«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
1994, ТОМ 25, ВЫП. 6

УДК 539.142

THE REALIZATION OF THE RESONATING GROUP METHOD ALGEBRAIC VERSION FOR THREE-CLUSTER SYSTEMS

G.F.Filippov, A.V.Nesterov, I.Yu.Rybkin, S.V.Korennov

Bogoliubov Institute for Theoretical Physics, Kiev, Ukraine

The harmonic oscillator basis for the description of the three-cluster systems is constructed in the Fock—Bargmann representation. The transformation of basis functions accompanying the nucleon permutation operation is found. The problem of constructing the allowed states is solved. An algorithm of calculation of the Hamiltonian matrix elements in the basis of allowed states is developed. The matrix elements of the kinetic and potential energy operators are obtained for the ^{6}He(^{4}He $+ n + n$) system in the state with zero orbital angular momentum.

В представлении Фока — Баргмана сконструирован гармонический осцилляторный базис, позволяющий описывать трехкластерные системы. Найдено преобразование базисных функций, отвечающее операции перестановки нуклонов. Решена проблема построения разрешенных состояний. Развит алгоритм вычисления матричных элементов гамильтониана на базисе разрешенных состояний. Получены матричные элементы операторов кинетической и потенциальной энергии для системы ^{6}He(^{4}He $+ n + n$) в состоянии с орбитальным угловым моментом 0.

1. INTRODUCTION

The microscopic theory of light nuclei collisions that takes into account only binary entry channels has proved its efficiency in a number of reaction calculations. Its realization, at least in simple cases, with the resonating group method (RGM) [1—4] or its algebraic version [5—7], does not meet now any principal difficulties. However, the problem becomes much more complicated if one tries to include into consideration three-body exit channels.

There appeared, however, papers where the three-cluster problems of the discrete [8—11] and the continuous [12] spectrum were successfully solved. But in each case clusters were taken as structureless particles whose interaction was modelled by some effective *cluster-cluster* potential and the Pauli principle was taken into account only approximately. Of course, it allowed to simplify the

calculations, but if we try to take into account other possible cluster partitions of a nucleus (e.g., the channel $t+t$ together with $^4\mathrm{He}+n+n$ in $^6\mathrm{He}$), the number of effective potentials needed for calculations becomes, as it seems, excessive. Therefore, the natural desire appears, to solve the problems of this sort using the same *nucleon-nucleon* potential. The RGM algebraic version could provide this possibility.

In the present work we present the basic ideas of the RGM algebraic version generalization for the three-cluster systems, discuss the appearing difficulties and outline ways to overcome them. The analytical results will be obtained for the simplest three-cluster system, $^6\mathrm{He}$ ($^4\mathrm{He}+n+n$) in the state with zero orbital angular momentum. Nevertheless, the ideas discussed below are applicable to any system of three s-clusters.

Let us recall that this approach is based on the expansion of a nuclear wave function over the multiparticle harmonic oscillator basis. We begin with the construction of such oscillator basis that a) takes into account the intrinsic structure of three clusters, b) reproduces the dynamics of their relative motion, c) has a certain permutation symmetry. The latter restriction, due to the Pauli principle, causes the major part of computational difficulties.

As known, in the case of binary channels, the Pauli principle excludes from the complete set of the harmonic oscillator basis states those which vanish acted on by the antisymmetrization operator and, therefore, cannot be used in calculations (these states are called the *forbidden* states). In the case of three-body channels, a number of basis states are also forbidden but their identification becomes more complicated [13—15]. If the number of oscillator quanta is not less than the minimal one allowed by the Pauli principle, the allowed states can be identified as the eigenvectors of a density matrix with non-zero eigenvalues, while the forbidden states correspond to its zero eigenvalues.

The realization of the RGM algebraic version is based on the extensive use of the *generalized coherent states* (GCS) [16,17,5] which are the generating functions of the harmonic oscillator basis. The GCS also perform a transformation from the coordinate space to the so-called *Fock—Bargmann space* [18—20], where the basis functions have a simpler form making the derivations less cumbersome.

The main steps in the implementation of the approach remain the same as in the binary-channel case. First we construct the generating functions of the three-cluster oscillator basis. Then we calculate their overlap integral and perform its projection onto the basis states. On this step we also discuss the choice of the quantum numbers for the classification of basis states and calculate the transition matrices between different bases. After that we obtain the matrix elements of the kinetic and potential energy operators of the six-

nucleon system between the generating invariants. Finally, we find the Hamiltonian matrix elements between the basis states. The analytical calculations conclude with the discussion of the Fourier coefficients asymptotics and the derivation of the RGM algebraic version dynamical equations.

2. GENERATING FUNCTIONS OF THE THREE-CLUSTER OSCILLATOR BASIS

We shall write the wave function of a system of N_{cl} clusters in a traditional for the RGM form

$$\Psi(\{\mathbf{r}_i, \sigma_i, \tau_i\}) = \mathcal{A}\{\varphi(1)\ldots\varphi(N_{cl}) f(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1})\}. \tag{1}$$

Here $\{\mathbf{r}_i, \sigma_i, \tau_i, \quad i = 1,\ldots,A\}$ are the spatial, spin and isospin coordinates of A nucleons, $\mathcal{A}$ is the antisymmetrization operator; $\varphi(1),\ldots,\varphi(N_{cl})$ are the functions describing the internal structure of each cluster, these functions are fixed; $f(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1})$ is the function describing the relative motion of clusters depending on a set of $N_{cl}-1$ translation-invariant Jacobi vectors of a N_{cl}-cluster system. The latter are the *dynamical variables* and the wave function f depending upon them should be found by a solution of the integrodifferential equation obtained from the Schrödinger equation after the substitution in it of the trial function of the above form and the integration over the rest of Jacobi vectors (intrinsic cluster coordinates).

In the framework of the RGM algebraic version the relative function $f(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1})$ is expanded in the basis states of the multiparticle harmonic oscillator

$$f(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1}) = \sum_{\{n\}} C_{\{n\}} f_{\{n\}}(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1}) \tag{2}$$

and the dynamical equations are reduced to the set of linear algebraic equations with respect to the expansion coefficients $C_{\{n\}}$. Here $\{n\}$ is the set of quantum numbers characterizing the basis states. The basis functions $f_{\{n\}}(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{cl}-1})$ can be written as some linear combinations of the products of $N_{cl}-1$ single-particle oscillator functions

$$f_{nlm}(\mathbf{q}) = (-1)^n \sqrt{\frac{2n!}{r_0\Gamma(n+l+3/2)}}\, x^l e^{-x^2/2} L_n^{l+1/2}(x^2) Y_{lm}(\Omega_q), \tag{3}$$

where $x = q/r_0$, r_0 is the oscillator length, the parameter of the basis, $L_n^k(x)$ are the Laguerre polynomials, $Y_{lm}(\Omega_g)$ are the spherical functions. The exact form of those linear combinations and the selection of the quantum numbers $\{n\}$ will be discussed in Sect.3.

The expansion (2) of f is equivalent to the following expansion of Ψ

$$\Psi(\{r_i, \sigma_i, \tau_i\}) = \sum_{\{n\}} C_{\{n\}} \Psi_{\{n\}}(\{r_i, \sigma_i, \tau_i\}), \tag{4}$$

where the basis states $\Psi_{\{n\}}$ are introduced in analogy to (1)

$$\Psi_{\{n\}}(r_i, \sigma_i, \tau_i\}) = \mathcal{A}\{\varphi(1)\ldots \varphi(N_{\rm cl}) f_{\{n\}}(\mathbf{q}_1,\ldots,\mathbf{q}_{N_{\rm cl}-1})\}. \tag{5}$$

As it was mentioned, the idea of using the generating functions in work with the oscillator basis turned out to be very productive for the calculations of nuclear processes involving binary channels (see the reviews [5—7], [3,4]). The well-known example of the generating function is that of Hermite polynomials [21]

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x).$$

In our case, the generating functions for the basis states (3) are the generalization of the above function. They are constructed as the Slater determinants composed of the single-particle Brink orbitals [22]

$$\varphi(\mathbf{R}_k, \mathbf{r}) = \exp\left\{-\frac{1}{2}\mathbf{r}^2 + \sqrt{2}\,\mathbf{R}_k\mathbf{r} - \frac{1}{2}\mathbf{R}_k^2\right\} \zeta(\sigma, \tau), \quad k = 1,\ldots, N_{\rm cl}. \tag{6}$$

Here $\zeta(\sigma, \tau)$ are the spin-isospin functions, $\mathbf{R}_k$, $k = 1,\ldots, N_{\rm cl}$ are the generating parameters. The generating function

$$\Phi(\{\mathbf{R}_k\}, \{\mathbf{r}_i, \sigma_i, \tau_i\}) = \det \| \varphi(\mathbf{R}_k, \mathbf{r}_i) \| \tag{7}$$

is not translationally invariant, but it allows an easy separation of the factor describing the center-of-mass motion (see [23])

$$\Phi(\{\mathbf{R}_k\}, \{\mathbf{r}_i, \sigma_i, \tau_i\}) = \varphi(\{\mathbf{Q}_k\}, \{\mathbf{r}_i, \sigma_i, \tau_i\})\, \Phi_{\rm CM}(\mathbf{R}_{\rm CM}, \mathbf{r}_{\rm CM}) \tag{8}$$

by introducing the Jacobi vectors $\{\mathbf{q}_k\}$ and the corresponding generating coordinates $\{\mathbf{Q}_k\}$ $(k = 1,\ldots, N_{\rm cl} - 1)$. The function $\varphi(\{\mathbf{Q}_k\}, \{r_i, \sigma_i, \tau_i\})$, as can be shown, has the form similar to (1)

$$\varphi(\{\mathbf{Q}_k\}, \{\mathbf{r}_i, \sigma_i, \tau_i\}) = \mathcal{A}\{\varphi(1)\ldots\varphi(N_{\text{cl}})f(\{\mathbf{Q}_k\}, \{\mathbf{q}_k\})\}, \tag{9}$$

where

$$\varphi(k) = \exp\left\{-\frac{1}{2}\rho_k^2\right\}\zeta_k$$

$$\rho_k^2 = \sum_{i\in A_k}(\mathbf{r}_i - \mathbf{R}_k^{\text{CM}})^2, \quad \zeta_k = \prod_{i\in A_k}\zeta(\sigma_i, \tau_i) \tag{10}$$

(the summation and the multiplication run over the coordinates of nucleons belonging to the k-th cluster),

$$f(\{\mathbf{Q}_k\}, \{\mathbf{q}_k\}) = \prod_{k=1}^{N_{\text{cl}}-1}\exp\left\{-\frac{1}{2}\mathbf{q}_k^2 + \sqrt{2}\,\mathbf{Q}_k\mathbf{q}_k - \frac{1}{2}\mathbf{Q}_k^2\right\}. \tag{11}$$

The function $\varphi(\{\mathbf{Q}_k\}, \{\mathbf{r}_i, \sigma_i, \tau_i\})$, which we have just constructed, is a generalized coherent state [16,17,5] for the nuclear system under consideration. In this quality it is a wave packet generating the harmonic oscillator basis describing the dynamics of cluster relative motion. Indeed, as can be shown, the function under the product sign in Eq.11 is the generating function for the basis states (3)

$$\exp\left\{-\frac{1}{2}\mathbf{q}^2 + \sqrt{2}\,\mathbf{Q}\mathbf{q} - \frac{1}{2}\mathbf{Q}^2\right\} =$$

$$= \sum_{nlm}\left[\frac{\pi^3 2^{-2n-l+1}}{n!\,\Gamma(n+l+3/2)}\right]^{1/2} Q^{2n+l}Y_{lm}^*(\Omega_Q)f_{nlm}(\mathbf{q}). \tag{12}$$

Furthermore, the function $\varphi(\{\mathbf{Q}_k\}, \{r_i, \sigma_i, \tau_i\})$ is a kernel of an integral transformation which maps the wave function in the coordinate space into the Fock—Bargmann space [18—20], where it depends only on $N_{\text{cl}}-1$ vector generating parameters (as we mentioned, the intrinsic cluster functions are fixed within this approach).

Below, using the generating function (9), we shall find the Fock—Bargmann images of the allowed by the Pauli principle basis states and, remaining in the Fock—Bargmann space, calculate all necessary matrix elements. The special attention will be paid to the overlap integral of the generating functions with different generating parameters since it allows one to get the comprehensive information on the harmonic oscillator basis for a three-cluster system.

Although the expressions for the basis functions will be obtained in the Fock—Bargmann space, this does not hinder to achieve our main objective, to obtain the Schrödinger equation in the harmonic oscillator representation.

