«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
1999, ТОМ 30, ВЫП. 1

УДК 539.12.01

WIGNER FUNCTIONS OF ESSENTIALLY NONEQUILIBRIUM SYSTEMS

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The aim of the article is to discuss the S-matrix interpretation of perturbation theory for the Wigner functions generating functional at a finite temperature. For the sake of definiteness, the concrete problem from particle physics of high-temperature initial states dissipation into cold one is considered from experimental and theoretical points of view. The temperature is introduced in the theory by typical for the microcanonical description way. The perturbation theory contains two-temperature (of initial and final states) Green functions. Two possible boundary conditions are considered. One of them is usual in a field theory vacuum boundary condition. Corresponding generating functional of Wigner functions can be used in the particle physics. Another type of the boundary condition assumes that the system under consideration is in environment of the black-body radiation. This leads to the usual in statistics Kubo–Martin–Schwinger boundary condition at the equilibrium (one-temperature) limit. The comparison of the S-matrix approach with Schwinger–Keldysh real-time finite-temperature field theory and with nonstationary statistical operator approach of Zubarev are considered. The range of applicability of the finite-temperature description of dissipation processes is shown.

Цель данной работы – описать S-матричную интерпретацию теории возмущений для производящего функционала функций Вигнера при конечных температурах. Для определенности будет рассмотрен с экспериментальной и теоретической точек зрения конкретный процесс диссипации горячего начального состояния в холодное, типичный для физики частиц. Температура состояний будет введена в формализм характерным для микроканонического описания образом. Теория возмущений содержит функции Грина, зависящие от двух температур (отдельно для начального и конечного состояний). Рассмотрены два типа граничных условий. Первое соответствует обычному для теории поля вакуумному граничному условию. Соответствующие производящие функционалы функций Вигнера могут быть использованы в физике частиц. Другой тип граничных условий предполагает, что система окружена излучением черного тела. Это приводит к обычным в статистической физике граничным условиям Кубо–Мартина–Швингера в однотемпературном пределе. Мы сравним наш S-матричный подход с реально-временной теорией Швингера–Келдыша при конечных температурах и с нестационарным статистическим оператором Зубарева. Показана область применимости температурного описания диссипативных процессов.

1. INTRODUCTION

At the very beginning of this century, couple P. and T.Ehrenfest had offered a model to visualize Boltzman's interpretation of irreversibility phenomena in

statistics. The model is extremely simple and fruitful [1]. It considers two boxes with $2N$ numerated balls. Choosing number $l = 1, 2, ..., 2N$ $randomly$ one must take the ball with label l from one box and put it into another one. Starting from the highly "nonequilibrium" state with all balls in one box it is seen a tendency to equalization of balls number in the boxes. So, there is seen irreversible* flow toward preferable (equilibrium) state. One can hope [1] that this model reflects a physical reality of nonequilibrium processes with initial state very far from equilibrium. A theory of such processes with (nonequilibrium) flow toward a state with maximal entropy should be sufficiently simple to give definite theoretical predictions.

In order to do the consideration less formal we will be connected with concrete physical problem. For instance, the particles creation processes are good laboratory for investigation of general properties of relativistic nonequilibrium processes. Indeed, considering the multiplicity n as the characteristics of final state entropy we can choose the asymptotically large $n >> \bar{n}(s)$, where mean multiplicity $\bar{n}(s)$ naturally defines the scale of n. Then one can expect, noting the above-mentioned general property of the nonequilibrium flow, that the theory of processes with practically total dissipation of initial-state kinetic energy into particles masses should be extremely simple. By this reason it is natural to start consideration from region $n >> \bar{n}(s)$. We would construct the theory permanently taking into account just this condition.

