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THE FAMILY PROBLEM IN THE 4D SUPERSTRING GRAND UNIFIED THEORIES

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The one of the main points of the investigations in high energy physics is to study the next chain: a law of the quark and lepton mass spectra $\rightarrow$ the puzzles of the quark and lepton family mixing $\rightarrow$ a possible new family dynamics.

The new family symmetry dynamics might be connected to the existence of some exotic gauge or matter fields or something else. For this, it will be better to study the possibilities of the appearance of this gauge symmetry in the framework of the Grand Unified String Theories. In the framework of the four-dimensional heterotic superstring with free fermions we investigate the rank eight Grand Unified String Theories (GUST) which contain the $SU(3)_H$-gauge family symmetry. We explicitly construct GUST with gauge symmetry $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H$ and $G = SO(10) \times (SU(3) \times U(1))_H \subset SO(16)$ or $E(6) \times SU(3)_H \subset E(8)$ in free complex fermion formulation. As the GUSTs originating from Kac-Moody algebras (KMA) contain only low-dimensional representations, it is usually difficult to break the gauge symmetry. We solve this problem by taking for the observable gauge symmetry the diagonal subgroup G^{sym} of the rank 16 group $G \times G \subset SO(16) \times SO(16)$ or $(E(6) \times SU(3)_H)^2 \subset E(8) \times E(8)$. We discuss the possible fermion matter and Higgs sectors in these models. In these GUST, there has to exist «superweak» light chiral matter $(m_H^f < M_W)$. The understanding of quark and lepton mass spectra and family mixing leaves a possibility for the existence of an unusually low mass breaking scale of the $SU(3)_H$ family gauge symmetry (some TeV).

Одно из основных направлений исследований в физике высоких энергий — это изучение следующей цепочки взаимосвязей: закономерности спектра масс кварков и лептонов $\rightarrow$ загадка смешивания кварк-лептонных поколений $\rightarrow$ возможная новая динамика поколений.

Новая динамика симметрии поколений может быть связана с существованием экзотических полей материи и калибровочных полей. Поэтому полезно изучить возмож-

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ность возникновения такой калибровочной симметрии в Струнных Теориях Великого Объединения (СТВО). В рамках 4-мерной гетеротической суперструны со свободными фермионами исследуются СТВО ранга 8, содержащие $SU(3)_H$ калибровочную симметрию поколений. Мы явно конструируем СТВО с калибровочной симметрией $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H$ и $G = SO(10) \times (SU(3) \times U(1))_H \subset SO(16)$ или $E(6) \times SU(3)_H \subset E(8)$ в формулировке свободных комплексных фермионов. Так как СТВО, основанные на Кац-Муди алгебрах (КМА), содержат только представления низких размерностей, то обычно имеется трудность с нарушением калибровочной симметрии. Мы решаем эту проблему выбирая в качестве наблюдаемой калибровочной симметрии диагональную подгруппу $G^{\rm sym}$ группы ранга 16 $G \times G \subset SO(16) \times SO(16)$ или $(E(6) \times SU(3)_H)^2 \subset E(8) \times E(8)$. Мы обсуждаем допустимые сектора фермионов материи и Хиггса в этих моделях. В таких СТВО возникает «суперслабая» легкая киральная материя $(m_H^f < M_W)$. Анализ кварк-лептонного массового спектра и смешивания поколений оставляет возможность существования необычно низкого масштаба нарушения $SU(3)_H$ калибровочной симметрии поколений (несколько ТэВ).

1. THEORETICAL TRENDS BEYOND THE STANDARD MODEL

1.1. The Family Mixing State in SM and Quark and Lepton Mass Origin. There are no experimental indications which would impel one to go beyond the framework of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (SM) with three generations of quarks and leptons. None of the up-to-date experiments contradicts, within the limits of accuracy, the validity of the SM predictions for low energy phenomena. The fermion mass origin and generation mixing, *CP*-violation problems are among most exciting theoretical puzzles in SM.

One has ten parameters in the quark sector of the SM with three generations: six quark masses, three mixing angles and the Kobayashi — Maskawa (KM) *CP*-violation phase $(0 < \delta^{KM} < \pi)$. The CKM (Cabibbo — Kobayashi — Maskawa) matrix in Wolfenstein parametrization is determined by the four parameters — Cabibbo angle $\lambda \approx 0.22$, A, ρ and η:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - 1/2\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - 1/2\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (1)$$

In the complex plane the point (ρ, η) is a vertex of the unitarity triangle and describes the *CP*-violation in SM. The unitarity triangle is constructed from the following unitarity condition of $V_{\rm CKM}$: $V_{ub}^* + V_{td} \approx A\lambda^3$.

Recently, the interest in the *CP*-violation problem was excited again due to the data on the search for the direct *CP*-violation effects in neutral *K*-mesons [1,2]:

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (7.4 \pm 6)\cdot 10^{-4}, \qquad (2)$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (23 \pm 7)\cdot 10^{-4}. \qquad (3)$$

The major contribution to the *CP*-violation parameters ε_K and ε'_K (K^0-decays), as well as to the $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d = \frac{\Delta m_{(B_d)}}{\Gamma_{(B_d)}}$ is due to the large *t*-quark mass contribution. The same statement holds also for some amplitudes of *K*- and *B*-meson rare decays. The CDF collaboration gives the following region for the top quark mass: $m_t = 174 \pm 25$ GeV [3]. The complete fit which is based on the low energy data as well as the latest LEP and SLC data and comparing with the mass indicated by CDF measurements gives $m_t = 162 \pm 9$ GeV [4].

The main drawbacks of SM now are going from our non-understanding the generation problem, their mixing and hierarchy of quark and lepton mass spectra. For example, for quark masses $\mu \approx 1$ GeV we can get the following approximate relations [5]:

$$m_{i_k} \approx (q_H^u)^{2k} m_0, \quad k = 0,1,2; \quad i_0 = u, \quad i_1 = c, \quad i_2 = t,$$

$$m_{i_k} \approx (q_H^d)^{2k} m_0, \quad k = 0,1,2; \quad i_0 = d, \quad i_1 = s, \quad i_2 = b, \qquad (4)$$

where $q_H^u \approx (q_H^d)^2$, $q_H^d \approx 4 - 5 \approx 1/\lambda$ and $\lambda \approx \sin\theta_C$.

Here we used the conventional ratios of the «running» quark masses [6]

$$m_d/m_s = 0.051 \pm 0.004, \quad m_u/m_c = 0.0038 \pm 0.0012,$$

$$m_s/m_b = 0.033 \pm 0.011, \quad m_c(\mu = 1\ \mathrm{GeV}) = (1.35 \pm 0.05)\ \mathrm{GeV}, \qquad (5)$$

$$\text{and } m_t^{\mathrm{phys}} \approx 0.6 m_t(\mu = 1\ \mathrm{GeV}).$$

This phenomenological formula (6) predicts the following value for the *t*-quark mass:

$$m_t^{\mathrm{phys}} \approx 180\text{—}200\ \mathrm{GeV}. \qquad (6)$$

In SM these mass matrices and mixing come from the Yukawa sector:

$$L_Y = QY_u\bar{q}_u h^* + QY_d\bar{q}_d h + LY_e\bar{l}_e h + \mathrm{h.c.}, \qquad (7)$$

where Q_i and L_i are three quark and lepton isodoublets, q_{u_i}, q_{d_i} and e_i are three right-handed antiquark and antilepton isosinglets, respectively, h is the ordinary Higgs doublet. In SM, the 3×3-family Yukawa matrices, $(Y_u)_{ij}$ and $(Y_d)_{ij}$, have no any particular symmetry. Therefore, it is necessary to find some additional mechanisms or symmetries beyond the SM which could diminish the number of the independent parameters in Yukawa sector L_Y. These new structures can be used for the determination of the mass hierarchy and family mixing.

To understand the generation mixing origin and fermion mass hierarchy several models beyond the SM suggest special forms for the mass matrix of «up» and «down» quarks (Fritzsch ansatz, «improved» Fritzsch ansatz, «Democratic» ansatz, etc. [7]). These mass matrices have less than ten independent parameters or they could have some matrix elements equal to zero («texture zeroes») [8]. This allows us to determine the diagonalizing matrix U_L and D_L in terms of quark masses:

$$Y_d^{\rm diag} = D_L Y_d D_R^+, \quad Y_u^{\rm diag} = U_L Y_d U_R^+. \tag{8}$$

For simplicity the symmetric form of Yukawa matrices has been taken, therefore: $D_L = D_R^*$, $U_L = U_R^*$. These ansatzes or zero «textures» could be checked experimentally in predictions for the mixing angles of the CKM matrix: $V_{\rm CKM} = U_L D_L^+$. For example, one can consider the following approximate form at the scale M_X for the symmetric «texture» used in paper [8]:

$$\mathrm{Y}_u = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix} \quad \mathrm{Y}_d = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix}. \tag{9}$$

Given these conditions it is possible to evolve down to low energies via the renormalization group equations all quantities including the matrix elements of Yukawa couplings $Y_{u,\,d}$, the values of the quark masses (see (4)) and the CKM matrix elements (see (1)). Also, using these relations we may compute U_L (or D_L) in terms of CKM matrix and/or of quark masses.

In GUT extensions of the SM with the family gauge symmetry embedded Yukawa matrices can acquire particular symmetry or an ansatz, depending on the Higgs multiplets to which they couple. The family gauge symmetry could help us to study in an independent way the origin of the up- (U) and down- (D) quark mixing matrices and consequently the structure of the CKM matrix

$V_{\rm CKM} = UD^{+}$. Due to the local gauge family symmetry a low energy breaking scale gives us a chance to define the quantum numbers of quarks and leptons and thus establishes a link between them in families. For the mass fermion ansatz considered above in the extensions of SM there could exist the following types of the $SU(3) \times SU(2_L)$ Higgs multiplets: (1,2), (3,1), (8,1), (3,2), (8,2), (1,1),..., which in turn could exist in the spectra of the String Models.

In the framework of the rank eight Grand Unified String Theories we will consider an extension of SM due to local family gauge symmetry, $G_H = SU(3)_H$, $SU(3)_H \times U(1)_H$ models and thier developments and their possible Higgs sector. Thus, for understanding the quark mass spectra and the difference between the origins of the up- (or down) quark and charged lepton mass matrices in GUSTs we have to study the Higgs content of the model, which we must use from the one hand for breaking the GUT-, Quark-Lepton-, $G_H = SU(3)_H$,...-, $SU(2)_L \times U(1)$- symmetries and from the other hand — for Yukawa matrix constructions. The vital question arising here is the nature of the ν mass.

2. TOWARDS A LOW ENERGY «EXACTLY SOLVABLE» GAUGE FAMILY SYMMETRY

2.1. The «Bootstrap» Gauge Family Models. The underlying analysis for this family symmetry breaking scale is lying on the modern experimental probability limitations for the typical rare flavour-changing processes. The estimates for the family symmetry breaking scale have certain regularities depending on the particular symmetry breaking schemes and generation mixing mechanisms (different ansatzes for quark and lepton mass matrices with 3_H or $3_H + 1_H$ generations have been discussed in [5]). As noted there, the current understanding of quark and lepton mass spectra leaves place for the existence of an unusually low mass breaking scale of non-abelian gauge $SU(3)_H$ or $(SU(3) \otimes U(1))_H$ family symmetry ~ *some* TeV. Some independent experiments for verifying the relevant hypotheses can be considered: light (π, K), heavy (B, D)-meson and charged lepton flavour changing rare decays [24, 25, 26, 5], family symmetry violation effects in e^+e^-, *ep*- and *pp*-collider experiments (LEP, HERA, FNAL, LHC).

The including into the model of the Higgs fields which are transformed under the $SU(3)_H \times SU(2)_L$ symmetry, like $H^a = (\underline{8}, \underline{1})$ (or $H^a_p = (\underline{8}, \underline{2})$, $p = 1,2$) and $X^i = (\underline{3},\underline{1})$ (or $X^i_p = (\underline{3},\underline{2})$, $p = 1,2$), gives the following contribution to the family gauge boson mass matrix:

$$(M_H^2)_{\underline{8}}^{ab} = g_H^2 \sum_{d=1}^{8} f^{adc} f^{bdc'} \langle H^c \rangle \langle H^{c'} \rangle^*, \tag{10}$$

$$(M_H^2)_{\underline{3}}^{ab} = g_H^2 \sum_{k=1}^{3} \frac{\lambda_{ik}^a}{2} \frac{\lambda_{kj}^b}{2} \langle X^i \rangle \langle X^j \rangle^*. \tag{11}$$

The lowest bound on M_H can be obtained from the analysis of the branching ratios of μ, π, K, D, B, ... rare decays ($\mathrm{Br} \geq 10^{-15-17}$).

In the paper [5] we investigated the samples of different scenarios of $SU(3)_H$-breakings down to the $SU(2)_H \times U(1)_{3H}$, $U(1)_{3H} \times U(1)_{8H}$ and $U(1)_{8H}$-subgroups, as well as the mechanism of the complete breaking of the base group $SU(3)_H$. We tried to realize the SUSY conserving program on the scales where the relevant gauge symmetry is broken. In the framework of these versions of the gauge symmetry breaking, we were searching for the spectra of horizontal gauge bosons and gauginos and calculated the amplitudes of some typical rare processes. Theoretical estimates for the branching ratios of some rare processes obtained from these calculations have been compared with the experimental data on the corresponding values. Further we have got some bounds on the masses of H_μ-bosons and the appropriate H-gauginos. Of particular interest was the case of the $SU(3)_H$-group which breaks completely on the scale M_{H_0}. We calculated the splitting of eight H-boson masses in a model dependent fashion. This splitting, depending on the quark mass spectrum, allows us to reduce considerably the predictive ambiguity of the model — «almost exactly solvable model».

We assume that when the $SU(3)_H$-gauge symmetry of quark-lepton generations is violated, all of the 8 gauge bosons acquire the same mass equal to M_{H_0}. Such a breaking is not difficult to get by, say, introducing the Higgs fields transforming in accordance with the triplet representation of the $SU(3)_H$ group. These fields are singlet under the Standard Model symmetries: ($z \in$ (3, 1, 1, 0) and $\bar{z} \in$ ($\bar{3}$, 1, 1, 0), $\langle \bar{z}^{i\alpha} \rangle_0 = \delta^{i\alpha} V$, $\langle z_i^\alpha \rangle_0 = \delta_i^\alpha V$, $i, \alpha = 1,2,3$, where $V = M_{H_0}$). We understand that here we need a more beautiful way to break this symmetry like dynamical way. But at this stage it is very important to establish a link between the mass spectra of the horizontal gauge bosons and known heavy fermions like t-quark. The degeneracy of the masses of 8 gauge horizontal vector bosons is eliminated by using the VEV's of the Higgs fields violating the electroweak symmetry and determining the mass matrix of up- and down-quarks (leptons). Thus, there is a set of the Higgs fields (see corresponding Table 1): $H(8, 2)$, $h(8, 2)$, $Y(\bar{3}, 2)$, $X(3, 2)$, $\kappa_{1,2}(1, 2)$ which could violate the $SU(2) \times U(1)$ symmetry and could determine the mass matrix of up- and down-quarks. On the other hand, in order to calculate the splitting between the masses of horizontal gauge bosons, one has to take into account the VEV's of this set of the Higgs fields.

Now we can come to constructing the horizontal gauge boson mass matrix M^2_{ab} $(a, b = 1,2,...,8)$:

$$(M^2_H)_{ab} = M^2_{H_0} \delta_{ab} + (\Delta M^2_d)_{ab} + (\Delta M^2_u)_{ab} \,. \tag{12}$$

Here (ΔM^2_d) and $(\Delta M^2_u)_{ab}$ are the «known» functions of heavy fermions, $(\Delta M^2_{u,d})_{ab} = F_{ab}(m_t, m_b, ...)$, which mainly get the contributions due to the vacuum expectations of the Higgs bosons that were used for construction of the mass matrix ansatzes for d-(u-)quarks.

For example in the case of $N_g = \underline{3} + \underline{1}$ families with Fritzsch ansatz for quark mass matrices and using $SU(3_H) \times SU(2)$ Higgs fields, (8,2), [5], we can write down some rough relations between the masses of horizontal gauge bosons («bootstrap» solution):

$$M^2_{H_1} \approx M^2_{H_2} \approx M^2_{H_3} \approx M^2_{H_0} + \frac{g^2_H}{4}\left[\frac{1}{\lambda^2}\frac{m_c m_t}{1 - m_t / m_{t'}}\right] + ...,$$

$$M^2_{H_4} \approx M^2_{H_5} \approx M^2_{H_6} \approx M^2_{H_7} \approx M^2_{H_0} + \frac{g^2_H}{4}\left[\frac{1}{\tilde{\lambda}^2} m_t m_{t'}\right] + ..., \tag{13}$$

$$M^2_{H_8} \approx M^2_{H_0} + \frac{g^2_H}{3}\left[\frac{1}{\tilde{\lambda}^2} m_t m_{t'}\right] + ...,$$

where λ and $\tilde{\lambda}$ are Yukawa couplings.

We were interested in how the unitary compensation for the contributions of horizontal forces to rare processes [5] depends on different versions of the $SU(3)_H$-symmetry breaking. The investigation of this dependence allows one, firstly, to understand how low the horizontal symmetry breaking scale M_H may be, and, secondly, how this scale is determined by a particular choice of a mass matrix ansatz both for quarks and leptons.

We would like to stress a possible existing of a local family symmetry with a low energy symmetry breaking scale, i.e., the existence of rather light H-bosons: $m_H \geq (1—10)$ TeV [5]. We have analyzed, in the framework of the «minimal» horizontal supersymmetric gauge model, the possibilities of obtaining a satisfactory hierarchy for quark masses and of connecting it with the splitting of horizontal gauge boson masses. We expect that due to this approach the horizontal model will become more definite since it will allow one to study the amplitudes of rare processes and the *CP*-violation mechanism more thoroughly. In this way we hope to get a deeper insight into the nature of interdependence between the generation mixing mechanism and the local horizontal symmetry breaking scale.

2.2. The $N=1$ SUSY Character of the $SU(3_H)$-Gauge Family Symmetry. We will consider the supersymmetric version of the Standard Model extended by the family (horizontal) gauge symmetry (and if one will need, we will also extend this model by the $G_R = SU(2)_R$ right-hand gauge group). The supersymmetric Lagrangian of strong electroweak and horizontal interactions, based on the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_{H...}$ (where the G_R-gauge group and the Abelian gauge factor $U(1)_H$ also can be taken into consideration), has the general form:

$$\mathcal{L}^{N=1} = \int d^2\theta \, \mathrm{Tr}(W^k W^k) + \int d^4\theta S_I^+ \, e^{\sum_k 2g_k \hat{V}_k} S_I +$$
$$+ \int d^4\theta \, \mathrm{Tr}(\Phi^+ e^{2g_H \hat{V}_H} \Phi \, e^{-2g_H \hat{V}_H})$$
$$+ \int d^4\theta \, \mathrm{Tr}(H_y^+ e^{2g_2 \hat{V}_2 + y2g_1 \hat{V}_1} e^{2g_H \hat{V}_H} H_y \, e^{-2g_H \hat{V}_H})$$
$$+ (\int d^2\theta \, P(S_i, \Phi, H_y, \eta, \xi, \ldots) + \mathrm{h.c.}) \tag{14}$$

(see for comparing $\mathcal{L}^{N=2}$ in Appendix A). In formula (14) the index k runs over all the gauge groups: $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, $SU(3)_H$, $\hat{V} = \mathbf{T}^a V^a$, where V^a are the real vector superfields, and $\mathbf{T}^a$ are the generators of the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, $SU(3)_H$-groups; S_I are left-chiral superfields from fundamental representations, and $I = i, 1, 2$; $S_i = Q, u^c, d^c, L, e^c, \nu^c$ are matter superfields, $S_1 = \eta$, $S_2 = \xi$ are Higgs fundamental superfields; the Higgs left chiral superfield Φ is transformed as the adjoint representation of the $SU(3)_H$-group, the Higgs left chiral superfields H_y: $H_{Y=+\frac{1}{2}} = H$, $H_{Y=-\frac{1}{2}} = h$ are transformed nontrivially under the horizontal $SU(3)_H$- and electroweak $SU(2)_L$-symmetries (see Table 1). P in formula (14) is a superpotential to be specified below. To construct it, we use the internal $U(1)_R$-symmetry which is habitual for a simple $N=1$ supersymmetry.

Table 1. The Higgs superfields with their $SU(3_H)$, $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ (and possible $U(1)_H$-factor) quantum numbers

	H	C	L	Y	Y_H
Φ	8	1	1	0	0
H	8	1	2	–1/2	$-y_{H_1}$
h	8	1	2	1/2	y_{H_1}
ξ	$\bar{3}$	1	1	0	0
η	3	1	1	0	0
Y	$\bar{3}$	1	2	1/2	$-y_{H_2}$
X	3	1	2	–1/2	y_{H_2}
κ_1	1	1	1 (2)	0 (1/2)	$-y_{H_3}$
κ_2	1	1	1 (2)	0 (–1/2)	y_{H_3}

In models with a global supersymmetry it is impossible to have simultaneously a SUSY breaking and a vanishing cosmological term. The reason is the semipositive definition of the scalar potential in the rigid supersymmetry approach (in particular, in the case of a broken SUSY we have $V_{\min} > 0$). The problem of supersymmetry breaking, with the cosmological term $\Lambda = 0$ vanishing, is solved in the framework of the $N=1$ SUGRA models. This may be done under an appropriate choice of the Kaehler potential, in particular, in the frames of «mini-maxi»- or «maxi» type models [27]. In such approaches, the spontaneous breaking of the local SUSY is due to the possibility to get nonvanishing VEVs for the scalar fields from the «hidden» sector of SUGRA [27]. The appearance in the observable sector of the so-called soft breaking terms comes as a consequence of this effect.