Now we consider the construction of the generating functions and the calculation of their overlap integral for the ^{6}He, which we regard as a system of three clusters, the alpha-particle and two neutrons. For each cluster we introduce the Brink orbitals with the corresponding generator parameter or, in more detail, four single-particle states for the alpha cluster

$$|1\rangle \equiv \phi_1 |p\uparrow\rangle, \quad |2\rangle \equiv \phi_1 |p\downarrow\rangle, \quad |3\rangle \equiv \phi_1 |n\uparrow\rangle, \quad |4\rangle \equiv \phi_1 |n\downarrow\rangle; \tag{13}$$

and two states for two neutron clusters

$$|5\rangle \equiv \phi_2 |n\uparrow\rangle, \quad |6\rangle \equiv \phi_3 |n\downarrow\rangle, \tag{14}$$

where ϕ_k is the spatial part of $\phi(\mathbf{R}_k, \mathbf{r})$, spin-isospin functions are denoted in an obvious manner. We shall use also the states with other generating parameters

$$\tilde{\phi}_k \zeta(\sigma, \tau) = \phi(\tilde{\mathbf{R}}_k, \mathbf{r}), \quad k = 1,2,3. \tag{15}$$

Let now

$$\Phi(\mathbf{R}_1,\mathbf{R}_2,\mathbf{R}_3;\{\mathbf{r}_i,\sigma_i,\tau_i\}) = \begin{vmatrix} \phi_1(\mathbf{r}_1)|p\uparrow\rangle & \phi_1(\mathbf{r}_2)|p\uparrow\rangle & \cdots & \phi_1(\mathbf{r}_6)|p\uparrow\rangle \\ \phi_1(\mathbf{r}_1)|p\downarrow\rangle & \phi_1(\mathbf{r}_2)|p\downarrow\rangle & \cdots & \phi_1(\mathbf{r}_6)|p\downarrow\rangle \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \phi_3(\mathbf{r}_1)|n\downarrow\rangle & \phi_3(\mathbf{r}_2)|n\downarrow\rangle & \cdots & \phi_3(\mathbf{r}_6)|n\downarrow\rangle \end{vmatrix}. \tag{16}$$

Then the overlap integral of two wave packets, Φ and $\tilde{\Phi}$, where

$$\tilde{\Phi} = \Phi(\tilde{\mathbf{R}}_1, \tilde{\mathbf{R}}_2, \tilde{\mathbf{R}}_3; \{\mathbf{r}_i, \sigma_i, \tau_i\}),$$

can be calculated as a determinant of single-particle overlaps [24]

$$\langle \Phi | \tilde{\Phi} \rangle = \begin{vmatrix} \langle 1|\tilde{1}\rangle & \langle 1|\tilde{2}\rangle & \cdots & \langle 1|\tilde{6}\rangle \\ \langle 2|\tilde{1}\rangle & \langle 2|\tilde{2}\rangle & \cdots & \langle 2|\tilde{6}\rangle \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \langle 6|\tilde{1}\rangle & \langle 6|\tilde{2}\rangle & \cdots & \langle 6|\tilde{6}\rangle \end{vmatrix}. \tag{17}$$

Most of its elements vanish because of the orthogonality of spin-isospin functions. Then

$$\langle \Phi | \tilde{\Phi} \rangle = \begin{vmatrix} \langle 1|\tilde{1}\rangle & 0 & 0 & 0 & 0 & 0 \\ 0 & \langle 2|\tilde{2}\rangle & 0 & 0 & 0 & 0 \\ 0 & 0 & \langle 3|\tilde{3}\rangle & \langle 3|\tilde{5}\rangle & 0 & 0 \\ 0 & 0 & \langle 5|\tilde{3}\rangle & \langle 5|\tilde{5}\rangle & 0 & 0 \\ 0 & 0 & 0 & 0 & \langle 4|\tilde{4}\rangle & \langle 4|\tilde{6}\rangle \\ 0 & 0 & 0 & 0 & \langle 6|\tilde{4}\rangle & \langle 6|\tilde{6}\rangle \end{vmatrix} =$$

$$= \langle 1|\tilde{1}\rangle \langle 2|\tilde{2}\rangle \times \begin{vmatrix} \langle 3|\tilde{3}\rangle & \langle 3|\tilde{5}\rangle \\ \langle 5|\tilde{3}\rangle & \langle 5|\tilde{5}\rangle \end{vmatrix} \begin{vmatrix} \langle 4|\tilde{4}\rangle & \langle 4|\tilde{6}\rangle \\ \langle 6|\tilde{4}\rangle & \langle 6|\tilde{6}\rangle \end{vmatrix}. \tag{18}$$

Thus, the generating overlap integral has a multiplicative form. Calculating the overlaps of Brink orbitals

$$\langle \phi(\mathbf{R}) | \tilde{\phi}(\tilde{\mathbf{R}}) \rangle = \exp(\mathbf{R}\tilde{\mathbf{R}}),$$

where the constant factor of $\pi^{3/2}$ is dropped, we obtain the overlap integral of the generating wave packets

$$\begin{aligned} \langle \Phi | \tilde{\Phi} \rangle = & \exp\{4\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_2\tilde{\mathbf{R}}_2 + \mathbf{R}_3\tilde{\mathbf{R}}_3\} - \\ & - \exp\{3\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_2 + \mathbf{R}_2\tilde{\mathbf{R}}_1 + \mathbf{R}_3\tilde{\mathbf{R}}_3\} - \\ & - \exp\{3\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_2\tilde{\mathbf{R}}_2 + \mathbf{R}_1\tilde{\mathbf{R}}_3 + \mathbf{R}_3\tilde{\mathbf{R}}_1\} + \\ & + \exp\{2\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_2 + \mathbf{R}_2\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_3 + \mathbf{R}_3\tilde{\mathbf{R}}_1\}. \end{aligned} \tag{19}$$

In the calculations we used the generating functions with the fixed total isospin of a system $T = 1$, while with respect to spin coordinates it is a superposition of singlet and triplet states ($S = 0, 1$). Later on we shall show how to separate basis functions with the definite spin.

To separate the center-of-mass motion, we introduce new generating parameters, which we denote $(\mathbf{a}, \mathbf{b}, \mathbf{R}_{CM})$ and $(\tilde{\mathbf{a}}, \tilde{\mathbf{b}}, \tilde{\mathbf{R}}_{CM})$:

$$\begin{aligned} \mathbf{a} &= \frac{2}{\sqrt{3}}\left(\mathbf{R}_1 - \frac{1}{2}(\mathbf{R}_2 + \mathbf{R}_3)\right), \quad \tilde{\mathbf{a}} = \frac{2}{\sqrt{3}}\left(\tilde{\mathbf{R}}_1 - \frac{1}{2}(\tilde{\mathbf{R}}_2 + \tilde{\mathbf{R}}_3)\right), \\ \mathbf{b} &= \frac{1}{\sqrt{2}}(\mathbf{R}_2 - \mathbf{R}_3), \quad \tilde{\mathbf{b}} = \frac{1}{\sqrt{2}}(\tilde{\mathbf{R}}_2 - \tilde{\mathbf{R}}_3), \\ \mathbf{R}_{CM} &= \frac{1}{\sqrt{6}}(4\mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3), \quad \tilde{\mathbf{R}}_{CM} = \frac{1}{\sqrt{6}}(4\tilde{\mathbf{R}}_1 + \tilde{\mathbf{R}}_2 + \tilde{\mathbf{R}}_3). \end{aligned} \tag{20}$$

After a transformation to new parameters, the overlap integral becomes

$$\langle \Phi | \tilde{\Phi} \rangle = \exp(\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM}) \langle \mathbf{ab} | \tilde{\mathbf{a}}\tilde{\mathbf{b}} \rangle, \tag{21}$$

$$\langle \mathbf{ab} \,|\, \tilde{\mathbf{a}}\,\tilde{\mathbf{b}} \rangle = \exp(\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}) -$$

$$- \exp\{1/4\mathbf{a}\tilde{\mathbf{a}} + \sqrt{6}/4\mathbf{a}\tilde{\mathbf{b}} + \sqrt{6}/4\mathbf{b}\tilde{\mathbf{a}} + 1/2\mathbf{b}\tilde{\mathbf{b}}\} -$$

$$- \exp\{1/4\mathbf{a}\tilde{\mathbf{a}} - \sqrt{6}/4\mathbf{a}\tilde{\mathbf{b}} - \sqrt{6}/4\mathbf{b}\tilde{\mathbf{a}} + 1/2\mathbf{b}\tilde{\mathbf{b}}\} + \exp\{-1/2\mathbf{a}\tilde{\mathbf{a}}\}, \qquad (22)$$

thus factoring out the center-of-mass motion. In the Fock—Bargmann space, the generating parameters, vectors $\mathbf{a}$ and $\mathbf{b}$ ($\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$), correspond to the Jacobi vectors of a three-body system in the coordinate space describing the relative position of two neutrons ($\mathbf{a}$) and the relative position of the alpha-particle and the center-of-mass of two neutrons ($\mathbf{b}$).

Let us now examine the calculation of the overlap integral from a different point of view, by explicitly considering the antisymmetrization operator $\mathcal{A}$. Among all $6! = 720$ nucleon permutations composing $\mathcal{A}$ one should take into account only those that involve nucleons in the same spin-isospin state. There are four such permutations, the identical one, the transpositions P_{35}, P_{46} and their product $P_{35}P_{46}$. The antisymmetrization operator becomes

$$\mathcal{A} = 1 - P_{35} - P_{46} + P_{35}P_{46}$$

and we have four terms in the overlap (19) corresponding to these permutations. If we write the first term (it is an overlap of unantisymmetrized generating functions) as

$$\exp\{\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_1\tilde{\mathbf{R}}_2 + \mathbf{R}_1\tilde{\mathbf{R}}_2 + \mathbf{R}_3\tilde{\mathbf{R}}_3\},$$

the other terms can be obtained from it by interchanging *one* of $\mathbf{R}_1$ by $\mathbf{R}_2$ or $\mathbf{R}_3$ or simultaneously *two* of $\mathbf{R}_1$ by $\mathbf{R}_2$ and $\mathbf{R}_3$.

The action of the antisymmetrization operator $\mathcal{A}$ in the space of the translation-invariant generating parameters $\mathbf{a}$ and $\mathbf{b}$ is reduced to some linear transformation of $\mathbf{a}$ and $\mathbf{b}$. It can be shown (see [23]) that the matrices of this transformation corresponding to the permutations P_{35}, P_{46} and $P_{35}P_{46}$ are

$$T^{(1)} = \begin{pmatrix} \frac{1}{4} & \sqrt{\frac{3}{8}} \\ \sqrt{\frac{3}{8}} & \frac{1}{2} \end{pmatrix}, \quad T^{(2)} = \begin{pmatrix} \frac{1}{4} & -\sqrt{\frac{3}{8}} \\ -\sqrt{\frac{3}{8}} & \frac{1}{2} \end{pmatrix}, \quad T^{(3)} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix}, \qquad (23)$$

respectively. Then, the overlap integral can be formally rewritten as a sum of four similar terms

$$\langle \mathbf{ab} \,|\, \tilde{\mathbf{a}}\tilde{\mathbf{b}} \rangle = \exp\{\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}\} - \exp\{\bar{\mathbf{a}}\tilde{\mathbf{a}} + \bar{\mathbf{b}}\tilde{\mathbf{b}}\} - \exp\{\bar{\bar{\mathbf{a}}}\tilde{\mathbf{a}} + \bar{\bar{\mathbf{b}}}\tilde{\mathbf{b}}\} + \exp\{\bar{\bar{\bar{\mathbf{a}}}}\tilde{\mathbf{a}} + \bar{\bar{\bar{\mathbf{b}}}}\tilde{\mathbf{b}}\}, \quad (24)$$

where

$$\bar{\mathbf{a}} = \frac{1}{4}\mathbf{a} + \sqrt{\frac{3}{8}}\,\mathbf{b}, \quad \bar{\mathbf{b}} = \sqrt{\frac{3}{8}}\,\mathbf{a} + \frac{1}{2}\mathbf{b}.$$

$$\bar{\bar{\mathbf{a}}} = \frac{1}{4}\mathbf{a} - \sqrt{\frac{3}{8}}\,\mathbf{b}, \quad \bar{\bar{\mathbf{b}}} = -\sqrt{\frac{3}{8}}\,\mathbf{a} + \frac{1}{2}\mathbf{b}. \tag{25}$$

$$\bar{\bar{\bar{\mathbf{a}}}} = -\frac{1}{2}\mathbf{a}, \quad \bar{\bar{\bar{\mathbf{b}}}} = 0.$$

We see that to calculate the overlap integral we need only to know the overlap of the unantisymmetrized generating functions and the transformation of generating parameters induced by the operation of nucleon permutation, i.e., the representation of the antisymmetrization operator $\mathcal{A}$ in the Fock—Bargmann space. This idea will be applied also to the calculation of the matrix elements of other operators, namely the operators of kinetic and potential energy.