The theory of dissipative processes has general significance from thermodynamical point of view and we would concentrate our attention on this important problem. There is also practical side of the problem considered. At $n >> \bar{n}(s)$ the cross sections $\sigma_n(s)$ fall down rapidly and are too small ($< nb$). Noting also a problem of triggering such rear final state, the experimenters must have enough arguments to examine them. The main arguments are as follows: at $n >> \bar{n}(s)$ we have unique chance (i) to examine
– $pure$ (practically without admixture of hadrons),
– $cold$ (it is a best condition for investigation of collective phenomena in a system),
– $dense$ (in this case the QCD interaction constant α_s is small)
quarks plasma (CQGP) and (ii) to realize experimentally the decay of very hot (at high energies) initial state in the "inflational" regime, with "freezed" nonperturbative degrees of freedom of hadrons system.

It is known from hadrons high-energy experiments that the cross sections σ_n have a maximum at $n \sim \bar{n}(s)$, where $1 << \bar{n}(s) << n_{\max}$ and $n_{\max} = \sqrt{s}/m_h$ is the maximally available multiplicity at given energy $\sqrt{s}$ (m_h is the hadrons characteristic mass). This testify to the statement that in hadron processes the

*"What never *(is the time-reversed flow)?* No never! What never? Well, hardly ever." [2]

nonequilibrium flow is not equal to zero ($\bar{n}(s) >> 1$), but the mostly probable processes did not lead to the state with maximal entropy ($\bar{n}(s) << n_{\max}$). (The early model was based on the assumption that the final state of inelastic hadron processes has maximal entropy $\bar{n}(s) \sim n_{\max}$ [3].)

The preferable at $n \sim \bar{n}$ processes are indebted for excitation of hadrons nonperturbative degrees of freedom described by creation of hadrons constituents from vacuum: the kinetic motion of partons leads to increasing, because of confinement phenomenon, polarization of vacuum and to its instability concerning quarks creation [4]. In other words, there is a long-range correlation among hadrons constituents. Under this special correlations the conservation laws constraints were implied. They are important in dynamics since each conservation law decreases the number of the dynamical degrees of freedom at least by one unite, i.e., it has nonperturbative effect (this must explain why $\bar{n}(s) << n_{\max}$). Moreover, in the so-called integrable systems each independent integral of motion (in involution) reduces number of degrees of freedom by two units. In result there is not stochastization in such systems [5], i.e., the nonequilibrium flow is equal to zero. But it will be argued that at the very high multiplicities this effect is negligible. So, if

$$\bar{n}(s) << n < n_{\max}$$

we will see that the particles creation processes are close to Markovian in accordance with Boltzman's idea. The reason of this phenomenon is the more fast falling down of soft channel of hadrons creation compared with hard channels in asymptotics over n.

Rejecting nonperturbative effects creation of the high-multiplicity final state can be described using standard methods of QCD. We will show dominance of processes with minimal number of QCD jets in the high-multiplicities region. This means that the high-multiplicity processes are stationary Markovian*. This result is in agreement with Boltzman's general idea concerning nonequilibrium flows.

So, the high-multiplicity processes are "unshadowed" by nonperturbative and complicated perturbation effects. This will allow one to investigate not only new state of the pure colored plasma but also the structure of fundamental Lagrangian. This conclusions are not evident and we start consideration from brief review of arguments.

It must be noted that the experimental investigation of high-multiplicity processes in deep asymptotics over n seems unreal. But considering moderate $n >> \bar{n}$ we cannot be sure that the final state is equilibrium. Investigation of fractal dimensions in the multiparticle hadron processes at high energies shows

*The vertices renormalization takes into account the time-reversed fluctuations in the nonequilibrium flow.

presence of considerable fluctuations [6]. This leads to necessity to have the theory of dissipation processes with nonequilibrium final state.

There is also another side of the problem. Today understanding of hadron processes is far from ability to give any quantitative prediction. The above-announced prediction concerning absence of nonperturbative contributions into hadron processes "works" in the deep asymptotics over n only. So, at moderate $n >> \bar{n}$ we cannot be sure that they do not have important influence. That is why we will concentrate our attention in this paper on the searching for economic (thermodynamical) description of the dissipative processes, trying to find the connections of our S-matrix approach with other ones. It is important to note that the offered formalism allows one to separate the dynamical side of question from the pure descriptional one (see also concluding Section).