In the «flat» limit, i.e., neglecting gravity, one is left with lagrangian (14) and soft SUSY breaking terms, which on the scales $\mu \ll M_{Pl}$ have the form:

$$\mathcal{L}_{SB} = \frac{1}{2}\sum_i m_i^2 |\phi_i|^2 + \frac{1}{2} m_1^2 \,\mathrm{Tr}|h|^2 + \frac{1}{2} m_2^2 \,\mathrm{Tr}|H|^2 +$$

$$+ \frac{1}{2}\mu_1^2 |\eta|^2 + \frac{1}{2}\mu_2^2 |\xi|^2 + \frac{1}{2} M^2 \,\mathrm{Tr}|\Phi|^2 +$$

$$+ \frac{1}{2}\sum_k M_k \lambda_k^a \lambda_k^a + \mathrm{h.c.} + \text{trilinear terms}, \qquad (15)$$

where $H_1 = H$, $H_2 = h$ and i runs over all the scalar matter fields $\tilde{Q}$, $\tilde{u}^c$, $\tilde{d}^c$, $\tilde{L}$, $\tilde{e}^c$, $\tilde{\nu}^c$ and k runs over all the gauge groups: $SU(3)_H$, $SU(3)_C$, $SU(2)_L$, $U(1)_Y$. At the energies close to the Plank scale all the masses, as well as the gauge coupling, are correspondingly equal (this is true if the analytic kinetic function satisfies $f_{\alpha\beta} \sim \delta_{\alpha\beta}$) [27], but at low energies they have different values depending on the corresponding renormgroup equation (RGE). The squares of some masses may be negative, which permits the spontaneous gauge symmetry breaking.

Considering the SUSY version of the $SU(3)_H$-model, it is natural to ask: why do we need to supersymmetrize the model? Basing on our present-day knowledge of the nature of supersymmetry [27, 28], the answer will be:

(a) First, it is necessary to preserve the hierarchy of the scales: $M_{EW} < M_{\rm SUSY} < M_H < ...?... < M_{\rm GUT}$. Breaking the horizontal gauge symmetry, one has to preserve SUSY on that scale. Another sample of hierarchy to be considered is: $M_{EW} < M_{\rm SUSY} \sim M_H$. In this case, the scale M_H should be rather low ($M_H \leq$ a few TeV).

(b) To use the SUSY $U(1)_R$ degrees of freedom for constructing the superpotential and forbidding undesired Yukawa couplings.

(c) Super-Higgs mechanism — it is possible to describe Higgs bosons by means of massive gauge superfields [28].

(d) To connect the vector-like character of the $SU(3)_H$-gauge horizontal model and $N = 2$ SUSY (see Appendix A).

Since the expected scale of the horizontal symmetry breaking is sufficiently large: $M_H >> M_{EW}$, $M_H >> M_{\rm SUSY}$ (where M_{EW} is the scale of the electroweak symmetry breaking, and $M_{\rm SUSY}$ is the value of the splitting into ordinary particles and their superpartners), it is reasonable to search for the SUSY-preserving stationary vacuum solutions.

Let us construct the gauge invariant superpotential P of Lagrangian (14). With the fields given in Table 1, the most general superpotential will have the form

$$P = \lambda_0 \left[\frac{1}{3} {\rm Tr}(\hat{\Phi}^3) + \frac{1}{2} M_I \,{\rm Tr}(\hat{\Phi}^2) \right] + \lambda_1 [\, \eta\hat{\Phi}\xi + M'\eta\xi \,] +$$
$$+ \lambda_2 {\rm Tr}(\hat{h}\hat{\Phi}\hat{H}) + (\text{Yukawa couplings}) + (\text{Majorana terms } \nu^c), \qquad (16)$$

where Yukawa Couplings could be constructed, for example, using the Higgs fields, H and h, transforming under $SU(3)_H \times SU(2_L)$, like (8,2):

$$P_Y = \lambda_3 Q\hat{H}d^c + \lambda_4 L\hat{H}e^c + \lambda_5 Q\hat{h}u^c. \qquad (17)$$

Also, one can consider another type of superpotential P_Y, using the Higgs fields from Table 1.

Note, the fields Φ, H, h can be obtained on the level 2 of Kac-Moody algebra g or effectively on the level 1 of algebras g^{I}, g^{II} after «integration» over heavy fields, when $G^{\mathrm{I}} \times G^{\mathrm{II}} \to G^{\mathrm{sym}}$ (see section 3). Higgs fields X and Y are very important in models with fourth $SU(3)_H$-singlet generation. In the construction of the stationary solutions, only the following contributions of the scalar potential are taken into account:

$$V = \sum_i |F_i|^2 + \sum_a |D^a|^2 = V_F + V_D \geq 0, \tag{18}$$

where

$$V_F = \sum \left| \frac{\partial P_F}{\partial F_i} \right|^2 = \left| \frac{\partial P_F}{\partial F_{\Phi^a}} \right|^2 + \left| \frac{\partial P_F}{\partial F_{\xi_i}} \right|^2 + \left| \frac{\partial P_F}{\partial F_{\eta_i}} \right|^2. \tag{19}$$

The case $\langle V \rangle = 0$ of supersymmetric vacuum can be realized within different gauge scenarios [5]. By switching on the SUGRA, the vanishing scalar potential is no more required to conserve the supersymmetry with the necessity. The different gauge breaking scenarios do not result in obligatory vacuum degeneracy, as in the case of the global SUSY version. Let us write down each of the terms of formula (19):

$$P_F(\Phi, \xi, \eta) = \lambda_0 \left[\frac{i}{4\times 3} f^{abc} \Phi^a \Phi^b \Phi^c + \frac{1}{4\times 3} d^{abc} \Phi^a \Phi^b \Phi^c + \frac{1}{4} M_I \Phi^c \Phi^c \right]_F +$$
$$+ \lambda_1 \left[\eta_i (T^c)^i_j \xi^j \Phi^c + M' \eta_i \xi^i \right]_F +$$
$$+ \lambda_2 \left[\frac{i}{4} f^{abc} h^a_i \Phi^b H^c_j \varepsilon^{ij} + \frac{d^{abc}}{4} h^a_i \Phi^b H^c_j \varepsilon^{ij} \right]_F + \text{h.c.} \tag{20}$$

The contribution of D-terms into the scalar potential will be:

$$V_D = g_H^2 |\eta^+ T^a \eta - \xi^+ T^a \xi + i/2 f^{abc} \Phi^b \Phi^{c+} + i/2 f^{abc} h^b h^{c+} + i/2 f^{abc} H^b H^c|^2 +$$
$$+ g_2^2 |h^+ \tau^i / 2 h + H^+ \tau^i / 2 H|^2 + (g')^2 |1/2 h^+ h - 1/2 H^+ H|^2. \tag{21}$$

The SUSY-preserving condition for scalar potential (18) is determined by the flat F_i and D^a directions: $\langle F_i \rangle_0 = \langle D^a \rangle_0 = 0$. It is possible to remove the degeneracy of the supersymmetric vacuum solutions taking into account the interaction with supergravity, which was endeavored in SUSY GUT's, e.g., in the $SU(5)$ one [27] ($SU(5) \to SU(5)$, $SU(4) \times U(1)$, $SU(3) \times SU(2) \times U(1)$.

The horizontal symmetry spontaneous breaking to the intermediate subgroups in the first three cases of [5] can be realized, using the scalar components of the chiral complex superfields Φ, which are singlet under the standard gauge group. The Φ-superfield transforms as the adjoint representation of $SU(3)_H$. The intermediate scale M_I can be sufficiently large: $M_I > 10^5 - 10^6$ GeV. The complete breaking of the remnant symmetry group V_H on the scale M_H will occur due to the nonvanishing VEV's of the scalars from the chiral superfields $\eta(3_H)$ and $\xi(\bar{3}_H)$. The $V_{\min}$, again, corresponds to the flat directions: $\langle F_{\eta,\xi}\rangle_0 = 0$. The version (iv) corresponds to the minimum of the scalar potential in the case when $\langle\Phi\rangle_0 = 0$.

As for the electroweak breaking, it is due to the VEV's of the fields h and H, providing masses for quarks and leptons. Note that VEV's of the fields h and H must be of the order of M_W as they determine the quark and lepton mass matrices. On the other hand, the masses of physical Higgs fields h and H , which mix generations, must be some orders higher than M_W, so that not to contradict the experimental restrictions of FCNC. As a careful search for the Higgs potential shows, this is the picture that can be attained.

2.3. The Superweak-Like Source of CP-Violation, the Baryon Stability and Neutrino Mass Problems in GUST with the Non-Abelian Gauge Family Symmetry. The existence of horizontal interactions might be closely connected to the CP-violation problem [5]. This interaction is described by the relevant part of the SUSY $SU(3)_H$-Lagrangian and has the form

$$\mathcal{L}_H = g_H \overline{\Psi}_d \Gamma_\mu \left(D \frac{\Lambda^a}{2} D^+ \right) \psi_d O_{ab} Z^b_\mu. \tag{22}$$

Here we have $(a, b = 1,2,\ldots,8)$. The matrix O_{ab} determines the relationship between the bare, H^b_μ, and physical Z^b_μ, gauge fields and is calculated for the mass matrix $(M^2_H)_{ab}$ diagonalized; $\psi_d = (\psi_d, \psi_s, \psi_b)$; g_H is the gauge coupling of the $SU(3)_H$ group.

After the calculations in «bootstrap» model with the Higgs fields $\langle H\rangle = (\lambda_a \varphi_a)/2$, $\langle h\rangle = (\lambda_a \tilde{\varphi}_a)/2$ the expressions for the $(K^0_L - K^0_s)$, $(B^0_{dL} - B^0_{dS})$, $(B^0_{sL} - B^0_{sS})$, $(D^0_L - D^0_S)$,... meson mass differences (pure quark processes) at tree level take the following general forms:

$$\left[\frac{(M_{12})^K_{ij}}{m_K}\right]_H = \frac{1}{2}\frac{g^4_H}{2M^4_{H_0}} \left\{ \left[\tilde{\varphi}_a \left(D \frac{\lambda^a}{d} D^+ \right)_{ij} \right]^2 + \left[\varphi_a \left(D \frac{\lambda^a}{d} D^+ \right)_{ij} \right]^2 \right\} f^2_{K_{ij}} R_K ,$$

$$\left[\frac{(M_{12})_{ij}^{D}}{m_D}\right]_H = \frac{1}{2}\frac{g_H^4}{2M_{H_0}^4}\left\{\left[\tilde{\varphi}_a\left(U\frac{\lambda^a}{2}U^+\right)_{ij}\right]^2 + \left[\varphi_a\left(U\frac{\lambda^a}{2}U^+\right)_{ij}\right]^2\right\} f_{D_{ij}}^2 R_{D_{ij}}, \tag{23}$$

where $i, j = (1,2), (1,3), (2,3)$ — the K or D, B_d or T_u, B_s or T_c-meson systems.

The coefficients in formulas (23) are calculated from (22) using formula (10) and the following useful relation

$$\sum_a (DT^aD^+)_{ik}(DT^aD^+)_{mn} = \frac{1}{2}\left(\delta_{in}\delta_{km} - \frac{1}{3}\delta_{ik}\delta_{mn}\right). \tag{24}$$

For example, for K-meson systems we find the following contribution (if $D_L = D_R = D$)

$$g_H^2/4(D\lambda^a O^{ab}D^+)_{12}\frac{1}{M_0^2+\Delta M_b^2}(D\lambda^c O^{cb}D^+)_{12} =$$

$$= \frac{g_H^2}{4M_0^2}(D\lambda^a O^{ab}D^+)_{12}\left(1 - \frac{\Delta M_b^2}{M_0^2}\right)(D\lambda^c O^{cb}D^+)_{12} =$$

$$= -\frac{g_H^2}{4M_0^4}(D\lambda^a D^+)_{12}\Delta M_{ac}^2(D\lambda^c D^+)_{12} =$$

$$= -\frac{g_H^4}{4M_0^4}(D\lambda^a D^+)_{12} f^{kal} f^{kcl'}\varphi^l\bar{\varphi}^{l'}(D\lambda^c D^+)_{12} =$$

$$= \frac{g_H^4}{4M_0}(D[\lambda^k\lambda^m]D^+)_{12}\,\varphi^l\bar{\varphi}^{l'}(D[\lambda^k\lambda^{m'}]D^+)_{12} = \frac{g_H^4}{M_0^4}(D\varphi D^+)_{12}\,(D\varphi^+D^+)_{12}. \tag{25}$$

It is interesting, that if a difference between the gauge boson's masses is generated by Higgs fields in representation (3, 2) (see (11)), then the contribution in $\left[\frac{\Delta m}{m}\right]_H$ is equal to zero in considering order (for case $D_L = D_R$), since we will use Higgs fields (8, 2) for these evaluations. However, for processes including three equivalent indices (like $\mu \to 3e$), Higgs fields (3, 2) give nonzero contribution $\sim (\varphi D^+)_i \cdot (D\bar{\varphi})_j$.

Note, that formula (25) is true for the case when D_L differs from D_R by diagonal phase multiply too. For us the case $D_L = -D_R$ which corresponds to axial-vector terms is important. In general if $D_L \neq D_R$ (or $U_L \neq U_R$) then in formulas (23) there is a quadratic term $g_H^2/M_0^2\,(D_L D_R^+)_{ij}\,(D_R D_L^+)_{ij}$, $i \neq j$.

Substituting in formula (23) the expressions for φ, $\tilde{\varphi}$ and the elements d_{ij} of the D mixing matrix («bootstrap» solution), we can obtain the lower limit for the value M_{H_0} ($M_{H_0} < O$ (some TeV)). So, we could analyse the ratios (similar for $B_{d,s}$-meson system):

$$\left[\frac{\Delta m_K}{m_K}\right]_H = \frac{g_H^2}{M_{H_0}^2}\,\mathrm{Re}[C_K] f_K^2 R_K < 7\cdot 10^{-15} \tag{26}$$

and

$$\left[\frac{\mathrm{Im} M_{12}}{m_K}\right]_H = \frac{1}{2}\,\frac{g_H^2}{2M_{H_0}^2}\,\mathrm{Im}[C_K] f_K^2 R_K < 2\cdot 10^{-17}. \tag{27}$$

In these formulas the expressions for $C_{K,D}$ are known functions in «bootstrap» models [5], namely

$$C_{K,D} = \frac{g_H^2}{2\lambda_Y^2}\,\frac{m_t^2}{M_{H_0}^2} \times f\left(\frac{m_i^{\mathrm{up}}}{m_j^{\mathrm{up}}};\ \frac{m_k^{\mathrm{down}}}{m_l^{\mathrm{down}}}\right), \tag{28}$$

where f's are known complex functions and their forms depend on quark fermion mass ansatzes [5].

Here noteworthy are the following two points: a) The appearance of the phase in the CKM mixing matrix may be due to new dynamics working at short distances $\left(r << \dfrac{1}{M_W}\right)$. Horizontal forces may be the source of this new dynamics [5]. Using this approach, we might have the CP-violation effects — both due to electroweak and horizontal interactions.

(b) The CP is conserved in the electroweak sector ($\delta^{KM} = 0$), and its breaking is provided by the structure of the horizontal interactions. Let us consider the situation when $\delta^{KM} = 0$. In the SM, such a case might be realized just accidentally. The vanishing phase of the electroweak sector ($\delta^{KM} = 0$) might arise spontaneously due to some additional symmetry. Again, such a situation might occur within the horizontal extension of the electroweak model.

In particular, this model gives rise to a rather natural mechanism of superweak-like CP-violation due to the ($CP = -1$) part of the effective Lagrangian of horizontal interactions — $(\varepsilon'/\varepsilon)_K \leq 10^{-4}$. That part of $\mathcal{L}_{\mathrm{eff}}$ includes the product of the $SU(3)_H$-currents $I_{\mu i}$; $I_{\mu j}$ ($i = 1,4,6,3,8$; $j = 2,5,7$ or, vice versa, $i \leftrightarrow j$) [5].

In the case of a vector-like $SU(3)_H$-gauge model the CP-violation could be only due to the charge symmetry breaking.

In electroweak and horizontal interactions we might also have two CP-violating contributions to the amplitudes of B-meson decays. But it is possible to construct a scheme where CP-violation will occur only in the horizontal interactions. The last fact might lead to a very interesting CP-violation asymmetry $A_f(t)$ for the decays of neutral B_d^0- and $\bar{B}_d^0$ -mesons to final hadron CP-eigenstates, for example, to $f=(J/\Psi K_s^0)$ or $(\pi\pi)$

$$A_f(t) \approx \sin(\Delta m_{B_d} t)\, \mathrm{Im}\left(\frac{p}{q} \times \rho_f\right), \quad \rho_f = \frac{A(\bar{B} \to f)}{A(B \to f)}. \tag{29}$$

In the standard model with the Kobayashi — Maskawa mechanism of CP-violation the asymmetry of the decay of B_d^0- and $\bar{B}_d^0$ -meson to the $J/\Psi K_S^0$ averaged by time is:

$$A(J/\Psi K_S^0) \approx \eta_f \frac{x_d}{1+x_d^2} \sin 2\phi_3 = -\frac{x_d}{1+x_d^2} \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2},$$

where $\eta_f = -1$ for a CP-odd $J/\Psi K_S^0$-final state; $\phi_3 = \arg V_{td}$ is one of the angles (ϕ_i, $i=1,2,3$) of the unitary triangle. Let us compare this asymmetry with the analogous asymmetry of the B^0 and $\bar{B}^0$-decays to the CP-even final state (π^+, π^-), the latter being known to depend on the phase magnitudes of V_{ub} and V_{td}. Then:

$$A(\pi^+\pi^-) \approx -\eta_f \frac{x_d}{1+x_d^2} \sin 2\phi_2 = -\frac{x_d}{1+x_d^2} \frac{2\eta[(\rho^2+\eta^2)-\rho]}{[(1-\rho)^2+\eta^2][\rho^2+\eta^2]},$$

where $\phi_2 = \pi - \phi_1 - \phi_3$ and $\phi_1 = \arg V_{ub}^* = \delta_{13}(\delta^{KM} = \phi_1 + \phi_3)$.

The contributions of CP-violating horizontal interactions to the asymmetries for both B^0-decays could vary large (10% — 30%). They are identical for both decays but the signs differ.

The space-time structure of horizontal interactions depends on the $SU(3)_H$ quantum numbers of quark and lepton superfields and their C-conjugate superfields. One can obtain vector (axial)-like horizontal interactions as far as the G_H particle quantum numbers are conjugate (equal) to those of antiparticles. The question arising in these theories is how such horizontal interactions are related with strong and electroweak ones. All these interactions can be unified within one gauge group, which would allow one to calculate the value of the coupling constant of horizontal interactions. Thus, unification of horizontal, strong and electroweak interactions might rest on the GUTs $\tilde{G} \equiv G \times SU(3)_H$ (where, for example, $\tilde{G} \equiv E(8)$, $G \equiv SU(5)$, $SO(10)$ or E_6), which may be further broken down to $SU(3)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$. For including «vector»-like horizontal gauge symmetry into GUT we have to introduce «mirror» superfields. Speaking more definitely, if we want to construct GUTs of the $\tilde{G} \equiv G \times SU(3)_H$ type, each generation must encompass double G-matter supermultiplets, mutually conjugate under the $SU(3)_H$-group. In this approach the first supermultiplet consists of the superfields f and $f_m^c \in 3_H$, while the second is constructed with the help of the supermultiplets f^c and $f_m \in \bar{3}_H$. In this scheme, proton decays are only possible in the case of mixing between ordinary and «mirror» fermions. In its turn, this mixing must, in particular, be related with the $SU(3)_H$-symmetry breaking.

The GUSTs spectra also predict the existing of the new neutral neutrino-like particles interacting with the matter only by «superweak»-like coupling. It is possible to estimate the masses of these particles, and, as will be shown further, some of them have to be light (superlight) to be observed in modern experiment.

A variant for unusual nonuniversal family gauge interactions of known quarks and leptons could be realized if for each generation we introduce new heavy quarks ($F = U, D$), and leptons (L, N) which are singlets (it is possible to consider doublets also) under $SU(2)_L$- and triplets under $SU(3)_H$-groups. (This fermion matter could exist in string spectra. See all the three models with $SU(3_H) \times SU(3_H)$ family gauge symmetry). Let us consider for concreteness a case of charged leptons: $\Psi_l = (e, \mu, \tau)$ and $\Psi_L = (E, M, T)$. Primarily, for simplicity we suggest that the ordinary fermions do not take part in $SU(3)_H$-interactions («white» color states). Then the interaction is described by the relevant part of the SUSY $SU(3)_H$-Lagrangian and gets the form

$$\mathcal{L}_H = g_H \overline{\Psi}_{\mathcal{L}} \gamma_\mu \frac{\Lambda^a_{6\times6}}{2} \Psi_{\mathcal{L}} O_{ab} Z^b_\mu, \tag{30}$$

where

$$\Lambda^a_{6\times6} = \begin{pmatrix} S(L\lambda^a L^+)S & -S(L\lambda^a L^+)C \\ -C(L\lambda^a L^+)S & C(L\lambda^a L^+)C \end{pmatrix}.$$

Here we have $\Psi_{\mathcal{L}} = (\Psi_l ; \Psi_L)$. The matrix O_{ab} ($a, b = 1,2,3..8$) determines the relationship between the bare, H^b_μ, and physical, Z^b_μ, gauge fields. The diagonal 3×3 matrices $S = \mathrm{diag}(s_e, s_\mu, s_\tau)$ and $C = \mathrm{diag}(c_e, c_\mu, c_\tau)$ define the nonuniversal character for lepton horizontal interactions, as the elements s_i depend on the lepton masses, like $s_i \sim \sqrt{m_i / M_0}$ $(i = e, \mu, \tau)$. The same suggestion we might accept for local quark family interactions.