To make clear the subsequent transformations of the overlap integral $\langle \mathbf{ab} | \tilde{\mathbf{a}}\tilde{\mathbf{b}} \rangle$, let us recall some facts on the generating wave packages. The translation-invariant generating function φ introduced by (9) generates the harmonic oscillator basis describing the dynamics of relative motion of clusters. On the other hand, as it was mentioned, it is a kernel of the integral transformation

$$\Psi(\mathbf{a}, \mathbf{b}) = \int \ldots \int \varphi(\mathbf{a}, \mathbf{b}, \{\mathbf{r}_i, \sigma_i, \tau_i\})\Psi(\{\mathbf{r}_i, \sigma_i, \tau_i\})d\mathbf{r}_1 \ldots d\mathbf{r}_6, \tag{26}$$

which maps a wave function of a system in the coordinate space $\Psi(\{\mathbf{r}_i, \sigma_i, \tau_i\})$ onto its image $\Psi(\mathbf{a},\mathbf{b})$ in the Fock—Bargmann space. Indeed,

$$\varphi(\mathbf{a}, \mathbf{b}, \{\mathbf{r}_i, \sigma_i, \tau_i\}) = \sum_{\{n\}} \varphi(\{n\}; \mathbf{a},\mathbf{b})\Psi^*_{\{n\}}(\{\mathbf{r}_i, \sigma_i, \tau_i\}), \tag{27}$$

where $\{\Psi_{\{n\}}\}$ are the orthonormalized harmonic oscillator basis states (5), $\varphi(\{n\}; \mathbf{a},\mathbf{b})$ are their images. Therefore, the expansion (4) can be rewritten in the Fock—Bargmann space

$$\Psi(\mathbf{a}, \mathbf{b}) = \sum_{\{n\}} C_{\{n\}}\varphi(\{n\}; \mathbf{a},\mathbf{b}), \tag{28}$$

and we come to the problem of constructing the image $\Psi(\mathbf{a},\mathbf{b})$ of wave functions in terms of their expansion coefficients $C_{\{n\}}$ and the subsequent return to the original

$$\Psi(\{\mathbf{r}_i, \sigma_i, \tau_i\}) = \sum_{\{n\}} C_{\{n\}}\Psi_{\{n\}}(\{\mathbf{r}_i, \sigma_i, \tau_i\}).$$

Of course, the original replaced by its image $\Psi(\mathbf{a},\mathbf{b})$ would be only the approximate solution of the Schrödinger equation. However, that solution gives us an adequate information on nuclear phenomena that do not involve the process of the alpha cluster decay.

The sense of introducing the image $\varphi(\{n\};\mathbf{a},\mathbf{b})$ is its simplicity and clarity of representation compared with the original $\Psi_{\{n\}}$. Just compare the simplest oscillator function in the coordinate space (Eq.3) and its Fock—Bargmann image (extracted from (12))

$$a_{nl}Q^{2n+l}Y_{lm}(\Omega_Q), \quad \text{where} \quad a_{nl}=\left[\frac{\pi^3 2^{-2n-l+1}}{n!\Gamma(n+l+3/2)}\right]^{1/2}.$$

For the three-cluster system, the image is a function of two vectors which can be written explicitly for most cases, whereas $\Psi_{\{n\}}(\{r_i,\sigma_i,\tau_i\})$ depends on all Jacobi vectors and spin-isospin variables of the coordinate space being a quite complicated construction. Various features of the latter become clear after its image is found. Working with the basis $\varphi(\{n\};\mathbf{a},\mathbf{b})$ effectively reduces our six-particle problem to the three-particle one, making thus possible to use some techniques developed for the three-body problem [25—27].

So far we have not shown yet an algorithm of constructing the basis functions $\varphi(\{n\};\mathbf{a},\mathbf{b})$, although we have obtained the expression for $\langle \mathbf{ab}\,|\,\tilde{\mathbf{a}}\tilde{\mathbf{b}}\rangle$ which permits one to formulate this algorithm.

From the expansion (27) and the orthonormality of basis functions $\Psi_{\{n\}}$ it follows that

$$\langle \mathbf{ab}\,|\,\tilde{\mathbf{a}}\tilde{\mathbf{b}}\rangle=\sum_{\{n\}}\varphi(\{n\};\mathbf{a},\mathbf{b})\,\varphi(\{n\};\tilde{\mathbf{a}},\tilde{\mathbf{b}}), \tag{29}$$

i.e., the overlap integral (22) may be written as a convolution of the basis functions to be found. This fact will be used later when we shall obtain the expressions for $\varphi(\{n\};\mathbf{a},\mathbf{b})$ for the states with zero orbital angular momentum.

3. ON THE CHOICE OF THE OSCILLATOR BASIS

Having determined our objective as the derivation of the Schrödinger equation for a three-cluster system in the representation of oscillator basis states $\{\Psi_{\{n\}}\}$, we should specify what classification of the basis states will be used. Among quantum numbers labeling the basis state there are two evident ones, the total number of oscillator excitation quanta n and the orbital angular

momentum of clusters relative motion L. At the first stage we shall restrict ourselves to the case $L=0$. Then only three dynamical degrees of freedom remain and, therefore, besides n, two other quantum numbers must be specified. Their choice is determined by the Pauli principle and the necessity to simplify as much as possible the calculations of Hamiltonian matrix element between the basis states. The Pauli principle requires that the basis states should be antisymmetric with respect to transposition of nucleons. Therefore, it is desirable to have quantum numbers which — along with n — remain unchanged under nucleon permutations.

The choice of these quantum numbers is based upon the classification of harmonic oscillator basis states provided by the $SU(3)$ group [28]. It is known that the space spanned by the harmonic oscillator states with a given n is a direct sum of the subspaces spanned by states with a fixed $SU(3)$symmetry $(\lambda\mu)$. For $L=0$, n, λ and μ must be even, so the following pairs of indices are possible

$$\lambda = n,\ \mu = 0;\ \lambda = n-4,\ \mu = 2,;...;\ \lambda = n-4m,\ \mu = 2m;\ \ldots .$$

If n is fixed, only one of $(\lambda\,\mu)$ is independent, for instance μ. Letting $\mu = 2m$ we take m as the second quantum number of a basis function. It can take any integer value from 0 to $[n/4]$.

For the states with $(\lambda\,\mu) = (n-4m2m)$ we should also introduce a third quantum number, since there are $n-2m+1$ states for each fixed n and m, with the following n_1 and n_2

$$n_1 = 2m,\ n_2 = n-2m;\ n_1 = 2m+1,\ n_2 = n-2m-1;...;\ n_1 = n-2m,\ n_2 = 2m.$$

This quantum number could be $n_1 (2m \le n_1 \le n-2m)$ or $n_2 (2m \le n_2 \le n-2m)$, but we choose another quantum number. For this purpose we consider the action of the antisymmetrization operator on the functions $|n_1 n_2 2m\rangle$, which have no certain permutation symmetry but maintain a simple algorithm of calculation of the potential energy operator matrix elements. We shall work with the basis functions $|n_1 n_2 2m\rangle$ in the Fock—Bargmann representation

$$\langle abt | n_1 n_2 2m \rangle = \Psi(n_1, n_2, 2m; a, b, t), \tag{30}$$

where $a = |\mathbf{a}|$, $b = |\mathbf{b}|$, t is the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$. The exact expressions for the functions $\Psi(n_1, n_2, 2m; a, b, t)$ can be found in Sect.4, now we only say that these functions are proportional to a^{n_1}, b^{n_2} and $|[\mathbf{ab}]|^{2m}$.

Let us list the most important features of the basis functions (30). First, the matrix elements of the potential energy operator in this basis have comparatively simple form (they are reduced to two-fold sums, see Sect.6).

Second, a transformation of these functions under nucleon permutations is also simple, so that it is not difficult to find such linear combinations of the functions (30), which are the eigenvectors of the antisymmetrization operator. As for the third quantum number of the antisymmetric basis functions, it is convenient to identify it with an eigenvalue of the antisymmetrization operator matrix in the space of the basis functions (30).

This is not the only possibility of the classification of oscillator basis states. The other choice, used in [23], is based upon the quantum numbers of two single-particle oscillators $\{n_1, l_1, n_2, l_2, L, M\}$ with their angular momenta coupled into the total angular momentum L. This classification has clearer physical interpretation, but the calculations with such a basis are more complicated.

4. HARMONIC OSCILLATOR BASIS STATES IN THE GENERATOR PARAMETER SPACE

The overlap integral (22) of generating wave packets is a natural way of constructing the harmonic oscillator basis states in the Fock—Bargmann representation, i.e., in the space of the generator parameters $\mathbf{a}$, $\mathbf{b}$ ($\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$). In order to construct these states we first consider the expansion in powers of $\mathbf{a}$, $\mathbf{b}$ ($\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$) of the first term in the overlap integral:

$$\exp(\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}) = \sum_{n=0}^{\infty} \left(\sum_{n_1+n_2=n} \frac{1}{n_1!\, n_2!} (\mathbf{a}\tilde{\mathbf{a}})^{n_1} (\mathbf{b}\tilde{\mathbf{b}})^{n_2} \right). \tag{31}$$

Each term of this sum is a homogeneous polynomial in the components of vectors $\mathbf{a}$, $\mathbf{b}$, $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$ of the same homogeneity degree with respect to $\mathbf{a}$, $\mathbf{b}$ and with respect to $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$. This polynomial may be considered as an overlap integral of basis functions with n oscillator quanta. Naturally, the basis functions of states with different n are orthogonal. Moreover, term in Eq.31 corresponds to a convolution of basis states classified by the number of quanta. Meanwhile, our aim is to construct a basis having such quantum numbers as total number n, $SU(3)$ symmetry indices $(\lambda\, \mu)$, orbital angular momentum L and its projection M.

The following important relation gives us a key to solving the problem of constructing the basis states with the required quantum numbers:

$$(\mathbf{a}\tilde{\mathbf{a}})^{n_1} (\mathbf{b}\tilde{\mathbf{b}})^{n_2} =$$

$$= \sum_{m=0}^{\min(n_1/2, n_2/2)} B_m^2(n_1, n_2) a^{n_1-m} b^{n_2-m} |[\mathbf{ab}]|^m I(n_1, n_2, m) \tilde{a}^{n_1-m} \tilde{b}^{n_2-m} |[\tilde{\mathbf{a}}\tilde{\mathbf{b}}]|^m, \tag{32}$$

where the normalizing coefficients

$$B_m^2(n_1,n_2) = \frac{(n_1+n_2+1-2m)!n_1!\,n_2!}{m!(n_1+n_2+1-m)!(n_1-m)!(n_2-m)!} \tag{33}$$

determine the weight of overlap integrals

$$I(n_1,n_2,m) = \langle\, n_1n_2(n_1+n_2-2mm) \,|\, \overline{n_1n_2(n_1+n_2-2mm)}\,\rangle \tag{34}$$

of wave packets $(\lambda\,\mu) = (n_1+n_2-2mm)$ of the $SU(3)$ irreducible representations when these integrals form the product $(\mathbf{a}\tilde{\mathbf{a}})^{n_1}(\mathbf{b}\tilde{\mathbf{b}})^{n_2}$. The overlap integrals (34) are normalized so that when $t(\tilde{t})$, the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$ ($\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$), equals 1, they take the well-known form first introduced by Elliot [28],

$$I(n_1,n_2,m) = d_{11}^{\,n_1+n_2-2m} d_{33}^{\,m} \tag{35}$$

where d_{33} is the cosine of an angle between the normals to the planes passing through $\mathbf{a}$, $\mathbf{b}$ and $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$ (these normals coincide with the axes ζ and $\tilde{\zeta}$ of the intrinsic coordinate frames spanned by vectors $\mathbf{a}$, $\mathbf{b}$ and $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, respectively), d_{11} is the cosine of the angle between the axes ξ and $\tilde{\xi}$ of these coordinate frames.