This central problem of formalism can be solved noting that our dissipative problem contains element of dynamics since it crucially depends on boundary condition. Therefore, we adopt S-matrix formalism which is natural for dissipative systems time evolution description. For this purpose the amplitudes

$$< (p)_m|(q)_n >= a_{n,m}(p_1, p_2, ..., p_m; q_1, q_2, ..., q_n)$$

of the m- into n-particles transition will be introduced. (The in- and out-states must be composed from mass-shell particles [7].) Moreover, to incorporate the boundary condition $n >> m$ we should calculate the probability integrating over particles momenta:

$$r(P; n.m) \sim \int |a_{n,m}|^2 = \int < (p)_m|(q)_n >< (q)_n|(p)_m >$$

since the amplitude a_{nm} is the function of too many variables, $(p_1, p_2, ..., p_m; q_1, q_2, ..., q_n)$. This standard method of particle physics practically solves our problem.

Nevertheless, it is desirable to use the thermodynamical language as the most economic one, i.e., the formalism which uses minimal number of parameters (temperature, chemical potential, etc.) for description of the system.

The field-theoretical description of statistical systems at a finite temperature is usually based on the formal analogy between imaginary time and inverse temperature β ($\beta = 1/T$) [8]. This approach is fruitful [9] for description of the static properties of a system, but it demands a complicated mathematical apparatus for the analytic continuation to the real time [10], if we want to clear up dynamical aspects. The first important quantitative attempt to build the real-time finite-temperature field theory [11] discovers a problem of the pinch-singularities. Further investigation of the theory has allowed one to demonstrate the cancellation mechanism of these unphysical singularities [12]. This was attained by doubling of the degrees of freedom [13, 14]: the Green functions of the theory represent

2×2 matrix. It surely makes the theory more complicated, but the operator formalism of the thermofield dynamics [15] shows the unavoidable character of this complication.

The Schwinger–Keldysh real-time finite-temperature field-theoretical description [13,14] of statistical systems is based on the Kubo–Martin–Schwinger (KMS) [16, 17] boundary condition for a field:

$$\Phi(t) = \Phi(t - i\beta).$$

This formal trick introduces into formalism the temperature $T = 1/\beta$ but, without fail, leads to the $equilibrium$ fluctuation-dissipation conditions [18] (see also [19]). Beside this we should have the two-temperature theory describing kinetic energy dissipation process (for initial and final states separately). It is evident that in such theory with two temperatures it is impossible to use the KMS boundary condition.

In the S-matrix approach finite-temperature description can be introduced (e.g., [20] and references cited therein) taking into account that, for instance,

$$d\Gamma_n = |a_{n,m}| \prod_1^n \frac{d^3 q_i}{(2\pi)^3 2\epsilon(q_i)}, \quad \epsilon(q) = (q^2 + m_h^2)^{1/2},$$

is the differential measure of final state. Then we can define the temperature as the function of initial energy through the equation of state, i.e., proportional to the mean energy of created particles. Such introduction of temperatures as the Lagrange multiplier is obvious for microcanonical approach [16]. The initial-state temperature will be introduced by the same way. Using standard terminology [21], we will deal with the "mechanical" perturbations only [22] and it will not be necessary to divide the perturbations on "thermal" and "mechanical" ones [23].

Introducing temperature as the Lagrange multiplier we should assume that the temperature fluctuations are small (Gaussian). In opposite case the notion of temperature looses its sense. The "working" idea concerning nonequilibrium processes is based on the assumption that evolution of a system goes through few phases. In the first "fast" phase the s-particle distribution functions $\mathbf{D}_s$, $s > 1$, strongly depend on initial conditions. But at the end of this phase the system forgets the initial-state information. Second phase is the "kinetic" one. One can expect that the space-time fluctuations of thermodynamical parameters in this phase are large scale, i.e., there are macroscopical domains in which the subsystems are equilibrium, with Gaussian fluctuations of thermodynamical parameters. In the last "hydrodynamical" phase the whole system is described by macroscopical parameters. We will see that the Schwinger–Keldysh [10, 13, 14] formalism is applicable for "hydrodynamical" phase only.