For the family mixing we might suggest the next scheme. The primary 3×3 mass matrix for the light ordinary fermions is equal to zero: $M^0_{ff} \approx 0$. The 3×3-mass matrix for heavy fermions is approximately proportional to unit $M^0_{FF} \approx M^Y_0 \times 1$, where $M^Y_0 \approx 0.5 - 1.0$ TeV and might be different for F_{up}-, F_{down}-quarks and for F_L-leptons. We assume that the splitting between new heavy fermions in each class F_Y (Y = up, down, L) is small and, at least in quark sector, might be described by the t-quark mass. Thus we think that at the first approximation it is possible to neglect the heavy fermion mixing. The mixing in the light sector is completely explained by the coupling of the light fermions with the heavy fermions. As a result of this coupling the 3×3-mass matrix M^0_{fF} could be constructed by «democratic» way which could lead to the well-known mass family hierarchy:

$$M^0_{6\times 6} = \begin{pmatrix} M^0_{ff} & M^0_{fF} \\ M^0_{Ff} & M^0_{FF} \end{pmatrix},$$

where

$$M^0_{fF} \approx M^{\mathrm{dem}}_{fF} + M^{\mathrm{corr}}_{fF}. \tag{31}$$

The diagonalization of the M^0_{fF}-mass matrix $X M^0_{fF} X^+$ ($X = L$-, D-, U-mixing matrices) gives us the eigenvalues, which define the family mass hierarchy — $n^Y_1 << n^Y_2 << n^Y_3$ and the following relations between the masses of the known light fermions and a new heavy mass scale:

$$n^Y_i = \sqrt{m_i M^Y_0}, \quad i = 1_g, 2_g, 3_g; \quad Y = \text{up-, down-fermions.}$$

In this «see-saw» mechanism the common mass scale of new heavy fermions might be not very far from the energy ~ 1 TeV, and as a consequence of it the mixing angles s_i-might be not too small. There is another interesting relation between the mass scales n^Y_i that might be in this mechanism, at least for the quark case:

$$n_t / n_c = n_c / n_u = q^u_H, \quad q^u_H \approx 1/\lambda^2,$$

$$n_b / n_s = n_s / n_d = q^d_H, \quad q^d_H \approx 1/\lambda.$$

An explicit example of nonuniversal $SU(3_H) \times SU(3_H)$ local family interactions will be considered later (see model 3 section 2).

3. THE HETEROTIC SUPERSTRING THEORY WITH RANK 8 AND 16 GRAND UNIFIED GAUGE GROUPS

3.1. Conformal Symmetry in Heterotic Superstring. In the heterotic string theory in left-moving (supersymmetric) sector there are $d-2$ (in the light-cone gauge) real fermions ψ^{μ}, their bosonic superpartners X^{μ}, and $3(10-d)$ real fermions χ^I. In the right-moving sector there are $d-2$ bosons $\overline{X}^{\mu}$ and $2(26-d)$ real fermions.

In the supersymmetric sector world-sheet supersymmetry is non-linearly realized via the supercharge

$$T_F = \psi^{\mu}\partial X_{\mu} + f_{IJK}\chi^I\chi^J\chi^K, \tag{32}$$

where f_{IJK} are the structure constants of a semi-simple Lie group G of dimension $3(10-d)$.

The possible Lie algebras of dimension 18 for $d=4$ are $SU(2)^6$, $SU(3)\times SO(5)$, and $SU(2)\times SU(4)$. However, $N=1$ space-time supersymmetry cannot be attained in two last cases [34].

If we take the moments of the energy-momentum operator we will get the conformal generators with the following Virasoro algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}\, n(n^2-1)\,\delta_{n,-m}. \tag{33}$$

Using Virasoro algebra we can construct representations of the conformal group where the highest weight state is specified by two quantum numbers, conformal weight h, and central charge c, such that:

$$L_0|h, c\rangle = h|h, c\rangle$$

$$L_n|h, c\rangle = 0, \quad n = 1,2,3... \tag{34}$$

For massless state the conformal weight $h=1$.

A Sugawara — Sommerfeld construction of the Virasoro algebra in terms of bilinears in the Kac-Moody generators [17, 18] allows one to get the following expression for the central Virasoro «charge»:

$$c_g = \frac{2k \dim g}{2k + Q_{\psi}} = \frac{x \dim g}{x + \tilde{h}}. \tag{35}$$

In heterotic string theories [9, 10] $(N=1\ \mathrm{SUSY})_{\mathrm{left}}$ with $(N=0\ \mathrm{SUSY})_{\mathrm{right}} \oplus \mathcal{M}_{c_L;c_R}$ with $d \le 10$, the conformal anomalies of the space-time sector are canceled by the conformal anomalies of the internal sector $\mathcal{M}_{c_L;c_R}$, where $c_L = 15 - 3d/2$ and $c_R = 26 - d$ are the conformal anomalies in the left- and right-moving string sectors, respectively.

In the fermionic formulation of the four-dimensional heterotic string theory in addition to the two transverse bosonic coordinates X_μ, $\bar{X}_\mu$ and their left-moving superpartners ψ_μ, the internal sector $\mathcal{M}_{c_L;c_R}$ contains 44 right-moving ($c_R = 22$) and 18 left-moving ($c_L = 9$) real fermions (each real world-sheet fermion has $c_f = 1/2$).

For a couple of years superstring theories, and particularly the heterotic string theory, have provided an efficient way to construct the Grand Unified Superstring Theories (GUST) of all known interactions, despite the fact that it is still difficult to construct unique and fully realistic low energy models resulting after decoupling of massive string modes. This is because of the fact that only 10-dimensional space-time allows existence of two consistent (invariant under reparametrization, superconformal, modular, Lorentz and SUSY transformations) theories with the gauge symmetries $E(8) \times E(8)$ or $spin(32)/Z_2$ [9,10] which after compactification of the six extra space coordinates (into the Calabi — Yau [11, 12] manifolds or into the orbifolds) can be used for constructing GUSTs. Unfortunately, the process of compactification to four dimensions is not unique and the number of possible low energy models is very large. On the other hand, constructing the theory directly in 4-dimensional space-time requires including a considerable number of free bosons or fermions into the internal string sector of the heterotic superstring [13,14,15, 16]. This leads to as large internal symmetry group such as, e.g., rank 22 group. The way of breaking this primordial symmetry is again not unique and leads to a huge number of possible models, each of them giving different low energy predictions.

Because of the presence of the affine Kac-Moody algebra (KMA) $\hat{g}$ (which is a 2-dimensional manifestation of gauge symmetries of the string itself) on the world sheet, string constructions yield definite predictions concerning representation of the symmetry group that can be used for low energy models building [17, 18]. Therefore the following longstanding questions have a chance to be answered in this kind of unification schemes:

1. How are the chiral matter fermions assigned to the multiplets of the unifying group?

2. How is the GUT gauge symmetry breaking realized?

3. What is the origin and the form of the fermion mass matrices?

The first of these problems is, of course, closely connected to the quantization of the electromagnetic charge of matter fields. In addition, string constructions can shed some light on the questions about the number of generation and possible existence of mirror fermions which remain unanswered in conventional GUTs [19].

There are not so many GUSTs describing the observable sector of Standard Models. They are well known: the SM gauge group, the Pati-Salam $(SU(4)\times SU(2)\times SU(2))$ gauge group, the flipped $SU(5)$ gauge group and $SO(10)$ gauge group, which includes flipped $SU(5)$ [16].

There are good physical reasons for including the horizontal $SU(3)_H$- group into the unification scheme. Firstly, this group naturally accomodates three fermion families presently observed (explaining their origin) and, secondly, can provide correct and economical description of the fermion mass spectrum and mixing without invoking high dimensional representation of conventional $SU(5)$, $SO(10)$ or $E(6)$ gauge groups. Construction of a string model (GUST) containing the horizontal gauge symmetry provides additional strong motivation to this idea. Moreover, the fact that in GUSTs high dimensional representations are forbidden by the KMA is a very welcome feature in this context.

3.2. The Possible Ways of $E(8)$-GUST Breaking Leading to the $N_G=3$ or $N_G=3+1$ Families. All this leads us naturally to consider possible forms for horizontal symmetry G_H, and G_H quantum number assignments for quarks (anti-quarks) and leptons (anti-leptons) which can be realized within GUSTs framework. To include the horizontal interactions with three known generations in the ordinary GUST it is natural to consider rank eight gauge symmetry. We can consider $SO(16)$ (or $E(6)\times SU(3)$) which is the maximal subgroup of $E(8)$ and which contains the rank eight subgroup $SO(10)\times(U(1)\times SU(3))_H$ [20]. We will be, therefore, concerned with the following chains (see Fig. 1):

$$E(8)\to SO(16)\to SO(10)\times(U(1)\times SU(3))_H\to$$

$$\to SU(5)\times U(1)_{Y_5}\times(SU(3)\times U(1))_H$$

or

$$E(8)\to E(6)\times SU(3)\to(SU(3))^{\times 4}.$$

According to this scheme one can get $SU(3)_H\times U(1)_H$ gauge family symmetry with $N_g=3+1$ (there are also other possibilities as, e.g., $E(6)\times SU(3)_H\subset E(8)$ $N_g=3$ generations can be obtained due to the second way of $E(8)$ gauge symmetry breaking via $E(6)\times SU(3)_H$, see Fig.1), where the

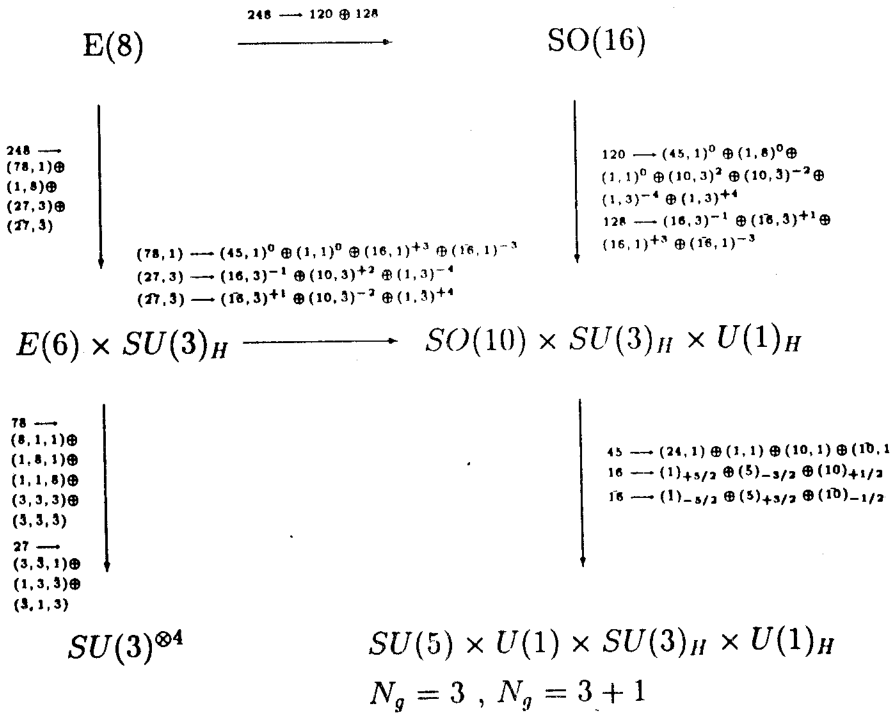

Fig.1. The possible ways of $E(8)$ gauge symmetry breaking leading to the 3 + 1 or 3 generations

possible additional fourth massive matter superfield could appear from $\underline{78}$ as a singlet of $SU(3)_H$ and transforms as $\underline{16}$ under the $SO(10)$ group.

In this note starting from the rank 16 grand unified gauge group (which is the minimal rank allowed in strings) of the form $G \times G$ [21, 22] and making use of the KMA which select the possible gauge group representations we construct the string models based on the diagonal subgroup $G^{\text{sym}} \subset G \times G \subset SO(16) \times$ $\times SO(16)$ $(\subset E_8 \times E_8)$ [21]. We discuss and consider $G^{\text{sym}} = SU(5) \times U(1) \times$ $\times (SU(3) \times U(1))_H \subset SO(16)$ where the factor $(SU(3) \times U(1))_H$ is interpreted as the horizontal gauge family symmetry. We explain how the unifying gauge symmetry can be broken down to the Standard Model group. Furthermore, the horizontal interaction predicted in our model can give an alternative description of the fermion mass matrices without invoking high dimensional Higgs representations. In contrast with other GUST constructions, our model does not contain particles with exotic fractional electric charges [23, 21]. This important virtue of the model is due to the symmetric construction of the electromagnetic charge Q_{em} from Q^{I} and Q^{II} — the two electric charges of each of the $U(5)$ groups [21]:

$$Q_{em} = Q^{II} \oplus Q^{I}. \tag{36}$$

We consider the possible forms of the $G_H = SU(3)_H$, $SU(3)_H \times U(1)$, $G_{HL} \times G_{HR} \ldots$-gauge family symmetries in the framework of Grand Unification Superstring Approach. Also we will study the matter spectrum of these GUST, the possible Higgs sectors. The form of the Higgs sector is very important for GUST-, G_H- and SM-gauge symmetries breaking and for constructing Yukawa couplings.

3.3. The Superstring Theory Scale of Unification and the Estimates on the Horizontal Coupling Constant. Really, the estimates on the M_{H_0}-scale depend on the value of the family gauge coupling. These estimates can be made in GUST using the string scale

$$M_{SU} \approx 0{,}73 g_{\text{string}} \cdot 10^{18}\ \text{GeV} \tag{37}$$

and the renormalization group equations (RGE) for the gauge couplings, α_{em}, α_3, α_2, to the low energies:

$$\alpha_{em}(M_Z) \approx 1/128,$$

$$\alpha_3(M_Z) \approx 0.11,$$

$$\sin^2\theta_W(M_Z) \approx 0.233. \tag{38}$$

The string unification scale could be contrasted with the $SU(3^c) \times SU(2) \times U(1)$ naive unification scale, $M_{GUT} \approx 10^{16}$ GeV, obtained by running the SM particles and their SUSY-partners to high energies. The simplest solution to this problem is the introduction of the new heavy particles with SM quantum numbers, which can exist in string spectra [16].

However there are some other ways to explain the difference between scales of string (M_{SU}) and ordinary (M_{GUT}) unifications. If one uses the breaking scheme $G^{I} \times G^{II} \to G^{\text{sym}}$ (where $G^{I,II} = U(5) \times U(3)_H \subset E_8$) described above, then unification scale $M_{GUT} \sim 10^{16}$ GeV is the scale of breaking the $G \times G$ group, and string unification does supply the equality of coupling constant $G \times G$ on the string scale $M_{SU} \sim 10^{18}$ GeV. Otherwise, we can have an addition scale of the symmetry breaking $M_{\rm sym} > M_{GUT}$. In any case on the scale of breaking $G \times G \to G^{\rm sym}$ the gauge coupling constants satisfy the equation

$$g^2_{\rm sym} = 1/4(g^2_{\rm I} + g^2_{\rm II}). \qquad (39)$$

Thus in this scheme knowing of scales M_{SU} and M_U gives us a principal possibility to trace the evolution of coupling constant of the original group $G \times G$ to the low energy and estimate the value of horizontal gauge constant g_{3H}.

The coincidence of $\sin^2\theta_W$ with experiment will show how realistic this model is.

Let us consider some relations which determine the value of $\sin^2\theta_W$ for different unification groups and for different ways of the breaking.

Firstly let us consider the case of $SO(10) \times U(3) \to SU(5) \times U(1) \times \times [SU(3) \times U(1)]_H$ breaking, which can be illustrated by Model 2. In this case matter fields are generated by world-sheet fermions with periodic boundary conditions. Consequently all representations of matter fields can be considered as the result of destruction of $\underline{16}$ and $\underline{\overline{16}}$ representations of the $SO(10)$ group.

If we write the general expansion for a world-sheet fermion in the form

$$f(\sigma, \tau) = \sum_{n=0}^{\infty} \left[b^+_{n+\frac{1-\alpha}{2}} \exp\left[-i\left(n + \frac{1-\alpha}{2} \right)(\sigma + \tau) \right] + \right.$$

$$\left. + d_{n+\frac{1+\alpha}{2}} \exp\left[i\left(n + \frac{1+\alpha}{2} \right)(\sigma + \tau) \right] \right], \qquad (40)$$

where the quantization conditions are given by the anti-commutation relations

$$\left\{ b^+_a, b_b \right\} = \left\{ d^+_a, d_b \right\} = \delta_{ab},$$

then the representation $\underline{\overline{16}}$ of $SO(10)$ in terms of the creation (b_0^{i+}) and annihilation (b_0^i) operators will have the form

$$\underline{\overline{16}} = \underline{1} + \underline{\overline{10}} + \underline{5} = (1 + b_0^{i+} b_0^{j+} + b_0^{i+} b_0^{j+} b_0^{k+} b_0^{l+})\,|0, \qquad (41)$$

where $i, j, k, l = 1,\ldots 5$.

The Clifford algebra is realized via the γ-matrix for $SO(10)$ group $\gamma_k = (b_k + b_k^+)$ and $\gamma_{5+k} = -i(b_k - b_k^+)$. Generators of the $U(5)$ subgroup can be written in terms b_0^i as $T[U(5)] = 1/2\,[b_i, b_j^+]$. Then the operator of the $U(1)_5$ hypercharge is

$$Y_5 = 1/2 \sum_i [b_i, b_i^+] = 5/2 - \sum_i b_i^+, b_i. \tag{42}$$

But this generator is not normalized, since $Y_5(\underline{1}, \overline{\underline{10}}, \underline{5}) = 5/2,\ 1/2,\ -3/2$, correspondingly, and $\mathrm{Tr}_{\underline{16}} Y_5^2 = 20$.

In our scheme the electromagnetic charge is

$$Q_{EM} = T_5 - 2/5\,Y_5, \tag{43}$$

where $T_5 = \mathrm{diag}(1/15,\ 1/15,\ 1/15,\ 2/5,\ -3/5)$. For representation $\underline{5}$ of the $SU(5)$ this means that

$$\begin{aligned} Q_{EM}(\underline{5}) &= [\mathrm{diag}\,(0^3, 1/2, -1/2) + \mathrm{diag}\,(2/30^3, -3/30^2)] - 2/5\cdot(-3/2) = \\ &= \mathrm{diag}\,(0^3,\ 1/2,\ -1/2) + 1/2[\mathrm{diag}\,(2/15^3, -3/15^2) + 6/5] = \\ &= t_3 + 1/2[\tilde{t}_0 - 4/5\,Y_5] = t_3 + 1/2\mathrm{diag}\,(4/3^3,\ 1^2) = t_3 + 1/2y, \end{aligned} \tag{44}$$

where y is the electroweak hypercharge.

Now let us write down the principal equation for coupling constants

$$g_5 t_0 A_\mu + (kg_5) Y A'_\mu = g_1 y B_\mu + g'_1 y' B'_\mu. \tag{45}$$

In this equation (kg_5) is $U(1)_5$ coupling constant on the scale, where $U(5)$ is breaking down (on the $SO(10) \to U(5)$ scale $k = 1$); operators $t_0 \sim \tilde{t}_0$, $Y \sim Y_5$ and have equal norm; A and B are gauge fields.

Diagonal generators can be written in terms of creation-annihilation operators as

$$\mathrm{diag}\,(A_i) = \sum_{i=1}^{5} A_i(1 - b_i^+ b_i) = -\sum A_i b_i^+ b_i. \tag{46}$$

Consequently $\mathrm{Tr}_{\underline{16}}\, \tilde{t}_0^2 = 8/15$. If we shall normalize generators as $\mathrm{Tr}_{\underline{16}} t_0^2 = \mathrm{Tr}_{\underline{16}} Y^2 = 8$, then $Y(\underline{5}) - \sqrt{2/5}\,Y_5(\underline{5}) = -3/\sqrt{10}$ and $t_0 = 1/\sqrt{15}\,\mathrm{diag}\,(2^3, -3^2)$. Now after rewriting the equation (45) separately for three up components and two down components, and substitution $B_\mu = cA_\mu + sA_\mu'$, $B_\mu' = -sA_\mu + cA_\mu'$ where $c^2 + s^2 = 1$, we find from equation (45)

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2 + g_5^2} = \left.\frac{15k^2}{16k^2 + 24}\right|_{k^2=1} = \frac{3}{8}. \tag{47}$$

Now let us consider the breaking $E_6 \to U(5) \times U(1)$, which corresponds to models like Model 4. The expansion of matter representation $\underline{27}$ of the E_6 group under the group $SU(5) \times U(1)_5$ is

$$\underline{27} = (\underline{\bar{5}}_{3/2} + \underline{10}_{-1/2}\, \underline{1}_{-5/2}) + (\underline{5}_1 + \underline{\bar{5}}_{-1}) + \underline{1}_0 =$$

$$= [(b_0^{i+} + b_0^{i+} b_o^{j+} b_0^{k+} + b_0^{i+} b_0^{j+} b_0^{k+} b_0^{l+} b_0^{m+}) + (d_{1/2}^{i+} + b_{1/2}^{i+}) + 1]|0\rangle. \qquad (48)$$

The generalization of the formula (42) for the case when representation contains states from different sectors with different boundary conditions is

$$Y_5 = \sum_i \left(\frac{\alpha_i}{2} + \sum_E^{\infty} [d_E^+(f_i) d_E(f_i) - b_E^+(f_i^*) b_E(f_i^*)] \right) \qquad (49)$$

and analogically for formula (46)

$$\text{diag}\,(A_i) = \sum_i \left[A_i \cdot \sum_E^{\infty} (d_E^+(f_i) d_E(f_i) - b_E^+(f_i^*) b_E(f_i^*)) \right], \quad \left(\sum A_i = 0 \right).$$

Now we have $\text{Tr}_{\underline{27}} Y_5^2 = 30$ and $\text{Tr}_{\underline{27}} \tilde{t}_0^2 = 4/5$. By comparison with preceding case both norms are 1.5 times greater, hence formula (47) is true for this case, too. But now the B'_μ is some linear combination of gauge fields.

Further, let us consider the Model 1. This case corresponds to breaking $SO(16) \to U(8) \to U(5) \times U(3)$. The matter fields arise from sectors with $\alpha = \pm 1/2$ and correspond to chips of the $SU(8)$ representations

$$\left.\begin{array}{l} 8 \to [(1, 3)] + (5,1) \\ \underline{56} \to [(1, 1) + (1\underline{0}, 3)] + (\overline{10}, 1) + (\underline{5}, \bar{3}) \\ \underline{56} \to [(10, 1) + (\underline{5}, 3)] + (1, 1) + (\overline{10}, \bar{3}) \\ 8 \to [(5, 1)] + (1, \bar{3}) \end{array}\right\} \sim 128_{SO(16)},$$

where only the fields in square brackets survive after the GSO projection.