In the present paper (as well as in [29]) we restrict ourselves to the consideration of states with $L=0$ reserving generalization on the case of arbitrary L for the next paper. The $L=0$ states contain only the terms with even powers of expansion (31). Retaining only these terms and denoting other by dots we rewrite (31) as

$$\exp(\mathbf{a}\tilde{\mathbf{a}}+\mathbf{b}\tilde{\mathbf{b}}) = \sum_{n=0}^{\infty} \sum_{n_1+n_2=2n} \frac{1}{n_1!n_2!} (\mathbf{a}\tilde{\mathbf{a}})^{n_1}(\mathbf{b}\tilde{\mathbf{b}})^{n_2} + \ldots =$$

$$= \sum_{n=0}^{\infty} \sum_{n_1+n_2=2n} N^2(n_1,n_2;2m) a^{n_1-2m} b^{n_2-2m} [\mathbf{ab}]^{2m} \Phi(n_1,n_2;2m;t) \times$$

$$\times \tilde{a}^{n_1-2m} \tilde{b}^{n_2-2m} \,|\, [\tilde{\mathbf{a}}\tilde{\mathbf{b}}] \,|^{2m} \Phi(n_1,n_2,2m;\tilde{t}) + \ldots, \tag{36}$$

where

$$N^2(n_1,n_2;2m) = \frac{1}{n_1!n_2!} B_{2m}^2(n_1,n_2) C_{2m}^2(n_1,n_2),$$

$$C^2_{2m}(n_1,n_2)=\frac{\Gamma\left(\frac{1}{2}(n_1+n_2)-m+1\right)\Gamma\left(\frac{1}{2}(n_1+n_2)-2m+\frac{1}{2}\right)\Gamma\left(m+\frac{1}{2}\right)}{2\sqrt{\pi}\,m!\,\Gamma\left(\frac{1}{2}(n_1+n_2)-2m+1\right)\Gamma\left(\frac{1}{2}(n_1+n_2)-m+\frac{3}{2}\right)}. \quad (37)$$

The functions $\Phi(n_1,n_2,2m,t)$ are the eigenfunctions of the second-order Casimir operator of the $SU(3)$ group. They were obtained in Refs.30,31. For even n_1 and n_2

$$\Phi(n_1,n_2,2m;t)=F(-n_1/2+m,-n_2/2+m;-(n_1+n_2)/2+2m+1/2;1-t^2), \quad (38)$$

where $F(\alpha,\beta;\gamma;z)$ is the hypergeometric function [21]. For odd n_1, and n_2

$$\Phi(n_1,n_2,2m;t)=$$

$$=tF(-(n_1-1)/2+m,-(n_2-1)/2+m;-(n_1+n_2)/2+2m+1/2;1-t^2). \quad (39)$$

So the basis functions are chosen in the form

$$\Psi(n_1,n_2;2m)=N(n_1,n_2;2m)a^{n_1-2m}b^{n_2-2m}[\mathbf{ab}]^{2m}\Phi(n_1,n_2,2m;t). \quad (40)$$

They are the orthonormalized basis functions $|n_1n_2 2m\rangle$ in the Fock—Bargmann representation discussed in Sect.3. Their $SU(3)$ symmetry is $(n_1+n_2-4m,2m)$ and the angular momentum $L=0$. Besides, we have now a new expansion for the first term of the overlap integral (22), since it is found terms of this expansion (polynomials of a homogeneity degree n) can be written as a convolution of basis functions

$$\frac{(\mathbf{a}\tilde{\mathbf{a}})^{n_1}(\mathbf{b}\tilde{\mathbf{b}})^{n_2}}{n_1!n_2!}=\sum_{m=0}^{\min(n_1/2,n_2/2)}\Psi(n_1,n_2,2m;a,b,t)\Psi(n_1,n_2,2m;\tilde{a},\tilde{b},\tilde{t})+\ldots, \quad (41)$$

where dots indicate states with $L\neq 0$.

Under the operation of nucleon permutation the basis states (40), as well as $|n_1n_2 2m\rangle$, are subjected to a linear transformation that conserves their $SU(3)$ symmetry and mixes only functions with different values of n_1-n_2 but not n_1+n_2 and m. The transformation of states (40) is induced by a linear transformation of **a** and **b**. The states (40), hence, do not have fixed permutation symmetry, so our next task is to find such their linear combinations that have permutation symmetry satisfying the Pauli principle (allowed states) and to eliminate the other (forbidden) states. In order to do that, we expand the three last terms of the overlap integral $\langle \mathbf{ab}|\tilde{\mathbf{a}}\tilde{\mathbf{b}}\rangle$ (Eq.22) in powers of vector

generating parameters. Then, in a polynomial of an even homogeneity degree (n) we retain only terms with $L=0$ and a fixed m. In this case, we obtain the following bilinear form with respect to basis functions (40)

$$\sum_{n_1+n_2=n} [\Psi(n_1,n_2,2m;\overline{\overline{a}},\overline{\overline{b}},\overline{\overline{t}}) - \Psi(n_1,n_2,2m;\bar{a},\bar{b},\bar{t}) -$$

$$- \Psi(n_1,n_2,2m;\overline{\overline{a}},\overline{\overline{b}},\overline{\overline{t}}) + \Psi(n_1,n_2,2m;\overline{\overline{a}},\overline{\overline{b}},\overline{\overline{t}})]\Psi(n_1,n_2,2m;\tilde{a},\tilde{b},\tilde{t}). \quad (42)$$

Since $\overline{\overline{\overline{b}}}=0$, the term with triple-barred a, b, t vanishes for $m=0$; its contribution for $m=0$ is

$$\left(-\frac{1}{2}\right)^n N(n,0,0)a^n N(n,0,0)\,\tilde{a}^n.$$

In order to find the transformation of basis functions under the nucleon permutation we first consider the transformation of basis functions induced by an arbitrary linear transformation of **a** and **b** performed by the matrix $\|\alpha\|$.

$$\begin{pmatrix}\bar{\mathbf{a}}\\ \bar{\mathbf{b}}\end{pmatrix}=\begin{pmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}.$$

This transformation, obtained in [32], is not very complicate

$$\Psi(\tilde{n}_1,\tilde{n}_2,2m;\bar{a},\bar{b},\bar{t})= \sum_{n_1+n_2=\tilde{n}_1+\tilde{n}_2} \langle n_1n_2 2m \,|\, \alpha|\, \tilde{n}_1\tilde{n}_2 2m\rangle\Psi(n_1,n_2,2m;a,b,t), \quad (43)$$

$$\langle n_1n_2 2m|\alpha\,|\tilde{n}_1\tilde{n}_2 2m\rangle =$$

$$= (\det\alpha)^{2m}\frac{N(\tilde{n}_1,\tilde{n}_2,2m)}{N(n_1,n_2,2m)}\sum_k \frac{(\tilde{n}_1-2m)!\,(\tilde{n}_2-2m)!}{k!(n_1-2m-k)!\,(\tilde{n}_1-2m-k)!\,(n_2-\tilde{n}_1+k)!}\times$$

$$\times(\alpha_{11})^k(\alpha_{21})^{n_1-2m-k}(\alpha_{12})^{\tilde{n}_1-2m-k}(\alpha_{22})^{n_2-\tilde{n}_1+k}. \quad (44)$$

Applying these results to three matrices $T^{(p)}$, given by Eq.23, we obtain the matrix of the antisymmetrization operator

$$\|\langle n_1n_2 2m\,|\,\mathcal{A}\,|\,\tilde{n}_1\tilde{n}_2 2m\rangle\|\,, = \|\langle n_1n_2 2m\,|\,1-T^{(1)}-T^{(2)}+T^{(3)}\,|\,\tilde{n}_1\tilde{n}_2 2m\rangle\|\,, \quad (45)$$

which gives us the expression for the overlap integral (42)

$$\langle \mathbf{ab} | \widetilde{\mathbf{ab}} \rangle =$$

$$= \sum_{n_1+n_2=n} \sum_{\tilde{n}_1+\tilde{n}_2=n} \Psi(n_1,n_2 2m; a,b,t) \langle n_1 n_2 2m | \mathcal{A} | \tilde{n}_1 \tilde{n}_2 2m \rangle \Psi(\tilde{n}_1,\tilde{n}_2 2m; a,b,t). \quad (46)$$

Later the transformation (43) of basis functions will be used to obtain the matrix elements of the potential energy operator.

Now we can return to the problem of separating the singlet and triplet spin states. The matrix (45) is symmetric, it has a block-diagonal structure with two blocks corresponding to even and odd values of n_1, n_2, $\tilde{n}_1$, $\tilde{n}_2$. Basis functions with odd n_1, n_2 are antisymmetric with respect to the transpositions of spatial coordinates of neutron clusters and, therefore, belong to the singlet spin state. The spin of a neutron pair is an integral of motion and the Hamiltonian does not mix triplet and singlet states.

Here are some examples of the matrices (45) for simple cases.

For $n = n_1 + n_2 = \tilde{n}_1 + \tilde{n}_2 = 2$; $m = 0$

$$\begin{array}{c} \\ \langle 200| \\ \langle 020| \end{array} \begin{array}{c} \begin{array}{cc} |200\rangle & |020\rangle \end{array} \\ \begin{pmatrix} \frac{9}{8} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{2} \end{pmatrix} \end{array} . \quad (47)$$

For $n = 4$; $m = 0$

$$\begin{array}{c} \\ \langle 400| \\ \langle 220| \\ \langle 040| \end{array} \begin{array}{c} \begin{array}{ccc} |400\rangle & |220\rangle & |040\rangle \end{array} \\ \begin{pmatrix} \frac{135}{128} & -\frac{3\sqrt{6}}{64} & -\frac{9}{32} \\ -\frac{3\sqrt{6}}{64} & \frac{5}{16} & -\frac{3\sqrt{6}}{16} \\ -\frac{9}{32} & -\frac{3\sqrt{6}}{16} & \frac{7}{8} \end{pmatrix} \end{array} . \quad (48)$$

The diagonalization of the matrix (45) solves the problem of eliminating the forbidden states and constructing the orthonormalized basis of allowed states, which are obtained as linear combinations of basis functions $\Psi(n_1,n_2,m;a,b,t)$.

The problem of forbidden states in many-cluster systems, in general features, has already been studied in [13,14] where some important results have been obtained. The next task is to construct the basis of allowed states and to find the Hamiltonian matrix elements in this basis. However, first we shall

clarify the relations between the basis of allowed states and its generating functions.

We have chosen as a generating function the wave packet $\Phi(\mathbf{R}_1,\mathbf{R}_2,\mathbf{R}_3;\{\mathbf{r}_i,\sigma_i,\tau_i\})$ constructed as a Slater determinant composed of six one-particle Brink orbitals (see Eq.7). In the center-of-mass system we have already introduced the generating function .

$$\varphi(\mathbf{a},\mathbf{b};\{\mathbf{r}_i,\sigma_i,\tau_i\}),$$

and now we introduce the corresponding unantisymmetrized function φ_0

$$\varphi(\mathbf{a},\mathbf{b};\{\mathbf{r}_i,\sigma_i,\tau_i\}) = \mathcal{A}\varphi_0(\mathbf{a},\mathbf{b};\{\mathbf{r}_i,\sigma_i,\tau_i\}). \tag{49}$$

It is also a generating function, but, compared to φ, it generates the basis of unantisymmetrised functions $\{|n\,\gamma\rangle\}$, which contains both the allowed and the forbidden states

$$\varphi_0(\mathbf{a},\mathbf{b};\{\mathbf{r}_i,\sigma_i,\tau_i\}) = \sum_{n,\gamma} \varphi_0(n,\gamma;\mathbf{a},\mathbf{b})|n\,\gamma\rangle. \tag{50}$$

Here n is a number of oscillator excitation quanta, γ is a set of additional quantum numbers, $\varphi(n,\gamma;\mathbf{a},\mathbf{b})$ is a Fock—Bargmann image of the basis states $|n\,\gamma\rangle$.

We cannot content ourselves with the basis $\{|n\,\gamma\rangle\}$ and construct the Hamiltonian matrix elements in it immediately for the Shrödinger equation since this basis contains the forbidden states. Each of the functions $|n\,\gamma\rangle$ has no permutation symmetry and, in general, is a superposition of allowed and forbidden states. Transition from the basis $\{|n\,\gamma\rangle\}$ to the orthonormalized basis of allowed states (i.e., states which satisfy the Pauli principle) is an orthogonal transformation of the basis $\{|n\,\gamma\rangle\}$

$$|n\,\nu\rangle = \sum_{\gamma} |n\,\gamma\rangle\langle n\,\gamma|n\,\nu\rangle. \tag{51}$$

Now we list the order of the procedures necessary to construct the Shrödinger equation

$$\sum_{n',\nu'} [\langle n\,\nu|\hat{H}|n'\nu'\rangle - E\,\delta_{n'\nu'}]C_{n'\nu'} = 0 \tag{52}$$

in the representation of the allowed states of the harmonic oscillator basis $\{|n\,\nu\rangle\}$ for the general case. Let us remind that $\{|n\,\gamma\rangle\}$ is the basis of states with no definite permutation symmetry and γ denotes all quantum numbers other than $n.$, On the other hand, $\{|n\,\nu\rangle\}$ is the basis of allowed states (here the quantum numbers additional to n are denoted by ν). For $L=0$

$$|n\,\gamma\rangle = |n_1 n_2 2m\rangle$$

$$|n\,\nu\rangle = |n_1 2m; \lambda_\nu^3\rangle \quad \text{and} \quad |n\,\nu\rangle = |n_1 2m; \lambda_\nu^1\rangle \quad \nu = 1,2,\ldots,$$

where indices 1 and 3 of λ correspond to singlet and triplet spin states.