The above described S-matrix finite-temperature description can be realized not only for uniform temperature distribution (we have done first step in this direc-

tion wishing to introduce initial and final temperatures separately). So, introducing cells of measuring device (calorimeter) and introducing the energy-momentum shells of each cell separately we can introduce the individual temperatures in each cell. This can be done since in the S-matrix theory the measurement performed by free (mass-shell) particles, i.e., the measurement of energy (and momentum) can be performed in each cell separately. This allows one to capture the "kinetic" phase also (if the number of calorimeter cells is high enough). In this phase multiparticle distribution functions $\mathbf{D}_s$, $s > 1$, are functionals of one-particle distribution function $\mathbf{D}_1$ only. This means the 'shortened" description of the non-equilibrium medium [24]. We will return to this question in Sec.4 considering the dissipative processes thermal descriptions applicability range.

The microcanonical description assumes that the energy of system is known with arbitrary accuracy. Introducing the measurement cells and corresponding energy shells we assume that the energy in each cell can be measured with arbitrary accuracy. That is why we should work in the frame of Wigner functions formalism [25].

E.Wigner had offered the function $W(q, R)$ for the quantum states phase space description [26]:

$$W(q, R) = \int dr e^{iqr} \Psi(R + r/2)\Psi^*(R - r/2),$$

where $\Psi(x)$ is the wave function of state. The existence of other approaches must be mentioned [27]. But as will be seen below the Wigner description is mostly natural for us.

In the classical limit $\hbar = 0$ the function $W(q, r)$ coincides with the phase space probability distribution function. It obeys the equation [28]:

$$\dot{W} = \{W, H\} + O(\hbar),$$

which coincides with the Liouville equation only in the classical limit $\hbar = 0$.

The extension of Wigner's idea on the relativistic case uses the connection between Wigner's approach and inclusive description of inelastic scattering processes [25, 29]. But the Wigner functions are not directly measurable quantities because of the quantum uncertainty principle $\Delta q \Delta r \sim \hbar$. Just this restriction leads to impossibility of taking the measurement (calorimeter) cells 4-dimension Δr arbitrary small and defines the natural boundary of Wigner-functions approach applicability. Wishing to use the Wigner-functions description of experiments the corresponding theory must take into account this restriction. The discussion of this question is given in Sec.4.

So, in our terms one can use the thermodynamical formalism if the non-stationary mediums "shortened" description may be applied: in this case mean values of correlation functions over the space-time are negligible and the fluctuations of thermodynamical parameters are small (Gaussian). Other approach

should be mentioned also. Proposing [32] that the equilibrium in the nonuniform nonstationary medium may be attained in small regions more quickly than in the whole system, the entropy maximalness in this restricted domains of a system can be used for construction of the "local equilibrium density matrix" (LDM) [32]. But LDM is applicable for description of processes in which dissipation may be disregarded [30]. Nevertheless, if the energy-momentum density of nonstationary flow is considerably smaller than the energy density of matter, then the first one can be taken into account perturbatively considering LDM as an initial condition. This modifies LDM to "nonstationary density matrix" (NDM) of Zubarev [32] introducing an infinitesimal interaction with a heat bath to get the increasing entropy. We will return to this question in Sec.5.

The S matrix will be introduced phenomenologically, using ordinary in a quantum field theory reduction formalism. This leads naturally to necessity to introduce the boundary conditions for interacting fields $\Phi(\sigma_\infty)$, where σ_∞ is the infinitely far hypersurface, e.g., [31]. The value of $\Phi(\sigma_\infty)$ specifies the environment of a system.

We start with vacuum boundary condition $\Phi(\sigma_\infty) = 0$ familiar for a field theory. This theory can be applied in the particle physics. The simplest choice of $\Phi(\sigma_\infty) \neq 0$ assumes that the system under consideration is surrounded by black-body radiation. Just this "boundary condition" restores the Schwinger–Keldysh [10] real-time finite-temperature field theory [12] from S-matrix formalism in the "hydrodynamical" phase and gives the dynamical interpretation of the KMS periodic boundary condition.

One should admit also that last choice of boundary condition is not unique: one can consider another organization of the environment of considered system. The S-matrix interpretation is able to show the way of adoption of formalism to the arbitrary environment*. It should broaden the potentialities of the real-time finite-temperature field-theoretical methods, for instance, for heavy nucleus high energy interactions. The special interest represent also the topological effects, but, by above-mentioned reason, in this paper consideration will be performed in the perturbation theory framework only (see also concluding Section).

The central purpose of this review paper is to describe connections between ordinary S-matrix description and popular in the modern literature real-time finite-temperature field theories. We wish to discuss:

– The QCD jets dominance in deep asymptotics over n (Sec.2);

In this section we would like to show why the $real\text{-}time$ formalism is needed for our dissipative process description.

– The S-matrix interpretation of Schwinger–Keldysh theory (Sec.3);

In this section the uniform temperature description of the state will be introduced

*This question was considered also in [29].

in the spirit of microcanonical description. There is presented the way of explicit calculations to show coincidence of used microcanonical description and ordinary (Gibbs) canonical formalism.
– The range of applicability of finite-temperature description (Sec.4);
In this section the necessity and sufficiency of Bogoliubov's "shortened" description is discussed.
– The S-matrix description of media with nonuniform temperature distribution (Sec.5);
In this section the Wigner functions formalism is introduced. The range of its applicability to describe an experiment is shown.
– The comparison of our S-matrix approach with "nonstationary statistical operator" of Zubarev [32] (Sec.6).
In this section the main distinction between S-matrix (microcanonical) and Zubarev's (canonical) perturbation theories is shown.
– Concluding remarks (Sec.7).
In this section the way the nonperturbative effects may be included in the formalism is discussed.

2. PHENOMENOLOGY

To build the phenomenology [33] of high multiplicity processes let us introduce the classification of asymptotics over n. For this purpose it is useful to consider the "big partition function":

$$T(z,s)=\sum_n z^n\sigma_n(s),\quad T(1,s)=\sigma_{\rm tot}(s).$$

Strictly speaking, summation over n is performed up to $n_{\max}$. But we can extend summation up to infinity* if the weight z is sufficiently small, $0<z<z_{\max}$. So, $T(z,s)$ can be considered as the nontrivial function of z with sufficient accuracy. Note that $z_{\max}>1$ since $\sigma_n(s)$ decreases with n. If we know $T(z,s)$, then $\sigma_n(s)$ is defined by inverse Mellin transformation. This gives (usual in thermodynamics) equation (of state):

$$n=z\frac{\partial}{\partial z}\ln T(z,s). \tag{2.1}$$

Solving this equation we can estimate the asymptotics of σ_n:

$$\sigma_n(s)\sim {\rm e}^{-n\ln\bar z(n,s)}, \tag{2.2}$$

where $1<\bar z(n,s)<<z_{\max}$ is the smallest solution of Eq. (2.1).

*That is, wishing to consider the "thermodynamical limit".

It follows from (2.2) that at $n \to \infty$ the solution of (2.1) must tend to singularity z_s of $T(z,s)$ and the character of singularity is not important. So, we must consider three possibilities:

$$a)\ z_s = z_a = 1, \quad b)\ z_s = z_b = \infty, \quad c)\ z_s = z_c, \ \ 1 < z_c < \infty.$$

Following Lee and Yang [34] there are no singularities at $0 < z < 1$.

Let us consider now the physical content of this classification.

a) $z_s = 1$.

It is known that the singularity $z_s = 1$ reflects the first order phase transition [34]. To find σ_n for this case we would adopt Langer's analyses [35]. Introducing the temperature $1/\beta$ instead of total energy $\sqrt{s}$ we can use the isomorphism with the Ising model. For this purpose we divide the space volume on cells and if there is particle in the cell, we will write (-1), in opposite case (+1). It is the model of lattice gas well described by the Ising model. We can regulate the number of down-looking spins, i.e., the number of created particles, by the external magnetic field $\mathbf{H}$. Therefore, $z = \exp\{-\beta\mathbf{H}\}$ and $\mathbf{H}$ is the chemical potential.