For these model it is necessary to change $Y_5 \to \tilde{Y}_5 = -1/4(Y_5 + 5Y_3)$ in formula (44). Now we can calculate the norms of $\tilde{t}_0$ and $\tilde{Y}_5$ operators for this model.

$$\text{Tr}_{128} \tilde{Y}_5^2 = 160 = 20 \times 8,$$

$$\text{Tr}_{128} \tilde{t}_0^2 = \frac{64}{15} = \frac{8}{15} \times 8.$$

Hence we find again formula (47), but A'_μ is linear combination of gauge fields, which corresponds to $\tilde{Y}_5$ hypercharge and kg_5 is his coupling constant.

Finally, let us consider the model with following chain of the gauge group breaking $E_6 \times SU(3)_H \to SU(3)^3 \times SU(3)_H$. Then representation $\underline{27}$ of E_6 group looks like $27 = (1, 3, \bar{3}) + (3, \bar{3}, 1) + (\bar{3}, 1, 3)$.

Let us write down quantum numbers for breaking: $SU(3)_C \times SU(3)_L \times \times SU(3)_R \to SU(3)_C \times [SU(2) \times U(1)]_L \times U(1)^n_R$ (we consider one generation). For this construction the electromagnetic charge is

$$Q_{EM} = t_3^L + 1/2(Y_L + Y_R) = t_3^L + 1/2 y_{EW},$$

where $t_3^L = \mathrm{diag}\,(1/2, -1/2, 0)$, $Y_L = \mathrm{diag}\,(-1/3, -1/3, 2/3)$, $Y_R = \mathrm{diag}\,(2/3, 2/3, -4/3)$. This corresponds to the following set of fields:

$$(1, 3, \bar{3}) \sim \begin{pmatrix} (\nu & l)_L & \nu_R^c \\ L_1^U & L_1^D & L \\ L_2^U & L_2^D & e_R^c \end{pmatrix}, \quad (3, \bar{3}, 1) \sim 3 \times [(d-u)_L\, D_L],$$

$$(\bar{3}, 1, 3) \sim \bar{3} \times \begin{pmatrix} D_R^c \\ d_R^c \\ u_R^c \end{pmatrix},$$

where capital letters correspond to additional heavy quarks and leptons.

Now we can calculate norms of Y_L, Y_R

$$\mathrm{Tr}_{27}\, Y_L^2 = 4, \quad \mathrm{Tr}_{27} Y_R^2 = 16,$$

and write down the equation for gauge coupling constants

$$g_L Y_L A_L^\mu + 1/2 g_R Y_R A_R^\mu = g_1 y_{EW} B^\mu + g'_1 y' B'^\mu, \tag{50}$$

where $g_R = k g_L$ and on the scale of the breaking $SU(3)_L \times SU(3)_R \to \to SU(2)_L \times U(1)_L \times U(1)_R$ $k = 1$.

For example, for $(\nu,\ e,\ \nu_R^c)$ we have $Y_L = \mathrm{diag}\,(-1/3, -1/3, 2/3)$, $Y_R = -2/3$ and $y_{EW} = \mathrm{diag}\,(-1, -1, 0)$. Then from equation (50) (analogically with (45, 47)) we find

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2 + g_L^2} = \frac{k^2}{2k^2 + 4}. \tag{51}$$

The analysis of RG-equations allows one to state that horizontal coupling constant g_{3H} does not exceed electro-weak one g_2.

For example, if below the M_{GUT} scale in non-horizontal sector we have effectively the standard model with four generations and two Higgs doublets (like Model 1, 2) then the evolution of gauge coupling constants is described by equations

$$\alpha_s^{-1}(\mu) = \alpha_5^{-1}(M_{GUT}) + 8\pi b_3 \ln (\mu/M_{GUT}) \tag{52}$$

$$\alpha^{-1}(\mu) \sin^2\theta_W = \alpha_5^{-1}(M_{GUT}) + 8\pi b_2 \ln (\mu/M_{GUT}) \tag{53}$$

$$\frac{15}{k^2+24}\,\alpha^{-1}(\mu) \cos^2\theta_W = (k^2\alpha_5(M_{GUT}))^{-1} + 8\pi b_1 \ln (\mu/M_{GUT}), \tag{54}$$

where

$$b_3 = \frac{1}{16\pi^2}, \quad b_2 = -\frac{3}{16\pi^2}, \quad b_1 = -\frac{21}{40\pi^2}.$$

From these equations and from (38) we can find

$$M_{GUT} = 1.3\cdot 10^{16}\,\text{GeV}, \quad k^2 = 0.9, \quad \alpha_5^{-1} = 14. \tag{55}$$

Now we can get the relation between $g_{\text{str}} \equiv g$ and M_{sym} from RG equations for gauge running constants $g_5^{\text{sym}} \equiv g_5$, g_5^{I} and g_5^{II} on $M_{GUT} - M_{SU}$ scale. For example, for Model 1

$$b_5^{\text{sym}} = -\frac{3}{4\pi^2}, \quad b_5^{\text{I}} = -\frac{5}{16\pi^2}; \quad b_5^{\text{II}} = \frac{3}{16\pi^2},$$

and from RGE we find the following relation

$$\frac{g_5^2}{4\pi^2 + 6g_5^2 \ln (M_{GUT}/M_{\text{sym}})} =$$

$$= g^2 \frac{8\pi^2 - g^2 \ln (M_{\text{sym}}/M_{SU})}{[8\pi^2 - 5g^2 \ln (M_{\text{sym}}/M_{SU})]\cdot[8\pi^2 + 3g^2 \ln (M_{\text{sym}}/M_{SU})]}. \tag{56}$$

According to this equation we obtain that if $M_{SU} \approx 10^{18}$ GeV and the scale of breaking down to symmetric subgroup changes in region $M_{\text{sym}} \div 1.5\cdot 10^{16}$ GeV — 10^{18} GeV, then $g_{\text{str}} \sim O(1)$. Note that these values agree with formula (37).

Using RG equations for the running constant g_{3H} and the value of the string coupling constant g_{str} we can estimate a value of the horizontal coupling constant at low energies. For Model 1 we have

$$b_{3H}^{\text{sym}} = -\frac{5}{2\pi^2}, \quad b_{3H}^{\text{I}} = -\frac{21}{16\pi^2}, \quad b_5^{\text{II}} = -\frac{13}{16\pi^2},$$

and taking into account the relation (39), we find from RGE for g_{3H} that

$$g_{3H}^2(O(1\text{ TeV})) \approx 0.05,$$

and this value depends very slightly on the scale M_{sym}. However, note that for all our estimations the presence of the breaking $G \times G$ group to diagonal subgroup G_{sym} played the crucial role.

The above calculations show that for evaluation of intensity of processes with a gauge horizontal bosons at low energies we can use inequality

$$\alpha_{3H}(\mu) \le \alpha_2(\mu).$$

4. WORLD-SHEET KAC-MOODY ALGEBRA AND MAIN FEATURES OF RANK EIGHT GUST

4.1. The Representations of Kac-Moody Algebra and Vertex Operators. Let us begin with a short review of the KMA results [17, 18]. In heterotic string, the KMA is constructed by the operator product expansion (OPE) of the fields J^a of the conformal dimension (0,1):

$$J^a(z)J^b(\omega) = \frac{1}{z-\omega^2}k\delta^{ab} + \frac{1}{z-\omega}if^{abc}J^c + \ldots . \tag{57}$$

The structure constants f^{abc} for the group g are normalized so that

$$f^{acd}f^{bcd} = Q_\psi \delta^{ab} = \tilde{h}\psi^2\delta^{ab}, \tag{58}$$

where Q_ψ and ψ are the quadratic Casimir and the highest weight of the adjoint representation and $\tilde{h}$ is the dual Coxeter number. The $\frac{\psi}{\psi^2}$ can be expanded as in integer linear combination of the simple roots of g:

$$\frac{\psi}{\psi^2} = \sum_{i=1}^{\text{rank } g} m_i\alpha_i . \tag{59}$$

The dual Coxeter number can be expressed through the integers numbers m_i

$$\tilde{h} = 1 + \sum_{i=1}^{\text{rank } g} m_i \tag{60}$$

and for the simply laced groups (all roots are equal and $\psi^2 = 2$): A_n, D_n, E_6, E_7, E_8 they are equal to $n+1$, $2n-2$, 12, 18 and 30, respectively.

The KMA $\hat{g}$ allows one to grade the representations R of the gauge group by a level number x (a non-negative integer) and by a conformal weight $h(R)$. An irreducible representation of the affine algebra $\hat{g}$ is characterized by the vacuum representation of the algebra g and the value of the central term k, which is connected to the level number by the relation $x = 2k/\psi^2$. The value of the level number of the KMA determines the possible highest weight unitary representations which are present in the spectrum in the following way

$$x = \frac{2k}{\psi^2} \geq \sum_{i=1}^{\text{rank } g} n_i m_i \,, \tag{61}$$

where the sets of non-negative integers $\{m_i = m_1, \ldots, m_r\}$ and $\{n_i = n_1, \ldots, n_r\}$ define the highest root and the highest weight of a representation R, respectively [17,18]:

$$\mu_0 = \sum_{i=1}^{\text{rank } g} n_i \alpha_i \,. \tag{62}$$

In fact, the KMA on the level one is realized in the 4-dimensional heterotic superstring theories with free world sheet fermions which allow a complex fermion description [14,15,16]. One can obtain KMA on a higher level working with real fermions and using some tricks [29]. For these models the level of KMA coincides with the Dynkin index of representation M to which free fermions are assigned

$$x = x_M = \frac{Q_M \dim M}{\psi^2 \dim g} \tag{63}$$

(Q_M is a quadratic Casimir eigenvalue of representation M) and equals one in cases when real fermions form vector representation M of $SO(2N)$, or when the world sheet fermions are complex and M is the fundamental representation of $U(N)$ [17,18].

Thus, in strings with KMA on the level one realized on the world-sheet, only very restricted set of unitary representations can arise in the spectrum:

1. singlet and totally antisymmetric tensor representations of $SU(N)$ groups, for which $m_i = (1, \ldots, 1)$;

2. singlet, vector, and spinor representations of $SO(2N)$ groups with $m_i = (1,2,2,\ldots 2,1,1)$;

3. singlet, $\underline{27}$ and $\underline{\overline{27}}$-plets of $E(6)$ corresponding to $m_i = (1,2,2,3,2,1)$;

4. singlet of $E(8)$ with $m_i = (2,3,4,6,5,4,3,2)$.

Therefore only these representations can be used to incorporate matter and Higgs fields in GUSTs with KMA on the level 1.

In principle it might be possible to construct explicitly an example of level-1 KMA-representation of the simply laced algebra $\hat{g}$ (A-, D-, E-types) from the level-one representations of the Cartan subalgebra of g. This construction is achieved using the vertex operator of string, where these operators are assigned to a set of lattice point corresponding to the roots of a simply-laced Lie algebra g. In heterotic string approach the vertex operator for a gauge boson with momentum p and polarization ζ is a primary field of conformal dimension $(1/2, 1)$ and could be written in the form:

$$V^a = \zeta_\mu \psi_\mu(\bar{z})\, J^a \exp(ipX), \quad p_\mu p^\mu = \zeta_\mu p^\mu = 0. \tag{64}$$

X_μ is the string coordinate and ψ^μ is a conformal dimension $(1/2, 0)$ Ramond-Neveu-Schwartz fermion.

4.2. The Features of the Level-One KMA in Matter and Higgs Representations in Rank 8 and 16 GUST Constructions. For example, to describe chiral matter fermions in GUST with the gauge symmetry group $SU(5) \times \times U(1) \subset SO(10)$ the following sum of the level-one complex representations: $\underline{1}(-5/2) + \underline{\bar{5}}(+3/2) + \underline{10}(-1/2) = \underline{16}$ can be used. On the other side, as real representations of $SU(5) \times U(1) \subset SO(10)$, from which Higgs fields can arise, one can take, for example, $\underline{5} + \underline{\bar{5}}$ representations arising from real representation $\underline{10}$ of $SO(10)$. Also, real Higgs representations like $\underline{10}(-1/2) + \underline{\overline{10}}(+1/2)$ of $SU(5) \times U(1)$ originating from $\underline{16} + \underline{\overline{16}}$ of $SO(10)$, which has been used in Ref.[6] for further symmetry breaking, are allowed.

Another example is provided by the decomposition of $SO(16)$ representations under $SU(8) \times U(1) \subset SO(16)$. Here, only singlet $v = \underline{16}$, $s = \underline{128}$ and $s' = \underline{128'}$ representations of $SO(16)$ are allowed by the KMA ($s = \underline{128}$ and $s' = \underline{128'}$ are the two nonequivalent, real spinor representations with the highest weights $\pi_{7,8} = 1/2(\varepsilon_1 + \varepsilon_2, + \ldots + \varepsilon_7 \mp \varepsilon_8)$, $\varepsilon_i\, \varepsilon_j = \delta_{ij}$) . From item 2, we can obtain the following $SU(8) \times U(1)$ representations: singlet, $\underline{8} + \underline{\bar{8}}$ $(= \underline{16})$, $\underline{8} + \underline{56} + \underline{\overline{56}} + \underline{\bar{8}}$ $(= \underline{128})$, and $\underline{1} + \underline{28} + \underline{70} + \underline{\overline{28}} + \underline{\bar{1}}$ $(= \underline{128'})$. The highest weights of $SU(8)$ representations $\pi_1 = \underline{8}$, $\pi_7 = \underline{\bar{8}}$ and $\pi_3 = \underline{56}$, $\pi_5 = \underline{\overline{56}}$ are:

$$\begin{aligned}
\pi_1 &= 1/8(\underbrace{7\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5}\, \underbrace{- \varepsilon_6 - \varepsilon_7 - \varepsilon_8}), \\
\pi_7 &= (1/8\underbrace{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5}\, \underbrace{+ \varepsilon_6 + \varepsilon_7 - 7\varepsilon_8}), \\
\pi_3 &= 1/8(\underbrace{5\varepsilon_1 + 5\varepsilon_2 + 5\varepsilon_3 - 3\varepsilon_4 - 3\varepsilon_5}\, \underbrace{- 3\varepsilon_6 - 3\varepsilon_7 - 3\varepsilon_8}), \\
\pi_5 &= 1/8(\underbrace{- 3\varepsilon_1 - 3\varepsilon_2 - 3\varepsilon_3 - 3\varepsilon_4 - 3\varepsilon_5}\, \underbrace{+ 5\varepsilon_6 + 5\varepsilon_7 + 5\varepsilon_8}) .
\end{aligned} \tag{65}$$

Similarly, the highest weights of $SU(8)$ representations $\pi_2 = \underline{28}$, $\pi_6 = \underline{\overline{28}}$ and $\pi_4 = \underline{70}$ are:

$$\pi_2 = 1/4(\underbrace{3\varepsilon_1 + 3\varepsilon_2 - \varepsilon_3 - \varepsilon_4 - \varepsilon_5} - \underbrace{\varepsilon_6 - \varepsilon_7 - \varepsilon_8}),$$

$$\pi_6 = 1/4(\underbrace{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 + \varepsilon_5} + \underbrace{\varepsilon_6 - 3\varepsilon_7 - 3\varepsilon_8}), \quad (66)$$

$$\pi_4 = 1/2(\underbrace{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 - \varepsilon_5} - \underbrace{\varepsilon_6 - \varepsilon_7 - \varepsilon_8}).$$

However, as we will demonstrate, in each of the string sectors the generalized Gliozzi — Scherk — Olive projection (the GSO projection in particular guarantees the modular invariance and supersymmetry of the theory and also gives some nontrivial restrictions on gauge groups and their representations) necessarily eliminates either $\underline{128}$ or $\underline{128'}$. It is therefore important that, in order to incorporate chiral matter in the model, only one spinor representation is sufficient. Moreover, if one wants to solve the chirality problem applying further GSO projections (which break the gauge symmetry), the representation $\underline{\overline{10}}$ which otherwise, together with $\underline{10}$, could form real Higgs representation, also disappears from this sector. Therefore, the existence of $\underline{\overline{10}}_{-1/2} + \underline{10}_{1/2}$, needed for breaking $SU(5) \times U(1)$ is incompatible (by our opinion) with the possible solution of the chirality problem for the family matter fields.

Thus, in the rank eight group $SU(8) \times U(1) \subset SO(16)$ with Higgs representations from the level-one KMA only, one cannot arrange for further symmetry breaking. Moreover, construction of the realistic fermion mass matrices seems to be impossible. In old-fashioned GUTs (see, e.g., [19]), not originating from strings, the representations of the level-two were commonly used to solve these problems.

The way out from this difficulty is based on the following important observations. Firstly, all higher-dimensional representations of (simple laced) groups like $SU(N)$, $SO(2N)$ or $E(6)$, which belong to the level two representation of the KMA (according to equation 61), appear in the direct product of the level-one representations:

$$R_G(x = 2) \subset R_G(x = 1) \times R'_G\,(x = 1). \quad (67)$$

For example, the level-two representations of $SU(5)$ will appear in the corresponding direct products of

$$\underline{15}, \underline{24}, \underline{40}, \underline{45}, \underline{50}, \underline{75} \subset \underline{5}, \underline{5}, \underline{5} \times \underline{\bar{5}}, \underline{5} \times \underline{10}, \text{etc.} \quad (68)$$

In the case of $SO(10)$ the level-two representations can be obtained by the suitable direct products:

$$\underline{45}, \underline{54}, \underline{120}, \underline{126}, \underline{210}, \underline{144} \subset \underline{10} \times \underline{10},$$

$$\underline{\overline{16}} \times \underline{10}, \underline{\overline{10}} \times \underline{16}, \underline{16} \times \underline{16}, \underline{\overline{16}} \times \underline{16}. \quad (69)$$

The level-two representations of $E(6)$ are the corresponding factors of the decomposition of the direct products:

$$\underline{78}, \underline{351}, \underline{351'}, \underline{650} \subset \overline{\underline{27}} \times \underline{27} \quad \text{or} \quad \underline{27} \times \underline{27}. \tag{70}$$

The only exception from this rule is the $E(8)$ group, two level-two representations ($\underline{248}$ and $\underline{3875}$) of which cannot be constructed as a product of level-one representations [20].

Secondly, the diagonal (symmetric) subgroup G^{sym} of $G \times G$ effectively corresponds to the level-two KMA $g(x = 1) \oplus g(x = 1)$ [21,22] because taking the $G \times G$ representations in the form (R_G, R_G') of the $G \times G$, where R_G and R_G' belong to the level-one of G, one obtains representations of the form $R_G \times R_G'$ when one considers only the diagonal subgroup of $G \times G$. This observation is crucial, because such a construction allows one to obtain level-two representations. (This construction has implicitly been used in [22] (see also [21] where we have constructed some examples of GUST with gauge symmetry realized as a diagonal subgroup of direct product of two rank eight groups $U(8) \times U(8) \subset SO(16) \times SO(16)$).

In strings, however, not all level-two representations can be obtained in that way because, as we will demonstrate, some of them become massive (with masses of order of the Planck scale). The condition ensuring that states in the string spectrum transforming as a representation R are massless reads:

$$h(R) = \frac{Q_R}{2k + Q_{ADJ}} = \frac{Q_R}{2Q_M} \leq 1, \tag{71}$$

where Q_i is the quadratic Casimir invariant of the corresponding representations, and M has been already defined before (see eq.63). Here the conformal weight is defined by $L_0 | \, 0 \, \rangle = h(R) \, | \, 0 \, \rangle$,

$$L_0 = \frac{1}{2k + Q_\psi} \left(\sum_{a=1}^{\dim g} \left(T_0^a T_0^a + 2 \sum_{n=1}^{\infty} T_{-n}^a T_n^a \right) \right), \tag{72}$$

where $T_n^a | \, 0 \, \rangle = 0$ for $n > 0$. The condition (71), when combined with (61), gives a restriction on the rank of GUT's group ($r \leq 8$), whose representations can accommodate chiral matter fields. For example, for antisymmetric representations of $SU(n = l + 1)$ we have the following values correspondingly: $h = p(n - p)/(2n)$. More exactly, for $SU(8)$ group: $h(\underline{8}) = 7/16$, $h(\underline{28}) = 3/4$, $h(\underline{56}) = 15/16$, $h(\underline{70}) = 1$; for $SU(5)$, correspondingly $h(\underline{5}) = 2/5$ and $h(\underline{10}) = 3/5$; for $SU(3)$ group $h(\underline{3}) = 1/3$ although for adjoint representation of $SU(3)$ — $h(\underline{8}) = 3/4$; for $SU(2)$ doublet representation we have $h(\underline{2}) = 1/4$. For vector representation of orthogonal series D_l, $h = 1/2$, and, respectively, for spinor — $h\,(spinor) = l/8$.

There are some other improtant cases. The values of conformal weights for $G = SO(16)$ or $E(6) \times SU(3)$, representations $\underline{128}$, $(\underline{27}, \underline{3})$ ($h(\underline{128}) = 1$, $h(\underline{27}, \underline{3}) = 1$), respectively, satisfy both conditions. Obviouslyy, these (important for incorporation of chiral matter) representations will exist in the level-two KMA of the symmetric subgroup of the group $G \times G$.

In general, condition (71) severely constrains massless string states transforming as $(R_G(x = 1), R_G'(x = 1))$ of the direct product $G \times G$. For example, for $SU(8) \times SU(8)$ and for $SU(5) \times SU(5)$ constructed from $SU(8) \times SU(8)$ only representations of the form

$$R_{N,N} = ((\underline{N}, \underline{N}) + \text{h.c.}), \qquad ((\underline{N}, \overline{\underline{N}}) + \text{h.c.}), \tag{73}$$

with $h(R_{N,N}) = (N - 1)/N$, where $N = 8$ or 5, respectively, can be massless. For $SO(2N) \times SO(2N)$ massless states are contained only in representations

$$R_{v,v} = (\underline{2N}, \underline{2N}) \tag{74}$$

with $h(R_{v,v}) = 1$. Thus, for the GUSTs based on a diagonal subgroup $G^{\,sym} \subset G \times G$, $G^{\,sym}$ high dimensional representations, which are embedded in $R_G(x = 1) \times R_G'(x = 1)$ are also severely constrained by condition (71).

For spontaneous breaking of $G \times G$ gauge symmetry down to $G^{\,sym}0$ (rank $G^{\,sym}$ = rank G) one can use the direct product of representations $R_G(x = 1) \times R_G(x = 1)$, where $R_G(x = 1)$ is the fundamental representation of $G = SU(N)$ or vector representation of $G = SO(2N)$. Furthermore, $G^{\,sym} \subset G \times G$ can subsequently be broken down to a smaller dimension gauge group (of the same rank as $G^{\,sym}$) through the VEVs of the adjoint representations which can appear as a result of $G \times G$ breaking. Alternatively, the real Higgs superfields (73) or (74) can directly break the $G \times G$ gauge symmetry down to a $G_1^{\,sym} \subset G^{\,sym}$ (rank $G_1^{\,sym} \leq$ rank $G^{\,sym}$). For example when $G = SU(5) \times U(1)$ or $SO(10) \times U(1)$, $G \times G$ can directly be broken in this way down to $SU(3^c) \times G^{\rm I}_{EW} \times G^{\rm II}_{EW} \times \ldots$.

The above examples show clearly, that within the framework of GUSTs with the KMA one can get interesting gauge symmetry breaking chains including the realistic ones when $G \times G$ gauge symmetry group is considered. However the lack of the higher dimension representations (which are forbidden by 71) on the level-two KMA prevents the construction of the realistic fermion mass matrices. That is why we consider an extended grand unified string model of rank eight $SO(16)$ or $E(6) \times SU(3)$ of $E(8)$.

The full chiral $SO(10) \times SU(3) \times U(1)$ matter multiplets can be constructed from $SU(8) \times U(1)$-multiplets

$$(\underline{8} + \underline{56} + \underline{\overline{8}} + \underline{\overline{56}}) = \underline{128} \quad (75)$$

of $SO(16)$. In the 4-dimensional heterotic superstring with free complex world-sheet fermions, in the spectrum of the Ramond sector there can appear also representations which are factros in the decomposition of $\underline{128'}$, in particular, $SU(5)$-decouplets $(\underline{10} + \underline{\overline{10}})$ from $(\underline{28} + \underline{\overline{28}})$ of $SU(8)$. However their $U(1)_5$ hypercharge does not allow one to use them for $SU(5) \times U(1)_5$-symmetry breaking. Thus, in this approach we have only singlet and $(\underline{5} + \underline{\overline{5}})$ Higgs fields which can break the grand unified $SU(5) \times U(1)$ gauge symmetry. Therefore it is necessary (as we already explained) to construct rank eight GUST based on a diagonal subgroup $G^{\mathrm{sym}} \subset G \times G$ primordial symmetry group, where in each rank eight group G the Higgs fields will appear only in singlets and in the fundamental representations as in (see 73).

A comment concerning $U(1)$ factors can be made here. Since the available $SU(5) \times U(1)$ decouplets have non-zero hypercharges with respect to $U(1)_5$ and $U(1)_H$, these $U(1)$ factors may remain unbroken down to the low energies in the model considered which seems to be very interesting.

5. MODULAR INVARIANCE IN GUST CONSTRUCTION WITH NON-ABELIAN GAUGE FAMILY SYMMETRY

5.1. Spin-Basis in Free World-Sheet Fermion Sector. The GUST model is completely defined by a set Ξ of spin boundary conditions for all these world-sheet fermions (see Appendix B). In a diagonal basis the vectors of Ξ are determined by the values of phases $\alpha(f) \in (-1, 1]$, fermions f acquire $(f \to -\exp(i\pi\alpha(f))\, f)$ when parallel transported around the string.

To construct the GUST according to the scheme outlined at the end of the previous section we consider three different bases each of them with six elements $B = b_1, b_2, b_3, b_4 \equiv S, b_5, b_6$ (See Tables 2, 5, 8).

Following [15] (see Appendix B) we construct the canonical basis in such a way that the vector $\bar{1}$, which belongs to Ξ, is the first element b_1 of the basis. The basis vector $b_4 = S$ is the generator of supersymmetry [16] responsible for the conservation of the space-time SUSY.

In this chapter we have chosen a basis in which all left movers $(\psi_\mu; \chi_i, y_i, \omega_i; i = 1,\ldots 6)$ (on which the world-sheet supersymmetry is realized nonlinearly) as well as 12 right movers $(\bar{\varphi}_k; k = 1,\ldots 12)$ are real whereas (8 + 8) right movers $\overline{\Psi}_A, \overline{\Phi}_M$ are complex. Such a construction corresponds to $SU(2)^6$ group of automorphisms of the left supersymmetric sector of a string. Right- and left-moving real fermions can be used for breaking $G^{\rm comp}$ symmetry [16]. In order to have a possibility to reduce the rank of the compactified group $G^{\rm comp}$, we have to select the spin boundary conditions for the maximal possible number, $N_{LR} = 12$, of left-moving, $\chi_{3,4,5,6}$, $y_{1,2,5,6}$, $\omega_{1,2,3,4}$, and right-moving, $\bar{\phi}^{1,\ldots 12}$ ($\bar{\phi}^{p} = \overline{\varphi}_p$, $p = 1,\ldots 12$) real fermions. The KMA based on 16 complex right-moving fermions gives rise to the «observable» gauge group $G^{\rm obs}$ with:

$$\text{rank}\,(G^{\rm obs}) \leq 16. \tag{76}$$

The study of the Hilbert spaces of the string theories is connected to the problem of finding all possible choices of the GSO coefficients $C\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$ (see Appendix B), such that the one-loop partition function

$$Z = \sum_{\alpha,\,\beta} C\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \prod_f Z\begin{bmatrix}\alpha_f\\ \beta_f\end{bmatrix} \tag{77}$$

and its multiloop counterparts are all modular invariant. In this formula $C\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$ are GSO coefficients, α and β are $(k + l)$-component spin-vectors $\alpha = [\alpha(f_1^r),\ldots, \alpha(f_k^r); \alpha(f_1^c),\ldots, \alpha(f_l^c)]$, the components α_f, β_f specify the spin structure of the f-th fermion and $Z[\ldots]$ — corresponding one-fermion partition functions on torus: $Z[\ldots] = \text{Tr}\exp[2\pi i H_{(\rm sect)}]$.

The physical states in the Hilbert space of a given sector α are obtained acting on the vacuum $|\,0\,\rangle_\alpha$ with the bosonic and fermionic operators with frequencies

$$n(f) = 1/2 + 1/2\alpha(f), \qquad n(f^*) = 1/2 - 1/2\alpha(f^*) \tag{78}$$

and subsequently applying the generalized GSO projections. The physical states satisfy the Virasoro condition:

$$M_L^2 = -\,1/2 + 1/8(\alpha_L\cdot\alpha_L) + N_L = -\,1 + 1/8(\alpha_R\cdot\alpha_R) + N_R = M_R^2, \tag{79}$$

where $\alpha = (\alpha_L, \alpha_R)$ is a sector in the set Ξ, $N_L = \sum_L$ (frequencies) and $N_R = \sum_R$ (freq.).

We keep the same sign convention for the fermion number operator F as in [16]. For complex fermions we have $F_\alpha(f) = 1$, $F_\alpha(f^*) = -1$ with the exception of the periodic fermions for which we get $F_{\alpha=1}(f) = -1/2(1-\gamma_{5f})$, where $\gamma_{5f}|\Omega\rangle = |\Omega\rangle$, $\gamma_{5f} b_0^+|\Omega\rangle = -b_0^+|\Omega\rangle$.

The full Hilbert space of the string theory is constructed as a direct sum of different sectors $\sum_i m_i b_i$, $(m_i = 0, 1, \ldots, N_i)$, where the integers N_i define additive groups $Z(b_i)$ of the basis vectors b_i. The generalized GSO projection leaves in sectors α those states, whose b_i-fermion number satisfies:

$$\exp(i\pi b_i F_\alpha) = \delta_\alpha C^*\begin{bmatrix}\alpha\\ b_i\end{bmatrix}, \tag{80}$$

where the space-time phase $\delta_\alpha = \exp(i\pi\alpha(\psi_\mu))$ is equal to -1 for the Ramond sector and $+1$ for the Neveu-Schwarz sector.

5.2. $SU(5)\times U(1)\times SU(3)\times U(1)$-Model 1. Model 1 is defined by 6 basis vectors given in Table 2 which generates the $Z_2\times Z_4\times Z_2\times Z_2\times Z_8\times Z_2$ group under addition.

Table 2. Basis of the boundary conditions for all world-sheet fermins. Model 1

Vectors	$\psi_{1,2}$	$\chi_{1,\ldots,6}$	$y_{1,\ldots,6}$	$\omega_{1,\ldots,6}$	$\bar\varphi_{1,\ldots,12}$	$\Psi_{1,\ldots,8}$	$\Phi_{1,\ldots,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1/2^8$	0^8
b_3	11	111100	000011	000000	$0^4\ 1^8$	0^8	1^8
$b_4 = S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	001100	000000	110011	1^{12}	$1/4^5 - 3/4^3$	$-1/4^5\ 3/4^3$
b_6	11	110000	000011	001100	$1^2\ 0^4\ 1^6$	1^8	0^8

In our approach the basis vector b_2 is constructed as a complex vector with the $1/2$ spin-boundary conditions for the right-moving fermions Ψ_A, $A = 1,\ldots 8$. Initially it generates chiral matter fields in the $\underline{8}+\underline{56}+\overline{\underline{56}}+\overline{\underline{8}}$ representations of $SU(8)\times U(1)$, which subsequently are decomposed under $SU(5)\times U(1)\times SU(3)\times U(1)$ to which $SU(8)\times U(1)$ gets broken by applying the b_5 GSO projection.

Generalized GSO projection coefficients are originally defined up to fifteen signs but some of them are fixed by the supersymmetry conditions. Below, in Table 3, we present a set of numbers

$$\gamma\begin{bmatrix} b_i \\ b_j \end{bmatrix} = \frac{1}{i\pi} \log C \begin{bmatrix} b_i \\ b_j \end{bmatrix},$$

which we use as a basis for our GSO projections.

Table 3. The choice of the GSO basis $\gamma[b_i, b_j]$. Model 1 (i, numbers rows; and j, columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	1	1	1	0
b_2	1	1/2	0	0	1/4	1
b_3	1	– 1/2	0	0	1/2	0
b_4	1	1	1	1	1	1
b_5	0	1	0	0	– 1/2	0
b_6	0	0	0	0	1	1

In our case of the $Z_2^4 \times Z_4 \times Z_8$ model, we initially have 256×2 sectors. After applying the GSO-projections we get only 49×2 sectors containing massless states, which depending on the vacuum energy values, $E_L^{\rm vac}$ and $E_R^{\rm vac}$, can be naturally divided into some classes and which determine the GUST representations.

Generally RNS (Ramond — Neveu — Schwarz) sector (built on vectors b_1 and $S = b_4$) has high symmetry including $N = 4$ supergravity and gauge $SO(44)$ symmetry. Corresponding gauge bosons are constructed as follows:

$$\psi^{\mu}_{1/2} \mid 0 \rangle_L \otimes \Psi^I_{1/2} \Psi^J_{1/2} \mid 0 \rangle_R,$$

$$\psi^{\mu}_{1/2} \mid 0 \rangle_L \otimes \Psi^I_{1/2} \Psi^{*J}_{1/2} \mid 0 \rangle_R, \qquad I, J = 1,\ldots, 22. \tag{81}$$

While $U(1)_J$ charges for Cartan subgroups are given by formula $Y = \frac{\alpha}{2} + F$ (where F — fermion number, see (80)), it is obvious that states (81) generate root lattice for $SO(44)$:

$$\pm \varepsilon_I \pm \varepsilon_J \ (I \neq J); \qquad \pm \varepsilon_I \mp \varepsilon_J . \tag{82}$$

The other vectors break $N = 4$ SUSY to $N = 1$ and gauge group $SO(44)$ to $SO(2)^3_{1,2,3} \times SO(6)_4 \times [SU(5) \times U(1) \times SU(3)_H \times U(1)_H]^2$, see Figure 1.

$$\begin{array}{llcccccc} \text{N=2 SUSY}: & V & = & (1,\tfrac{1}{2}) & + & (\tfrac{1}{2},0) & & SU(8) \\ \Downarrow & & & & & & & \Downarrow \\ \text{N=1 SUSY}: & V_{N=2} & \rightarrow & V_{N=1} & + & S_{N=1} & & SU(5)\times SU(3)\times U(1) \end{array}$$

	J=1	J=1/2	J=1/2	J=0
E_{vac} = [−1/2; −1] NS sector	(63)	—	—	(63)
E_{vac} = [0; −1] SUSY sector		(63) × 2	(63) × 2	—
	Gauge multiplets			

⇓ b_5 GSO projection

	J=1	J=1/2	J=1/2	J=0
E_{vac} = [−1/2; −1] NS sector	(24,1)+(1,1)+(1,8)	—	—	$(5,\bar{3})+(\bar{5},3)$
E_{vac} = [0; −1] SUSY sector		((24,1)+(1,1)+(1,8))× 2	$((5,\bar{3})+(\bar{5},3))\times 2$	—
	Gauge multiplets		Higgs multiplets	

Fig.2. Supersymmetry breaking

Generally, additional basis vectors can generate extra vector bosons and extend gauge group that remains after applying GSO-projection to RNS-sector. In our case dangerous sectors are: $2b_2 + nb_5$, $n = 0, 2, 4, 6$; $2b_5$; $6b_5$. But our choice of GSO coefficients cancels all the vector states in these sectors. Thus gauge bosons in this model appear only from RNS-sector.

In NS sector the b_3 GSO projection leaves $(5, \bar{3}) + (\bar{5}, 3)$ Higgs superfields (see Figure 2):

$$\chi^{1,2}_{1/2}|\,\Omega\,\rangle_L \otimes \Psi^a_{1/2}\,\Psi^{i*}_{1/2};\ \Psi^{a*}_{1/2}\,\Psi^i_{1/2}|\,\Omega\,\rangle_R \tag{83}$$

and exchange $\Psi \rightarrow \Phi$, where $a, b = 1,\ldots, 5$, $i, j = 1, 2, 3$.

Four $(3_H + 1_H)$ generations of chiral matter fields from $(SU(5)\times SU(3))_I$ group forming $SO(10)$-multiplets $(\underline{1}, \underline{3}) + (\underline{\bar{5}}, \underline{3}) + (\underline{10}, \underline{3})$; $(\underline{1}, \underline{1}) + (\underline{\bar{5}}, \underline{1}) + (\underline{10}, \underline{1})$ are contained in b_2 and $3b_2$ sectors. Applying b_3 GSO projection to the $3b_2$ section yields the following massless states:

$$b^+_{\Psi_{12}}\,b^+_{\chi_{34}}\,b^+_{\chi_{56}}\,|\,\Omega\,\rangle_L \otimes \left\{\Psi^{i*}_{3/4},\ \Psi^a_{1/4}\Psi^b_{1/4}\Psi^c_{1/4},\ \Psi^a_{1/4}\Psi^i_{1/4}\Psi^j_{1/4}\right\}|\,\Omega\,\rangle_R\,,$$

$$b^+_{\chi_{12}} b^+_{\chi_{34}} b^+_{\chi_{56}} \mid \Omega \rangle_L \otimes \left\{ \Psi^{a*}_{3/4}, \Psi^a_{1/4}\Psi^b_{1/4}\Psi^i_{1/4}, \Psi^i_{1/4}\Psi^j_{1/4}\Psi^k_{1/4} \right\} \mid \Omega \rangle_R \quad (84)$$

with the space-time chirality $\gamma_{5\psi_{12}} = -1$ and $\gamma_{5\psi_{12}} = 1$, respectively. In these formulae the Ramond creation operators $b^+_{\psi_{1,2}}$ and $b^+_{\chi_{\alpha,\beta}}$ of the zero modes are built of a pair of real fermions (as indicated by double indices): $\chi_{\alpha,\beta}$, $(\alpha, \beta) = (1, 2), (3, 4), (5, 6)$. Here, as in (83) indices take values $a, b = 1,\dots, 5$ and $i, j = 1, 2, 3$, respectively.

We stress that without using the b_3 projection we would get matter supermultiplets belonging to real representations only, i.e., «mirror» particles would remain in the spectrum. The b_6 projection instead, eliminates all chiral matter superfields from $U(8)^{\mathrm{II}}$ group.

Since the matter fields form the chiral multiplets of $SO(10)$, it is possible to write down $U(1)_{Y_5}$-hypercharges of massless states. In order to construct the right electromagnetic charges for matter fields we must define the hypercharges operators for the observable $U(8)^{\mathrm{I}}$ group as

$$Y_5 = \int_0^{\pi} d\sigma \sum_a \Psi^{*a}\Psi^a, \qquad Y_3 = \int_0^{\pi} d\sigma \sum_i \Psi^{*i}\Psi^i \quad (85)$$

and analogously for the $U(8)^{\mathrm{II}}$ group.

Then the orthogonal combinations

$$\tilde{Y}_5 = \frac{1}{4}(Y_5 + 5Y_3), \qquad \tilde{Y}_3 = \frac{1}{4}(Y_3 - 3Y_5) \quad (86)$$

play the role of the hypercharge operators of $U(1)_{Y_5}$ and $U(1)_{Y_H}$ groups, respectively. In Table 4 we give the hypercharges $\tilde{Y}^{\mathrm{I}}_5, \tilde{Y}^{\mathrm{I}}_3, \tilde{Y}^{\mathrm{II}}_5, \tilde{Y}^{\mathrm{II}}_3$.

The full list of states in this model is given in Table 4. For fermion states only sectors with positive (left) chirality are written. Superpartners arise from sectors with $S = b_4$-component changed by 1. Chirality under hidden $SO(2)^3_{1,2,3} \times SO(6)_4$ is defined as $\pm_1, \pm_2, \pm_3, \pm_4$, respectively. Lower signs in items 5 and 6 correspond to sectors with components given in brackets.

In the next section we discuss the problem of rank eight GUST gauge symmetry breaking. The matter is that according to the results of section 4 the Higgs fields $(\underline{10}_{1/2} + \overline{\underline{10}}_{-1/2})$ do not appear.

Table 4. The list of quantum numbers of the states. Model 1

No.	$b_1, b_2, b_3,$ b_4, b_5, b_6	SO_{hid}	$U(5)^{\text{I}}$	$U(3)^{\text{I}}$	$U(5)^{\text{II}}$	$U(3)^{\text{II}}$	$\tilde{Y}_5^{\text{I}}$	$\tilde{Y}_3^{\text{I}}$	$\tilde{Y}_5^{\text{II}}$	$\tilde{Y}_3^{\text{II}}$
1	RNS		5	3	1	1	-1	-1	0	0
			1	1	5	$\bar{3}$	0	0	-1	-1
$\hat{\Phi}$	02012(6)0		5	1	5	1	-1	0	-1	0
			1	3	1	3	0	1	0	1
			5	1	1	3	-1	0	0	1
			1	3	5	1	0	1	-1	0
2	010000		1	3	1	1	5/2	$-1/2$	0	0
$\hat{\Psi}$			$\bar{5}$	3	1	1	$-3/2$	$-1/2$	0	0
			10	1	1	1	1/2	3/2	0	0
	030000		1	1	1	1	5/2	3/2	0	0
			$\bar{5}$	1	1	1	$-3/2$	3/2	0	0
			10	3	1	1	1/2	$-1/2$	0	0
3	001130	-1 ± 2	1	1	1	3	0	$-3/2$	0	$-1/2$
	001170	-1 ± 2	1	$\bar{3}$	1	1	0	1/2	0	3/2
$\hat{\Psi}^H$	021130	$+1\pm2$	1	$\bar{3}$	1	3	0	1/2	0	$-1/2$
	021170	$+1\pm2$	1	1	1	1	0	$-3/2$	0	$+3/2$
4	111011	$\mp1\pm3$	1	1	1	$\bar{3}$	0	$-3/2$	0	1/2
	111051	$\mp1\pm3$	1	$\bar{3}$	1	1	0	1/2	0	$-3/2$
$\hat{\Phi}^H$	131011	$\mp1\pm3$	1	$\bar{3}$	1	$\bar{3}$	0	1/2	0	1/2
	131051	$\mp1\pm3$	1	1	1	1	0	$-3/2$	0	$-3/2$
5	01(3)102(6)1	-1 ± 3	1	$3(\bar{3})$	1	1	$\pm5/4$	$\pm1/4$	$\pm5/4$	$\mp3/4$
		$+1\pm3$	$5(\bar{5})$	1	1	1	$\pm1/4$	$\mp3/4$	$\pm5/4$	$\mp3/4$
$\hat{\phi}$	01(3) 1041	-1 ± 3	1	1	1	$3(\bar{3})$	$\pm5/4$	$\mp3/4$	$\pm5/4$	$\pm1/4$
		$+1\pm3$	1	1	$5(\bar{5})$	1	$\pm5/4$	$\mp3/4$	$\pm1/4$	$\mp3/4$
6	12003(5)1	$\pm1-4$	1	1	1	1	$\pm5/4$	$\pm3/4$	$\mp5/4$	$\mp3/4$
$\hat{\sigma}$	11(3)015(3)1	$+1\mp4$	1	1	1	1	$\pm5/4$	$\pm3/4$	$\pm5/4$	$\pm3/4$
	00102(6)0	$\mp3+4$	1	1	1	1	$\pm5/4$	$\mp3/4$	$\pm5/4$	$\mp3/4$

5.3. ***SU*(5) × *U*(1) × *SU*(3) × *U*(1)-Model 2.** Consider then another $[U(5)\times U(3)]^2$ model which after breaking gauge symmetry by Higgs mechanism leads to the spectrum similar to Model 1.

This model is defined by basis vectors given in Table 5 with the $Z_2^4\times Z_6\times Z_{12}$ group under addition.

GSO coefficients are given in Table 6.

Table 5. Basis of the boundary conditions for Model 2

Vectors	$\psi_{1,2}$	$\chi_{1,...,6}$	$y_{1,...,6}$	$\omega_{1,...,6}$	$\bar{\varphi}_{1,...,12}$	$\Psi_{1,...,8}$	$\Phi_{1,...,8}$
b_1	11	1^6	1^6	1^6	1^{12}	1^8	1^8
b_2	11	1^6	0^6	0^6	0^{12}	$1^5\ 1/3^3$	0^8
b_3	11	$1^2\ 0^2\ 1^2$	0^6	$0^2\ 1^2\ 0^2$	$0^8\ 1^4$	$1/2^5\ 1/6^3$	$-1/2^5\ 1/6^3$
$b_4 = S$	11	$1^2\ 0^4$	$0^2\ 1^2\ 0^2$	$0^4\ 1^2$	0^{12}	0^8	0^8
b_5	11	$1^4\ 0^2$	$0^4\ 1^2$	0^6	$1^8\ 0^4$	$1^5\ 0^3$	$0^5\ 1^3$
b_6	11	$0^2\ 1^2\ 0^2$	$1^2\ 0^4$	$0^4\ 1^2$	$1^2\ 0^2\ 1^6\ 0^2$	1^8	0^8

Table 6. The choice of the GSO basis $\gamma[b_i, b_j]$. Model 2 (i, numbers rows; and j, columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	1/2	0	0	0
b_2	0	2/3	− 1/6	1	0	1
b_3	0	1/3	5/6	1	0	0
b_4	0	0	0	0	0	0
b_5	0	1	− 1/2	1	1	1
b_6	0	1	1/2	1	0	1

The given model corresponds to the following chain of the gauge symmetry breaking:

$$E_8^2 \to SO(16)^2 \to U(8)^2 \to [U(5) \times U(3)]^2.$$

Here the breaking of $U(8)^2$-group to $[U(5) \times U(3)]^2$ is determined by basis vector b_5, and the breaking of $N = 2$ SUSY $\to N = 1$ SUSY is determined by basis vector b_6.

It is interesting to note that in the absense of vector b_5 $U(8)^2$ gauge group is restored by sectors $4b_3$, $8b_3$, $2b_2$ + c.c. and $4b_2$ + c.c.

The full massless spectrum for the given model is given in Table 7. By analogy with Table 4 only fermion states with positive chirality are written and obviously vector supermuliplets are absent. Hypercharges are determined by formula:

$$Y_n = \sum_{k=1}^{n} (\alpha_k/2 + F_k).$$

Table 7. The list of quantum numbers of the states. Model 2

№	$b_1, b_2, b_3, b_4, b_5, b_6$	SO_{hid}	$U(5)^{I}$	$U(3)^{I}$	$U(5)^{II}$	$U(3)^{II}$	Y_5^{I}	Y_3^{I}	Y_5^{II}	Y_3^{II}
1	RNS	$6_1\ 2_2$	1	1	1	1	0	0	0	0
		$2_3\ 2_4$	1	1	1	1	0	0	0	0
			5	1	$\bar{5}$	1	1	0	− 1	0
	004100		1	3	1	3	0	− 1	0	− 1
	008100		1	$\bar{3}$	1	$\bar{3}$	0	1	0	1
2	010000		5	$\bar{3}$	1	1	− 3/2	− 1/2	0	0
			1	$\bar{3}$	1	1	5/2	− 1/2	0	0
	030000		$\overline{10}$	1	1	1	1/2	3/2	0	0
3	0110000		1	1	$\overline{10}$	3	0	0	1/2	1/2
	036000		1	1	5	1	0	0	− 3/2	− 3/2
			1	1	1	1	0	0	5/2	− 3/2
4	023000	− 3 ± 4	1	3	1	1	− 5/4	− 1/4	5/4	3/4
5	003000	+ 3 ± 4	1	1	$\bar{5}$	1	− 5/4	3/4	1/4	3/4
6	009000	+ 3 ± 4	1	1	5	1	5/4	− 3/4	− 1/4	− 3/4
7	049000	− 3 ± 4	1	$\bar{3}$	1	1	5/4	1/4	− 5/4	− 3/4
8,9	050101	− 1 ± 3	1	3	1	1	0	− 1	0	0
	030101	+ 1 + 3	5	1	1	1	1	0	0	0
		+ 1 − 3	$\bar{5}$	1	1	1	− 1	0	0	0
		− 1 + 3	1	1	5	1	0	0	1	0
		− 1 − 3	1	1	$\bar{5}$	1	0	0	− 1	0
	058101	+ 1 + 3	1	1	1	$\bar{3}$	0	0	0	1
10	033001	+ 1 ± 4	1	1	1	1	− 5/4	3/4	5/4	3/4
11	103001	± 2 − 3	1	1	5	1	− 1/4	3/4	− 5/4	− 3/4
	1211001	± 2 − 3	1	1	1	$\bar{3}$	− 5/4	3/4	− 5/4	1/4
12	109001	± 2 + 3	$\bar{5}$	1	1	1	1/4	− 3/4	5/4	3/4
	149001	± 2 + 3	1	$\bar{3}$	1	1	5/4	1/4	5/4	3/4
13	000111	± 2 + 3	1	1	1	1	0	− 3/2	0	3/2
	020111	± 2 − 3	1	3	1	1	0	1/2	0	3/2
	028111	± 2 − 3	1	1	1	$\bar{3}$	0	− 3/2	0	− 1/2
	048111	± 2 + 3	1	3	1	$\bar{3}$	0	1/2	0	− 1/2
	103111	+ 1 + 3	1	1	1	1	5/4	3/4	− 5/4	3/4
	109111	+ 1 + 3	1	1	1	1	− 5/4	− 3/4	5/4	− 3/4
	133011	− 1 − 3	1	1	1	1	− 5/4	− 3/4	− 5/4	3/4
	139011	− 1 + 3	1	1	1	1	5/4	3/4	5/4	− 3/4

The given model possesses the hidden gauge symmetry $SO(6)_1 \times SO(2)^3_{2,3,4}$. The corresponding chirality is given in column SO_{hid}. The sectors are divided by horizontal lines and without including the b_5-vector form $SU(8)$-multiplets.

For example, let us consider row No.2. In sectors b_2, $5b_2$ in addition to the states $(1, \bar{3})$ and $(5, \bar{3})$, the state (10, 3) appears, and in sector $3b_2$ besides the state (10, 1), the states (1, 1) and $(\bar{5}, 1)$ survive, too. All these states form $\bar{8} + 56$ representation of the $SU(8)^{\text{I}}$ group.

Analogically we can get the full structure of the theory according to the $U(8)^{\text{I}} \times U(8)^{\text{II}}$-group. (For correct restoration of the $SU(8)^{\text{II}}$-group we must invert 3 and $\bar{3}$ representations).

In Model 2 matter fields appear both in $U(8)^{\text{I}}$ and $U(8)^{\text{II}}$ groups. This is the main difference between this model and Model 1. However, note that in the Model 2 similarly to the Model 1 all gauge fields appear in RNS-sector only and $10 + \overline{10}$ representation (which can be the Higgs field for gauge symmetry breaking) is absent.

5.4. $SO(10) \times SU(4)$-Model 3. As an illustration we can consider the GUST construction involving $SO(10)$ as GUT gauge group. We consider the set consisting of six vectors $B = b_1, b_2, b_3, b_4 \equiv S, b_5, b_6$ given in Table 8.

Table 8. Basis of the boundary conditions for the Model 3

Vectors	$\psi_{1,2}$	$\chi_{1,\dots,6}$	$y_{1,\dots,6}$	$\omega_{1,\dots,6}$	$\bar{\varphi}_{1,\dots,12}$	$\Psi_{1,\dots,8}$	$\Phi_{1,\dots,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1^5\ 1/3^3$	0^8
b_3	11	000000	111111	000000	$0^8\ 1^4$	$0^5\ 1^3$	$0^5\ 1^3$
$b_4 = S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	111111	000000	000000	0^{12}	0^8	$1^5\ 1/3^3$
b_6	11	001100	110000	000011	$1^2\ 0^2\ 1^6\ 0^2$	1^8	0^8

GSO projections are given in Table 9. It is interesting to note that in this model the horizontal gauge symmetry $U(3)$ extends to $SU(4)$. Vector bosons which are needed for this appear in sectors $2b_2(4b_2)$ and $2b_5(4b_5)$. For further breaking $SU(4)$ to $SU(3) \times U(1)$ we need an additional basis spin-vector.

So, the given model possesses gauge group $G^{\text{comp}} \times [SO(10) \times SU(4)]^2$ and matter fields appear both in the first and in the second group symmetrically.

Table 9. The choice of the GSO basis $\gamma[b_i, b_j]$. Model 3 (i, numbers rows; and j, columns)

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	0	0	1	0
b_2	0	2/3	1	1	1	1
b_3	0	1	0	1	1	1
b_4	0	0	0	0	0	0
b_5	0	1	1	1	2/3	0
b_6	0	1	0	1	1	1

Sectors $3b_2$ and $5b_2$ + c.c. give the matter fields $(\underline{16}, \underline{4}; \underline{1}, \underline{1})$ (first group) and sectors $3b_5$ and $5b_5$ + c.c. give the matter fields $(\underline{1}, \underline{1}; \underline{16}, \underline{4})$ (second group).

Of course for getting a realistic model we must add some basis vectors which give additional GSO-projections.

The condition of generation chirality in this model results in the choice of Higgs fields as vector representations of $SO(10)$ ($\underline{16} + \overline{16}$ are absent). According to conclusion (74) the only Higgs fields $(\underline{10}, \underline{1}; \underline{10}, \underline{1})$ of $(SO(10) \times SU(4))^{\times 2}$ appear in the model (from RNS-sector) which can be used for GUT gauge symmetry.

6. MORE EXPLICIT METHODS OF MODEL BUILDING

In the previous models we had to guess how to obtain certain algebra representation and select boundary conditions vectors and GSO coefficients basing only on basis building rules. Below we will develop some methods that help to build models for more complicated cases such as $E_6 \times SU(3)$ and $SU(3) \times SU(3) \times SU(3) \times SU(3)$.

6.1. Normalization of Algebra Roots. As is known, $U(1)$ eigenvalue when acting on state in sector α is $\alpha_i / 2 + F_i$. So square of a root represented by the state in sector α is $\sum_i (\alpha_i / 2 + F_i)$.

Consider then a mass condition. It reads (for right mass only)

$$M_R^2 = -1 + \frac{1}{8}(\alpha_R \cdot \alpha_R) + N_R = 0.$$

In general we can write n_f as

$$n(f) = F^2 \frac{1 + F\alpha(f)}{2} = \frac{F^2}{2} + F\frac{\alpha}{2}$$

for any $F = 0, \pm 1$ ($F^3 = F$ for that values).

Now M_R^2 formulae read

$$M_R^2 = -1 + \frac{1}{8}\sum_{i=1}^{22}(\alpha_i^2) + \sum_{i=1}^{22}\left(\frac{F_i^2}{2} + F_i\frac{\alpha_i}{2}\right).$$

Hence

$$2 = \sum_{i=1}^{22}\left(\frac{\alpha}{2} + F\right)^2.$$

Clearly it is the square of algebra root and it equals 2 for any massless state. Obviously for massive states normalization will differ from that.

On the other hand, if we desire to obtain gauge group like $G^{\rm I} \times G^{\rm II} \times G^{\rm hid}$, then sectors that give gauge group should be like this

$$(0^{10} \mid \alpha^{\rm I}\;\; 0^8\;\; 0^6)\;\; \text{for}\;\; G^{\rm I},$$

$$(0^{10} \mid 0^8\;\; \alpha^{\rm II}\;\; 0^6)\;\; \text{for}\;\; G^{\rm II},$$

$$(0^{10} \mid 0^8\;\; 0^8\;\; \alpha^{\rm hid})\;\; \text{for}\;\; G^{\rm hid},$$

where we assume that both of $G^{\rm I,\,II}$ are rank eight groups (| divides left and right movers). With other vectors we will get roots that mix some of our algebras.

With all this in mind, we can develop some methods of building GSO-projectors (vectors that apply appropriate GSO-projection on the states in order to obtain certain representation) for several interesting cases.

6.2. Building GSO-Projectors for a Given Algebra. As we follow certain breaking chain of E_8, then it is very natural to take E_8 construction as a starting point. Note that root lattice of E_8 arises from two sectors: NS sector gives $\underline{120}$ of $SO(16)$ while sector with 1^8 gives $\underline{128}$ of $SO(16)$. This corresponds to the following choice of simple roots

$$\pi_1 = -e_1 + e_2,$$

$$\pi_2 = -e_2 + e_3,$$

$$\pi_3 = -e_3 + e_4,$$

$$\pi_4 = -e_4 + e_5,$$

$$\pi_5 = -e_5 + e_6,$$

$$\pi_6 = - e_6 + e_7,$$

$$\pi_7 = - e_7 + e_8,$$

$$\pi_8 = \frac{1}{2}(e_1 + e_2 + e_3 + e_4 + e_5 - e_6 - e_7 - e_8).$$

Basing on this choice of roots it is very clear how to build basis of simple roots for any subalgebra of E_8. One can just find out appropriate vectors π_i of the form as in E_8 with needed scalar products or build weight diagram and break it in a desirable fashion to find roots corresponding to certain representation in terms of E_8 roots.

After the basis of simple roots is written down one can build GSO-projectors in a following way.

GSO-projection is defined by operator $(b_i{\cdot}F)$ acting on given state. The goal is to find those b_i that allow only states from algebra lattice to survive. Note that $F_i = \gamma_i - \alpha_i/2$ (γ_i — components of a root in basis of e_i), so value of GSO-projector for sector α depends on γ_i only. So, if scalar products of all simple roots that arise from a given sector with vector b_i is equal to mod 2, then they surely will survive GSO-projection. Taking several such vectors b_i one can eliminate all extra states that do not belong to a given algebra.

Suppose that simple roots of the algebra are in the form

$$\pi_i = \frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5 \pm e_6 \pm e_7 \pm e_8),$$

$$\pi_j = (\pm e_k \pm e_m).$$

In this choice we have to find vectors b which gives 0 or 1 in a scalar product with all simple roots. Note that $(b{\cdot}\pi_i) = (b{\cdot}\pi_j)$ mod 2 for all i, j so $c_i = (b{\cdot}\pi_i)$ either all equal 0 mod 2 or equal 1 mod 2 (for $\pi_j = (\pm e_k \pm e_m)$ it should be 0 mod 2 because they arise from NS sector). Value 0 or 1 is taken because if root $\pi \in$ algebra lattice, then $-\pi$ is a root also. With such choice of simple roots and scalar products with b all states from sector like 1^8 will have the same projector value. Roots like $\pm e_i \pm e_j$ rise from NS sector and are sum of roots like $\pi_i = \frac{1}{2}(\pm e_1 \pm e_2 \pm e_3 \pm e_4 \pm e_5 \pm e_6 \pm e_7 \pm e_8)$ and therefore have scalar products equal to 0 mod 2 as is needed for NS sector.

Now vectors b are obtained very simple. Consider

$$c_i = (b{\cdot}\pi_i) = b_j A_{ji},$$

where $A_{ji} = (\pi_i)_j$ — matrix of roots component in e_j basis. Hence $b = A^{-1} \cdot c$, where either all $c_j = 0$ mod 2 or $c_j = 1$ mod 2. One has to try some combination of c_j to obtain appropriate set of b. The next task is to combine those b_i that satisfy modular invariance rules and do not give extra states to the spectrum.

6.3. Breaking Given Algebra Using GSO-Projectors. It appears that this method of constructing GSO-projectors allows one to break a given algebra down to its subalgebra.

Consider root system of a simple Lie algebra. It is well known that if $\pi_1, \pi_2 \in \Delta$, where Δ is a set of positive roots, then $(\pi_1 - \pi_2(\pi_1 \cdot \pi_2)) \in \Delta$ also. For simply laced algebras it means that if π_i , $\pi \in \Delta$ and $(\pi_i \cdot \pi) = -1$, where π_i is a simple root, then $\pi + \pi_i$ is a root also. This rule is hold automatically in string construction: if a sector gives some simple roots, then all roots of algebra and only they also exist (but part of them may be found in another sector). Because square of every root represented by a state is 2, then if $(\pi_i \cdot \pi) \neq -1$, then $(\pi + \pi_i)^2 \neq 2$. So one must construct GSO-projectors checking only simple roots. On the other hand, if one cuts out some of simple roots, then algebra will be broken. For example, if a vector b has non-integer scalar product with simple root π_1 of E_6, then we will obtain algebra $SO(10) \times U(1)$ $((b \cdot \pi_1)$ even could be 1 if other products are equals 0 mod 2).

More complicated examples are $E_6 \times SU(3)$ and $SU(3) \times SU(3) \times SU(3) \times$ $\times SU(3)$. For the former we must forbid the π_2 root but permit it to form $SU(3)$ algebra. Note that in E_8 root system there are two roots with $3\pi_2$. We will use them for $SU(3)$. So the product $(b \cdot \pi_2)$ must be 2/3 while others must be 0 mod 2.

We can also get GSO-projectors for all interesting subgroups of E_8 in such a way, but so far choosing of constant for scalar products (c_i in a previous subsection) is rather experimental, so it is more convenient to follow certain breaking chain.

Below we will give some results for $E_6 \times SU(3)$, $SU(3) \times SU(3) \times SU(3) \times$ $\times SU(3)$ and $SO(10) \times U(1) \times SU(3)$. We will give algebra basis and vectors that give GSO-projection needed for obtaining this algebra.

$E_6 \times SU(3)$. This case follows from E_8 using root basis from a previous subsection and choosing

$$c_i = \left(-2, -\frac{2}{3} ; 0, 2, -2, -2, 2, 0 \right).$$

This gives GSO-projector of the form

$$b_1 = \left(1, 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right).$$

Basis of simple roots arises from sector with 1^8 in right part and reads

$$\begin{aligned}
\pi_1 &= \frac{1}{2}(+e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8),\\
\pi_2 &= \frac{1}{2}(+e_1 + e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - e_8),\\
\pi_3 &= \frac{1}{2}(+e_1 - e_2 - e_3 - e_4 + e_5 + e_6 - e_7 + e_8),\\
\pi_4 &= \frac{1}{2}(-e_1 + e_2 + e_3 - e_4 - e_5 + e_6 + e_7 - e_8),\\
\pi_5 &= \frac{1}{2}(+e_1 - e_2 + e_3 + e_4 + e_5 - e_6 - e_7 - e_8),\\
\pi_6 &= \frac{1}{2}(-e_1 + e_2 - e_3 + e_4 - e_5 + e_6 - e_7 + e_8),\\
\pi_7 &= \frac{1}{2}(+e_1 - e_2 + e_3 - e_4 - e_5 - e_6 + e_7 + e_8),\\
\pi_8 &= \frac{1}{2}(-e_1 + e_2 - e_3 - e_4 + e_5 - e_6 + e_7 + e_8).
\end{aligned} \tag{87}$$

$SO(10) \times U(1) \times SU(3)$. This case follows from $E_6 \times SU(3)$. In addition to b_1 we must find a vector that cuts out π_3. Using

$$c_i = (0, 0, 1, 0, 0, 0, 0, 0)$$

and inverse matrix of $E_6 \times SU(3)$ basis we get GSO-projector of the form

$$b_2 = \left(0, 0, \frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{2}{3}, \frac{1}{3} \right).$$

Basis of simple roots is the same as for $E_6 \times SU(3)$ excluding π_3.

$SU(3) \times SU(3) \times SU(3) \times SU(3)$. Using $E_6 \times SU(3)$ basis inverse matrix with

$$c_i = \left(1, -1, -1, \frac{1}{3}, 1, \frac{1}{3}, -1, -1 \right)$$

We get GSO-projector of the form

$$b_2 = \left(-\frac{1}{3}, \frac{1}{3}, 1, 1, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, -\frac{1}{3} \right).$$

Easy to see that such a c_i cuts out π_4 and π_6 roots but due to appropriate combination in E_6 root system two $SU(3)$ groups will remain. Basis of simple roots is

$$\begin{aligned}
\pi_1 &= \frac{1}{2}(+e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8),\\
\pi_2 &= \frac{1}{2}(+e_1 + e_2 - e_3 - e_4 - e_5 - e_6 - e_7 - e_8),\\
\pi_3 &= \frac{1}{2}(+e_1 - e_2 + e_3 - e_4 - e_5 - e_6 + e_7 + e_8),\\
\pi_4 &= \frac{1}{2}(-e_1 + e_2 + e_3 - e_4 + e_5 + e_6 - e_7 - e_8),\\
\pi_5 &= \frac{1}{2}(+e_1 - e_2 + e_3 + e_4 + e_5 - e_6 - e_7 - e_8),\\
\pi_6 &= \frac{1}{2}(-e_1 + e_2 - e_3 - e_4 + e_5 - e_6 + e_7 + e_8),\\
\pi_7 &= \frac{1}{2}(-e_1 + e_2 + e_3 + e_4 - e_5 - e_6 + e_7 - e_8),\\
\pi_8 &= \frac{1}{2}(+e_1 - e_2 - e_3 - e_4 + e_5 + e_6 + e_7 - e_8).
\end{aligned} \tag{88}$$

Using all this methods we could construct a model described in the next section.

6.4. $E_6 \times SU(3)$ Three Generations Model — Model 4. This model illustrates a branch of E_8 breaking $E_8 \to E_6 \times SU(3)$ and is an interesting result on a way to obtain three generations with gauge horizontal symmetry. Basis of the boundary conditions (see Table 10) is rather simple but there are some subtle points. In [34] the possible left parts of basis vectors were worked out, see it for details. We just use the notation given in [34] (hat on left part means complex fermion, other fermions on the left sector are real, all of the right movers are complex) and an example of commuting set of vectors.

A construction of an $E_6 \times SU(3)$ group caused us to use rational left boundary conditions. It seems that it is the only way to obtain such a gauge group with appropriate matter contents.

The model has $N = 2$ SUSY. We can also construct model with $N = 0$ but according to [34] using vectors that can give rise to $E_6 \times SU(3)$ (with realistic matter fields) one cannot obtain $N = 1$ SUSY.

Table 10. Basis of the boundary conditions for the Model 4

Vectors	$\psi_{1,2}$	$\chi_{1,\dots,9}$	$\omega_{1,\dots,9}$	$\bar{\varphi}_{1,\dots,6}$	$\Psi_{1,\dots,8}$	$\Phi_{1,\dots,8}$
b_1	11	1^9	1^9	1^6	1^8	1^8
b_2	11	$\widehat{\frac{1}{3}}, 1; -\widehat{\frac{2}{3}}, 0,0, \widehat{\frac{2}{3}}$	$\widehat{\frac{1}{3}}, 1; -\widehat{\frac{2}{3}}, 0,0, \widehat{\frac{2}{3}}$	$\frac{2^3}{3} - \frac{2^3}{3}$	$0^2 - \frac{2^6}{3}$	$1^2 \, \frac{1^6}{3}$
b_3	00	0^9	0^9	0^6	1^8	0^8
b_4	11	$\hat{1}, 1; \hat{0}, 0, 0, \hat{0}$	$\hat{1}, 1; \hat{0}, 0, 0, \hat{0}$	0^6	0^8	0^8

Table 11. The choice of the GSO basis $\gamma[b_i, b_j]$. Model 4. (i, numbers rows; and j, columns)

	b_1	b_2	b_3	b_4
b_1	0	1/3	1	1
b_2	1	1	1	1
b_3	1	1	0	1
b_4	1	1/3	1	1

Let us give a brief review of the model contents. First notice that all superpartners of states in sector α are found in sector $\alpha + b_4$ as in all previous models. Although the same sector may contain, say, matter fields and gauginos simultaneously.

The observable gauge group $(SU(3)^{\mathrm{I}}_H \times E^{\mathrm{I}}_6) \times (SU(3)^{\mathrm{II}}_H \times E^{\mathrm{II}}_6)$ and hidden group $SU(6) \times U(1)$ are rising up from sectors NS, b_3 and $3b_2 + b_4$. Matter fields in representations $(\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \overline{\mathbf{27}})$ for each $SU(3)_H \times E_6$ group are found in sectors $3b_2$, $b_3 + b_4$ and b_4. Also there are some interesting states in sectors b_2, $b_2 + b_3$, $2b_2 + b_3 + b_4$, $2b_2 + b_4$ and $5b_2$, $5b_2 + b_3$, $4b_2 + b_3 + b_4$, $4b_2 + b_4$ that form representation $(\bar{\mathbf{3}}, \mathbf{3})$ and $(\mathbf{3}, \bar{\mathbf{3}})$ of the $SU(3)^{\mathrm{I}}_H \times SU(3)^{\mathrm{II}}_H$ group. These states are singlets under both E_6 groups but have nonzero $U(1)_{\text{hidden}}$ charge.

We suppose that the model permits further breaking of E_6 down to other grand unification groups, but problem with breaking supersymmetry $N = 2 \to N = 1$ is a great obstacle on this way.

7. GUST SPECTRUM (MODEL 1)

7.1. Gauge Symmetry Breaking. Let us consider Model 1 in detail. In Model 1 there exists a possibility to break the GUST group $(U(5) \times U(3))^{\mathrm{I}} \times$

$\times (U(5) \times U(3))^{II}$ down to the symmetric group by the ordinary Higgs mechanism [9]:

$$G^{I} \times G^{II} \to G^{sym} \to \ldots \qquad (89)$$

To achieve such breaking one can use nonzero vacuum expectation values of the tensor Higgs fields (see Table 4, row No.1), contained in the $2b_2 + 2(6)b_5(+S)$ sectors which transform under the $(SU(5) \times U(1) \times SU(3) \times \times U(1))^{sym}$ group in the following way:

$$(\underline{5}, \underline{1}; \underline{5}, \underline{1})_{(-1,0;\,-1,0)} \to (\underline{24}, \underline{1})_{(0,0)} + (\underline{1}, \underline{1})_{(0,0)};$$

$$(\underline{1}, \underline{3}; \underline{1}, \underline{3})_{(0,1;\,0,1)} \to (\underline{1}, \underline{8})_{(0,0)} + (\underline{1}, \underline{1})_{(0,0)}, \qquad (90)$$

$$(\underline{5}, \underline{1}; \underline{1}, \underline{3})_{(-1,0;\,0,1)} \to (\overline{\underline{5}}, \underline{3})_{(1,1)};$$

$$(\underline{1}, \underline{3}; \underline{5}, \underline{1})_{(0,1;\,-1,0)} \to (\underline{5}, \overline{\underline{3}})_{(-1,-1)}. \qquad (91)$$

The diagonal vacuum expectation values for Higgs fields (90) break the GUST group $(U(5) \times U(3))^{I} \times (U(5) \times U(3))^{II}$ down to the «skew»-symmetric group with the generators Δ_{sym} of the form:

$$\Delta_{sym}(t) = -t^{*} \times 1 + 1 \times t. \qquad (92)$$

The corresponding hypercharge of the symmetric group reads:

$$\overline{Y} = \tilde{Y}^{II} - \tilde{Y}^{I}. \qquad (93)$$

Similarily, for the electromagnetic charge we get:

$$Q_{em} = Q^{II} - Q^{I} = (T_5^{II} - T_5^{I}) + \frac{2}{5}(\tilde{Y}_5^{II} - \tilde{Y}_5^{I}) = \overline{T}_5 + \frac{2}{5}\overline{Y}_5, \qquad (94)$$

where $T_5 = \mathrm{diag}\left(\frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{2}{5}, -\frac{3}{5}\right)$. Note, that this charge quantization does not lead to exotic states with fractional electromagnetic charges (e.g., $Q_{em} = \pm 1/2, \pm 1/6$).

Thus, in breaking scheme (92) it is possible to avoid colour singlet states with fractional electromagnetic charges, to achieve desired GUT breaking and moreover to get the usual value for the weak mixing angle at the unification scale (see (47)).

Adjoint representations which appear on the r.h.s. of (90) can be used for further breaking of the symmetric group. This can lead to the final physical symmetry

$$(SU(3^c) \times SU(2_{EW}) \times U(1_Y) \times U(1)') \times (SU(3_H) \times U(1_H)) \qquad (95)$$

with low-energy gauge symmetry of the quark-lepton generations with an additional $U(1)'$-factor.

Note, that when we use the same Higgs fields as in (90), there exists also another interesting way of breaking the $C^{\mathrm{I}} \times G^{\mathrm{II}}$ gauge symmetry:

$$G^{\mathrm{I}} \times G^{\mathrm{II}} \to SU(3^c) \times SU(2)^{\mathrm{I}}_{EW} \times SU(2)^{\mathrm{II}}_{EW} \times U(1_{\bar{Y}}) \times$$
$$\times SU(3_H)^{\mathrm{I}} \times SU(3_H)^{\mathrm{II}} \times U(1_{\bar{Y}_H}) \to \ldots \tag{96}$$

It is attractive because it naturally solves the Higgs doublet-triplet mass splitting problem with rather low energy scale of GUST symmetry breaking [30].

In turn, the Higgs fields $\hat{h}_{(\Gamma, N)}$ from the NS sector

$$(\underline{5}, \bar{\underline{3}})_{(-1,-1)} + (\bar{\underline{5}}, \underline{3})_{(1,1)} \tag{97}$$

are obtained from $N = 2$ SUSY vector representation $\underline{63}$ of $SU(8)^{\mathrm{I}}$ (or $SU(8)^{\mathrm{II}}$) by applying the b_5 GSO projection (see Fig.2 and Appendix A). These Higgs fields .(and fields (91)) can be used for constructing chiral fermion (see Table 4, row No.2) mass matrices.

The b spin boundary conditions (Table 2) generate chiral matter and Higgs fields with the GUST gauge symmetry $G_{\mathrm{comp}} \times (G^{\mathrm{I}} \times G^{\mathrm{II}})_{\mathrm{obs}}$ (where $G_{\mathrm{comp}} = U(1)^3 \times SO(6)$ and $G^{\mathrm{I,II}}$ have been already defined). The chiral matter spectrum, which we denote as $\hat{\Psi}_{(\Gamma, N)}$ with ($\Gamma = \underline{1}, \bar{\underline{5}}, \underline{10}$; $N = \underline{3}, \underline{1}$), consists of $N_g = 3_H + 1_H$ families. See Table 4, row No.2 for the $((SU(5) \times U(1)) \times \times (SU(3) \times U(1))_H)^{\mathrm{sym}}$ quantum numbers.

The $SU(3_H)$ anomalies of the matter fields (row No.2) are naturally canceled by the chiral «horizontal» superfields forming two sets: $\hat{\Psi}^H_{(1,N;\,1,N)}$ and $\hat{\Phi}^H_{(1,N;\,1,N)}$, $\Gamma = \underline{1}$, $N = \underline{1}, \underline{3}$, (with both $SO(2)$ chiralities, see Table 4, row No.3, 4, respectively).

The horizontal fields (No.3, 4) cancel all $SU(3)^{\mathrm{I}}$ anomalies introduced by the chiral matter spectrum (No.2) of the $(U(5) \times U(3))^{\mathrm{I}}$ group (due to b_6 GSO projection the chiral fields of the $(U(5) \times U(3))^{\mathrm{II}}$ group disappear from the final string spectrum). Performing the decomposition of fields (No.3) under $(SU(5) \times SU(3))^{\mathrm{sym}}$ we get (among others) three «horizontal» fields $\hat{\Psi}^H$:

$$2 \times (\underline{1}, \bar{\underline{3}})_{(0,-1)}, \quad (\underline{1}, \underline{1})_{(0,-3)}, \quad (\underline{1}, \bar{\underline{6}})_{(0,1)}, \tag{98}$$

coming from $\hat{\Psi}^H_{(\underline{1},\bar{\underline{3}};\underline{1},\underline{1})}$ (and $\hat{\Psi}^H_{(\underline{1},\underline{1};\underline{1},\underline{3})}$), $\hat{\Psi}^H_{(\underline{1},\underline{1};\underline{1},\underline{1})}$ and $\hat{\Psi}^H_{(\underline{1},\bar{\underline{3}};\underline{1},\underline{3})}$, respectively, which make the low energy spectrum of the resulting model (96) $SU(3_H)^{sym}$-anomaly free. The other fields arising from rows No.3, 4, Table 4 form anomaly-free representations of $(SU(3_H) \times U(1_H))^{sym}$:

$$2(\underline{1}, \underline{1})_{(0,0)}, \quad (\underline{1}, \bar{\underline{3}})_{(0,2)} + (\underline{1}, \underline{3})_{(0,-2)}, \quad (\underline{1}, \underline{8})_{(0,0)}. \tag{99}$$

The superfields $\hat{\phi}_{(\Gamma, N)}$ + h.c., where ($\Gamma = \underline{1}, \underline{5}$; $N = \underline{1}, \underline{3}$) from Table 4, row No.5 forming representations of $(U(5) \times U(3))^{I,II}$ have either $Q^{\,I}$ or $Q^{\,II}$ exotic fractional charges. Because of the strong $G^{\,comp}$ gauge forces these fields may develop the double scalar condensate $\langle \hat{\phi}, \hat{\phi} \rangle$, which can also serve for $U(5) \times U(5)$ gauge symmetry breaking. For example, the composite condensate $\langle \hat{\phi}_{(5,1;1,1)}, \hat{\phi}_{(1,1;\bar{5},1)} \rangle$ can break the $U(5) \times U(5)$ gauge symmetry down to the symmetric diagonal subgroup with generators of the form

$$\Delta_{sym}(t) = t \times 1 + 1 \times t, \tag{100}$$

so for the electromagnetic charges we would have the form

$$Q_{em} = Q^{\,II} + Q^{\,I}, \tag{101}$$

leading again to no exotic, fractionally charged states in the low-energy string spectrum.

The superfields which transform nontrivially under the compactified group $G^{\,comp} = SO(6) \times SO(2)^{\times 3}$ (denoted as $\hat{\sigma}$ + h.c.), and which are singlets of $(SU(5) \times SU(3)) \times (SU(5) \times SU(3))$, arise in three sectors, see Table 4, row No.6. The superfields $\hat{\sigma}$ form the spinor representations $\underline{4} + \bar{\underline{4}}$ of $SO(6)$ and they are also spinors of one of the $SO(2)$ groups. They have following hypercharges $\tilde{Y}_5^{I,II}$, $\tilde{Y}_3^{I,II}$:

$$\tilde{Y} = (5/4, \mp 3/4; 5/4, \mp 3/4), \quad \tilde{Y} = (5/4, 3/4; -5/4, -3/4). \tag{102}$$

With respect to the diagonal $G^{\,sym}$ group with generators given by (92) or (100), some fields $\hat{\sigma}$ are of zero hypercharges and can, therefore, be used for breaking the $SO(6) \times SO(2)^{\times 3}$ group.

Note, that for the fields $\hat{\phi}$ and for the fields $\hat{\sigma}$ any other electromagnetic charge quantization different from (94) or (101) would lead to «quarks» and «leptons» with the exotic fractional charges, for example, for the $\underline{5}$- and $\underline{1}$-multiplets according to the values of hypercharges (see eqs. 102) the generator

$Q^{\rm II}$ (or $Q^{\rm I}$) has the eigenvalues $(\pm 1/6, \pm 1/6, \pm 1/6, \pm 1/2, \mp 1/2)$ or $\pm 1/2$, respectively.

Scheme of the breaking of the gauge group to the symmetric subgroup, which is similar to the scheme of Model 1, works for Model 2, too. In this case vector-like multiplets $(\underline{5}, \underline{1}; \underline{\bar{5}}, \underline{1})$ from RNS-sector and $(\underline{1}, \underline{3}, \underline{1}, \underline{3})$ from $4b_3$ $(8b_3)$ play the role of Higgs fields. Then generators of the symmetric subgroup and electromagnetic charges of particles are determined by formulas:

$$\Delta^{(5)}_{\rm sym} = t^{(5)} \times 1 \oplus 1 \times t^{(5)},$$

$$\Delta^{(3)}_{\rm sym} = (-t^{(3)}) \times 1 \oplus 1 \times t^{(3)},$$

$$Q_{em} = t_5^{(5)} - 2/5Y^5, \quad \text{where} \quad t_5^{(5)} = (1/15, 1/15, 1/15, 2/5, -3/5). \qquad (103)$$

After this symmetry breaking matter fields (see Table 7 rows No.2, 3) as usual for flip models take place in representations of the $U(5)$-group and form four generations $(\underline{1} + \underline{5} + \underline{\overline{10}}; \underline{\bar{3}} + \underline{1})_{\rm sym}$. And Higgs fields form adjoint representation of the symmetric group, similar to Model 1, which is necessary for breaking of the gauge group to the Standard group. Besides, due to quantization of the electromagnetic charge according to the formula (103), states with exotic charges in low energy spectrum also do not appear in this model.

7.2. Superpotential. The ability of making a correct description of the fermion masses and mixings will, of course, constitute the decisive criterion for selection of a model of this kind. Therefore, within our approach one has to

1. study the possible nature of the G_H horizontal gauge symmetry $(N_g = 3_H$ or $3_H + 1_H)$,

2. investigate the possible cases for G_H-quantum numbers for quarks (anti-quarks) and leptons (anti-leptons), i.e., whether one can obtain vector-like or axial-like structure (or even chiral $G_{HL} \times G_{HR}$ structure) for the horizontal interactions.

3. find the structure of the sector of the matter fields which are needed for the $SU(3)_H$ anomaly cancelation (chiral neutral «horizontal» or «mirror» fermions),

4. write out all possible renormalizable and relevant non-renormalizable contributions to the superpotential W and their consequences for fermion mass matrices.

All these questions are currently under investigation. Here we restrict ourselves to some general remarks only.

With the chiral matter and «horizontal» Higgs fields available in Model 1 constructed in this paper, the possible form of the renormalizable (trilinear) part of the superpotential responsible for fermion mass matrices is well restricted by the gauge symmetry:

$$W_1 = g\sqrt{2}\Big[\hat{\Psi}_{(1,3)}\hat{\Psi}_{(\bar{5},1)}\hat{h}_{(5,\bar{3})} + \hat{\Psi}_{(1,1)}\hat{\Psi}_{(\bar{5},3)}\hat{h}_{(5,\bar{3})} + \\ + \hat{\Psi}_{(10,3)}\hat{\Psi}_{(\bar{5},3)}\hat{h}_{(\bar{5},3)} + \hat{\Psi}_{(10,3)}\hat{\Psi}_{(10,1)}\hat{h}_{(5,\bar{3})}\Big]. \qquad (104)$$

From the above form of the Yukawa couplings follows that two (chiral) generations have to be very light (comparing to M_W scale).

The construction of realistic quarks and leptons mass matrices depends, of course, on the nature of the horizontal interactions. In the construction described in Sec.5 there is a freedom of choosing spin boundary conditions for $N_R = 12$ right fermions in the basis vectors b_3, b_5, b_6,..., which in the Ramond sector $2b_2$, may yield another Higgs fields, denoted as $\tilde{h}_{(\Gamma,\,N)}$ and transforming as $(\underline{5}, \underline{3})_{(-1,1)} + (\underline{\bar{5}}, \underline{\bar{3}})_{(1,-1)} \subset \underline{28} + \overline{\underline{28}}$ of $SU(8)$. Using these Higgs fields we get the following alternative form of the renormalizable part of the superpotential W:

$$W_1' = g\sqrt{2}\Big[\hat{\Psi}_{(1,3)}\hat{\Psi}_{(\bar{5},3)}\tilde{h}_{(5,3)} + \hat{\Psi}_{(10,1)}\hat{\Psi}_{(\bar{5},3)}\tilde{h}_{(\bar{5},\bar{3})} + \\ + \hat{\Psi}_{(10,3)}\hat{\Psi}_{(10,3)}\tilde{h}_{(5,3)} + \hat{\Psi}_{(10,3)}\hat{\Psi}_{(\bar{5},1)}\tilde{h}_{(\bar{5},\bar{3})}\Big]. \qquad (105)$$

To construct the realistic fermion mass matrices one has to also use Higgs fields (90, 91) and (Table 4, No.5) and also to take into account all relevant non-renormalizable contributions [16].

Higgs fields (90) can be used for constructing Yukawa couplings of the horizontal superfields (No.3 and 4). The most general contribution of these fields to the superpotential is:

$$W_2 = g\sqrt{2}\Big[\hat{\Phi}^H_{(1,1;1,\bar{3})}\hat{\Phi}^H_{(1,\bar{3};1,1)}\hat{\Phi}_{(1,3;1,3)} + \hat{\Phi}^H_{(1,1;1,1)}\hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(1,3;1,3)} + \\ + \hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}^H_{(1,\bar{3};1,\bar{3})}\hat{\Phi}_{(1,\bar{3};1,\bar{3})} + \hat{\Psi}^H_{(1,\bar{3};1,1)}\hat{\Psi}^H_{(1,\bar{3};1,3)}\hat{\Phi}_{(1,\bar{3};1,\bar{3})} + \\ + \hat{\Psi}^H_{(1,1;1,3)}\hat{\Psi}^H_{(1,\bar{3};1,3)}\hat{\Phi}_{(1,3;1,3)}\Big]. \qquad (106)$$

From this expression it follows that some of the horizontal fields in (99) (No.3, 4) remain massless at the tree-level. This is a remarkable prediction: fields (99) interact with the ordinary chiral matter fields only through the $U(1_H)$ and $SU(3_H)$ gauge boson and therefore are very interesting in the context of the experimental searches for the new gauge bosons.

Finally, we remark that the Higgs sector of out GUST allows conservation of the G_H gauge family symmetry down to the low energies (~ $\mathcal{O}$ (1 TeV) [5]). Thus in this energy region we can expect new interesting physics (new gauge bosons, new chiral matter fermions, superweak-like *CP*-violation in *K*-, *B*-, *D*-meson decays with $\delta_{KM} < 10^{-4}$ [5]).

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8. APPENDIX A. SHORT INTRODUCTION TO $N = 2$ SUSY MODELS

Let us consider a gauge interaction of the vector (gauge) hypermultiplet and Fayet-Sohnius (matter) hypermultiplet. On the language of $N = 1$ SUSY these hypermultiplets consist of $N = 1$ superfields as follows:

vector hypermultiplet — vector superfield $\mathbf{V} = (V_m; \lambda)$ and chiral superfield $\mathbf{\Phi} = (N; \phi)$;

Fayet-Sohnius hypermultiplet — two chiral superfields: $\mathbf{X} = (X, \psi)$ and mirror $\mathbf{Y} = (Y; \chi)$. (In brackets we have written bosonic components on the first place).

Suppose, we have matter multiplets in some representations of a gauge group G with generators t_a:

$$\mathrm{Tr}\,(t_a t_b) = k\delta_{ab}, \qquad [t_a t_b] = if_{abc}\, t_c.$$

Then $N = 2$ gauge Lagrangian on the language of $N = 1$ superfields looks like

$$\mathcal{L}^{N=2} = \left[\frac{1}{16kg^2}\,\mathrm{Tr}\, W^{\alpha} W_{\alpha}\right]_F + \mathrm{h.c.} + [1/k\ \mathrm{Tr}\,(\mathbf{\Phi}^{+} e^{2g\mathbf{V}}\, \mathbf{\Phi} e^{-2g\mathbf{V}}) +$$

$$+ \mathbf{X}_j^{+} e^{2g\mathbf{V}} \mathbf{X}^{j} + \mathbf{Y}_j^{+} e^{-2g\mathbf{V}^T} \mathbf{Y}^{j}]_D + [i\sqrt{2}g\ \mathbf{Y}_j^{T} \mathbf{\Phi}\, \mathbf{X}^{j}]_F + \mathrm{h.c.}, \qquad (107)$$

where superstrength $W_\alpha = -1/4\bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha} e^{-2g\mathbf{V}} D_\alpha e^{2g\mathbf{V}}$, $\mathbf{V} = \mathbf{V}^a t_a$ and analogically for $\mathbf{\Phi}$. It is interesting that the Yukawa coupling of $N = 2$ theories is entirely determined by the gauge structure.

Under following gauge transformations this Lagrangian is invariant

$$e^{2g\mathbf{V}} \to \exp\,(ig\Lambda_a^{+} t_a)\, e^{2g\mathbf{V}} \exp\,(-ig\Lambda_b t_b),$$

$$\mathbf{\Phi} \to \exp\,(ig\Lambda_a t_a)\, \mathbf{\Phi} \exp\,(-ig\Lambda_b t_b),$$

$$\mathbf{X} \to \exp\,(ig\Lambda_a t_a)\, \mathbf{X}, \qquad \mathbf{Y} \to \exp\,(-ig\Lambda_a t_a^{*})\, \mathbf{Y}. \qquad (108)$$

We can see that ordinary fields (superfields $\mathbf{X}$) and mirror fields (superfields $\mathbf{Y}$) are transformed as mutual conjugating representations of a gauge group.

After excluding of auxilary fields the Lagrangian $\mathcal{L}^{N=2}$ looks like (we suppose that $k = 1/2$):

$$\mathcal{L}^{N=2} = \mathrm{Tr}\,(-1/2V_{mn}V^{mn} - 2i\,\lambda\sigma^m\nabla_m\bar\lambda + 2N^{+}\nabla^2 N + 2i\,\nabla_m\phi\sigma^m\bar\phi +$$

$$+ 2\sqrt{2}ig\lambda[\phi N^{+}] + 2\sqrt{2}ig\bar\lambda[\bar\phi N] - g^2[NN^{+}]^2) +$$

$$+ X^{+}\nabla^2 X + i\nabla_m\bar\psi\,\bar\sigma^m\,\psi + Y^{+}\nabla^2 Y + i\nabla_m\,\bar\chi\,\bar\sigma^m\,\chi +$$

$$+ \sqrt{2}ig(X^{+}\lambda\psi - Y^{+}\lambda^T\chi - \chi^T\phi X - Y^T\phi\psi - \chi^T N\psi) + \mathrm{h.c.} -$$

$$- g^2 X^{+}\{N^{+}N\}X - g^2 Y^T\{NN^{+}\}Y^{*} - g^2/4(X_i^{+}X_j)(X_j^{+}X_i) -$$

$$g^2/4(Y_i^{+}Y_j)(Y_j^{+}Y_i) + g^2/2(Y_j^T X_i)(X_i^{+}Y_j^{*}) - g^2(T_i^T Y_j^{*})(X_j^{+}X_i), \qquad (109)$$

where i and j indices are numbers of matter hypermultiplets. Note, that covariant derivative for $\mathbf{X}$-fields $\nabla_m^X = \partial_m + igV_m$, but for $\mathbf{Y}$-fields $\nabla_m^Y = \partial_m - igV_m^T$. Since the $N = 2$ SUSY is present, this Lagrangian possesses the hidden global internal $SU(2)$ symmetry. The component fields $(\lambda, -\phi)$ and

(X, Y^*) are doublets under internal $SU(2)$ group and remaining fields are singlets.

The attractive feature of the $N = 2$ SUSY theory is that the β-function differs from zero at the one-loop level only..

$$\beta(g_A) = \frac{g^3}{8\pi^2}\left(\sum_\sigma T_A(R_\sigma) - C_2(G_A)\right), \tag{110}$$

where

$$f_{ijk}\, f_{ljk} = C_2(G)\,\delta_{il}\,, \qquad \mathrm{Tr}\; t_a^{(\sigma)} t_b^{(\sigma)} = T(R_\sigma)\,\delta_{ab},$$

and we suppose that gauge group is like $G = \prod_A \otimes G_A$, and R_σ are representations for chiral superfields.

We can see that this theory can be made finite for some gauge group through a certain choice of representations of matter fields if the following relation is true [31]:

$$C_2(G_A) = \sum_\sigma T_A(R_\sigma). \tag{111}$$

Let us write down representations of some subgroup of E_8 which warrant the finitness of the $N = 2$ theory [32]:

$SU(5)$ __ p, q and r matter multiplets in representations $(5 + \bar{5})$, $(10 + \overline{10})$ and $(15 + \overline{15})$ correspondingly, for which $p + 3q + 7r = 10$.

$SU(10)$ p and q matter multiplets in representations $(10 + 10)$ and $(16 + \overline{16})$ correspondingly for which $p + 2q = 8$.

E_6 4 multiplets in representation $(27 + \overline{27})$.

E_7 3 multiplets in representation $(56 + 56)$.

E_8 one multiplet in the lowest (adjoint) representation $(248 + 248)$. This means presence of $N = 4$ SUSY.

There are five types of soft SUSY-breaking operators, and their addition to the Lagrangian does not destroy the finiteness of the theory [32,33].

1) Any gauge invariant $N = 1$ supersymmetric mass addition. For example $m\,\mathrm{Tr}\,\mathbf{\Phi}^2|_F$ + h.c., $m\,\mathbf{Y}_i^T\mathbf{X}_i|_F$ + h.c. First addition can be written in component fields as

$$\mathrm{Tr}\,(-m\phi\phi - m^*\overline{\phi\phi} - 2|m|^2 N^+N) + i\sqrt{2}gmX^+NY^* - i\sqrt{2}gm^*Y^TN^+X. \tag{112}$$

This addition breaks $N = 2$ to $N = 1$ SUSY.

2) Any gauge invariant masses for scalar fields of view $A^2 - B^2$. We suppose that scalar is $\frac{A + iB}{\sqrt{2}}$.

3) Certain mass terms of view $A^2 + B^2$.

$$U_1 N_a^* N_a + \sum_i (U_2^i X_i^+ X_i + U_3^i Y_i^+ Y_i). \tag{113}$$

If for each $iU_1 + U_2^i + U_3^i = 0$.

4) Certain combination of mass addition and three-linear scalar addition.

$$\mathrm{Tr}\,(- m\lambda\lambda - m^*\overline{\lambda\lambda} - 2|m|^2 N^+ N) - i\sqrt{2} gmY^T NX + i\sqrt{2} gm^* X^+ N^+ Y^*. \tag{114}$$

This combination is simply the addition 1) under the transformation of internal $SU(2)$ group. It breaks $N = 2$ to $N = 1$ SUSY, too.

5) Gauge invariant scalar three-linear operators of view

a) $k^{ijk}(X_i X_j X_k + Y_i^* Y_j^* Y_k^*)$ + h.c.,

b) $k^{ijk}(X_i X_j X_k + Y_i^* Y_j^* Y_k^*)$ + h.c., joint with certain set of scalar mass $A^2 + B^2$ and scalar three-linear addition Tr N^3 + h.c., where X and Y don't lie in adjoint representation and the gauge group must be $SU(n)$ with $n \geq 3$ or their direct production. However, these terms lead to unbounded below potential.

9. APPENDIX B. RULES FOR CONSTRUCTING CONSISTENT STRING MODELS OUT OF FREE WORLD-SHEET FERMIONS

The partition function of the theory is a sum over terms corresponding to world-sheets of different genus g. For consistence of the theory we must require that partition function to be invariant under modular transformation, which is reparametrization not continuously connected to the identity. For this we must sum over the different possible boundary conditions for the world-sheet fermions with appropriate weights [35].

If the fermions are parallel transported around a nontrivial loop of a genus-g world-sheet M_g, they must transform into themselves:

$$\chi^I \to L_g(\alpha)^I_J \chi^J \tag{115}$$

and similar for the right-moving fermions. The only constraints on $L_g(\alpha)$ and $R_g(\alpha)$ are that it be orthogonal matrix representation of $\pi_1(M_g)$ to leave the energy-momentum current invariant and supercharge (32) invariant up to a sign. It means that

$$\psi^\mu \to - \delta_\alpha \psi^\mu, \qquad \delta_\alpha = \pm 1, \tag{116}$$

$$L^I_{gI'}\, L^J_{gJ'}\, L^K_{gK'}\, f_{IJK} = -\,\delta_\alpha f_{I'J'K'} \tag{117}$$

and consequently $-\delta_\alpha L_g(\alpha)$ is an automorphism of the Lie algebra of G.

Further, the following restrictions on $L_g(\alpha)$ and $R_g(\alpha)$ are imposed:

(a) $L_g(\alpha)$ and $R_g(\alpha)$ are abelian matrix representations of $\pi_1(M_g)$. Thus all of the $L_g(\alpha)$ and all of the $R_g(\alpha)$ can be simultaneously diagonalized in some basis.

(b) There is commutativity between the boundary conditions on surfaces of different genus.

When all of the $L(\alpha)$ and $R(\alpha)$ have been simultaneously diagonalized the transformations like (116) can be written as

$$f \to -\exp\,(i\pi\alpha_f)\, f. \tag{118}$$

Here and in eqs.(117), (118) the minus signs are conventional.

The boundary conditions (116), (117) are specified in this basis by a vector of phases

$$\alpha = [\alpha(f_1^L),\ldots,\alpha(f_k^L) \mid \alpha(f_1^R),\ldots,\alpha(f_l^R)]. \tag{119}$$

For complex fermions and $d = 4$, $k = 10$ and $l = 22$. The phases in this formula are reduced mod (2) and are chosen to be in the interval $(-1, +1]$.

Modular transformations mix spin-structures amongst one another within a surface of a given genus. Thus, requiring the modular invariace of the partition function imposes constraints on the coefficients $C\begin{bmatrix}\alpha_1 & \ldots & \alpha_g\\ \beta_1 & \ldots & \beta_g\end{bmatrix}$ (weights in the partition function sum, for example, see eq.(77)) which in turn imposes constraints on what spin-structures are allowed in a consistent theory. According to the assumptions (a) and (b) these coefficients factorize:

$$C\begin{bmatrix}\alpha_1 & \ldots & \alpha_g\\ \beta_1 & \ldots & \beta_g\end{bmatrix} = C\begin{bmatrix}\alpha_1\\ \beta_1\end{bmatrix} C\begin{bmatrix}\alpha_2\\ \beta_2\end{bmatrix} \ldots C\begin{bmatrix}\alpha_g\\ \beta_g\end{bmatrix}. \tag{120}$$

The requirement of modular invariance of the partition function thus gives rise to constraints on the one-loop coefficients C and hence on the possible spin structures (α, β) on the torus.

For rational phases $\alpha(f)$ (we consider only this case) the possible boundary conditions α comprise a finite additive group $\Xi = \sum_{i=1}^{k} \oplus Z_{N_i}$ which is generated by a basis $(b_1,\ldots, b_k)$, where N_i is the smallest integer for which $N_i\, b_i = 0$ mod(2). A multiplication of two vectors from Ξ is defined by

$$\alpha\cdot\beta = (\alpha_L^i\,\beta_L^i - \alpha_R^j\,\beta_R^j)_{\text{complex}} + 1/2(\alpha_L^k\,\beta_L^k - \alpha_R^l\,\beta_R^l)_{\text{real}}. \qquad (121)$$

The basis satisfies following conditions derived in [15]:

(A1) The basis $(b_1,\ldots, b_k)$ is chosen to be canonical:

$$\sum m_i b_i = 0 \Leftrightarrow m_i = 0 \bmod(N_i) \quad \forall\, i.$$

Then an arbitrary vector α from Ξ is a linear combination $\alpha = \sum a_i b_i$.

(A2) The vector b_1 satisfies $1/2N_1 b_1 = \mathbf{1}$. This is clearly satisfied by $b_1 = \mathbf{1}$.

(A3) $N_{ij}\, b_i \cdot b_j = 0 \bmod(4)$, where N_{ij} is the least common multiple of N_i and N_j .

(A4) $N_i\, b_i^2 = 0 \bmod(4)$; however, if N_i is even, we must have $N_i\, b_i^2 = 0 \bmod(8)$.

(A5) The number of real fermions that are simultaneously periodic under any four boundary conditions b_i , b_j , b_k, b_l is even, where i, j, k and l are not necessarily distinct. This implies that the number of periodic real fermions in any b_i be even.

(A6) The boundary condition matrix corresponding to each b_i is an automorphism of the Lie algebra that defines the supercharge (32). All such automorphisms must commute with one another, since they must be simultaneously diagonalizable.

For each group of boundary conditions Ξ there are a number of consistent choices for coefficients $C[\ldots]$, which are determined from requirement of invariance under modular transformation. The number of such theories corresponds to the number of different choices of $C\begin{bmatrix} b_i \\ b_j \end{bmatrix}$. This set must satisfy equations:

(B1) $$C\begin{bmatrix} b_i \\ b_j \end{bmatrix} = \delta_{b_i} e^{2\pi i n/N_j} = \delta_{b_j} e^{i\pi(b_i\cdot b_j)/2}\, e^{2\pi i m/N_i}.$$

(B2) $$C\begin{bmatrix} b_1 \\ b_1 \end{bmatrix} = \pm\, e^{i\pi b_1^2/4}.$$

The values of $C\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ for arbitrary α, $\beta \in \Xi$ can be obtained by means of the following rules:

(B3) $$C\begin{bmatrix} \alpha \\ \alpha \end{bmatrix} = e^{i\pi(\alpha\cdot\alpha + 1\cdot 1)/4}\, C\begin{bmatrix} \alpha \\ b_1 \end{bmatrix}^{N_1/2}.$$

(B4) $$C\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = e^{i\pi(\alpha\cdot\beta)/2}\, C\begin{bmatrix}\beta\\ \alpha\end{bmatrix}^{*}.$$

(B5) $$C\begin{bmatrix}\alpha\\ \beta+\gamma\end{bmatrix} = \delta_{\alpha} C\begin{bmatrix}\alpha\\ \beta\end{bmatrix} C\begin{bmatrix}\alpha\\ \gamma\end{bmatrix}.$$

The relative normalization of all the $C[\ldots]$ is fixed in these expressions conventionally to be $C\begin{bmatrix}0\\ 0\end{bmatrix} \equiv 1$.

For each $\alpha \in \Xi$ there is a corresponding Hilbert space of string states $\mathcal{H}_{\alpha}$ that potentially contribute to the one-loop partition function. If we write $\alpha = (\alpha_L \mid \alpha_R)$, then the states in $\mathcal{H}_{\alpha}$ are those that satisfy the Virasoro condition:

$$M_L^2 = -c_L + 1/8\alpha_L\cdot\alpha_L + \sum_{L-\text{mov.}} (\text{frequencies}) =$$

$$= -c_R + 1/8\alpha_R\cdot\alpha_R + \sum_{R-\text{mov.}} (\text{freq.}) = M_R^2. \tag{122}$$

Here $c_L = 1/2$ and $c_R = 1$ in the heterotic case. In $\mathcal{H}_{\alpha}$ sector the fermion $f(f^*)$ has oscillator frequencies

$$\frac{1 \pm \alpha(f)}{2} + \text{integer}. \tag{123}$$

The only states $\mid s\rangle$ in $\mathcal{H}_{\alpha}$ that contribute to the partition function are those that satisfy the generalized GSO conditions

$$\left\{ e^{i\pi(b_i\cdot F_{\alpha})} - \delta_{\alpha} C\begin{bmatrix}\alpha\\ b_i\end{bmatrix}^{*} \right\} \mid s\rangle = 0 \tag{124}$$

for all b_i, where $F_{\alpha}(f)$ is a fermion number operator. If α contains periodic fermions, then $\mid 0\rangle_{\alpha}$ is degenerate, transforming as a representation of an $SO(2n)$ Glifford algebra.

REFERENCES

1. **E731 Collaboration, Gibbons L.K. et al.** —Phys. Rev. Lett., 1993, vol. 70, p.1203.
2. **NA31 Collaboration, Barr G.D. et al.** —Phys. Lett., 1993, vol. B317, p.233.
3. **D0 Collaboration, Abachi F. et al.** — Phys. Rev. Lett., 1994, vol. 72, p.2138.
4. **Ellis J., Figli G.L., Lisi B.** — CERN-TH/177-94, 1994.
5. **Amaglobeli A.N., Kereselidze A.R., Liparteliani A.G., Volkov G.G.** —Phys. Lett., 1990, vol. B237, p.417; **Maslikov A.A., Volkov G.G.** — Towards a Low Quark-

Lepton Family with Non-Abelian Gauge Symmetry, ENSLAPP-A-382/92, 1992; **Monich V.A., Struminsky B.V., Volkov G.G.** — Phys. Lett., 1981, vol. B104, p.382; **Liparteliani A.G., Monich V.A., Nikitin Yu.P., Volkov G.G.** — Proc. of the Int. Conf. Neutrino-77, Baksan Valley, USSR, Moscow, Nauka, 1978, vol. 2, p.541; Sov. J. Nucl. Phys., 1978, vol. 27, p.735; **Maslikov A., Naumov I., Volkov G.** — Non-Abelian Gauge Family Symmetry in Rank 8 and 16 Grand Unified String Theories, INFN Sezione di Padova DF-PD/94/TH/46, 1994.

6. **Gasser J., Leutwyler H.** — Phys. Rep., 1982, 87, p.77; **Leitwyller H.** — Nucl. Phys., 1990, vol. B337, p.108.
7. **a) Fritzsch H., Plankl J.** — Phys. Lett., 1990, vol. 237, p.451; **b) Fritzsch H.** — Nucl. Phys., 1979, vol. B155, p.189; **c) Matumoto K.M., Matsuoka T.** — Progr. Theor. Phys. Progress Letters, 1990, vol. 83, p.373.
8. **a) Ramond P., Roberts R., Ross G.** —Nucl. Phys., 1993, vol. B406, p.19; **b) Dimopoulos S., Hall L., Raby S.** — Phys. Rev., 1992, vol. D45, p.4195.
9. **Green M.B., Schwarz J.H.** — Phys. Lett., 1984, vol. B149, p.117.
10. **Gross D.J., Harvey J.A., Martinec E., Rohm R.** — Nucl.Phys., 1985, vol. B256, p.253.
11. **Candelas P., Horowitz G.T., Strominger A., Witten E.** — Nucl., Phys., 1985, vol. B258, p.46; **Greene B., Kirklin K., Miron R., Ross G.** — Nucl. Phys., 1986, vol. B278, p.667.
12. **Witten E.** — Nucl. Phys., 1986, vol. B268, p.79.
13. **Narain K.S.** — Phys. Lett., 1986, vol. B169, p.246; **Lerche W., Lust D., Schellekens A.N.** — Nucl. Phys., 1987, vol. B287, p.477.
14. **Kawai H., Lewellen D., Tye S.-H.H.** — Phys. Rev. Lett., 1986, vol. 57, p.1832; Nucl. Phys., 1987, vol. 288B, p.1.
15. **Antoniadis I., Bachas C., Kounnas C.** — Nucl. Phys., 1988, vol. B289, p.87; **Antoniadis I., Bachas C.** — Nucl. Phys., 1988, vol. B298, p.586.
16. **Antoniadis I., Ellis J., Hagelin J.S., Nanopoulos D.V.** — Phys. Lett., 1987, vol. B194, p.231; 1988, vol. B208, p.209; **Ellis J., Lopez J.I., Nanoloulos D.V.** — Phys. Lett., 1990, vol. B245, p.375; **Kalara S., Lopez J.I., Nanopoulos D.V.** — Nucl. Phys., 1991, vol. B353, p.650; **Lopez J.I., Nanopoulos D.V., Zichichi A.** — CERN-TH 6934/93, 1993; **Antoniadis I., Leontaris G., Rizos J.** — Phys. Lett., 1990, vol. B245, p.161.
17. **Goddard P., Olive D.** — Int. J. Mod. Phys., 1986, vol. A1, p.1.
18. **Alvarez-Gaume L., Sierra G., Gomez C.** — Topics in Conformal Field Theory. In: Physics and Mathematics of Strings, World Scientific, Singapore, 1990, p.p.16—184.
19. **Langacker P.** — Phys. Rep., 1981, vol. 72, p.p.185—385.
20. **Vinberg E.B., Onichik A.L.** — Seminar on Group Lee. In: Russian Nauka, Moscow, 1988; **Slansky R.** — Phys. Rep., 1981, vol. 79, p.p.1—128.
21. **Sergeev S.M., Volkov G.G.** — Preprint DFPD/TH/51 1992; Rus. Nucl. Phys., 1994, vol. 57, No.1; **Maslikov A.A., Sergeev S.M., Volkov G.G.** — Phys. Lett., 1994, vol. B328, p.319; **Maslikov A.A., Sergeev S.M., Volkov G.G.** — Phys. Rev. D, 1994, vol. 50, p.7740.
22. **Font A., Ibanez L.E., Quevedo F.** — Nucl. Phys., 1990, vol. B345, p.389.
23. **Schellekens A.** — Phys. Lett., 1990 vol. B237, p.363.
24. **Cahn R.N., Harari H.** — Nucl. Phys., 1980, vol. B176, p.136.

25. **Liparteliani A.G., Monich V.A., Volkov G.G.** — Sov. J. Nucl. Phys., 1983, vol. 37, p.393; The Problems of CP Violation, Preprint IHEP 85-9, Serpukhov, 1985; **Volkov G.G., Liparteliani A.G., Monich V.A., Nikitin Yu.P.** — Yadernaya Fizika (Sov. J. Nucl. Phys.), 1981, vol. 34, p.435; **Volkov G.G. et al.** — Sov. J. Nucl. Phys., 1989, vol. 49.
26. **Jones D.R.T., Kane G.L., Leveille J.P.** — Nucl. Phys., 1982, vol. B198, p.45; **Frampton P.H., Kephart T.W.** — Nucl. Phys., 1983, vol. B211, p.239.
27. **Nilles H.P.** — Phys. Rep., 1984, vol. 110, p.1; **Lahanas A.B., Nanopoulos D.V.** — Phys. Rep., 1987, vol. 145, p.1.
28. **Fayet P.** — Nucl. Phys., 1984, vol. B237, p.367.
29. **Lewellen D.C.** — Nucl. Phys., 1990, vol. B337, p.61.
30. **Masiero A., Nanopoulos D.V., Tamvakis K., Yanagida T.** — Phys. Lett., 1982, vol. B115, p.380.
31. **Howe P., Stelle K., West P.** — Phys. Lett., 1983, vol. B124, p.55.
32. **Parkes A., West P.** — Phys. Lett., 1983, vol. B127, p.353.
33. **F. del Aguila et al.** —Nucl. Phys., 1985, vol. B250, p.225.
34. **Dreiner H., Lopez J.L., Nanopoulos D.V., Reiss D.** — Nucl. Phys., 1989, vol. B320, p.401.
35. **Seiberg N., Witten E.** —Nucl. Phys., 1986, vol. B276, p.272.