1. First, the overlap integral of *unantisymmetrized* generating functions $\langle \varphi_0|\tilde{\varphi}_0\rangle$ is calculated and the basis states $\{|n\,\gamma\rangle\}$ in the Fock—Bargmann representation ($\varphi_0(n,\gamma;\mathbf{a},\mathbf{b})$) are extracted

$$\langle \varphi_0 | \tilde{\varphi}_0\rangle = \exp(\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}) = \sum_{n,\gamma} \varphi_0^*(n,\gamma;\mathbf{a},\mathbf{b})\varphi_0(n,\gamma;\tilde{\mathbf{a}},\tilde{\mathbf{b}}). \tag{53}$$

2. The overlap integral of *antisymmetrized* generating functions $\langle \varphi | \tilde{\varphi}\rangle$ is calculated, then diagonalized in the basis $\{\varphi_0(n,\gamma;\mathbf{a},\mathbf{b})\}$

$$\langle \varphi | \tilde{\varphi}\rangle = \sum_{n,\gamma,\tilde{\gamma}} \varphi_0^*(n,\gamma;\mathbf{a},\mathbf{b})\langle n\,\gamma|\mathcal{A}|n\,\tilde{\gamma}\rangle\,\varphi_0(n,\tilde{\gamma};\tilde{\mathbf{a}}\tilde{\mathbf{b}}), \tag{54}$$

and is written as

$$\langle \mathbf{ab}\,|\,\tilde{\mathbf{a}}\tilde{\mathbf{b}}\rangle = \sum_{n,\nu} \lambda_\nu\,\varphi^*(n,\nu;\mathbf{a},\mathbf{b})\varphi(n,\nu;\tilde{\mathbf{a}},\tilde{\mathbf{b}}), \tag{55}$$

where λ_ν are the eigenvalues of the $\langle n\,\gamma|\mathcal{A}|n\,\tilde{\gamma}\rangle$ matrix. The transition matrix from $\{|n\,\gamma\rangle\}$ to $\{|n\,\nu\rangle\}$ basis

$$\varphi(n,\nu;\mathbf{a},\mathbf{b}) = \sum_{\gamma} \langle n\,\gamma\,|n\nu\rangle\varphi_0(n,\gamma;\mathbf{a},\mathbf{b}) \tag{56}$$

is found by the diagonalization procedure. Note that $\lambda_\nu(n) = 0$ for the forbidden states, therefore these states drop out from the expansion (53).

3. After that, the Hamiltonian matrix elements between the generating functions

$$\varphi_0(\mathbf{a},\mathbf{b};\{\mathbf{r}_i,\sigma_i,\tau_i\}) \quad \text{and} \quad \varphi_0(\tilde{\mathbf{a}},\tilde{\mathbf{b}}\,\{\mathbf{r}_i,\sigma_i,\tau_i\})$$

are calculated and their projecting onto the basis states $\varphi_0(n,\gamma;\mathbf{a},\mathbf{b})$ and $\varphi_0(\tilde{n},\tilde{\gamma};\tilde{\mathbf{a}},\tilde{\mathbf{b}})$ is carried out

$$\langle \varphi_0|\hat{H}|\tilde{\varphi}_0\rangle = \sum_{n\gamma}\sum_{\tilde{n}\tilde{\gamma}} \varphi_0^*(n,\gamma;\mathbf{a},\mathbf{b})\langle n\,\gamma|\hat{H}|\tilde{n}\,\tilde{\gamma}\rangle\varphi_0(\tilde{n},\tilde{\gamma};\tilde{\mathbf{a}},\tilde{\mathbf{b}}), \tag{57}$$

what yields the matrix elements $\langle n\gamma|\hat{H}|\tilde{n}\tilde{\gamma}\rangle$.

4. Finally, the Hamiltonian matrix elements in the basis $\{|n\nu\rangle\}$ are found

$$\langle n\nu|\hat{H}|\tilde{n}\tilde{\nu}\rangle = \sum_{\gamma}\sum_{\tilde{\gamma}} \langle n\nu|n\gamma\rangle\langle n\gamma|\hat{H}|\tilde{n}\tilde{\gamma}\rangle\langle\tilde{n}\tilde{\gamma}|\tilde{n}\tilde{\nu}\rangle. \tag{58}$$

Of course, to extract the Hamiltonian matrix elements from $\langle \varphi_0|\hat{H}|\tilde{\varphi}_0 \rangle$ is simpler than from $\langle \varphi|\hat{H}|\tilde{\varphi} \rangle$.

Now we shall show how to realize the described procedure for the specific case of $^4He + n + n$ system and states with $L = 0$.

We start from determining the allowed states. This problem is solved by a diagonalization of the symmetric matrix given above (45). We denote the matrix eigenvalues and eigenvectors for singlet states as

$$\lambda_\nu^1(m,2m), \quad \text{and} \quad \Phi_\nu^1(n,2m) \quad \nu = 1,2, ..., n/2 - 2m + 1,$$

and for triplet states as

$$\lambda_\nu^3(m,2m), \quad \text{and} \quad \Phi_\nu^3(n,2m) \quad \nu = 1,2, ..., n/2 - 2m + 1.$$

Then, instead of (46) we get

$$\langle \mathbf{ab}|\tilde{\mathbf{ab}} \rangle = \sum_\nu \lambda_\nu(n,2m)\Phi_\nu(n,2m)\Phi_\nu(n,2m). \tag{59}$$

The terms with zero eigenvalues of λ_ν drop out from this sum, they correspond to the forbidden states.

The simplest examples of the singlet eigenvalues and eigenfunctions are

$$\lambda_1^1(2,0) = \frac{13}{8},$$

$$\Phi_1^2(2,0) = \frac{3}{\sqrt{13}}|200\rangle - \frac{2}{\sqrt{13}}|020\rangle = \frac{1}{\sqrt{78}}(3a^2 - 2b^2). \tag{60}$$

$\Phi_1^1(2,0)$ is the image of the shell-model ground function

$$\lambda_1^1(4,0) = 0{,}936,$$

$$\Phi_1^1(4,0) = 0{,}706|400\rangle - 0{,}499|220\rangle + 0{,}502|040\rangle; \tag{61}$$

$$\lambda_2^1(4,0) = 1{,}306,$$

$$\Phi_2^1(4,0) = 0{,}671|400\rangle + 0{,}245|220\rangle - 0{,}699|040\rangle. \tag{62}$$

Transition from the basis $\Psi(n_1n_2,2m)$ to the basis $\Phi_\nu(n,2m)$ is carried out by the matrix$\langle n_1n_2 2m|n2m\nu \rangle$

$$\Phi_\nu(n,2m) = \sum_{n_1+n_2=n} \Psi(n_1,n_2 2m)\langle n_1n_2 2m|n2m\nu \rangle. \tag{63}$$

The matrix elements of $\hat{H}$ in the new basis are calculated by means of the same matrix

$$\langle n2m\nu|\hat{H}|\tilde{n}2\tilde{m}\tilde{\nu}\rangle =$$

$$= \sum_{n_1+n_2=n} \sum_{\tilde{n}_1+\tilde{n}_2=\tilde{n}} \langle n2m\nu|n_1n_2 2m\rangle\langle n_1n_2 2m|\hat{H}|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle\langle n_1n_2 2m|n2m\nu\rangle. \quad (64)$$

After that the Shrödinger equation in the representation $\{|n2m\nu\rangle\}$ becomes

$$\sum_{\tilde{n},\tilde{m},\tilde{\nu}} \langle n,2m,\nu|\hat{H}-E|\tilde{n},2\tilde{m},\nu\rangle C(\tilde{n},2\tilde{m},\tilde{\nu}) = 0. \quad (65)$$

In the end of this Section let us notice two important points:

1. Forbidden states correspond to zero eigenvalues of the antisymmetrization operator matrix. They are absent if $m \neq 0$. If $m = 0$ then for each n there are two forbidden states, one singlet and one triplet. Eigenvectors of forbidden states cannot be realized since they vanish after the antisymmetrization. One discovers that obtaining $\lambda_\nu = 0$.

2. Allowed states correspond to λ_ν which are not equal to unity in general case. Although all λ_ν of allowed states are close to unity, they lie between ~ 1.25 and ~ 0.9375, even if $n \to \infty$. So we see that the action of the Pauli principle results not only in the emergence of forbidden states, but also in the change of the normalization factors of allowed states in the Fock—Bargmann space. These factors differ from ones which would have been obtained if only the orthogonality of allowed and forbidden states were required (i.e., the normalization factors extracted from the overlap integral $\langle \varphi_0|\tilde{\varphi}_0\rangle$).

5. MATRIX ELEMENTS OF THE KINETIC ENERGY OPERATOR

For the kinetic energy operator in the center-of-mass system

$$\hat{T} = \frac{\hbar^2}{2Mr_0^2} \sum_{i=1}^{A} \nabla_i^2 - \frac{\hbar^2}{2AMr_0^2} \nabla_{\mathrm{CM}}^2 \quad (66)$$

(M is the nucleon mass, r_0 is the oscillator length) we first calculate the matrix elements of the single-particle operator $\frac{\hbar^2}{2Mr_0^2}\nabla^2$ between the Brink orbitals (7)

$$\langle \phi(\mathbf{R})|\frac{\hbar^2}{2Mr_0^2}\nabla^2|\phi(\tilde{\mathbf{R}})\rangle = \pi^{3/2}\frac{\hbar^2}{2Mr_0^2}[3-(\mathbf{R}-\tilde{\mathbf{R}})^2]\exp(\mathbf{R}\tilde{\mathbf{R}}). \quad (67)$$

After some transformations, dropping the factor $\pi^{3/2}$, we obtain the matrix elements of the operator (66) between the unantisymmetrized generating functions φ_0 and $\tilde{\varphi}_0$ for the case of ^{6}He

$$\langle \varphi_0 | \hat{T} | \tilde{\varphi}_0 \rangle = \frac{h^2}{2Mr_0^2} [15 - (\mathbf{a} - \tilde{\mathbf{a}})^2 - (\mathbf{b} - \tilde{\mathbf{b}})^2] \exp(\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}). \tag{68}$$

Then, for the diagonal matrix elements between the basis functions we get

$$\langle n_1 n_2 2m | \hat{T} | n_1 n_2 2m \rangle = \frac{h^2}{4Mr_0^2} (2n_1 + 2n_2 + 15) = \frac{h^2}{4Mr_0^2} (2n + 15). \tag{69}$$

There are eight different non-diagonal matrix elements, but it is sufficient to write here only four of them which are contained in the following identities

$$a^2 \Psi(n_1, n_2, 2m; a, b, t) = \frac{N(n_1, n_2, 2m)}{N(n_1 + 2, n_2, 2m)} \Psi(n_1 + 2, n_2, 2m; a, b, t) +$$

$$+ \frac{N(n_1, n_2, 2m)}{N(n_1 + 2, n_2, 2m + 2)} \frac{(n_2 - 2m - 1)(n_2 - 2m)}{(n_1 + n_2 - 4m - 1)(n_1 + n_2 - 4m + 1)} \times$$

$$\times \Psi(n_1 + 2, n_2, 2m + 2; a, b, t); \tag{70}$$

$$b^2 \Psi(n_1, n_2, 2m; a, b, t) = \frac{N(n_1, n_2, 2m)}{N(n_1, n_2 + 2, 2m)} \Psi(n_1, n_2 + 2, 2m; a, b, t) +$$

$$+ \frac{N(n_1, n_2, 2m)}{N(n_1, n_2 + 2, 2m + 2)} \frac{(n_1 - 2m - 1)(n_1 - 2m)}{(n_1 + n_2 - 4m - 1)(n_1 + n_2 - 4m + 1)} \times$$

$$\Psi((n_1, n_2 + 2, 2m + 2; a, b, t). \tag{71}$$

It is evident now that

$$\langle n_1 + 2n_2 2m | \hat{T} | n_1 n_2 2m \rangle = -\frac{\hbar^2}{4Mr_0^2} \frac{N(n_1, n_2, 2m)}{N(n_1 + 2, n_2, 2m)}; \tag{72}$$

$$\langle n_1 n_2 + 22m | \hat{T} | n_1 n_2 2m \rangle = -\frac{\hbar^2}{4Mr_0^2} \frac{N(n_1, n_2, 2m)}{N(n_1, n_2 + 2, n_2, 2m)}; \tag{73}$$

$$\langle n_1 + 2n_2 2m | \hat{T} | n_1 n_2 2m \rangle = -\frac{\hbar^2}{4Mr_0^2} \frac{N(n_1, n_2, 2m)}{N(n_1 + 2, n_2, 2m + 2)} \times$$

$$\times \frac{(n_2 - 2m - 1)(n_2 - 2m)}{(n_1 + n_2 - 4m - 1)(n_1 + n_2 - 4m + 1)}; \tag{74}$$

$$\langle n_1 n_2 + 22m + 2|\hat{T}|n_1 n_2 2m\rangle = -\frac{\hbar^2}{4Mr_0^2} \frac{N(n_1, n_2, 2m)}{N(n_1, n_2 + 2, 2m + 2)} \times$$

$$\times \frac{(n_1 - 2m - 1)(n_1 - 2m)}{(n_1 + n_2 - 4m - 1)(n_1 + n_2 - 4m + 1)}. \tag{75}$$

The matrix elements (72—75) are obtained as a result of the raising operators (a^2 and b^2) acting to the basis states $|n_1 n_2 2m\rangle$ (these operators add two to the total number of oscillator quanta). Similarly, the other four matrix elements correspond to the action of lowering operators ($\tilde{a}^2$ and $\tilde{b}^2$).

Transition to the matrix elements

$$\langle n2m\nu|\hat{T}|\tilde{n}2\tilde{m}\nu\rangle \tag{76}$$

is carried out by using the rules formulated in the previous section.

The basis $|n2m\nu\rangle$ of allowed states is, however, not very convenient for the study of continuous spectrum states of three-cluster systems. The classification of basis states optimal for the constructing the wave functions with the asymptotic behaviour characteristic to continuous spectrum is that provided by the basis of hyperspherical harmonics [25,33—36]. The hypermomentum quantum number K is unchanged under the kinetic energy operator, therefore, for the basis functions $|nK\kappa\rangle$ (κ is the additional quantum number of the hyperspherical basis) one gets

$$\hat{T}|nK\kappa\rangle = \langle n + 2K\kappa|\hat{T}|nK\kappa\rangle|n + 2K\kappa\rangle +$$

$$+ \frac{\hbar^2}{4Mr_0^2}(2n + 15)|nK\kappa\rangle + \langle n - 2K\kappa|\hat{T}|nK\kappa\rangle|n - 2K\kappa\rangle. \tag{77}$$

Transition from the basis $|n2m\nu\rangle$ to the basis $|nK\kappa\rangle$ is carried out by the orthogonal transformation

$$|nK\kappa\rangle = \sum_{m,\nu} |n2m\nu\rangle\langle n2m\nu|nK\kappa\rangle. \tag{78}$$

The transition matrix $\langle n2m\nu|nK\kappa\rangle$ should be determined at the final stage of constructing the Shrödinger equation in the representation of the hyperspherical harmonic oscillator

$$\sum_{\tilde{n}\tilde{K}\tilde{\kappa}} \langle nK\kappa|\hat{H} - E|\tilde{n}, \tilde{K}\tilde{\kappa}\rangle C(\tilde{n}, \tilde{K}\tilde{\kappa}) = 0. \tag{79}$$

6. MATRIX ELEMENTS OF THE POTENTIAL ENERGY OPERATOR

For the nucleon-nucleon potential with a Gaussian spatial dependence

$$V(\mathbf{r}_1-\mathbf{r}_2) = V_0 \exp\left\{-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{s^2}\right\} \tag{80}$$

calculations of the matrix elements between the generating functions begin from the evaluation of the integral

$$J = \int \exp\left\{-r_1^2 - r_2^2 + \sqrt{2}\,(\mathbf{R}_1 + \tilde{\mathbf{R}}_1, \mathbf{r}_1) + \sqrt{2}\,(\mathbf{R}_2 + \tilde{\mathbf{R}}_2, \mathbf{r}_2) - \right.$$

$$\left. - \frac{1}{2}(R_1^2 + S_1^2 + R_2^2 + S_2^2) - \frac{2r_0^2}{s^2}\left(\frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}\right)^2\right\} d\mathbf{r}_1 d\mathbf{r}_2. \tag{81}$$

After introducing the new vectors

$$\mathbf{r} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \quad \mathbf{q} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 + \mathbf{r}_2), \quad d\,\mathbf{r}_1 d\,\mathbf{r}_2 = d\,\mathbf{r} d\,\mathbf{q}, \tag{82}$$

taking into account the relations

$$\mathbf{r}_1^2 + \mathbf{r}_2^2 = \mathbf{r}^2 + \mathbf{q}^2,$$

$$\sqrt{2}\,(\mathbf{R}_1 + \tilde{\mathbf{R}}_1, \mathbf{r}_1) + \sqrt{2}\,(\mathbf{R}_2 + \tilde{\mathbf{R}}_2 \mathbf{r}_2) =$$

$$= (\mathbf{R}_1 + \tilde{\mathbf{R}}_1 + \mathbf{R}_2 + \tilde{\mathbf{R}}_2, \mathbf{q}) + (\mathbf{R}_1 + \tilde{\mathbf{R}}_1 - \mathbf{R}_2 - \tilde{\mathbf{R}}_2, \mathbf{r}),$$

the integrand is factorized and can be easily calculated

$$J = \pi^3 z^{3/2} \exp\left\{\frac{1}{2}(\mathbf{R}_1 + \mathbf{R}_2, \tilde{\mathbf{R}}_1 + \tilde{\mathbf{R}}_2) + \frac{1}{2} z(\mathbf{R}_1 - \mathbf{R}_2, \tilde{\mathbf{R}}_1 - \tilde{\mathbf{R}}_2) + \right.$$

$$\left. + \frac{1}{4}(z-1)\,[(\mathbf{R}_1 - \mathbf{R}_2)^2 + (\tilde{\mathbf{R}}_1 - \tilde{\mathbf{R}}_2)^2]\right\}, \tag{83}$$

where $z = \left(1 + \frac{2r_0^2}{s^2}\right)^{-1}$. (Below we shall omit the factor π^3 as we have been doing for the overlap integral).

The generating matrix element can be obtained easily after some transformation of the overlap integral (19). Considering first only spatial dependence of the nucleon-nucleon potential (80), we discuss the transformation of the first term in (19). For the matrix elements corresponding to the interaction of nucleons belonging to the alpha-cluster, this transformation is

reduced to the product of the exponential, the intensity V_0 of the Gaussian potential, the number of nucleon pairs and the factor $z^{3/2}$. Denoting the unantisymmetrized generating function before the separation of the center-of-mass motion as Φ_0 ($\Phi = \Phi_0$), for six nucleon pairs of the alpha-particle we get

$$\langle \Phi_0|\hat{U}_{11}|\tilde{\Phi}_0\rangle = 6z^{3/2}V_0 \exp\{4\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_2\tilde{\mathbf{R}}_2 + \mathbf{R}_3\tilde{\mathbf{R}}_3\} =$$

$$= 6z^{3/2}V_0 \exp\{\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM} + \mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}\}. \tag{84}$$

The calculation of matrix elements corresponding to the interaction of nucleons belonging to different clasters is more complicated. Again, it is necessary to determine how many nucleon pairs are there and to take into account the integral J. Then in the case of interaction of non-alpha neutrons (corresponding to generating vectors $\mathbf{R}_2$ and $\mathbf{R}_3$ ($\tilde{\mathbf{R}}_2$ and $\tilde{\mathbf{R}}_3$))

$$\langle \Phi_0|\hat{U}_{23}|\tilde{\Phi}_0\rangle = z^{3/2}V_0 \exp\{4\mathbf{R}_1\tilde{\mathbf{R}}_1\} \times$$

$$\times \exp\left\{\frac{1}{2}(\mathbf{R}_2 + \mathbf{R}_3, \tilde{\mathbf{R}}_2 + \tilde{\mathbf{R}}_3) + \frac{1}{2}z(\mathbf{R}_2 - \mathbf{R}_3, \tilde{\mathbf{R}}_2 - \tilde{\mathbf{R}}_3) + \right.$$

$$\left. + \frac{1}{4}(z-1)[(\mathbf{R}_2 - \mathbf{R}_3)^2 + (\tilde{\mathbf{R}}_2 - \tilde{\mathbf{R}}_3)^2]\right\} =$$

$$= z^{3/2}V_0 \exp\left\{\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM} + \mathbf{a}\tilde{\mathbf{a}} + z\,\mathbf{b}\tilde{\mathbf{b}} + \frac{1}{2}(z-1)(b^2 + \tilde{b}^2)\right\}. \tag{85}$$

The matrix element of interaction of alpha-cluster nucleons and the neutron differs only in minor details

$$\langle \Phi_0|\hat{U}_{12}|\tilde{\Phi}_0\rangle = 4z^{3/2}V_0 \exp\{3\mathbf{R}_1\tilde{\mathbf{R}}_1 + \mathbf{R}_3\tilde{\mathbf{R}}_3\} \times$$

$$\times \exp\left\{\frac{1}{2}(\mathbf{R}_1 + \mathbf{R}_2, \tilde{\mathbf{R}}_1 + \tilde{\mathbf{R}}_2) + \frac{1}{2}z(\mathbf{R}_1 - \mathbf{R}_2, \tilde{\mathbf{R}}_1 - \tilde{\mathbf{R}}_2) + \right.$$

$$\left. + \frac{1}{4}(z-1)[(\mathbf{R}_1 - \mathbf{R}_2)^2 + (\tilde{\mathbf{R}}_1 - \tilde{\mathbf{R}}_2)^2]\right\} =$$

$$= 4z^{3/2}V_0 \exp\left\{\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM} + \mathbf{a}'\tilde{\mathbf{a}}' + z\,\mathbf{b}'\tilde{\mathbf{b}}' + \frac{1}{2}(z-1)(b'^2 + \tilde{b}'^2)\right\}, \tag{86}$$

where in order to simplify both notation and subsequent calculation the Jacobi vectors corresponding to another choice of Jacobi tree are introduced

$$\mathbf{a}' = \sqrt{\frac{5}{6}}\left(\mathbf{R}_3 - \frac{4\mathbf{R}_1 + \mathbf{R}_2}{5}\right), \quad \mathbf{b}' = \frac{2}{\sqrt{5}}(\mathbf{R}_1 - \mathbf{R}_2). \tag{87}$$

These Jacobi vectors are obtained from the old ones by an orthogonal transformation

$$\mathbf{a}' = -\sqrt{\frac{2}{5}}\,\mathbf{a} - \sqrt{\frac{3}{5}}\,\mathbf{b}, \quad \mathbf{b}' = \sqrt{\frac{3}{5}}\,\mathbf{a} - \sqrt{\frac{2}{5}}\,\mathbf{b}. \tag{88}$$

The third set of Jacobi vectors

$$\mathbf{a}'' = \sqrt{\frac{5}{6}}\left(\mathbf{R}_2 - \frac{4\mathbf{R}_1 + \mathbf{R}_3}{5}\right), \quad \mathbf{b}'' = \frac{2}{\sqrt{5}}(\mathbf{R}_1 - \mathbf{R}_3); \tag{89}$$

$$\mathbf{a}'' = -\sqrt{\frac{2}{5}}\,\mathbf{a} + \sqrt{\frac{3}{5}}\,\mathbf{b}, \quad \mathbf{b}'' = \sqrt{\frac{3}{5}}\,\mathbf{a} + \sqrt{\frac{2}{5}}\,\mathbf{b} \tag{90}$$

is introduced for the calculation of the matrix elements corresponding to the interaction of alpha-cluster nucleons and the neutron $\mathbf{R}_3$

$$\langle \Phi_0 | \hat{U}_{12} | \tilde{\Phi}_0 \rangle =$$

$$= 4z^{3/2} V_0 \exp\left\{\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM} + \mathbf{a}''\tilde{\mathbf{a}}'' + \left(\frac{3}{8} + \frac{5}{8}z\right)\mathbf{b}''\tilde{\mathbf{b}}'' + \frac{5}{16}(z-1)(b''^2 + \tilde{b}''^2)\right\}. \tag{91}$$

(The tilded vectors $\tilde{\mathbf{a}}'$, $\tilde{\mathbf{b}}'$, $\tilde{\mathbf{a}}''$, $\tilde{\mathbf{b}}''$ are expressed similarly in terms of $\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$).

Functions Φ_0 and $\tilde{\Phi}_0$ are the products of single-particle orbitals, so the matrix elements between them are calculated easily. Since

$$\langle \Phi_0 | \hat{U}_{kl} | \tilde{\Phi}_0 \rangle = \exp\{\mathbf{R}_{CM}\tilde{\mathbf{R}}_{CM}\} \langle \varphi_0 | \hat{U}_{kl} | \tilde{\varphi}_0 \rangle, \tag{92}$$

the expression for the matrix elements $\langle \varphi_0 | \hat{U}_{kl} | \tilde{\varphi}_0 \rangle$ follows immediately from the Eqs.(84—86,91).

The generalization to the case of central exchange forces with components $V_{2S+1,2T+1}$ (S and T are the total spin and isospin of an interacting nucleon pair) is shown separately for each of the four terms in Eqs.(84—86,91).

$$\langle \varphi_0 | \hat{U}_{11} | \tilde{\varphi}_0 \rangle = z^{3/2}(3V_{31} + 3V_{13}) \exp\{\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}\}; \tag{93}$$

$$\langle \varphi_0 | \hat{U}_{23} | \tilde{\varphi}_0 \rangle = z^{3/2}\frac{1}{2}(V_{33} + V_{13}) \exp\{\mathbf{a}\tilde{\mathbf{a}} + B_{23}\mathbf{b}\tilde{\mathbf{b}} + D_{23}(\mathbf{b}^2 + \tilde{\mathbf{b}}^2)\} -$$

$$- z^{3/2}\frac{1}{2}(V_{33} + V_{13}) \exp\{\mathbf{a}\tilde{\mathbf{a}} + B'_{23}\mathbf{b}\tilde{\mathbf{b}} + D_{23}(\mathbf{b}^2 + \tilde{\mathbf{b}}^2)\}; \tag{94}$$

$$\langle \varphi_0 | \hat{U}_{12} | \tilde{\varphi}_0 \rangle =$$

$$= z^{3/2} \frac{1}{4}(9V_{33} + 3V_{31} + 3V_{13} + V_{11}) \exp\{\mathbf{a}'\tilde{\mathbf{a}}' + B_{12}\mathbf{b}'\tilde{\mathbf{b}}' + D_{12}(\mathbf{b}'^2 + \tilde{\mathbf{b}}'^2)\} -$$

$$- z^{3/2} \frac{1}{4}(5V_{33} + 3V_{31} + 3V_{13} + V_{11}) \exp\{\mathbf{a}'\tilde{\mathbf{a}}' + B_{12}\mathbf{b}'\tilde{\mathbf{b}}' + D_{12}(\mathbf{b}'^2 + \tilde{\mathbf{b}}'^2)\}; \quad (95)$$

$$\langle \varphi_0 | \hat{U}_{13} | \tilde{\varphi}_0 \rangle =$$

$$= z^{3/2} \frac{1}{4}(9V_{33} + 3V_{31} + 3V_{13} + V_{11}) \exp\{\mathbf{a}''\tilde{\mathbf{a}}'' + B_{13}\mathbf{b}''\tilde{\mathbf{b}}'' + D_{13}(\mathbf{b}''^2 + \tilde{\mathbf{b}}''^2)\} -$$

$$- z^{3/2} \frac{1}{4}(5V_{33} + 3V_{31} + 3V_{13} + V_{11}) \exp\{\mathbf{a}''\tilde{\mathbf{a}}'' + B_{13}\mathbf{b}''\tilde{\mathbf{b}}'' + D_{13}(\mathbf{b}''^2 + \tilde{\mathbf{b}}''^2)\}. \quad (96)$$

Here

$$B_{23} = z, \quad B'_{23} = -z, \quad D_{23} = (z-1)/2, \quad (97)$$

$$B_{12} = (3+5z)/8, \quad B'_{12} = (3-5z)/8, \quad D_{12} = 5(z-1)/16, \quad (98)$$

$$B_{13} = B_{12}, \quad B'_{13} = B'_{12}, \quad D_{13} = D_{12}. \quad (99)$$

The generating matrix element of the potential energy operator of a six-nucleon system $\hat{U}$ between the function φ_0 and $\tilde{\varphi}_0$ is a sum of the four matrix elements given above

$$\langle \varphi_0 | \hat{U} | \tilde{\varphi}_0 \rangle = \langle \varphi_0 | \hat{U}_{11} | \tilde{\varphi}_0 \rangle + \langle \varphi_0 | \hat{U}_{23} | \tilde{\varphi}_0 \rangle +$$

$$+ \langle \varphi_0 | \hat{U}_{12} | \tilde{\varphi}_0 \rangle + \langle \varphi_0 | \hat{U}_{13} | \tilde{\varphi}_0 \rangle . \quad (100)$$

Now we turn to the final stage of our calculations: the projection of the generating matrix elements $\langle \varphi_0 | \hat{U} | \tilde{\varphi}_0 \rangle$ onto the basis states $\Psi(n_1 n_2 2m)$ and $\tilde{\Psi}(\tilde{n}_1 \tilde{n}_2 2\tilde{m})$ and the subsequent extraction of the matrix elements $\langle n_1 n_2 2m | \hat{U} | \tilde{n}_1 \tilde{n}_2 2\tilde{m} \rangle$ of the three-cluster potential energy operator.

We start from writing Eq.(36) as

$$\exp\{\mathbf{a}\tilde{\mathbf{a}} + \mathbf{b}\tilde{\mathbf{b}}\} = \sum_{n_1 n_2 m} \Psi(n_1, n_2, 2m)\, \tilde{\Psi}(\tilde{n}_1, \tilde{n}_2, 2\tilde{m}) + \ldots, \quad (101)$$

where terms with $L \neq 0$ are omitted and $\Psi(n_1, n_2, 2m)$ are given by Eq.(40), $\tilde{\Psi}(\tilde{n}_1, \tilde{n}_2, 2\tilde{m})$ depends on tilded generating parameters **a** and **b**.

The exponential

$$\exp\{\mathbf{a}\tilde{\mathbf{a}} + B\,\mathbf{b}\tilde{\mathbf{b}} + D(\mathbf{b}^2 + \tilde{\mathbf{b}}^2)\} \tag{102}$$

is the basic block for all matrix elements of the potential energy operator. After being projected onto the basis states $\psi(n_1 n_2 2m)$ and $\tilde{\psi}(\tilde{n}_1\tilde{n}_2 2\tilde{m})$, the expansion of this block in powers of $\mathbf{a}$, $\mathbf{b}$, $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$ takes the form

$$\exp\{\mathbf{a}\tilde{\mathbf{a}} + B\,\mathbf{b}\tilde{\mathbf{b}} + D(\mathbf{b}^2 + \tilde{\mathbf{b}}^2)\} =$$

$$= \sum_{n_1 n_2 m} \sum_{\tilde{n}_1\tilde{n}_2\tilde{m}} \Psi(n_1,n_2,2m)\langle n_1 n_2 2m|u_{23}(B,D)|(\tilde{n}_1\tilde{n}_2 2\tilde{m})\rangle\, \tilde{\Psi}(\tilde{n}_1,\tilde{n}_2,2\tilde{m}) + \ldots \tag{103}$$

and our task is to find the matrix elements

$$\langle n_1 n_2 2m|u_{23}(B,D)|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle \tag{104}$$

which give us expressions for matrix elements of all terms of the nucleon-nucleon interaction potential energy operator for different clusters.

Notice first a simple consequence of the expansion (101) applied to the block (102)

$$\exp\{\mathbf{a}\tilde{\mathbf{a}} + B\,\mathbf{b}\tilde{\mathbf{b}} + D(\mathbf{b}^2 + \tilde{\mathbf{b}}^2)\} =$$

$$= \sum_{n=0}^{\infty} \sum_{n_1+n_2=2n} \sum_{m=0}^{\min(n_1/2,n_2/2)} e^{Db^2} N(n_1,n_2,2m)\, a^{n_1-2m} b^{n_2-2m} [\mathbf{ab}]^{2m} \Phi(n_1,n_2,2m;t) \times$$

$$\times e^{D\tilde{b}^2} B^{n_2} N(n_1,n_2,2m)\, \tilde{a}^{n_1-2m}\, \tilde{b}^{n_2-2m} [\tilde{\mathbf{a}}\tilde{\mathbf{b}}]^{2m} \Phi(n_1,n_2,2m;\tilde{t}) + \ldots . \tag{105}$$

The constant B appears here in power n_2 in each term of the expansion as well as moduli of vectors $\mathbf{b}$ and $\tilde{\mathbf{b}}$. The next step is the expansion of $\exp\{D\,\mathbf{b}^2\}$ and $\exp\{D\,\tilde{\mathbf{b}}^2\}$ in powers of $\mathbf{b}^2$ and $\tilde{\mathbf{b}}^2$, respectively. Then the factors $\mathbf{b}^{2k}$ and $\tilde{\mathbf{b}}^{2k}$ appear before the basis functions. The products

$$\mathbf{b}^{2k}\Psi((n_1,n_2,2m) \quad \text{and} \quad \tilde{\mathbf{b}}^{2k}\Psi((n_1,n_2,2m)$$

can be presented as a superposition of the basis functions

$$\Psi(n_1, n_2 + 2k, 2m + 2l) \quad \text{and} \quad \tilde{\Psi}(n_1, n_2 + 2k, 2m + 2l)$$

with different l

$$l = 0,1,\ldots, \min\left(\frac{n_1}{2} - m, \frac{n_2}{2} + k - m\right).$$

This fact can be proved using the Gauss recurrency relation for the hypergeometric functions [21]

$$F(\alpha, \beta, \gamma; z) = F(\alpha, \beta - 1, \gamma - 1; z) + \frac{\alpha(\gamma - \beta)}{\gamma(\gamma - 1)} zF(\alpha + 1, \beta, \gamma + 1; z) \quad (106)$$

from which follows the identity

$$b^{2k} a^{n_1 - 2m} b^{n_2 - 2m} [\mathbf{ab}]^{2m} \Phi(n_1, n_2, 2m; t) =$$

$$= \sum_{l=0}^{\bar{n}} \frac{k!}{l!(k-l)!} \frac{\Gamma(n_1/2 - m + 1)\,\Gamma(n_1/2 - m + 1/2)}{\Gamma(n_1/2 - m - l + 1)\,\Gamma(n_1/2 - m - l + 1/2)} \times$$

$$\times \frac{\Gamma(n/2 + k - 2m - 2l - 3/2)\,\Gamma(n/2 - 2m - l + 1/2)}{\Gamma(n/2 + k - 2m - l + 3/2)\,\Gamma(n/2 - 2m + 1/2)} \times$$

$$\times a^{n_1 - 2m - 2l} b^{n_2 + 2k - 2m - 2l} [\mathbf{ab}]^{2m+2l} \Phi(n_1, n_2 + 2k, 2m + 2l; t) =$$

$$= \sum_{l=0}^{\bar{n}} k!\, A(n_1, n_2, m; n_2 + 2k, m + l)\, a^{n_1 - 2m - 2l} b^{n_2 + 2k - 2m - 2l} \times$$

$$\times [\mathbf{ab}]^{2m+2l} \Phi(n_1, n_2 + 2k, 2m + 2l; t), \quad (107)$$

where $\bar{n} = \min\left(\frac{n_1}{2} - m, \frac{n_2}{2} + k - m\right)$, $n = n_1 + n_2$ and, by definition,

$$A(n_1, \bar{n}_2, \bar{m}; n_2, m) = \frac{2^{-2(m-\bar{m})}}{(m - \bar{m})!\left(\frac{n_2 - \bar{n}_2}{2} - m + \bar{m}\right)!} \times$$

$$\times \frac{(n_1 - 2\bar{m})!}{(n_1 - 2m)!} \frac{\Gamma\left(\frac{n_1 + \bar{n}_2}{2} - m - \bar{m} + \frac{1}{2}\right)\Gamma\left(\frac{n_1 + n_2}{2} - 2m + \frac{3}{2}\right)}{\Gamma\left(\frac{n_1 + \bar{n}_2}{2} - 2\bar{m} + \frac{1}{2}\right)\Gamma\left(\frac{n_1 + n_2}{2} - m - \bar{m} + \frac{3}{2}\right)}. \quad (108)$$

Finally, we can now write the matrix element (104). It has a form of a two-fold sum and is expressed through the coefficients $A(n_1,\bar{n}_2,\bar{m};n_2,m)$ and the normalizing factors $N(n_1,n_2,2m)$

$$\langle n_1 n_2 2m| u_{23}(B,D)| \tilde{n}_1 \tilde{n}_2 2\tilde{m} \rangle =$$

$$= \sum_{\tilde{n}_2=0}^{\min(n_2,\tilde{n}_2)} \sum_{\tilde{m}=0}^{\min(m,\tilde{m})} \frac{N^2(n_1, \bar{n}_2, 2\bar{m})}{N(n_1,n_2,2m)N(n_1,\tilde{n}_2,2\tilde{m})} B^{\bar{n}_2} D^{(n_2+\tilde{n}_2)/2-\bar{n}_2} \times$$

$$\times A(n_1, \bar{n}_2, \bar{m}; n_2, m)A(n_1, \bar{n}_2, \bar{m}; \tilde{n}_2, \tilde{m})\, \delta_{n_1 \tilde{n}_1}. \qquad (109)$$

Two order matrix elements similar to (109) can be extracted from the same block (102) if Jacobi vectors **a**, **b**, $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$ are substituted first by **a′**, **b′**, $\tilde{\mathbf{a}}'$, $\tilde{\mathbf{b}}'$ (See Eq.(88) and then by **a″**, **b″**, $\tilde{\mathbf{a}}''$, $\tilde{\mathbf{b}}''$ (See Eq.(90)) and the following definitions are made

$$\exp\{\mathbf{a}'\tilde{\mathbf{a}}' + B\tilde{\mathbf{b}}'\tilde{\mathbf{b}}' + D(\mathbf{b}'^2 + \tilde{\mathbf{b}}'^2)\} =$$

$$= \sum_{n_1 n_2 m} \sum_{\tilde{n}_1 \tilde{n}_2 \tilde{m}} \psi(n_1 n_2 2m) \langle n_1 n_2 2m|u'_{12}(B,D)|\tilde{n}_1 \tilde{n}_2 2\bar{m}\rangle \tilde{\Psi}(\tilde{n}_1, \tilde{n}_2, 2\tilde{m}) + \ldots, \qquad (110)$$

$$\exp\{\mathbf{a}''\tilde{\mathbf{a}}'' + B\tilde{\mathbf{b}}''\tilde{\mathbf{b}}'' + D(\mathbf{b}''^2 + \tilde{\mathbf{b}}''^2)\} =$$

$$= \sum_{n_1 n_2 m} \sum_{\tilde{n}_1 \tilde{n}_2 \tilde{m}} \psi(n_1 n_2 2m) \langle n_1 n_2 2m|u'_{13}(B,D)|\tilde{n}_1 \tilde{n}_2 2\bar{m}\rangle \tilde{\Psi}(\tilde{n}_1, \tilde{n}_2, 2\tilde{m}) + \ldots . \qquad (111)$$

To return to the basis of initial Jacobi vectors **a**, **b**; $\tilde{\mathbf{a}}$, $\tilde{\mathbf{b}}$, the orthogonal transformation of the matrices

$$\langle n_1 n_2 2m|u'_{12}(B,D)|\tilde{n}_1 \tilde{n}_2 2\tilde{m}\rangle \quad \text{and} \quad \langle n_1 n_2 2m|u'_{13}(B,D)|\tilde{n}_1 \tilde{n}_2 2\tilde{m}\rangle$$

should be made. The matrices of such transformation

$$\| \langle n_1 n_2 2m|\alpha'|n'_1 n'_2 2m\rangle \| \quad \text{and} \quad \| \langle n_1 n_2 2m|\alpha''|n'_1 n'_2 2m\rangle \|$$

are given by Eq.(44), where $\| \alpha' \|$ and $\| \alpha'' \|$ are the matrices transforming **a**, **b** into **a′**, **b′** (Eq.88) and **a**, **b** into **a″**, **b″**(Eq.90), respectively.

After the orthogonal transformation we get new matrices

$$\langle n_1 n_2 2m|u_{12}(B,D)|\tilde{n}_1 \tilde{n}_2 2\tilde{m}\rangle =$$

$$= \sum_{n'_1+n'_2=n_1+n_2} \sum_{\tilde{n}'_1+\tilde{n}'_2=\tilde{n}_1+\tilde{n}_2} \langle n_1 n_2 2m|\alpha'|n'_1 n'_2\ 2m\rangle \times$$

$$\times \langle n'_1 n'_2\ 2m|u'_{12}(B,D)|\tilde{n}_1 \tilde{n}_2 2\tilde{m}\rangle \langle \tilde{n}_1 \tilde{n}_2 2\tilde{m}|\alpha'|\tilde{n}'_1 \tilde{n}'_2\ 2\tilde{m}\rangle, \qquad (112)$$

$$\langle n_1 n_2 2m|u_{12}(B,D)|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle =$$

$$= \sum_{n_1''+n_2''=n_1+n_2} \sum_{\tilde{n}_1''+\tilde{n}_2''=\tilde{n}_1+\tilde{n}_2} \langle n_1 n_2 2m|\alpha''|n_1'' n_2'' 2m\rangle \times$$

$$\times \langle n_1'' n_2'' 2m|u_{12}'(B,D)|\tilde{n}_1''\tilde{n}_2'' 2\tilde{m}\rangle \langle \tilde{n}_1\tilde{n}_2 2\tilde{m}|\alpha''|\tilde{n}_1''\tilde{n}_2'' 2\tilde{m}\rangle. \quad (113)$$

Now, instead of the generating matrix elements (Eqs.(94)—(96)) we have the corresponding matrix elements between the basis functions

$$\langle n_1 n_2 2m|\hat{U}_{23}|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle =$$

$$= z^{3/2}\left\{\frac{1}{2}(V_{31}+V_{13})\langle n_1 n_2 2m|u_{23}(B_{23},D_{23})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle -\right.$$

$$\left. -\frac{1}{2}(V_{31}+V_{13}+V_{11})\langle n_1 n_2 2m|u_{23}(B_{23}',D_{23})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle\right\}; \quad (114)$$

$$\langle n_1 n_2 2m|\hat{U}_{12}|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle =$$

$$= z^{3/2}\left\{\frac{1}{4}(9V_{33}+3V_{31}+3V_{13}+V_{11})\langle n_1 n_2 2m|u_{12}(B_{12},D_{12})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle -\right.$$

$$\left. -\frac{1}{4}(5V_{33}+3V_{31}+3V_{13}+V_{11})\langle n_1 n_2 2m|u_{12}(B_{12}',D_{12})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle\right\}; \quad (115)$$

$$\langle n_1 n_2 2m|\hat{U}_{13}|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle =$$

$$= z^{3/2}\left\{\frac{1}{4}(9V_{33}+3V_{31}+3V_{13}+V_{11})\langle n_1 n_2 2m|u_{13}(B_{13},D_{13})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle -\right.$$

$$\left. -\frac{1}{4}(5V_{33}+3V_{31}+3V_{13}+V_{11})\langle n_1 n_2 2m|u_{13}(B_{13}',D_{13})|\tilde{n}_1\tilde{n}_2 2\tilde{m}\rangle\right\}. \quad (116)$$

With this the derivation of analytic formulae necessary for the construction of the Schrödinger equation is completed.

7. CONCLUSION

Summarizing the investigation we state the following basic results.

In the Fock—Bargmann space, the harmonic oscillator basis for a three-cluster system with zero orbital angular momentum is constructed using the

generating wave packets technique. The basis functions have a form of homogeneous polynomials in two Jacobi generator vectors and can be expressed in terms of the hypergeometric functions ${}_2F_1$. Their quantum numbers are the $SU(3)$ symmetry and the degree of homogeneity with respect to each Jacobi vector.

The rule of basis function transformation induced by the nucleon transpositions is established. Such linear combinations of basis states are found that do not vanish after the antisymmetrization (the allowed states).

The matrix elements of kinetic and potential energy operators between the generating wave packets are obtained, their projection onto the basis states is performed. The explicit expressions for the Hamiltonian matrix elements in the basis of allowed states and the Schrödinger equation for a three-cluster system in the harmonic oscillator representation are obtained. During this procedure the Pauli principle is taken into account accurately.

The research described in this publication was supported, in part, by the Ukrainian State Committee for Science and Technology, as well as by a Soros Foundation Grant awarded by the American Physical Society.

REFERENCES

1. **Wildermuth K., Tang Y.C.** — A Unified Theory of the Nucleus, Vieweg, Braunschweig, 1977.
2. **Tang Y.C., Le Mere M., Thompson D.K.** — Phys.Rep., 1978, 47, p.1.
3. **Microscopic Method for the Interactions Between Complex Nuclei.** — Prog. Theor. Phys. Suppl., 1977, 62.
4. **Comphrehensive Study of Structure of Light Nuclei.** — Prog. Theor. Phys. Suppl., 1977, 62.
5. **Filippov G.F., Vasilevsky V.S., Chopovsky L.L.** — Sov. J. Part. Nucl., 1984, 15, p.1338.
6. **Filippov G.F., Vasilevsky V.S., Chopovsky L.L.** — Ibid, 1985, 16, p.349.
7. **Filippov G.F.** — Riv. Nuovo Cim., 1989, No.9, p.1.
8. **Krasnopol'sky V.M., Kukulin V.M., Sazonov P.V., Voronchev V.T.** — Nucl. Phys., 1986, A453, p.365.
9. **Danilin B.V., Zhukov M.V., Korsheninnikov A.A. et al.** — Sov. J. Nucl. Phys., 1989, 49, pp.351, 359.
10. **Danilin B.V., Zhukov M.V., Korsheninnikov A.A., Chulkov L.V.** — Sov. J. Nucl. Phys., 1991, 53, p.71.
11. **Zhukov M.V., Danilin B.V., Fedorov D.V. et al.** — Phys. Rep., 1993, 231, p.151.
12. **Mikhelashvili T.Ya., Smirnov Yu.F., Shirokov A.M.** — Sov. J. Nucl. Phys., 1988, 48, p.969.
13. **Horiuchi H.** — Progr. Theor. Phys., 1977, 58, p.204.
14. **Kato K., Fukatsu K., Tanaka H.** — Progr. Theor. Phys., 1988, 80, p.663.
15. **Kato K., Filippov G.F.** — Progr. Theor. Phys., 1991, 85, p.257.

16. **Perelomov A.M.** — Generalized Coherent States and Their Applications, Springer, Berlin, 1987.
17. **Hecht K.T.** — Preprint MPI H-1987-V19, Heidelberg, 1987.
18. **Fock V.A.** — Zs. f. Phys., 1928, 49, p.339.
19. **Bargmann V.** — Comm. Pure Appl. Math., 1961, 14, p.187.
20. **Faddeev L.D., Yakubovsky O.F.** — Lectures on Quantum Mechanics for Students in Mathematics, Leningrad University, 1980 (in Russian).
21. **Abramovitz M., Stegun I.A., eds., Handbook of Mathematical Functions, Washington, NBS, 1964.**
22. **Brink D.M.** — The Alpha-Particle Model of Light Nuclei, in: International School of Physics «Enrico Fermi», Course 37, Varenna, Italy, N.Y.-London, Acad. Press, 1965.
23. **Nesterov A.V.** — Sov. J. Nucl. Phys., 1993, 56, p.35.
24. **Löwdin P.O.** — Phys. Rev., 1955, 97, p.1474.
25. **Zickendraht W.** — Ann. Phys., 1965, 35, p.18.
26. **Filippov G.F.** — Preprint ITP-67-49P, Kiev, 1967 (in Russian).
27. **Nyiri J., Smorodinsky Ya.A.** — Sov. J. Nucl. Phys., 1969, 9, p.882 (in Russian).
28. **Elliot J.P.** — Proc. Roy. Soc., 1958, 245, p.128; Ibid, 1958, 245, p.562.
29. **Filippov G.F., Vasilevsky V.S., Nesterov A.V., Rybkin I.Yu.** — Preprint ITP-92-33P, Kiev, 1992 (in Russian).
30. **Filippov G.F., Dotsenko I.S.** — Preprint ITP-93-4P, Kiev, 1993 (in Russian).
31. **Filippov G.F., Dotsenko I.S.** — Preprint ITP-93-27P, Kiev, 1993 (in Russian).
32. **Filippov G.F.. Korennov S.V.** — Preprint ITP-93-38E, Kiev, 1993.
33. **Simonov Yu.A.** — Sov. J. Nucl. Phys., 1966, 3, p.630 (in Russian).
34. **Badalyan A.M., Simonov Yu.A.** — Ibid, 1966,3, p.1032 (in Russian).
35. **Efros V.D.** — Ibid, 1972, 15, p.226 (in Russian).
36. **Dzhibuti R.I., Krupennikova N.B.** — The Hyperspherical Function Method in the Few-Body Quantum Mechanics, Metsniereba, Tbilisi, 1984 (in Russian).