The corresponding partition function in the continuous limit [35] (see also [36]) has the form:

$$R(\beta, z) = \int D\mu \mathrm{e}^{-\int dx\{\frac{1}{2}(\vec{\partial}\mu)^2 - \varepsilon\mu^2 + \alpha\mu^4 - \lambda\mu\}}, \tag{2.3}$$

where $\varepsilon \sim (1 - \frac{\beta_c}{\beta})$ and $\lambda \sim \mathbf{H}$, with critical temperature $1/\beta_c$.

If $\beta_c > \beta$ there is no phase transition and the potential has one minimum at $\mu = 0$. But if $\beta_c < \beta$, there are two degenerate minima at $\mu_\pm = \pm\sqrt{\varepsilon/2\alpha}$ if $\lambda = 0$. Switching on $\mathbf{H} < 0$ the left minimum at $\mu_- \sim -\sqrt{\varepsilon/2\alpha}$ becomes absolute and the system will tunnel into this minimum (see also [37]). This process describes particles creations as a process of spins flippings.

Eq. (2.1) gives at $n \to \infty$

$$\ln \bar{z} \sim n^{-1/3} > 0.$$

In result,

$$\sigma_n \sim \mathrm{e}^{-an^{2/3}} > 0(\mathrm{e}^{-n}), \quad a > 0,$$

i.e., decrease slower than e^{-n}. The quasi-classical calculation shows that the functional determinant is singular at $\mathbf{H} = 0$. It must be underlined that in the used Ising model description the chemical potential deforms the ground state. In result, the quasi-classical approximation is applicable since $\ln \bar{z} << 1$, i.e., since the processes of spin flippings are rear at high multiplicity region. It is easy to show in this approximation [35] that the functional determinant is singular at $\mathbf{H} = 0$, i.e., at $z = 1$.

Note that $\bar{z}$ *decreases* to one with n. This unusual phenomenon must be explained. Considered above mechanism of particles creation describes "fate of false vacuum" [37]. In the process of decay of unstable state the clusters of new phase of size X are created. If the cluster has dimension $X > X_c$ its size increases since the volume energy ($\sim X^3$) of the cluster becomes better than the surface tension energy ($\sim X^2$). This condition defines the value of X_c. The "critical" clusters wall accelerates, i.e., the work needed to add one particle into cluster decreases with $X > X_c$. This explains the reason why $\bar{z}$ decreases with n noting that $\ln \bar{z}$ is proportional to Gibbs free energy per one particle.

The described mechanism of particles creation assumes that we had prepared the *equilibrium* system in the unstable phase at $\mu_+ \sim +\sqrt{\varepsilon/2\alpha}$ and going to another state at $\mu_- \sim -\sqrt{\varepsilon/2\alpha}$ the system creates particles. The initial state may be the QGP and final state may be the hadrons system. Therefore, we must describe the way as the quarks system was prepared.

Following to Lee–Yang's picture of first order phase transition [34] (see also [36]) there is no phase transition in a finite system (the partition function cannot be singular for finite $n_{\max}$). This means that the multiplicity (and the energy) must be high enough to see described phenomena.

b) $z_s = \infty$.

Let us return to the integral (2.3) to investigate the case $\beta_c > \beta$. In this case the potential has one minimum at $\mu = 0$. The external field $\mathbf{H}$ creates the mean field $\bar{\mu} = \bar{\mu}(\mathbf{H})$ and the integral (2.3) should be calculated expanding it near $\mu = \bar{\mu}$. In result, in the quasi-classical approximation ($\bar{\mu}$ increases with increasing n),

$$\ln R(\beta, z) \sim (\ln z)^{4/3}.$$

This gives $\ln \bar{z} \sim n^3$ and $\ln \sigma_n \sim -n^4$, i.e.,

$$\sigma_n < 0(\mathrm{e}^{-n}).$$

There is also other possibility to interpret considered case b). For this case we can put

$$\ln T(z, s) = n_0(s) + \bar{n}(s)(z-1) + \mathrm{O}((z-1)^2) \tag{2.4}$$

at $|z-1| << 1$. By definition $n_0(s) = \ln \sigma_{\mathrm{tot}}$. The experimental distribution of $\ln T(z,s) - n_0(s)$ for various energies shows that the contributions of $O((z-1)^2)$ terms increase with energy [38]. The hadrons "standard model" (SM) assumes that

$$\ln t(z, s) = n_0(s) + \bar{n}(s)(z-1)$$

is the Born term in the perturbation series (2.4). There are various interpretations of this series, e.g., the multiperipheral model, the Regge pole model, the heavy color strings model, the QCD multiperipheral models, etc. In all these models $n_0 = a_1 + a_2 \ln s$, $0 \leq a_2 << 1$, and $\bar{n}(s) = b_1 + b_2 \ln s$, $b_2 > 0$. The

second ingredient of hadrons SM is the assumption that mean value of created particles transfers momentum $< k >$=const, i.e., is the energy (and multiplicity) independent. It can be shown that under these assumptions:

$$\ln T(z,s) = n_0(s) + \sum_n c_n(s)(z-1)^n, \quad c_1 \equiv \bar{n} \tag{2.5}$$

is $regular$ at finite values of z [38] and is able to give well confirmed by experiment predictions.

Inserting (2.5) into (2.1) we find that $\bar{z}(n,s)$ is the increasing function of n. Therefore,

$$\sigma_n < \mathrm{O}(\mathrm{e}^{-n}). \tag{2.6}$$

But the SM have a finite range of validity: beyond $n \sim \bar{n}^2$ the model must be changed since it is impossible to conserve $< k >$=const at higher multiplicities [39].

We should underline once more that only two possibilities a) and b) can be deduced from representation (2.3), see also [35]. But nevertheless there is other possibility:
c) $1 < z_s < \infty$.
Let us assume now that

$$T(z,s) \sim (1 - \frac{z-1}{z_c - 1})^{-\gamma}, \quad \gamma > 0. \tag{2.7}$$

Then, using normalization condition, $(\partial T(z,s)/\partial z)|_{z=1} = \bar{n}_j(s)$ we can find that $z_c(s) = 1 + \gamma/\bar{n}_j(s)$. The singular structure (2.7) is impossible in SM because of condition $< k >$=const. But if $|z-1| << 1$, we have estimation (2.4). The difference between SM and c) is seen only at $1 - (z-1)/(z_c - 1) << 1$, i.e., either in asymptotics over n or in asymptotics over energy. The singular structure is familiar for "logistic" equations of QCD jets, e.g., [40].

In considered case $\bar{z} = z_c + 0(\bar{n}_j/n)$ and at high energies ($\bar{n}_j(s) >> 1$)

$$\sigma_n \sim \mathrm{e}^{-\gamma n/\bar{n}_j} = O(\mathrm{e}^{-n}). \tag{2.8}$$

Therefore,
comparing (2.6) and (2.8) we can conclude that at sufficiently high energies, i.e., if $\bar{n}_j >> \bar{n}$, where $\bar{n}$ is the SM mean multiplicity, the mechanism c) must dominate in asymptotics over n.

It is the general, practically model independent, prediction. It has important, from experimental point of view, consequence that at high energies there is a wide range of multiplicities, where the SM mechanism of hadrons creation is negligible. In other words, the CQGP of high multiplicity processes is the dynamical consequence of jets and SM mechanisms. At transition region between

"soft" of SM and "hard" of jets one can expect the "semihard" processes of minijets dominance.

The multiplicity distribution in jets has interesting property noted many decades ago by Volterra in his mathematical theory of populations [41]. In our terms, if one-jet partition function has the singularity at $z_c^{(1)}(s) = 1 + \gamma/\bar{n}_j(s)$, then two-jet partition function must be singular at

$$z_c^{(2)}(s) = 1 + \frac{\gamma}{\bar{n}_j(s/4)} > z_c^{(1)}(s),$$

and so on. