}, \qquad \Delta^{(0)} u_m^{(j)} = 0, \tag{3.35}$$

$$\Delta^{(i)(j)} u_m^{[\pm 2]} = 0, \qquad \Delta^{(i)(j)} u_m^{(i')} = -2\delta^{i'[i} u_m^{(j)]}, \tag{3.36}$$

$$\Delta^{[+2](i)} u_m^{[-2]} = 2u_m^{(i)}, \qquad \Delta^{[+2](i)} u_m^{[+2]} = 0,$$

$$\Delta^{[+2](i)} u_m^{(j)} = \delta^{ij} u_m^{[+2]}, \tag{3.37}$$

$$\Delta^{[-2](i)} u_m^{[-2]} = 0, \qquad \Delta^{[-2](i)} u_m^{[+2]} = 2u_m^{(i)},$$

$$\Delta^{[-2](i)} u_m^{(j)} = \delta^{ij} u_m^{[-2]}. \tag{3.38}$$

In the notation (3.33) the Lorentz group algebra (2.18a), generated by the operators $\Delta_{(l)(k)}$, takes the form of the relations

$$[\Delta^{(0)}, \Delta^{(i)(j)}] = 0, \tag{3.39a}$$

$$[\Delta^{(i)(j)}, \Delta^{(i')(j')}] = 2\delta^{i[i'}\Delta^{(j')]j} - 2\delta^{j[i'}\Delta^{(j')]i}, \tag{3.39b}$$

$$[\Delta^{(0)}, \Delta^{[\mp 2](j)}] = \mp 2\Delta^{[\mp 2](j)}, \tag{3.39c}$$

$$[\Delta^{(i)(j)}, \Delta^{[\mp 2](i')}] = -\delta^{ii'}\Delta^{[\mp 2]j} + \delta^{ji'}\Delta^{[\mp 2]i}, \tag{3.39d}$$

$$[\Delta^{[+2](i)}, \Delta^{[-2](j)}] = \delta^{ij}\Delta^{(0)} + 2\Delta^{(i)(j)}. \tag{3.39e}$$

The nature of the operators $\Delta^{(0)}$, $\Delta^{[\mp 2](j)}$, $\Delta^{(i)(j)}$ becomes evident from the Eqs. (3.35)—(3.39). $\Delta^{(i)(j)}$ and $\Delta^{(0)}$ generate the $SO(8)$ and $SO(1,1)$ transformations, respectively. The operators $\Delta^{[\mp 2](j)}$ generate the transformations from the coset space $SO(1,9)/SO(1,1) \otimes SO(8)$. One of them may be associated with the «boost» symmetry appearing in the Lorentz harmonic approach to superparticle theory [20,21]. But in the discussed formulation of $D = 10$ superstring theory such symmetry is absent [22,23].

Now we are ready to discuss the equations of motion for $D = 10$ superstring theory (3.1). The procedure is similar to the one discussed in Section 2.

3.3 Equations of Motion. The equation of motion $\partial S/\partial e_f^{\mu} = 0$ gives the expression for the zweinbein e_μ^f in terms of the imbedding functions $x^m(\xi)$, $\theta^{\alpha I}(\xi)$ and the composed repere vectors $u_m^{(f)}$

$$e_\mu^{[\pm 2]} = \omega_\mu^m u_m^{[\pm 2]} / c(\alpha')^{1/2}. \tag{3.40}$$

This expression is similar to Eq. (2.6) since the zweinbein variables are absent in the expression for the additional Wess — Zumino term (3.2).

In the straightforward analogy with the repere bosonic string formulation (see Section 2, Eq. (2.28)), the equations of motion for the harmonic variables $v^+_{\alpha A}$, $v^-_{\alpha \dot{A}}$ can be presented in the form

$$\sum_{\pm} e\, e^{\mu [\pm 2]} \omega_\mu^m \Delta_{(k)(l)} u_m^{[\pm 2]} = 0 \tag{3.41}$$

using the admissible variations (3.28), which conserve the harmonicity conditions (3.6), (3.8).

Eqs. (3.41) are satisfied identically when $(k) = (i)$ and $(l) = (j)$ (see Eq. (3.10)). This results from the $SO(8)$ gauge symmetry of the action (3.1) generated by $\Delta^{(i)(j)}$ operators (3.33d). If $(k) = [+2]$, $(l) = [-2]$ (or vice versa), then Eqs. (3.41) are reduced to the relation (see Eq. (3.37))

$$\omega_\mu^m \left(e^{\mu[-2]} u_m^{[+2]} - e^{\mu[+2]} u_m^{[-2]}\right) = 0. \tag{3.42}$$

Taking into account Eq. (3.40) it may be justified that Eq. (3.42) is satisfied identically. This fact is associated with the gauge $SO(1,1)$ symmetry of the theory. However, the generator of the corresponding symmetry includes terms acting on the zweinbein fields in addition to the $\Delta^{(0)}$ operator (3.33a) (see Section 4).

Since, Eqs. (3.41) lead to the nontrivial results if and only if $(k) = [\pm 2]$, $(l) = (i)$ (or vice versa). In this case they are reduced to the equations (see Eqs. (3.37), (3.38))

$$e\, e^{\mu[\pm 2]} \omega_\mu^m u_m^{(i)} = 0, \tag{3.43}$$

which can be easily transformed into the form

$$\omega_\mu^m u_m^{(i)} = 0 \tag{3.44}$$

(comprise with Eqs. (2.31), (2.9)).

Taking into account Eqs. (3.40), (3.44), and the completeness conditions (which follow from the orthonormality ones (3.12))

$$\delta_m^n = \frac{1}{2} u_m^{[-2]} u^{n[+2]} + \frac{1}{2} u_m^{[+2]} u^{n[-2]} - u_m^{(i)} u^{n(i)},$$

the coefficients ω_μ^m (3.2) of the ω-form pull-back may be decomposed on the $u_m^{[\pm 2]}$ light-like vectors

$$\omega_\mu^m = \frac{1}{2} c(\alpha')^{1/2} (e_\mu^{[-2]} u^{m[+2]} + e_\mu^{[+2]} u^{m[-2]}) \tag{3.45}$$

and vice versa

$$u^{m[\pm 2]} = e^{\mu[\pm 2]} \omega_\mu^m / c(\alpha')^{1/2}. \tag{3.46}$$

Thus the vectors $u^{m[\pm 2]}$ are tangent to the superstring world-sheet on the shell, defined by the motion equations. Contrary, the vectors $u^{m(i)}$ are orthogonal to the world-sheet on this shell.

Using Eqs. (3.40), (3.45), (3.46), the classical equivalence of the discussed $D = 10$ superstring formulation with the standard Green — Schwarz one [1] can be justified easily. Substituting Eq. (3.46) into the functional (3.1) and using the definition of the world-sheet metric (2.4), we get the standard action functional [1] (comporise with Eqs. (2.9)—(2.13)).

The equation of motion for the $x^m(\xi)$ field, $\partial S / \partial x^m(\xi) = 0$, has the form

$$\partial_\mu \left(e \sum_{\pm} (e^{\mu[\pm 2]} u_m^{[\mp 2]}) \right) -$$
$$- \varepsilon^{\mu\nu} (\partial_\mu \theta^1 \sigma_m \partial_\nu \theta^1 - \partial_\mu \theta^2 \sigma_m \partial_\nu \theta^2 / c(\alpha')^{1/2} = 0, \tag{3.47}$$

which is similar to Eq. (2.12), except for the last term containing Grassmannian degrees of freedom, and may be easily reduced to the standard form [1]

$$\partial_\mu (\sqrt{-g}\, g^{\mu\nu} \omega_\nu^m) - \varepsilon^{\mu\nu} (\partial_\mu \theta^1 \sigma^m \partial_\nu \theta^1 - \partial_\mu \theta^2 \sigma^m \partial_\nu \theta^2) = 0, \tag{3.48}$$

using Eq. (3.48) (see also Eq. (2.1)).

The equations $\delta S/\delta\theta^{\alpha I}(\xi) = 0$ have the form

$$(\partial_\mu \theta^I \sigma^m)_\alpha \left(\sum_\pm e^{\mu[\pm 2]} u^{m[\mp 2]} - 2(-1)^I \varepsilon^{\mu\nu} \omega_\nu^m\right) = 0, \quad I = 1, 2, \quad (3.49)$$

which may be reduced to the standard one [1]

$$(\partial_\mu \theta^I \sigma^m)_\alpha (\sqrt{-g}\, g^{\mu\nu} - (-1)^I \varepsilon^{\mu\nu})\, \omega_\nu^m = 0, \quad I = 1, 2 \quad (3.50)$$

if Eqs.(3.46) are taken into account. However, it is interesting to use Eq.(3.45) and to exclude the fields ω_ν^m from the equations (3.49). Since we derive the following relations

$$e^{\mu[+2]} \partial_\mu \theta^{\alpha 1} v^-_{\alpha \dot{A}} = 0, \quad (3.51a)$$

$$e^{\mu[-2]} \partial_\mu \theta^{\alpha 2} v^+_{\alpha A} = 0, \quad (3.51b)$$

when the Eqs.(3.13a,b) are taken into account.

Therefore the equations of motion for the $D = 10$, $N = \mathrm{II}B$ superstring in twistor-like formulation (3.1) have the form of Eqs.(3.40), (3.44)—(3.47), (3.51). The relations (3.47), (3.51) are equivalent to the standard equations of motion (3.48), (3.50) [1], however they have more simple form. Thus the twistor-like formulation (3.1) is equivalent to the standard one [1] on the classical level [23] and simplifies the equations of motion essentially.

In the next sections the Hamiltonian formalism for the twistor-like $D = 10$ superstring formulation (3.1) is worked out. This formalism is necessary for the covariant superstring quantization using the BFV-BFF scheme [7].

4. HAMILTONIAN FORMALISM FOR D = 10 SUPERSTRING IN TWISTOR-LIKE FORMULATION

To simplify Hamiltonian formalism and to make the meaning of some constraints more clear, let us reformulate the action principle (3.1) in terms of the zweinbein densities

$$\rho_f^\mu \equiv \left(\frac{1}{2}(\rho^{\mu[-2]} + \rho^{\mu[+2]}), \frac{1}{2}(\rho^{\mu[-2]} - \rho^{\mu[+2]})\right) \equiv e\, e_f^\mu/(\alpha')^{1/2}, \quad (4.1)$$

$$e \equiv \det(e_\mu^f) = \frac{1}{2}\alpha' \varepsilon_{\mu\nu} \rho^{\mu[-2]} \rho^{\nu[+2]}, \quad (\varepsilon_{01} = -\varepsilon^{01} = -1), \quad (4.2)$$

instead of zweinbein e_μ^f, e_f^μ themselves

$$S = S_1 + S_{W-Z}, \quad (4.3)$$

$$S_1 = -\frac{1}{2}\int d\tau d\sigma [(\rho^{\mu[+2]}u_m^{[-2]} + \rho^{\mu[-2]}u_m^{[+2]})\,\omega_\mu^m + c\alpha'\varepsilon_{\mu\nu}\rho^{\mu[+2]}\rho^{\nu[-2]}\,] \equiv$$

$$\equiv -\frac{1}{2}\int d\tau d\sigma \left[\left(\rho^{\mu[+2]}\frac{1}{8}(v_A^-\tilde{\sigma}_m v_A^-) + \rho^{\mu[-2]}\frac{1}{8}(v_A^+\tilde{\sigma}_m v_A^+)\right)\omega_\mu^m + \right.$$

$$\left. + c\alpha'\varepsilon_{\mu\nu}\rho^{\mu[+2]}\rho^{\nu[-2]}\right], \tag{4.3a}$$

$$S_{W-Z} \equiv -(c\alpha')^{-1}\int d\tau d\sigma\, \varepsilon^{\mu\nu}\,[i\,\omega_\mu^m\,(\partial_\nu\theta^1\sigma_m\theta^1 - \partial_\nu\theta^2\sigma_m\theta^2) +$$

$$+ \partial_\mu\theta^1\sigma^m\theta^1\,\partial_\nu\theta^2\sigma_m\theta^2\,]. \tag{4.3b}$$

Here (see Eq. (3.2))

$$\omega_\mu^m = \partial_\mu x^m - i\,(\partial_\mu\theta^1\sigma^m\theta^1 + \partial_\mu\theta^2\sigma^m\theta^2).$$

Of course, the Wess — Zumino term (4.3b) is not modified. However, the twistor-like part of the action (4.3a) includes the terms which are dependent on the densities $\rho^{\mu[\pm 2]}$ in a linear or bilinear way. At the same time, their dependences on the inverse zweinbein variables $e^{\mu[\pm 2]}$ are the more complicated ones (see Eq. (3.1b)).

4.1 Primary Constraints and Covariant Momentum Density. The canonical momentum densities

$$P_M \equiv (P_m, \pi_\alpha^1, \pi_\alpha^2, P_A^{-\alpha}, P_A^{+\alpha}, P_{\alpha A}^+, P_{\alpha \dot{A}}^-, P_{(\rho)\mu}^{[\mp 2]}) \equiv$$

$$\equiv -(-1)^M \partial Z/\partial(\partial_\tau z^M) \tag{4.4}$$

are conjugated to the configurational space (target space) coordinates of the discussed superstring formulation (4.3)

$$z^M \equiv (x^m, \theta^{\alpha 1}, \theta^{\alpha 2}, v_{\alpha A}^+, v_{\alpha \dot{A}}^-, v_A^{-\alpha}, v_{\dot{A}}^{\alpha}, \rho^{[\pm 2]\mu}) \tag{4.5}$$

with respect to the standard Poisson brackets

$$[z^M(\sigma), P_N(\sigma')\,]_P = -(-1)^{MN}\,[P_N(\sigma'), z^M(\sigma)\,]_P = -\delta_N^M\,\delta(\sigma - \sigma'). \tag{4.6}$$

Here the multiplier $(-1)^{MN}$ is equal to (-1), if both the indices M and N belong to the fermionic variables, and is equal to $(+1)$ in any other case.

The action functional (4.3) is the first order one on the proper time derivatives (i.e., on the velocities). Hence all the expressions (4.4) for the canonical momentum densities lead to the primary constraints. For the nonharmonical variables such constraints are

$$\Phi_m(\sigma) \equiv P_m - \frac{1}{2}\rho^{\tau[+2]}u_m^{[-2]} - \frac{1}{2}\rho^{\tau[-2]}u_m^{[+2]} +$$

$$+ \frac{i}{c\alpha'}\sum_I (-1)^I \partial_\sigma\theta^I\sigma_m\theta^I \approx 0, \tag{4.7a}$$

$$D^I_\alpha(\sigma) \equiv -\pi^I_\alpha + i(\sigma^m\Theta^I)_\alpha [P_m - $$
$$-(-1)^I \frac{1}{\alpha'}(\partial_\sigma x_m - i\partial_\mu \theta^I \sigma_m \theta^I)] \approx 0, \qquad (4.7b)$$

$$P^{[\pm 2]}_{(\rho)\tau} \approx 0, \qquad (4.7c)$$

$$P^{[\pm 2]}_{(\rho)\sigma} \approx 0. \qquad (4.7d)$$

For the spinor harmonics

$$(v^+_{\alpha A}, v^-_{\alpha \dot{A}}) = v^a_\alpha, \qquad (v^{-\alpha}_A, v^{+\alpha}_{\dot{A}})^T = (v^{-1})^\alpha_a \equiv v^\alpha_a$$

the set of the primary constraints consists of the completely trivial relations

$$P^\alpha_a \approx 0: \qquad P^{-\alpha}_A \approx 0, \qquad P^{+\alpha}_{\dot{A}} \approx 0, \qquad (4.8a)$$

$$P^a_\alpha \approx 0: \qquad P^+_{\alpha A} \approx 0, \qquad P^-_{\alpha \dot{A}} \approx 0, \qquad (4.8b)$$

(which reflect the auxiliary character of the harmonic variables in the discussed formulation) and of the harmonicity conditions (3.6), (3.8), discussed as the «week» relations [50]

$$\Xi_{m_1 \ldots m_4} = u^{m(n)} \eta_{(n)(l)} \Xi^{(l)}_{m_1 \ldots m_4 m} \equiv$$

$$\equiv u^{m(n)} \eta_{(n)(l)} \mathrm{Sp}\,(v^T \tilde{\sigma}_{m_1 \ldots m_4 m} v \sigma^{(n)}) \approx 0, \qquad (4.9a)$$

$$\Xi_0 \equiv u^{[-2]}_m u^{m[+2]} - 2 \equiv \frac{1}{8}(v^-_{\dot{A}} \tilde{\sigma}_m v^-_{\dot{A}}) \frac{1}{8}(v^+_A \tilde{\sigma}_m v^+_A) - 2 \approx 0, \qquad (4.9b)$$

$$\Xi^{[0]}_{AB} \equiv v^{-\alpha}_A v^+_{\alpha B} - \delta_{AB} \approx 0, \qquad \Xi^{[-2]}_{A\dot{B}} \equiv v^{-\alpha}_A v^-_{\alpha \dot{B}} \approx 0,$$

$$\Xi^{[+2]}_{\dot{A}B} \equiv v^{+\alpha}_{\dot{A}} v^+_{\alpha B} \approx 0, \qquad \Xi^{[0]}_{\dot{A}\dot{B}} \equiv v^{+\alpha}_{\dot{A}} v^-_{\alpha \dot{B}} - \delta_{\dot{A}\dot{B}} \approx 0. \qquad (4.9c)$$

The expressions $\Xi^{(l)}_{m_1 \ldots m_5}$ [20,21] included in Eq.(49a) vanish as the result of Eq.(4.9a) [46] and my be specified as follows

$$\Xi^{[-2]}_{m_1 \ldots m_5} = \frac{1}{8} v^-_{\alpha \dot{A}} \tilde{\sigma}^{\alpha\gamma}_{m_1 \ldots m_5} v^-_{\gamma \dot{A}} \approx 0, \qquad (4.10a)$$

$$\Xi^{[+2]}_{m_1 \ldots m_5} = \frac{1}{8} v^+_{\alpha A} \tilde{\sigma}^{\alpha\gamma}_{m_1 \ldots m_5} v^+_{\gamma A} \approx 0, \qquad (4.10b)$$

$$\Xi^{(i)}_{m_1 \ldots m_5} = \frac{1}{8} v^+_{\alpha A} \gamma^{(i)}_{A\dot{A}} \tilde{\sigma}^{\alpha\gamma}_{m_1 \ldots m_5} v^-_{\gamma \dot{A}} \approx 0. \qquad (4.10c)$$

The relations (4.9), (4.10) are complicated ones. So it is evident, that the computation of the constraint algebra is a hard task if Eqs.(4.9) are understood as the «week» equality.

Hence, it is important to work out the method which allows one to exclude the conditions (3.6), (3.8) from the set of constraints and to discuss them as the «strong» relations [50]. Such method can be devised in the straightforward analogy with one discussed in Section 2 for the case of bosonic string repere formulation.

This means that the concept of covariant momentum density should be used. Now we discuss it for the case of the twistor-like superstring formulation (4.3) using the experience obtained in Section 2 (and hence omitting some technical details).

Let us remind some properties of the covariant momentum densities, which was discussed in Section 2. Firts of all they should have the vanishing (in the weak sense, see Eq.(2.56) Poisson brackets with the harmonicity conditions (3.6), (3.8). From the other hand, it is known that the harmonicity conditions (3.6), (3.8) are the realization of the relation (3.4) [20,21,46]. Since, the covariant momentum variables should be canonically conjugated to some parameters of SO(1,9) group included in the spinor harmonics $(v^{+}_{\alpha A}, v^{-}_{\alpha \dot{A}}) = v^{a}_{\alpha}$, $(v^{-\alpha}_{A}, v^{+\alpha}_{\dot{A}}) \equiv v^{\alpha}_{a}$. Therefore, the covariant momentum densities should be associated with the Lorentz group, too, and hence they should generate the $SO(1,9)$ group algebra on the Poisson brackets.

Another degrees of freedom included in the spinor harmonics $v^{+}_{\alpha A}$, $v^{-}_{\alpha \dot{A}}$, $v^{-\alpha}_{A}$, $v^{+\alpha}_{\dot{A}}$ are killed by the harmonicity conditions (3.6), (3.8). Henceforth, the harmonic momentum degrees of freedom, which cannot be reduced to the covariant ones, should be conjugated to the harmonicity conditions in the week sense (see Eqs.(2.46), (2.47) for the case of bosonic string repere formulation). Since we may understand the condition of vanishing of these variables, together with the harmonicity conditions (3.6), (3.8), as the «strong» equalities, if the corresponding Dirac brackets are used instead of Poisson ones (4.6). These Dirac brackets should be analogous to ones presented in Eq.(2.48). However, if we discuss the space of functions dependent on the covariant harmonic momentum densities only, these Dirac brackets coincide with the Poisson ones (4.6).

The discussed situation is similar to the case, where the second class constraints are solved explicitly (i.e., the superfluous momentum degrees of freedom vanish and the coordinates conjugated to them are expressed through the «physical» ones) [50]. The unique distinction is that the 256 + 256 harmonic variables are expressed through the 45 degrees of freedom associated with the $SO(1.9)$ group in an implicit way. Such implicit dependence is defined by the harmonicity conditions (3.6), (3.8). (See [22,23,46] for details).

Hence, the harmonicity conditions (3.6), (3.8) can be excluded from the set of Hamiltonian constraints without changing the Poisson brackets if we define the set of covariant momentum densities with the properties listed above and exclude all other harmonic momentum variables from the phase space.

The experience of studying the repere bosonic string formulation (2.1) gives us the prescription for the extracting of the covariant momentum densities from the set of canonical ones.

First of all, these densities are the classical analogs of the covariant derivatives (3.31)—(3.34) appearing in the expression (3.31) for the admissible variation, i.e., they may be derived from the expressions (3.31)—(3.34) by the simple replacement of the derivatives $\partial/\partial v^{a}_{\alpha}$, $\partial/\partial v^{\alpha}_{a}$ by the canonical momentum densities P^{α}_{a}, P^{a}_{α}.

From the other hand, it may be derived as the derivatives of the Lagrangian density L of the action (4.3) with respect to $\Omega^{(k)(l)}_{\tau}$ (where $\Omega^{(k)(l)}_{\tau}$ are the τ-coefficients of the pull-backs of the $SO(1,9)$ Cartan differential forms (3.32) on the world-sheet). Their form may be derived from Eqs. (3.25), (3.32) as follows

$$\Omega^{(k)(l)}_{\tau}= \Omega^{(k)(l)}(\partial_{\tau}) = u^{m(k)}\partial_{\tau}u^{(l)}_{m} \equiv -\frac{1}{8}\,\mathrm{Sp}(v^{-1}\partial_{\tau}v\sigma^{(k)(l)}) \qquad (4.11)$$

or, equivalently, using the following representations

$$\Omega^{(k)(l)}(d) = d\tau\,\Omega^{(k)(l)}_{\tau}+ d\sigma\,\Omega^{(k)(l)}_{\sigma} \qquad (4.12)$$

for the discussed pull-backs.

Thus the general expression for the covariant momentum densities in the whole phase space (4.5), (4.4) has the form

$$\Pi^{(l)(k)}\equiv -\,\partial L/\partial\,(\Omega_{\tau(k)(l)}) =$$

$$= -\frac{1}{2}\,\mathrm{Sp}\,(v\sigma^{(k)(l)}\partial L/\partial\,(\partial_{\tau}v) - \partial L/\partial\,(\partial_{\tau}v^{-1})\,\sigma^{(k)(l)}v^{-1}). \qquad (4.13)$$

The Poisson brackets of the covariant momentum density with any functional F, living on the configurational space of the discussed dynamical system, may be presented in the form

$$[\Pi^{(l)(k)}(\sigma), F[v, v^{-1}, x, \theta]]_{P} = \tilde{\Delta}^{(l)(k)}(\sigma)\,F[v, v^{-1}, x, \theta]. \qquad (4.14)$$

Here $\tilde{\Delta}^{(l)(k)}(\sigma)$ are the variational analogs of the covariant harmonic derivatives (3.29), (3.33). Thus these covariant derivatives play the same role for the covariant momentum variables, as the ordinary derivatives play for the canonical ones

$$[P_M(\sigma), F[z^N]]_P = \delta/\delta z^M(\sigma)\, F[z^N].$$

Moreover, the covariant momentum densities (4.13) generate the current algebra, associated with the Lorentz group algebra (2.18a), (3.39), on the Poisson brackets (4.6)

$$[\Pi^{(l_1)(l_2)}(\sigma), \Pi^{(k_1)(k_2)}(\sigma')]_P = [\Delta^{(l_1)(l_2)}, \Delta^{(k_1)(k_2)}]\Big|_{\Delta\to\Pi(\sigma)} \delta(\sigma-\sigma') \quad (4.15)$$

and have the vanishing Poisson brackets with the harmonicity conditions (3.6), (3.8)

$$[\Pi^{(l)(k)}(\sigma), \Xi(\sigma'),]_P\Big|_{\Xi=0} = 0, \quad (4.16)$$

when the same harmonicity conditions are taken into account.

Thus we should leave only the covariant harmonic momentum densities (4.13) in the phase space, which is parameterized now by following variables

$$(x^m(\sigma), P_m(\sigma); \theta^{\alpha I}(\sigma), \pi_{\alpha I}(\sigma);$$

$$v^+_{\alpha A}(\sigma), v^-_{\alpha \dot A}(\sigma), v_A^{-\alpha}(\sigma), v_{\dot A}^{+\alpha}(\sigma), \Pi^{(l)(k)}(\sigma)). \quad (4.17)$$

And only the primary constraints

$$\Pi^{(l)(k)}(\sigma) \approx 0 \quad (4.18)$$

should be taken into account besides ones presented in Eqs.(4.7). Eqs.(4.18) replace the whole set (4.8), (4.9) of constraints for harmonic variables in the discussed approach. The harmonicity conditions (3.6), (3.8) are understood as the strong equality.

The Poisson brackets are defined by Eqs.(4.14), (4.15) or by the basic relations

$$[\Pi^{(l)(k)}(\sigma),\ v^a_\alpha(\sigma'),]_P = \frac{1}{2}(v_\alpha\sigma^{(k)(l)})^a\,\delta(\sigma-\sigma'), \quad (4.19a)$$

$$[\Pi^{(l)(k)}(\sigma), v_a^\alpha(\sigma'),]_P = -\frac{1}{2}(\sigma^{(k)(l)}v^\alpha)_a\,\delta(\sigma-\sigma'), \quad (4.19b)$$

which lead to the Eqs.(4.15), when the Jacoby identities for the Poisson brackets are taken into account.

Now let us discuss the form of the canonical Hamiltonian H_0 density which is consistent with the definitions of the Poisson brackets and the Hamiltonian equations of motion

$$\partial_\tau f(\sigma) = [f(\sigma), \int d\sigma' H_0(\sigma')]_P. \tag{4.20}$$

The standard expression for the canonical Hamiltonian

$$H_0^{\text{stand}} = -(-1)^M \partial_\tau z^M P_M - L \tag{4.21}$$

has such consistency with Eqs.(4.6) and (4.20). This may be verified by the following formal manipulation. The use of the density (4.21) in Eq.(4.20) written for the simplest function $f = z^M(\tau, \sigma)$ leads to the identity

$$\partial_\tau z^M(\sigma) = [z^M(\sigma), \int d\sigma' H_0^{\text{stand}}(\sigma')]_P =$$
$$= -\int d\sigma' \partial_\tau z^N(\sigma') [z^M(\sigma), P_N(\sigma')]_P = \partial_\tau z^M(\sigma).$$

To achieve such consistency with Eqs.(4.14), (4.19), (4.20), we should define the canonical Hamiltonian density in terms of covariant harmonic momentum variables as follows

$$H_0 = -\partial_\tau x^m(\sigma) P_m(\sigma) + \partial_\tau \theta^{\alpha l}(\sigma) \pi_{\alpha l}(\sigma) -$$
$$-\frac{1}{2}\Omega_\tau^{(k)(l)}(\sigma)\Pi_{(l)(k)}(\sigma) - L(\sigma), \tag{4.22}$$

i.e., instead of the standard combination $\partial_\tau z^M P_M$ (which can be derived by the replacement $\delta \to \partial_\tau$, $\partial/\partial z^M \to P_M$ from the expression for arbitrary variation $\delta = \delta z^M \partial/\partial z^M$) the expression

$$\frac{1}{2}\Omega_\tau^{(k)(l)}(\sigma)\Pi_{(l)(k)}(\sigma)$$

(which can be derived by the replacement $\Omega^{(k)(l)}(\delta) \to \Omega_\tau^{(k)(l)}(\sigma)$, $\Delta_{(l)(k)} \to \Pi_{(l)(k)}(\sigma)$, from the expression for the admissible variation (3.28)) appears in the canonical Hamiltonian.

For the forthcoming discussion some specification of the relations (4.13), (4.18), (4.22) is necessary.

Let us introduce the covariant momentum densities $\Pi^{(0)}$, $\Pi^{[\mp 2](i)}$, $\Pi^{(i)(j)}$, which are the classical analogs of the covariant derivatives (3.33). In terms of the canonical momentum densities $P_v \equiv (P_A^{-\alpha}, P_A^{+\alpha})$ and $P_{(v^{-1})} \equiv (P_{\alpha A}^{+}, P_{\alpha A}^{-})$ they are defined by the relations

$$\Pi^{(l)(k)} = (-2\Pi^{(0)}, \Pi^{[\mp 2](i)}, \Pi^{(i)(j)}) =$$
$$= \frac{1}{2}\,\text{Sp}\,(v\,\sigma^{(k)(l)} P_v - P_{(v^{-1})}\sigma^{(k)(l)} v^{-1}), \tag{4.23}$$

$$\Pi^{(0)} \equiv -\frac{1}{2}\Pi^{[+2][-2]} = v_A^+ P_A^- - v_{\dot A}^- P_{\dot A}^+ - v_A^- P_A^+ + v_{\dot A}^+ P_{\dot A}^-, \qquad (4.23a)$$

$$\Pi^{[+2](i)} = v_A^+ \gamma^i_{A\dot A} P_{\dot A}^+ - v_{\dot A}^+ \tilde\gamma^i_{\dot A A} P_A^+, \qquad (4.23b)$$

$$\Pi^{[-2](i)} = v_{\dot A}^- \tilde\gamma^i_{\dot A A} P_A^- - v_A^- \gamma^i_{A\dot A} P_{\dot A}^-, \qquad (4.23c)$$

$$\Pi^{(i)(j)} = \frac{1}{2}(v_A^+ \gamma^{ij}_{AB} P_B^- + v_{\dot A}^- \tilde\gamma^{ij}_{\dot A\dot B} P_{\dot B}^+ + v_A^- \gamma^{ij}_{AB} P_B^+ + v_{\dot A}^+ \tilde\gamma^{ij}_{\dot A\dot B} P_{\dot B}^-). \qquad (4.23d)$$

It is evident (see Eqs.(4.15), (3.39)) that the densities $\Pi^{(0)}$ and $\Pi^{(i)(j)}$ generate (on the Poisson brackets) the Kac-Moody-like extensions of the $SO(1,1)$ and $SO(8)$ group algebras, respectively. The densities $\Pi^{[\mp 2](j)}$ are associated with the coset $SO(1,9)/[SO(1,1)\times SO(8)]$.

The Poisson brackets (4.6) may be written in the following form

$$[\tilde F, \tilde G]_P = \int d\sigma\,(\delta\tilde F/\delta P_m(\sigma)\,\delta\tilde G/\delta x^m(\sigma) - \delta\tilde F/\delta x^m(\sigma)\,\delta\tilde G/\delta P_m(\sigma)) -$$
$$- \int d\sigma\,(\delta\tilde F/\delta\theta^{\alpha I}(\sigma)\,\delta\tilde G/\delta\pi_{\alpha I}(\sigma) + \delta\tilde F/\delta\pi_{\alpha I}(\sigma)\,\delta\tilde G/\delta\theta^{\alpha I}(\sigma) +$$
$$+ \int d\sigma(\delta\tilde F/\delta\Pi^{(r)(l)}(\sigma)\,\tilde\Delta^{(r)(l)}(\sigma)\tilde G - \tilde\Delta^{(r)(l)}(\sigma)\tilde F\,\delta\tilde G/\delta\Pi^{(r)(l)}(\sigma)) \qquad (4.24)$$

for the functionals $\tilde F$ and $\tilde G$ living in the phase space (4.17) (comprise with Eq.(2.55)). All the expressions (4.23) vanish in the week sense (4.18). Hence the primary harmonic constraints have the following form in the discussed approach

$$\Pi^{(0)} \equiv -\,\partial L/\partial\Omega_\tau^{(0)} \approx 0, \qquad (4.25a)$$

$$\Pi^{[\mp 2](j)} \equiv -\,\partial L/\partial\Omega_\tau^{[\pm 2](j)} \approx 0, \qquad (4.25b)$$

$$\Pi^{(i)(j)} \equiv +\,\partial L/\partial\Omega_\tau^{(i)(j)} \approx 0. \qquad (4.25c)$$

At last, the expression (4.22) for the canonical Hamiltonian may be specified as follows

$$H_0 = -\,\partial_\tau x^m(\sigma) P_m(\sigma) + \partial_\tau\theta^{\alpha I}(\sigma)\,\pi_{\alpha I}(\sigma) -$$
$$- \Omega_\tau^{(0)}\Pi^{(0)} - \Omega_\tau^{[\pm 2](j)}\Pi^{[\mp 2](j)} + \frac{1}{2}\Omega_\tau^{(i)(j)}\Pi^{(i)(j)} - L, \qquad (4.26)$$

where L denotes Lagrangian density for the action (4.3).

Let us resume the results of this subsection which define the starting point for the next one.

Hence, the phase space of the discussed system is parameterized by the variables (4.17), Poisson brackets are defined by the standard relations (4.6) for the ordinary variables and by the relations (4.14), (4.15), (4.19) for the harmonic ones*). The canonical Hamiltonian is defined by the relation (4.26). And the set of primary constraints includes relations (4.7a-d) and (4.25).

4.2. Irreducible First Class Constraints for the $D = 10$, $N = 2B$ Green — Schwarz Superstring. The first class constraints can be extracted by means of the well-known Dirac procedure of checking the constraints conservation during evolution [50].

The evolution of the dynamical variables of system with constraints is defined by a generalized Hamiltonian, which is the sum of the canonical Hamiltonian and the products of the primary constraints on the corresponding Lagrange multipliers. For the discussed dynamical system, with the primary constraints (4.7a-d), (4.25) the generalized Hamiltonian has the following form

$$H' = \int d\sigma\, H'(\tau, \sigma),$$

$$\begin{aligned} H'(\tau, \sigma) = H_0(\tau, \sigma) + \xi^{+I}_A v^{-\alpha}_A D^I_\alpha(\sigma) + \xi^{-I}_A v^{+\alpha}_A D^I_\alpha(\sigma) + \\ + [a^{[+2]} u^{m[-2]} + a^{[-2]} u^{m[+2]} + a^{(i)} u^{m(i)}]\, \Phi_m + \\ + i a^{(0)} \Pi^{(0)} + \frac{1}{2} \alpha^{ij} \Pi^{ij} + \alpha^{[\mp 2] i} \Pi^{[\pm 2] i} + \beta^{\mu [\mp 2]} P^{[\pm 2]}_{(\rho)\mu}. \end{aligned} \tag{4.27}$$

Here the canonical Hamiltonian H_0 is defined by the general expression (4.26) for any dynamical system living on the phase space (4.17). For the discussed superstring formulation (4.3) it has the following form

$$H_0 \equiv \int d\sigma\, H_0(\tau, \sigma),$$

$$\begin{aligned} H_0(\tau, \sigma) \equiv - \partial_\tau x^m P_m + \partial_\tau \theta^{\alpha I} \pi_{\alpha I} - \frac{1}{2} \Omega^{(k)(l)}_\tau \Pi_{(l)(k)} - L = \\ = \frac{1}{2} (\rho^{[+2]\sigma} u^{[-2]}_m + \rho^{[-2]\sigma} u^{[+2]}_m)\, \omega^m_\sigma + \frac{c\alpha'}{2} \varepsilon_{\mu\nu} \rho^{\mu[+2]} \rho^{\nu[-2]}, \end{aligned} \tag{4.28}$$

$$\varepsilon_{01} = - \varepsilon_{10} = - 1.$$

The conditions of the constraint conservation

$$\frac{d}{d\tau} (\text{constraint}) \approx [(\text{constraint}), H']_P \approx 0 \tag{4.29}$$

should lead either to the restrictions for the Lagrange multipliers or to the appearance of the «secondary» constraints [50].

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*Of course, the simple expressions (4.23) and the initial Poisson brackets definition may be used for the calculations, because the Poisson brackets were not changed (see above).

So, the requirement of the conservation of the constraints $P^{[\pm 2]}_{(\rho)\sigma} \approx 0$ (4.7d) leads to the secondary constraints

$$\omega^{m}_{\sigma} u^{[\mp 2]}_{m} \pm c\alpha' \rho^{\tau [\mp 2]} \approx 0. \qquad (4.30a,b)$$

At the same time, the conservation of the constraints $P^{[\pm 2]}_{(\rho)\tau} \approx 0$ (4.7c) permits to express the Lagrange multipliers $a^{[\mp 2]}$ through the σ-components of the world-sheet vector densities $\rho^{\mu [\mp 2]}$ (or, more precisely, vice versa)

$$a^{[\mp 2]} = \mp \frac{c\alpha'}{2} \rho^{\sigma [\mp 2]}. \qquad (4.31a,b)$$

Owing to $SO(1,1) \otimes SO(8)$ gauge symmetry of the discussed superstring action (3.1), (4.3), the requirement of the conservation of the harmonic constraints $\Pi^{(0)} \approx 0$ and $\Pi^{ij} \approx 0$ (4.25a,d) has no the nontrivial consequences. (Let's remind, that $\Pi^{(0)}$ and Π^{ij} generate $SO(1,1)$ and $SO(8)$ transformations on the Poisson brackets). However, the conservation of the other 16 harmonic constraints $\Pi^{[\pm 2]i} \approx 0$ (4.25) has nontrivial consequences for the Lagrange multipliers

$$\rho^{\sigma [\mp 2]} \omega^{m}_{\sigma} u^{(i)}_{m} - a^{i} \rho^{\tau [\mp 2]} \approx 0. \qquad (4.32a,b)$$

This means, that the gauge symmetry under the tramsformations from the coset space $SO(1,9)/[SO(1,1) \otimes SO(8)]$ are absent in the discussed formulation. Such fact was discussed in Refs.22,23 in detail.

The consistency condition for Eqs.(4.32a) and (4.32b) leads to the relation

$$a^{i} e \equiv a^{i} (\rho^{\tau [+2]} \rho^{\sigma [-2]} - \rho^{\tau [-2]} \rho^{\sigma [+2]}) = 0,$$

which results in the vanishing of the $SO(8)$ vector Lagrange multiplier

$$a^{i} = 0, \qquad (4.33)$$

in the case of a nondegenerate world-sheet metric (or, more precisely, nondegenerate world-sheet moving frame). Using Eq.(4.33), we can see that Eqs.(4.32a,b) produce the following secondary constraint

$$\omega^{(i)}_{\sigma} \equiv \omega^{m}_{\sigma} u^{(i)}_{m} \approx 0, \qquad (4.34)$$

which is the σ-component of Eq.(3.44)*.

*It is of interest to note that for the case of a degenerate world-sheet metric, which corresponds to the null-superstrings [52,53,42,43,19] Eqs.(4.32a) and (4.32b) become consistent without using Eq.(4.33), and the secondary constraints (4.34) are absent (see [42,43,19]).

Together with Eqs. (4.30a,b), Eq. (4.34) gives the possibility of decomposing the component of the Cartan form pull-back ω^m_σ onto the basis of two vectors $u_m^{[\pm 2]}$ of the target space moving frame which, therefore, are tangent to the world-sheet on mass shell (i.e., on the shell defined by the equations of motion in the target space).

The requirement of the preservation of the Grassmannian spinor constraints $D^I_\alpha(\sigma) \approx 0$ (4.7b) gives the expression for the Grassmannian Lagrange multipliers ξ^{+2}_A and $\xi^{-1}_{\dot{A}}$ through the dynamical variables

$$\xi^{+2}_A = \frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}} \partial_\sigma \theta^{\alpha 2} v^+_{\alpha A}, \tag{4.35a}$$

$$\xi^{-1}_{\dot{A}} = \frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}} \partial_\sigma \theta^{\alpha 1} v^-_{\alpha \dot{A}}. \tag{4.35b}$$

The rest of the Grassmannian Lagrange multipliers ξ^{+1}_A and $\xi^{-2}_{\dot{A}}$ remains independent and plays the role of the parameters of the fermionic κ-symmetry in the framework of Hamiltonian formalism. We should stress that this symmetry is present in the theory only for the definite choice of the numerical coefficient a' included in the Wess — Zumino term (4.3b) of the superstring action (4.3). If this coefficient is different from $\pm \frac{1}{c\alpha'}$, the conservation conditions for the Grassmannian constraints (4.7b)

$$D^I_\alpha(\sigma) \equiv -\pi^I_\alpha + i(\sigma_m \theta^I)_\alpha (P_m - (-1)^I a'(\partial_\sigma x_m - i\partial_\sigma \theta^I \sigma_m \theta^I)) \approx 0$$

result in the relations

$$\xi^{+I}_A v^+_{\alpha A} u^{m[-2]} (P_m - (-1)^I a' (\partial_\sigma x_m - 2i\, \partial_\sigma \theta^I \sigma_m \theta^I)) +$$
$$+ \xi^{-I}_{\dot{A}} v^-_{\alpha \dot{A}} u^{m[+2]} (P_m + (-1)^{I+1} a' (\partial_\sigma x_m - 2i\, \partial_\sigma \theta^I \sigma_m \theta^I)) =$$
$$= \rho^{\sigma[-2]} \partial_\sigma \theta^{\beta 2} v^+_{\beta A} v^+_{\alpha A} (1-(-1)^{I+1} + \rho^{\sigma[+2]} \partial_\sigma \theta^{\gamma 1} v^-_{\gamma \dot{A}} v^-_{\alpha \dot{A}} (1+(-1)^{I+1}). \tag{4.36}$$

From Eq. (4.36) we may get the relations similar to Eqs. (4.35a,b) not only for the ξ^{+2}_A and $\xi^{-1}_{\dot{A}}$, but also for the remaining Grassmannian Lagrange multipliers ξ^{+1}_A and $\xi^{-2}_{\dot{A}}$. (The primary and secondary constraints (4.7a), (4.30) should be used in such computations). So the fermionic κ-symmetry is absent for the theory with the numerical coefficient in front of Wess — Zumino term $a' \neq \pm \frac{1}{c\alpha'}$.

If we choose this coefficient a' to be equal to $\left(-\frac{1}{c\alpha'}\right)$, instead of $+\frac{1}{c\alpha'}$, then the relation (4.36) gives the expressions like Eqs.(4.35a,b), but for the Lagrange multipliers ξ_A^{+1} and ξ_A^{-2}. The Lagrangian miltipliers ξ_A^{+2} and ξ_A^{-1}, which remain undetermined in this case, play the role of the parameters of the κ-symmetry transformations.

The conservation of the constraints (4.7a)

$$\Phi_m \equiv P_m - \frac{1}{2}\rho^{\tau[+2]}u_m^{[-2]} - \frac{1}{2}\rho^{\tau[-2]}u_m^{[+2]} + i\frac{1}{c\alpha'}(-1)^I \partial_\sigma \theta^I \sigma_m \theta^I \approx 0$$

yields the following relation

$$\begin{aligned} & u_m^{[+2]}\Big[\frac{1}{2}\beta^{\tau[-2]} - \frac{1}{2}\partial_\sigma \rho^{\sigma[-2]} - \Omega_\sigma^{(0)}\rho^{\sigma[-2]} + i\alpha^{(0)}\rho^{\tau[-2]} + \\ & \qquad + 2i\frac{1}{c\alpha'}(-1)^I \xi_A^{-I}\partial_\sigma \theta^{\alpha I} v_{\alpha A}^{-}\Big] + \\ & + u_m^{[-2]}\Big[\frac{1}{2}\beta^{\tau[+2]} - \frac{1}{2}\partial_\sigma \rho^{\sigma[+2]} - \Omega_\sigma^{(0)}\rho^{\sigma[+2]} - i\alpha^{(0)}\rho^{\tau[+2]} + \\ & \qquad + 2i\frac{1}{c\alpha'}(-1)^I \xi_A^{+I}\partial_\sigma \theta^{\alpha I} v_{\alpha A}^{+}\Big] + \\ & + u_m^{(i)}\,[(\alpha^{[+2](i)}\rho^{\tau[-2]} + \alpha^{[-2](i)}\rho^{\tau[+2]}) - \sum_{\pm} \rho^{\sigma[\mp 2]}\Omega_\sigma^{[\pm 2](i)} - \\ & - 2i\frac{1}{c\alpha'}(-1)^I (\xi_A^{+I}\gamma^i_{A\dot{B}}\partial_\sigma \theta^{\alpha I} v_{\alpha\dot{B}}^{-} + \xi_A^{-I}\gamma^i_{A\dot{B}}\partial_\sigma \theta^{\alpha I} v_{\alpha\dot{B}}^{+})\,] = 0. \end{aligned} \quad (4.37)$$

Here $\Omega_\sigma^{(0)}$, $\Omega_\sigma^{[\pm 2](i)}$, $\Omega_\sigma^{(i)(j)}$ are the coefficients of the pull-back of the $SO(1,9)$ Cartan forms (3.34) on the world-sheet. They are related to the $d\sigma$ differential and their form may be derived from Eqs.(3.34) using the relation

$$\Omega(d) = d\xi^\mu \Omega_\mu(\xi) = d\tau\, \Omega_\tau(\tau, \sigma) + d\tau\, \Omega_\sigma(\tau, \sigma).$$

The projections of Eq.(4.37) on the composed vectors $u_m^{[-2]}$, $u_m^{[+2]}$, $u_m^{(i)}$ (3.11) of the moving frame system (3.10) give us the following expressions for the Lagrange multipliers

$$\beta^{\tau[-2]} = \partial_\sigma \rho^{\sigma[-2]} + \Omega_\sigma^{(0)}\rho^{\sigma[-2]} - 2i\alpha^{(0)}\rho^{\tau[-2]} - 4i\frac{1}{c\alpha'}\xi_A^{-2}\partial_\sigma \theta^{\alpha 2} v_{\alpha A}^{-}, \quad (4.38a)$$

$$\beta^{\tau[+2]} = \partial_\sigma \rho^{\sigma[+2]} - \Omega^{(0)}_\sigma \rho^{\sigma[+2]} + 2i\alpha^{(0)}\rho^{\tau[+2]} + 4i\frac{1}{c\alpha'}\xi^{+1}_A \partial_\sigma\theta^{\alpha 1} v^+_{\alpha A}, \quad (4.38b)$$

$$\alpha^{[+2](i)}\rho^{\tau[-2]} + \alpha^{[-2](i)}\rho^{\tau[+2]} = \sum_\pm \rho^{\sigma[\mp 2]}\Omega^{[\pm 2](i)}_\sigma -$$

$$- 2i\frac{1}{c\alpha'}\xi^{+1}_A \gamma^i_{A\dot{B}}\partial_\sigma\theta^{\alpha 1} v^-_{\alpha\dot{B}} + 2i\frac{1}{c\alpha'}\xi^{-2}_A \gamma^i_{AB}\partial_\sigma\theta^{\alpha 2} v^+_{\alpha B} +$$

$$+ 2i\frac{1}{c\alpha'}\left[\frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\partial_\sigma\theta^{\alpha 1}\partial_\sigma\theta^{\beta 1} + \frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\partial_\sigma\theta^{\alpha 2}\partial_\sigma\theta^{\beta 2}\right] v^+_{\alpha A}\gamma^i_{A\dot{B}} v^-_{\beta\dot{B}}. \quad (4.38c)$$

At last, we should verify the conservation of the secondary constraints (4.30a,b) and (4.34). They may be presented as the projections of one constraint

$$\omega^m_\sigma - \frac{1}{2}c\alpha'\rho^{\tau[+2]}u^{[-2]}_m + \frac{1}{2}c\alpha'\rho^{\tau[-2]}u^{[+2]}_m \approx 0 \quad (4.39)$$

onto the moving frame vectors $u^{[-2]}_m$, $u^{[+2]}_m$, $u^{(i)}_m$. The requirement of the conservation of this constraint leads to the equation similar to (4.37). Moreover, the projections of the equation onto the moving frame vectors $u^{[-2]}_m$ and $u^{[+2]}_m$ coincide with Eqs.(4.38a) and (4.38b), respectively. However, its projection onto the moving frame vectors $u^{(i)}_m$ differs from Eq.(4.38c) and has the form

$$\alpha^{[+2](i)}\rho^{\tau[-2]} - \alpha^{[-2](i)}\rho^{\tau[+2]} =$$

$$= \frac{1}{2}[\rho^{\sigma[-2]}\Omega^{[+2](i)}_\sigma - \rho^{\sigma[+2]}\Omega^{[-2](i)}_\sigma] +$$

$$+ 2i\frac{1}{c\alpha'}\xi^{+1}_A \gamma^i_{A\dot{B}}\partial_\sigma\theta^{\alpha 1} v^-_{\alpha\dot{B}} + 2i\frac{1}{c\alpha'}\xi^{-2}_A \gamma^i_{AB}\partial_\sigma\theta^{\alpha 2} v^+_{\alpha B} -$$

$$- 2i\frac{1}{c\alpha'}\left[\frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}}\partial_\sigma\theta^{\alpha 1}\partial_\sigma\theta^{\beta 1} - \frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}}\partial_\sigma\theta^{\alpha 2}\partial_\sigma\theta^{\beta 2}\right] v^+_{\alpha A}\gamma^i_{A\dot{B}} v^-_{\beta\dot{B}}. \quad (4.40)$$

This relation corresponds to the requirement of the conservation of the secondary constraints (4.34) and thus is absent in the case of null-superstring (as well as the constraint (4.34) itself). Consequently the corresponding «boost» symmetry [20,21], which characterizes the superparticle [18,20,21] and null-superstring theory [19,42,43], is absent in the case of twistor-like superstring formulation. This is because the discussed superstring action (3.1), (4.3) contains spinor harmonic variables of both types: the $v^+_{\alpha A}$

harmonics as well as the $v^{-}_{\alpha\dot{A}}$ ones. The eight «boost» symmetries [20,21] consist in the shifting one of these harmonics by harmonics of another type

$$\delta v^{+}_{\alpha A} = b^{i[+2]} \gamma^{i}_{A\dot{A}} v^{-}_{\alpha\dot{A}} \qquad \text{or} \qquad \delta v^{-}_{\alpha\dot{A}} = b^{i[-2]} v^{+}_{\alpha A} \gamma^{i}_{A\dot{A}}.$$

It is clear that such symmetry is present in the theories which formulation contains only one of the types of the harmonics $v^{+}_{\alpha A}$ or $v^{-}_{\alpha\dot{A}}$. This property is satisfied not only in the twistor-like formulation of massless superparticle, null-supersting and null-super-p-branes [18—21,42,43], but also in the heterotic string formulations of the type of ones discussed in [32,34,36,38,40].*

Eqs.(4.38c) and (4.39) can be solved with respect to the Lagrange multipliers $\alpha^{[+2](i)}$ and $\alpha^{[-2](i)}$

$$\alpha^{[+2](i)} = \frac{1}{\rho^{\tau[-2]}} \left[\rho^{\sigma[-2]} \frac{1}{2} \Omega^{[+2](i)}_{\sigma} + 2i \frac{1}{c\alpha'} \xi^{-2}_{\dot{A}} \gamma^{i}_{A\dot{B}} \partial_{\sigma} \theta^{\alpha 2} v^{+}_{\alpha B} + \right.$$
$$\left. + 2i \frac{1}{c\alpha'} \frac{\rho^{\sigma[-2]}}{\rho^{\tau[-2]}} \partial_{\sigma} \theta^{\alpha 2} \partial_{\sigma} \theta^{\beta 2} v^{+}_{\alpha A} \gamma^{i}_{A\dot{B}} v^{-}_{\beta\dot{B}} \right], \qquad (4.41a)$$

$$\alpha^{[-2](i)} = \frac{1}{\rho^{\tau[+2]}} \left[\rho^{\sigma[+2]} \frac{1}{2} \Omega^{[-2](i)}_{\sigma} - 2i \frac{1}{c\alpha'} \xi^{+1}_{A} \gamma^{i}_{A\dot{B}} \partial_{\sigma} \theta^{\alpha 1} v^{-}_{\alpha\dot{B}} + \right.$$
$$\left. + 2i \frac{1}{c\alpha'} \frac{\rho^{\sigma[+2]}}{\rho^{\tau[+2]}} \partial_{\sigma} \theta^{\alpha 1} \partial_{\sigma} \theta^{\beta 1} v^{+}_{\alpha A} \gamma^{i}_{A\dot{B}} v^{-}_{\beta\dot{B}} \right]. \qquad (4.41b)$$

Thus the verification of the conservation of the constraints under the evolution is completed and, hence, the complete set of the first-class constraints is extracted (up to a transition to some linear combinations of them). They may be defined as the variations of the generalized Hamiltonian H' (4.27), (4.28) with respect to the generalized Lagrange multipliers, which set may contain the undetermined field parameters of the canonical Hamiltonian playing the role of the Lagrange multipliers for the secondary constraints (besides the original Lagrange multipliers). In the discussed case we may use as a gene-

*These formulations may be named half-twistor-like because only one of the Virasoro constraints is «twistorized» (i.e., is solved using twistor-like prescription) in them. And just this fact explains the presence of the (heterotic superstring) «boost» symmetry in them. Indeed, as it is easy to see from the discussed superstring formulation (3.10), (4.3), the inclusion of both types of harmonic variable is necessary just for the «twistorization» of both Virasoro constraints. And so, the formulations in which only one of them is «twistorized» may be constructed using only type of spinor harmonics and, consequently, may have the «boost» symmetry.

ralized Lagrange multipliers the moving frame density variables $\rho^{\sigma[\mp 2]}$, which are related with the original Lagrange multipliers $a^{[\mp 2]}$ by Eqs.(4.31a,b)*.

After the substitution of the expressions for the dependent Lagrange multipliers (4.31), (4.33), (4.35), (4.38a,b), (4.40) into the expressions (4.27) and taking into account (4.28), the generalized Hamiltonian may be written as follows

$$H'(\tau,\sigma) = \frac{c\alpha'}{2}\rho^{\sigma[+2]}L^{[-2]}(\tau,\sigma) - \frac{c\alpha'}{2}\rho^{\sigma[-2]}L^{[+2]}(\tau,\sigma) + \\ + \xi^{+1}_A D^-_A + \xi^{-2}_A D^+_A + ia^{(0)}D^{(0)} + \frac{1}{2}a^{ij}\Pi^{ij} + \beta^{\sigma[\mp 2]}P^{[\pm 2]}_{(\rho)\sigma}. \tag{4.42}$$

The first-class constraints has the following form

$$L^{[-2]} \equiv \frac{2}{c\alpha'}\frac{\partial H'}{\partial\rho^{\sigma[+2]}} = u^{[-2]m}L^1_m + \frac{2}{c\alpha'}\frac{1}{\rho^{[+2]\tau}}\partial_\sigma\Theta^{\alpha 1}v^-_{\alpha A}(v^{+\gamma}_A D^1_\gamma) + \\ + \frac{1}{c\alpha'}\frac{1}{\rho^{[+2]\tau}}\Omega^{[-2]i}_\sigma\Pi^{i[+2]} + 4i\,[c\alpha'\rho^{[+2]\tau}]^{-2}\times \\ \times\partial_\sigma\Theta^{\alpha 1}\partial_\sigma\Theta^{\gamma 1}v^+_{\alpha A}\gamma^i_{AB}v^-_{\gamma\dot B}\Pi^{i[+2]} - \frac{2}{c\alpha'}\partial_\sigma P^{[-2]}_{(\rho)\tau} - \frac{2}{c\alpha'}\Omega^{(0)}_\sigma P^{[-2]}_{(\rho)\tau} \approx 0, \tag{4.43a}$$

$$\hat{D}^-_A(\sigma) \equiv \frac{\partial H'}{\partial\xi^{+1}_A} = v^{\alpha-}_A D^1_\alpha(\sigma) - \frac{2i}{c\alpha'\rho^{[+2]\tau}}\gamma^i_{A\dot A}v^-_{\alpha\dot A}\partial_\sigma\Theta^{\alpha 1}\Pi^{[+2]i} + \\ + \frac{4i}{c\alpha'}\partial_\sigma\Theta^{\gamma 1}v^+_{\gamma A}P^{[-2]}_{(\rho)\tau} \approx 0, \tag{4.43b}$$

$$L^{[+2]} \equiv -\frac{2}{c\alpha'}\frac{\partial H'}{\partial\rho^{\sigma[-2]}} = \\ = u^{[+2]m}L^2_m - \frac{2}{c\alpha'}\frac{1}{\rho^{[-2]\tau}}\partial_\sigma\Theta^{\alpha 2}v^+_{\alpha A}[v^{-\gamma}_A D^2_\gamma] - \frac{1}{c\alpha'}\frac{1}{\rho^{[-2]\tau}}\Omega^{[+2]i}_\sigma\Pi^{[-2]i} -$$

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*It is important to note that the choice of the generalized Lagrange multipliers is a very delicate point. So, if we try to use the components of the world-sheet repere $e^{\sigma[\mp 2]}$ as the generalized Lagrange multipliers (instead of components of vector densities $\rho^{\sigma[\mp 2]} \equiv (\alpha')^{-1/2}\, e\, e^{\sigma[\mp 2]}$), then the extraction of the corresponding first-class constraint becomes problematic because of the nonlinear dependence of the resulting expression for the generalized Hamiltonian on $e^{\sigma[\mp 2]}$. In the discussed case such problem may be solved by using the relations (4.31) of the discussed repere variables with the original Lagrange multipliers and requiring that new generalized Lagrange multiplier must be expressed by linear relation through the original one.

$$- 4i\,[c\alpha'\rho^{[-2]\tau}]^{-2}\partial_\sigma\Theta^{\alpha 2}\partial_\sigma\Theta^{\gamma 2} v^{+}_{\alpha A}\gamma^{i}_{A\dot{B}} v^{-}_{\gamma\dot{B}}\Pi^{[-2]i} +$$

$$+ \frac{2}{c\alpha'}\partial_\sigma P^{[+2]}_{(\rho)\tau} - \frac{2}{c\alpha'}\Omega^{(0)}_\sigma P^{[+2]}_{(\rho)\tau} \approx 0, \qquad (4.43c)$$

$$\hat{D}^{+}_{A}(\sigma) \equiv \frac{\partial H'}{\partial \xi^{-2}_{A}} = v^{\alpha +}_{A} D^{2}_{\alpha}(\sigma) + \frac{2i}{c\alpha'\rho^{[-2]\tau}}\tilde{\gamma}^{i}_{\dot{A}A} v^{+}_{\alpha A}\partial_\sigma\Theta^{\alpha 2}\Pi^{[-2]i} -$$

$$- \frac{4i}{c\alpha'}\partial_\sigma\Theta^{\alpha 2} v^{-}_{\gamma A} P^{[+2]}_{(\rho)\tau} \approx 0, \qquad (4.43d)$$

$$D^{(0)} \equiv \Pi^{(0)} + 2\rho^{[+2]\tau} P^{[-2]}_{(\rho)\tau} - 2\rho^{[-2]\tau} P^{[+2]}_{(\rho)\tau} \approx 0, \qquad (4.43e)$$

$$D^{ij} \equiv \Pi^{ij} \approx 0, \qquad (4.43f)$$

$$P^{[\mp 2]}_{(\rho)\sigma} \approx 0, \qquad (4.43g)$$

where the expressions D^{I}_{α}, L^{I}_{m} $(I = 1,2)$ are defined by the relations

$$D^{I}_{\alpha}(\sigma) \equiv - \pi^{I}_{\alpha} + i(\sigma^{m}\theta^{I})_{\alpha}\left[P_m - (-1)^{I}\frac{1}{c\alpha'}(\partial_\sigma x_m - i\partial_\mu\theta^{I}\sigma_m\theta^{I})\right], \qquad (4.44)$$

$$L^{I}_{m} \equiv \left[P_m - (-1)^{I}\frac{1}{c\alpha'}\omega^{m}_{\sigma} + \frac{i}{c\alpha'}\sum_{J}(-1)^{J}\partial_\sigma\theta^{J}\sigma_m\theta^{J}\right], \quad I, J = 1, 2. \qquad (4.45)$$

The first-class constraints (4.43a,b) generate the reparameterization symmetry with parameters $\rho^{\sigma[\mp 2]}$ on Poisson brackets. The first-class constraints (4.43c,d), (4.43e), (4.43f) generate the κ-symmetry transformations (with parameters ξ^{+1}_{A} and ξ^{-2}_{A}), $SO(1,1)$ symmetry (with parameters $\alpha^{(0)}$), $SO(8)$ symmetry (with parameters α^{ij}) and, finally, the symmetry under the arbitrary of the repere density components $\rho^{\sigma[\mp 2]}$ (with parameters $\beta^{\sigma[\mp 2]}$). The last symmetry means the Lagrange multiplier nature of the variables $\rho^{\sigma[\mp 2]}$).

The connection of the reparameterization symmetry generators (4.43a,c) with the well-known Virasoro constraints should be discussed in the next section.

Thus the complete set of the covariant and irreducible first-class constraints for the $D = 10$, $N = IIB$ superstring in the twistor-like Lorentz harmonic formulation (3.1), (4.3) is derived.

5. ALGEBRA OF IRREDUCIBLE SYMMETRIES AND SECOND CLASS CONSTRAINT SYMPLECTIC STRUCTURE FOR GREEN-SCHWARZ SUPERSTRING

5.1. First-Class Constraints and Their Algebra. To simplify the algebra of the gauge symmetries generated by the first-class constraints (4.43), let us redefine them, using some linear transformations inside of the first-class constraints set. To formulate the results of such redefinition in a compact form, let us introduce the following bosonic and fermionic blocks

$$\tilde{L}^1_m \equiv L^1_m - \partial_\sigma A^1_m, \tag{5.1a}$$

$$\tilde{D}^1_\alpha(\sigma) \equiv D^1_\alpha(\sigma) + 2i\,(\sigma^m\theta^1)_\alpha A^1_m, \tag{5.1b}$$

$$\tilde{L}^2_m \equiv L^2_m - \partial_\sigma A^2_m, \tag{5.2a}$$

$$\tilde{D}^2_\alpha(\sigma) \equiv D^2_\alpha(\sigma) + 2i\,(\sigma^m\theta^2)_\alpha A^2_m, \tag{5.2b}$$

where

$$A^1_m \equiv u^{(+2)}_m P^{(-2)}_{(\rho)\tau} - u^{(i)}_m \Pi^{(+2)i}/\rho^{\tau(+2)} \approx 0, \tag{5.3a}$$

$$A^2_m \equiv u^{(-2)}_m P^{(+2)}_{(\rho)\tau} - u^{(i)}_m \Pi^{(-2)i}/\rho^{\tau(-2)} \approx 0, \tag{5.3b}$$

and expressions D^I_α, $L^I_m (I = 1,2)$ are defined by the relations (4.44), (4.45), or by the expressions

$$L^1_m \equiv \left[P_m + \frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\,\partial_\sigma\theta^1\sigma_m\theta^1)\right], \tag{5.4a}$$

$$D^1_\alpha(\sigma) \equiv -\pi^1_\alpha + i\,(\sigma^m\theta^1)_\alpha \left[P_m + \frac{1}{c\alpha'}(\partial_\sigma x_m - i\,\partial_\mu\theta^1\sigma_m\theta^1)\right], \tag{5.4b}$$

$$L^2_m \equiv \left[P_m - \frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\,\partial_\sigma\theta^2\sigma_m\theta^2)\right], \tag{5.5a}$$

$$D^2_\alpha(\sigma) \equiv -\pi^2_\alpha + i\,(\sigma^m\theta^2)_\alpha \left[P_m - \frac{1}{c\alpha'}(\partial_\sigma x_m - i\,\partial_\mu\theta^2\sigma_m\theta^2)\right]. \tag{5.5b}$$

The algebraic structure associated with blocks (5.4), (5.5) is very simple one

$$\{D^I_\alpha(\sigma), D^J_\beta(\sigma')\}_P = 2i\,\delta^{IJ}\sigma^m_{\alpha\beta}L^I_m\,\delta(\sigma-\sigma'), \tag{5.6a}$$

$$[D^I_\alpha(\sigma), L^J_n(\sigma')]_P = (-1)^I\delta^{IJ}\,4i\,(c\alpha')^{-1}\,(\partial_\sigma\theta^I\sigma_n)_\alpha\,\delta(\sigma-\sigma'), \tag{5.6b}$$

$$[L^I_m(\sigma), L^J_n(\sigma')\,]_P = 2\,(-1)^I\delta^{IJ}\,(c\alpha')^{-1}\eta_{mn}\partial_\sigma\delta\,(\sigma-\sigma'). \qquad (5.6c)$$

Their sets are decomposed naturally onto the two pieces (D^1_α, L^1_m) and (D^2_α, L^2_m), associated with the different light-like directions tangent to the superstring world-sheet. Inside of any of such pieces the bracket for two fermionic blocks produces the corresponding bosonic one (5.6a), the bracket for the bosonic block with the fermionic one produces the derivative of the corresponding Grassmannian variable (5.6b) and, finally, the bracket for two bosonic blocks is equal to the product of the flat space-time metric on the derivative of δ-function (5.6c). The brackets vanish for any two blocks belonging to the different sets (i.e., associated with different light-cone directions).

In distinction with Eqs.(5.6), the algebra of the blocks A^I_m has the vanishing brackets for any two blocks from the same set and complicated nonvanishing brackets for the pair A^1_m, A^2_m of blocks associated with the different light-like directions

$$[A^1_m(\sigma), A^1_n(\sigma')\,]_P = 0, \qquad [A^2_m(\sigma), A^2_n(\sigma')\,]_P = 0, \qquad (5.7a,b)$$

$$[A^1_m(\sigma), A^2_n(\sigma')\,]_P = (\rho^{\tau|+2|}\rho^{\tau|-2|})^{-1}\,[u^{(i)}_m u^{(j)}_n\,(\delta^{ij}D^{(0)} + 2D^{ij}) +$$
$$+\, u^{(i)}_m u^{|+2|}_n \Pi^{|-2|i} - u^{|-2|}_m u^{(j)}_n \Pi^{|+2|j}\,] \approx 0. \qquad (5.7c)$$

It is important that the brackets (5.7c) vanish in the week sense and include in its right-hand side the harmonical constraints (4.43e,f) (which are the first-class ones) and (4.25b) (which are the second-class ones) only.

Taking into account Eqs.(5.6), (5.7), we can see that the algebra of the blocks $\widetilde{L}^1_m(\sigma)$, $\widetilde{D}^1_\alpha(\sigma)$ is defined by following relations

$$\{\widetilde{D}^1_\alpha(\sigma), \widetilde{D}^1_\beta(\sigma')\}_P = 2i\,\sigma^m_{\alpha\beta}\,\widetilde{L}^1_m\,\delta\,(\sigma-\sigma'), \qquad (5.8a)$$

$$[\widetilde{D}^1_\alpha(\sigma), \widetilde{L}^1_n(\sigma')\,]_P = -\,4i\,(c\alpha')^{-1}(\partial_\sigma\theta^1\sigma_n)_\alpha\,\partial\,(\sigma-\sigma'), \qquad (5.8b)$$

$$[\widetilde{L}^1_m(\sigma), \widetilde{L}^1_n(\sigma')\,]_P = -\,2\,(c\alpha')^{-1}\eta_{mn}\,\partial_\sigma\delta\,(\sigma-\sigma') \qquad (5.8c)$$

and coincides with the algebra (5.6) written for $I = J = 1$.

The same is true for the algebra of $\widetilde{D}^2_\alpha(\sigma)$, $\widetilde{L}^2_n(\sigma')$,

$$\{\widetilde{D}^2_\alpha(\sigma), \widetilde{D}^2_\beta(\sigma')\}_P = 2i\,\sigma^m_{\alpha\beta}\,\widetilde{L}^2_m\,\delta\,(\sigma-\sigma'), \qquad (5.9a)$$

$$[\widetilde{D}^2_\alpha(\sigma), \widetilde{L}^2_n(\sigma')\,]_P = +\,4i\,(c\alpha')^{-1}(\partial_\sigma\theta^2\sigma_n)_\alpha\,\delta\,(\sigma-\sigma'), \qquad (5.9b)$$

$$[\tilde{L}^2_m(\sigma), \tilde{L}^2_n(\sigma')]_P = +2(c\alpha')^{-1}\eta_{mn}\,\partial_\sigma\delta(\sigma-\sigma') \tag{5.9c}$$

(see Eqs.(5.6) with $I = J = 2$).

However, the «crossing» terms have the more complicated form and are completely determined by the brackets (5.7c)

$$\{\tilde{D}^1_\alpha(\sigma), \tilde{D}^2_\beta(\sigma')\}_P =$$

$$= 4i\,(c\alpha')^{-2}\,(\partial_\sigma\theta^1\sigma_n)_\alpha\,(\partial_\sigma\theta^2\sigma_n)_\beta\,[A^1_m(\sigma), A^2_n(\sigma')]_P, \tag{5.10a}$$

$$[\tilde{D}^1_\alpha(\sigma), \tilde{L}^2_n(\sigma')]_P =$$

$$= 2i\,(c\alpha')^2\,(\partial_\sigma\theta^1\sigma_n)_\alpha(\sigma)\,\partial_{\sigma'}\,[A^1_n(\sigma), A^2_m(\sigma')]_P, \tag{5.10b}$$

$$[\tilde{L}^1_n(\sigma), \tilde{D}^2_\alpha(\sigma')]_P =$$

$$= 2i\,(c\alpha')^2\,(\partial_{\sigma'}\theta^2\sigma_n)_\alpha(\sigma')\,\partial_\sigma\,[A^1_m(\sigma), A^2_n(\sigma')]_P, \tag{5.10c}$$

$$[\tilde{L}^1_m(\sigma), \tilde{L}^2_n(\sigma')]_P = -(c\alpha')^{-2}\,\partial_\sigma\partial_{\sigma'}\,[A^1_m(\sigma), A^2_n(\sigma')]_P, \tag{5.10d}$$

as it is easy to see from Eqs.(5.6), (5.7).

The first class constraints $Y^J_M = (Y^1_M, Y^2_M)$ (4.43a-d), which generate κ-symmetry and reparameterization transformations, may be redefined as follows

$$Y^1_M(\sigma) \equiv (L^1(\sigma), \hat{D}^-_A(\sigma)):$$

$$L^1(\sigma) \equiv 2\rho^{\tau[+2]}\tilde{L}^{[-2]} \equiv$$

$$\equiv 2\rho^{\tau[+2]}u^{m[-2]}\tilde{L}^1_m(\sigma) + 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma), \tag{5.11a}$$

$$\hat{D}^-_A(\sigma) \equiv v^{-\alpha}_A(\sigma)\,\tilde{D}^1_\alpha(\sigma), \tag{5.11b}$$

$$Y^2_M(\sigma) \equiv (L^2(\sigma), \hat{D}^+_A(\sigma)):$$

$$L^2(\sigma) \equiv 2\rho^{\tau[-2]}\tilde{L}^{[+2]} \equiv$$

$$\equiv 2\rho^{\tau[-2]}u^{m[+2]}\tilde{L}^2_m(\sigma) - 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 2}\tilde{D}^2_\alpha(\sigma), \tag{5.12a}$$

$$\hat{D}^+_A(\sigma) \equiv v^{+\alpha}_A(\sigma)\,\tilde{D}^2_\alpha(\sigma). \tag{5.12b}$$

The distinctions in Eqs.(5.11), (5.12) with respect to (4.43a-d) lay i) in the adding of the expressions

$$+\frac{2}{c\alpha'}\frac{1}{\rho^{[+2]\tau}}\,\partial_\sigma\Theta^{\alpha 1}\,v^-_{\alpha\dot{A}}\,(v^{-\gamma}_A\tilde{D}^1_\gamma)$$

and

$$-\frac{2}{c\alpha'}\frac{1}{\rho^{[-2]\tau}}\partial_\sigma\Theta^{\alpha 2}v^{+}_{\alpha A}[v^{+\gamma}_{A}\tilde{D}^{2}_{\gamma}],$$

which are proportional to the first class constraints (5.11b), (5.12b) (or equivalently, to (4.43b), (4.43d)), to the constraints (4.43a), respectively and

ii) in the multiplying the resulting expressions on the overall factors $2\rho^{\tau[+2]}$ and $2\rho^{\tau[-2]}$, respectively.

The algebra of reparameterization and κ-symmetry transformations, associated with the same light-like direction tangent to the world-sheet, is realized in the form of the following bracket relations

$$[Y^1_M, Y^1_N]_P = C^{1K}_{MN}Y^1_N: \tag{5.13}$$

$$[L^1(\sigma), L^1(\sigma')]_P = -4(c\alpha')^{-1}(L^1(\sigma) + L^1(\sigma'))\,\partial_\sigma\delta(\sigma-\sigma'), \tag{5.13a}$$

$$\begin{aligned}[L^1(\sigma), \hat{D}^-_A(\sigma')]_P &= -4(c\alpha')^{-1}\,\hat{D}^-_A(\sigma)\,\partial_\sigma\delta(\sigma-\sigma') + \\ &+ 2(c\alpha')^{-1}\,\Omega^{(0)}_\sigma(\sigma)\,\hat{D}^-_A(\sigma)\,\delta(\sigma-\sigma'),\end{aligned} \tag{5.13b}$$

$$\begin{aligned}\{\hat{D}^-_A(\sigma), \hat{D}^-_B(\sigma')\}_P &= i\delta_{AB}(\rho^{\tau[+2]})^{-1}(L^1(\sigma) - \\ &- 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 1}v^{+}_{\alpha C}\hat{D}^-_C(\sigma))\,\delta(\sigma-\sigma'),\end{aligned} \tag{5.13c}$$

$$[Y^2_M, Y^2_N]_P = C^{2K}_{MN}Y^2_N: \tag{5.14}$$

$$[L^2(\sigma), L^2(\sigma')]_P = 4(c\alpha')^{-1}(L^2(\sigma) + L^2(\sigma'))\,\partial_\sigma\delta(\sigma-\sigma'), \tag{5.14a}$$

$$\begin{aligned}[L^2(\sigma), \hat{D}^+_A(\sigma')]_P &= 4(c\alpha')^{-1}\,\hat{D}^+_A(\sigma)\,\partial_\sigma\delta(\sigma-\sigma') + \\ &+ 2(c\alpha')^{-1}\,\Omega^{(0)}_\sigma(\sigma)\,\hat{D}^+_A(\sigma)\,\delta(\sigma-\sigma'),\end{aligned} \tag{5.14b}$$

$$\begin{aligned}\{\hat{D}^+_A(\sigma), \hat{D}^+_B(\sigma')\}_P &= i\delta_{\dot{A}\dot{B}}(\rho^{\tau[-2]})^{-1}(L^1(\sigma) + \\ &+ 4(c\alpha')^{-1}\partial_\sigma\theta^{\alpha 2}v^{-}_{\alpha\dot{C}}\,\hat{D}^+_C(\sigma))\,\delta(\sigma-\sigma'),\end{aligned} \tag{5.14c}$$

where $\Omega^{(0)}_\sigma(\sigma)$ is the σ-component of the $SO(1,1)$ Cartan form (3.32a); they transform as the connection (or gauge field) component under the gauge $SO(1,1)$ transformations.

The brackets of the reparameterization and κ-symmetry generators, associated with the different light-like world-sheet directions, have more complicated structure. However they are completely defined by the relations (5.7c)

$$[Y^1_M(\sigma),\, Y^2_N(\sigma')\,]_P = C^{12ij}_{MN}\,\delta(\sigma-\sigma')\,(\delta^{ij}D^{(0)} + 2D^{ij}) :$$

$$[L^1(\sigma),\, L^2(\sigma')\,]_P = -\,16(c\alpha')^{-1}\Omega^{[-2]i}_\sigma\Omega^{[+2]j}_\sigma\times$$

$$\times\,(\delta^{ij}\dot{D}^{(0)} + 2D^{ij})\,\delta(\sigma-\sigma'), \qquad (5.15a)$$

$$[L^1(\sigma),\, \hat{D}^+_A(\sigma')\,]_P = 4i((c\alpha')^2\rho^{\tau[-2]})^{-1}\Omega^{[-2]i}_\sigma\partial_\sigma\theta^{\alpha 2}v^+_{\alpha B}\gamma^j_{B\dot{A}}\times$$

$$\times(\partial^{ij}D^{(0)} + 2D^{ij})\,\delta\,(\sigma-\sigma'), \qquad (5.15b)$$

$$[\hat{D}^-_A(\sigma),\, L^2(\sigma')\,]_P = 4i((c\alpha')^2\rho^{\tau[+2]})^{-1}\Omega^{[+2]j}_\sigma\partial_\sigma\theta^{\alpha 1}v^-_{\alpha\dot{B}}\tilde{\gamma}^i_{\dot{B}A}\times$$

$$\times(\partial^{ij}D^{(0)} + 2D^{ij})\,\delta\,(\sigma-\sigma'), \qquad (5.15c)$$

$$\{\hat{D}^-_A(\sigma),\, \hat{D}^+_A(\sigma')\}_P = 4\,((c\alpha')^2\,\rho^{\tau[+2]}\,\rho^{\tau[-2]})^{-1}\,\partial_\sigma\theta^{\alpha 1}v^-_{\alpha\dot{B}}\gamma^i_{A\dot{B}}\times$$

$$\times\partial_\sigma\theta^{\alpha 2}v^+_{\alpha B}\,\gamma^j_{B\dot{A}}\,(\delta^{ij}D^{(0)} + 2D^{ij})\,\delta(\sigma-\sigma), \qquad (5.14d)$$

where $\Omega^{[+2]i}_\sigma$, $\Omega^{[-2]i}_\sigma$ are the components of the pull-backs of the covariant Cartan forms (4.32b,c) ($\Omega_\sigma \equiv \Omega\,(\partial_\sigma)$) and

$$D^{(0)} \equiv \Pi^{(0)} + 2\rho^{[+2]\tau}P^{[-2]}_{(\rho)\tau} - 2\rho^{[-2]\tau}P^{[+2]}_{(\rho)\tau} \approx 0, \qquad (5.16a)$$

$$D^{ij} \equiv \Pi^{ij} \approx 0, \qquad (5.16b)$$

are the first-class constraints (4.43e,f) generating the $SO(1,1)$ and $SO(8)$ gauge symmetries (on the Poisson brackets).

The fact of closure of the superparameterization symmetry algebra (i.e., the algebra of reparameterizations and κ-symmetry transformations) on the $SO(1,1)$ and $SO(8)$ gauge symmetry transformations is a significant one. It means that $SO(1,1)\times SO(8)$ gauge symmetry connects different light-like directions, tangent to the world-sheet.

The bracket relations of the $SO(1,1)$ and $SO(8)$ symmetry generators (5.16a,b) with another first-class constraints are defined by the $SO(1,1)$ weight and $SO(8)$ index structures of such constraints

$$[D^{(0)}(\sigma),\, Y^I_M(\sigma')\,]_P = w\,(Y^I_M)\,Y^I_M(\sigma)\,\delta\,(\sigma-\sigma') :$$

$$[D^{(0)}(\sigma),\, \hat{D}^-_A(\sigma')\,]_P = -\,\hat{D}^-_A(\sigma)\,\delta(\sigma-\sigma'), \qquad (5.17a)$$

$$[D^{(0)}(\sigma),\, \hat{D}^+_A(\sigma')\,]_P = +\,\hat{D}^+_A(\sigma)\,\delta(\sigma-\sigma'), \qquad (5.17b)$$

$$[D^{(0)}(\sigma), L^{1,2}(\sigma')]_P = 0, \tag{5.17c}$$

$$[D^{(0)}(\sigma), D^{ij}(\sigma')]_P = 0, \tag{5.17d}$$

$$[D^{ij}(\sigma), Y^I_M(\sigma')]_P = (\gamma^{ij})^N_M \, Y^I_N(\sigma)\,\delta(\sigma-\sigma'):$$

$$[D^{ij}(\sigma), \hat{D}^-_A(\sigma')]_P = -\gamma^{ij}_{AB}\,\hat{D}^-_B(\sigma)\,\delta(\sigma-\sigma'), \tag{5.18a}$$

$$[D^{ij}(\sigma), \hat{D}^+_A(\sigma')]_P = +\gamma^{ij}_{A\dot{B}}\hat{D}^+_B(\sigma)\,\delta(\sigma-\sigma'), \tag{5.18b}$$

$$[D^{ij}(\sigma), L^{1,2}(\sigma')]_P = 0, \tag{5.18c}$$

$$[D^{ij}(\sigma), D^{i'j'}(\sigma')]_P = 2\delta^{i[i'}D^{j']j}(\sigma)\,\delta(\sigma-\sigma'). \tag{5.18d}$$

So, the algebra of the first-class constraints

$$Y_\Lambda \equiv (Y^I_M(\sigma),\, Y^2_N(\sigma),\, D^{(0)}(\sigma),\, D^{ij}(\sigma),\, P^{[\pm 2]}_{(\rho)\sigma}) \equiv$$

$$\equiv (L^1(\sigma),\, \hat{D}^-_A(\sigma),\, L^2(\sigma),\, \hat{D}^+_A(\sigma'),\, D^{(0)},\, D^{ij}(\sigma),\, P^{[\pm 2]}_{(\rho)\sigma}),$$

$$[Y_\Lambda(\sigma), Y_\Sigma(\sigma')]_P = \int d\sigma''\, C^{\Pi}_{\Lambda\Sigma}(\sigma,\sigma'|\sigma'')\, Y_\Pi(\sigma'') \tag{5.19}$$

is completely specified by Eqs. (5.13)—(5.15), (5.18), except for the bracket relations of them with the rest two first-class constraints $P^{[\pm 2]}_{(\rho)\sigma} \approx 0$ (4.43g). All these brackets vanish because of the absence of the variables $\rho^{[\mp 2]\sigma}$ in the expressions for the first-class constraints

$$[P^{[\pm 2]}_{(\rho)\sigma}(\sigma), Y_\Sigma(\sigma')]_P = 0. \tag{5.20}$$

The symmetry generated by the constraints $P^{[\pm 2]}_{(\rho)\sigma} \approx 0$ indicates the Lagrange multiplier nature of the zweinbein densities $\rho^{[\mp 2]\sigma}$ in the discussed formulation.

5.2. Second-Class Constraints, Their Algebra and Symplectic Structure. The rest of the constraints (4.7), (4.25) are the second-class ones. They also may be decomposed naturally onto the two sets

$$S_f = (S^1_f, S^2_f) \approx 0 \tag{5.21}$$

associated with different light-like directions tangent to the world-sheet

$$S^1_f \approx 0: \tag{5.22}$$

$$L^{1(i)}(\sigma) \equiv u^{m(i)}(\sigma)\, L^1_m(\sigma) =$$

$$= u^{m(i)}\,[P_m + \frac{1}{c\alpha'}(\partial_\sigma x_m - 2i\,\partial_\sigma\theta^1\sigma_m\theta^1] \approx 0, \tag{5.22a}$$

$$v_A^{+\alpha}(\sigma)\, D^1_\alpha(\sigma) \approx 0, \tag{5.22b}$$

$$\Pi^{[+2](i)}(\sigma) \approx 0, \tag{5.22c}$$

$$\rho^{[+2]\tau} - \frac{1}{2}\, u^{m[+2]} L^1_m(\sigma) \approx 0, \tag{5.22d}$$

$$P^{[-2]}_{(\rho)\tau}(\sigma) \approx 0, \tag{5.22e}$$

$$S^2_f \approx 0: \tag{5.23}$$

$$L^{2(i)}(\sigma) \equiv u^{m(i)}(\sigma)\, L^2_m(\sigma) =$$

$$= u^{m(i)}\, [P_m - \frac{1}{c\alpha'}\, (\partial_\sigma x_m - 2i\, \partial_\sigma \theta^2 \sigma_m \theta^2\,] \approx 0, \tag{5.23a}$$

$$v_A^{-\alpha}(\sigma)\, D^2_\alpha(\sigma) \approx 0, \tag{5.23b}$$

$$\Pi^{[-2](i)}(\sigma) \approx 0, \tag{5.23c}$$

$$\rho^{[-2]\tau} - \frac{1}{2}\, u^{m[-2]} L^2_m(\sigma) \approx 0, \tag{5.23d}$$

$$P^{[+2]}_{(\rho)\tau}(\sigma) \approx 0. \tag{5.23e}$$

A nondegenerate symplectic structure Ω^{IJ}_{fg}

$$\{S^I_f, S^J_g\}_P \approx \Omega^{IJ}_{fg} \tag{5.24}$$

of the set of constraints (5.21), (5.22) is the block-diagonal one and is defined by the relations

$$\{S^1_f, S^1_g\}_P \approx \Omega^1_{fg}: \tag{5.25}$$

$$\{v_A^{+\alpha} D^1_\alpha(\sigma), v_B^{+\alpha} D^1_\alpha(\sigma')\}_P = 2i\delta_{A\dot{B}} u^{m[+2]} L^1_m \delta(\sigma - \sigma') \approx$$

$$\approx 4i\, \delta_{A\dot{B}}\, \rho^{[+2]\tau}\, \delta\, (\sigma - \sigma'), \tag{5.25a}$$

$$[L^{1(i)}(\sigma), v_B^{+\alpha} D^1_\alpha(\sigma')\,]_P = -\, 4i(c\alpha')^{-1} \partial_\sigma \theta^{\alpha 1} v^+_{\alpha A} \gamma^i_{A\dot{B}}\, \delta(\sigma - \sigma'), \tag{5.25b}$$

$$[L^{1(i)}(\sigma), L^{1(j)}(\sigma')\,]_P = -\, 2(c\alpha')^{-1} (\delta^{ij} \partial_\sigma - \Omega^{ij}_\sigma)\, \delta\, (\sigma - \sigma'), \tag{5.25c}$$

$$[\Pi^{[+2](i)}(\sigma), L^{1(j)}(\sigma')\,]_P = \delta^{ij} u^{m[+2]} L^1_m(\sigma)\, \delta(\sigma - \sigma') \approx$$

$$\approx 2\delta^{ij} \rho^{[+2]\tau}\, \delta\, (\sigma - \sigma'), \tag{5.25d}$$

$$\left[\left(\rho^{[+2]\tau}-\frac{1}{2}u^{m[+2]}L^{1}_{m}\right)(\sigma),\, L^{1(j)}(\sigma')\right]_{P}=$$

$$=(c\alpha')^{-1}\Omega^{[+2]i}_{\sigma}\,\delta(\sigma-\sigma'), \tag{5.25e}$$

$$\left[P^{[-2]}_{(\rho)\tau}(\sigma),\, \left(\rho^{[+2]\tau}-\frac{1}{2}u^{m[+2]}L^{1}_{m}\right)(\sigma')\right]_{P}=\delta(\sigma-\sigma'), \tag{5.25f}$$

$$\{S^{2}_{f},\, S^{2}_{g}\}_{P}\approx\Omega^{2}_{fg}: \tag{5.26}$$

$$\{v^{-\alpha}_{A}D^{2}_{\alpha}(\sigma),\, v^{-\alpha}_{B}D^{2}_{\alpha}(\sigma')\}_{P}=2i\delta_{AB}u^{m[-2]}L^{2}_{m}(\sigma)\,\delta(\sigma-\sigma')\approx$$

$$\approx 4i\,\delta_{AB}\rho^{[-2]\tau}\,\delta(\sigma-\sigma'), \tag{5.26a}$$

$$[L^{2(i)}(\sigma),\, v^{-\alpha}_{B}D^{2}_{\alpha}(\sigma')\,]_{P}=4i\,(c\alpha')^{-1}\partial_{\sigma}\theta^{\alpha 2}v^{-}_{\alpha\dot{A}}\gamma^{i}_{B\dot{A}}\,\delta(\sigma-\sigma'), \tag{5.26b}$$

$$[L^{1(i)}(\sigma),\, L^{1(j)}(\sigma')\,]_{P}=2\,(c\alpha')^{-1}(\delta^{ij}\partial_{\sigma}-\Omega^{ij}_{\sigma})\,\delta(\sigma-\sigma'), \tag{5.26c}$$

$$[\Pi^{[-2](i)}(\sigma),\, L^{2(j)}(\sigma')\,]_{P}=\delta^{ij}u^{m[-2]}L^{2}_{m}(\sigma)\,\delta(\sigma-\sigma')\approx$$

$$\approx 2\delta_{\dot{A}\dot{B}}\rho^{[-2]\tau}\,\delta(\sigma-\sigma'), \tag{5.26d}$$

$$\left[\left(\rho^{[-2]\tau}-\frac{1}{2}u^{m[-2]}L^{2}_{m}\right)(\sigma),\, L^{2(j)}(\sigma')\right]_{P}=$$

$$=(c\alpha')^{-1}\Omega^{[-2]i}_{\sigma}\,\delta(\sigma-\sigma'), \tag{5.26e}$$

$$\left[P^{[+2]}_{(\rho)\tau}(\sigma),\, \left(\rho^{[-2]\tau}-\frac{1}{2}u^{m[-2]}L^{2}_{m}\right)(\sigma')\right]_{P}=\delta(\sigma-\sigma'). \tag{5.26f}$$

All other brackets between the pairs of the constraints from the same set (either (5.20), or (5.21)) vanish in the strong sense. The brackets between the constraints from different sets are all equal to zero in the week sense

$$\{S^{1}_{f},\, S^{2}_{g}\}_{P}\approx 0. \tag{5.27}$$

All of these bracket relations, which are nonvanishing in the strong sense, involve the constraints (5.22c), or (5.23c)

$$\{S^{1}_{f},\, S^{2}_{g}\}_{P}\neq 0\ (\approx 0): \tag{5.28}$$

$$[\Pi^{[+2](i)}(\sigma),\, \Pi^{[-2](j)}(\sigma')\,]_{P}=(\delta^{ij}D^{(0)}+2D^{ij})\,\delta(\sigma-\sigma')\approx 0, \tag{5.28a}$$

$$[\Pi^{[+2](i)}(\sigma),\, v^{-\alpha}_{B}D^{2}_{\alpha}(\sigma')\,]_{P}=-\gamma^{i}_{B\dot{A}}\hat{D}^{+}_{\dot{A}}(\sigma)\,\delta(\sigma-\sigma')\approx 0, \tag{5.28b}$$

$$[v_{\dot A}^{+\alpha} D_\alpha^1(\sigma), \Pi^{[-2](j)}(\sigma')]_P = \gamma^i_{B\dot A} \widehat{D}^-_A(\sigma)\, \delta(\sigma - \sigma') \approx 0, \tag{5.28c}$$

$$[\Pi^{[+2](i)}(\sigma), L^{2(j)}(\sigma')]_P = \delta^{ij} u^{m[+2]} L^2_m(\sigma)\, \delta(\sigma - \sigma') \equiv$$

$$\equiv \delta^{ij}(2\rho^{[-2]\tau})^{-1} \Big[L^2(\sigma) + \frac{4}{c\alpha'} \partial_\sigma \Theta^{\alpha 2} v^-_{\alpha\dot A} \widehat{D}^+_{\dot A}(\sigma) +$$

$$+ \frac{4}{c\alpha'} \partial_\sigma \Theta^{\alpha 2} v^+_{\alpha A}(v_A^{-\gamma} D^2_\gamma) - \frac{4}{c\alpha'} \rho^{[-2]\tau}(\partial_\sigma - \Omega^{(0)}_\sigma)\, P^{[+2]}_{(\rho)\tau} +$$

$$+ \frac{4}{c\alpha'} \Omega_\sigma^{[+2]i'} \Pi^{[-2]i'} \Big]\, \delta(\sigma - \sigma') \approx 0, \tag{5.28d}$$

$$[L^{1(i)}(\sigma), \Pi^{[+2]j}(\sigma')]_P = \delta^{ij} u^{m[-2]} L^1_m(\sigma)\, \delta(\sigma - \sigma') \equiv$$

$$\equiv -\delta^{ij}(2\rho^{[+2]\tau})^{-1} \Big[L^1(\sigma) - \frac{4}{c\alpha'} \partial_\sigma \Theta^{\alpha 1} v^+_{\alpha A} \widehat{D}^-_A(\sigma) -$$

$$- \frac{4}{c\alpha'} \partial_\sigma \Theta^{\alpha 1} v^-_{\alpha\dot A}(v_{\dot A}^{+\gamma} D^1_\gamma) + \frac{4}{c\alpha'} \rho^{[+2]\tau}(\partial_\sigma + \Omega^{(0)}_\sigma)\, P^{[-2]}_{(\rho)\tau} +$$

$$- \frac{4}{c\alpha'} \Omega_\sigma^{[-2]i'} \Pi^{[+2]i'} \Big]\, \delta(\sigma - \sigma') \approx 0, \tag{5.28e}$$

$$\Big[\Pi^{[+2]i}(\sigma), \Big(\rho^{[-2]\tau} - \frac{1}{2} u^{m[-2]} L^2_m\Big)(\sigma')\Big]_P =$$

$$= - L^{2(i)}(\sigma)\, \delta(\sigma - \sigma') \approx 0, \tag{5.28f}$$

$$\Big[\Big(\rho^{[+2]\tau} - \frac{1}{2} u^{m[+2]} L^1_m\Big)(\sigma), \Pi^{[-2]i}(\sigma')\Big]_P =$$

$$= L^{1(i)}(\sigma)\, \delta(\sigma - \sigma') \approx 0. \tag{5.28g}$$

Hence, the symplectic structure (5.24) associated with the irreducible second-class constraints of the Green — Schwarz superstring is derived in the framework of the twistor-like Lorentz harmonic formulation [22,23]. Moreover, the second-class constraints algebra

$$[S^I_f, S^J_g]_P = C^{IJhK}_{fg} S^K_h + C^{IJ\Sigma}_{fg} Y_\Sigma + \Omega^{IJ}_{fg} \approx \Omega^{IJ}_{fg} \tag{5.29}$$

is described completely by the relations (5.25), (5.26), (5.28).

5.3. Reparameterization Generators and Virasoro Conditions. Let us clarify the relation of the first-class constraints (5.11a), (5.12a) with the well-known Virasoro conditions

$$V^1 \equiv [P_m + (c\alpha')^{-1} \partial_\sigma x_m]\, [P^m + (c\alpha')^{-1} \partial_\sigma x^m], \tag{5.30a}$$

$$V^2 \equiv [P_m - (c\alpha')^{-1}\partial_\sigma x_m]\,[P^m - (c\alpha')^{-1}\partial_\sigma x^m], \qquad (5.30b)$$

which generate the reparameterization symmetry transformation in the standard bosonic string formulation (see, for example, Ref.3). The expressions $[P_m + (c\alpha')^{-1}\partial_\sigma x_m]$ and $[P_m - (c\alpha')^{-1}\partial_\sigma x_m]$ are included in the blocks (5.4a), (5.5a) and may be identified with them, or with the blocks (5.1a), (5.2a), up to the harmonic and Grassmannian constraints. Thus we may discuss the forms of the Virasoro condition generalizations in terms of these blocks.

It is sufficient to discuss the Virasoro condition (5.30a) only.

The expression $\tilde{L}^1_m$ is included in the first constraint (5.11a) which may be reformulated as follows

$$u^{m|-2|}\tilde{L}^1_m(\sigma) + 2(c\alpha'\rho^{\tau|+2|})^{-1}\,\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma) \approx 0, \qquad (5.31)$$

and remains first-class constraint after multiplication on $u^{n|+2|}\tilde{L}^1_n(\sigma)$

$$u^{m|+2|}u^{n|-2|}\tilde{L}^1_m(\sigma)\,\tilde{L}^1_n(\sigma) +$$
$$+\,2(c\alpha'\rho^{\tau|+2|})^{-1}u^{n|+2|}\tilde{L}^1_n(\sigma)\,\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma) \approx 0. \qquad (5.32)$$

From the other hand, the second-class constraint (5.22a) may be transformed into the following one

$$\tilde{L}^1_m(\sigma)\,u^{m(i)} \approx 0, \qquad (5.33)$$

because the block $\tilde{L}^1_m(\sigma)$ (5.1a) differs from $L^1_m(\sigma)$ one (5.4a) by the sum of constraints (see Eq.(5.3a)). The square power of the second-class constraint (5.33) is the first-class constraint by definition. Thus, there is the following (dependent) first-class constraint

$$\tilde{L}^1_m(\sigma)\,u^{m(i)}u^{n(i)}\,\tilde{L}^1_n(\sigma) \approx 0 \qquad (5.34)$$

in the discussed dynamical system. The linear combination of Eqs.(5.32) and (5.34)

$$u^{m|+2|}u^{n|-2|}\,\tilde{L}^1_m(\sigma)\,\tilde{L}^1_n(\sigma) - \tilde{L}^1_m(\sigma)\,u^{m(i)}u^{n(i)}\tilde{L}^1_n(\sigma) +$$
$$+\,2(c\alpha'\rho^{\tau|+2|})^{-1}\,u^{n|+2|}\tilde{L}^1_n(\sigma)\,\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma) \approx 0 \qquad (5.35)$$

may be written in the form

$$\tilde{L}^1_m(\sigma)\,\tilde{L}^{m1}(\sigma) + 2(c\alpha'\rho^{\tau|+2|})^{-1}u^{n|+2|}\tilde{L}^1_n(\sigma)\,\partial_\sigma\theta^{\alpha 1}\tilde{D}^1_\alpha(\sigma) \approx 0 \quad (5.36)$$

(if the completeness conditions

$$\delta^n_m = \frac{1}{2} u^{[+2]}_m u^{n[-2]} + \frac{1}{2} u^{[-2]}_m u^{n[+2]} - u^{(i)}_m u^{n(i)} \qquad (5.37)$$

for the composed moving frame variables (1.8), (1.9) are taken into account).

It is easy to see that the first-class constraint (5.36) coincides with the Virasoro condition (5.30a) up to the Grassmannian and harmonic degrees of freedom (which are absent in the standard bosonic string formulation).

6. CONCLUSION

So the classical mechanics of the twistor-like Lorentz harmonic formulation of the $D = 10$, $N = IIB$ superstring [22,23] is built in the frameworks of Lagrangian and Hamiltonian approaches. The equations of motion are derived (Eqs.(3.40), (3.44), (3.47), (3.51)) using the concept of the admissible variation (3.31) for the harmonic variables. The complete sets of Lorentz covariant and irreducible first-class and second-class constraints are presented in Eqs.(4.43a-g) and Eqs.(5.28)—(5.33), respectively. The algebra of the gauge symmetries (Eqs.(5.13)—(5.15), (5.17)—(5.20) and symplectic structure associated with the set of second-class constraints (Eqs.(5.25), (5.26), (5.28), (5.29)) are calculated.

Thus we have developed the machinery of the component twistor approach necessary for the next steps towards covariant quantization of $D = 10$ superstring, which consist in the providing of the conversion [54—56,7] of the second-class constraints into the Abelian first-class ones and the construction of the classical BRST charge (see [42,43,19] for the case of null-super-p-branes in $D = 4$). These steps are under investigation now.

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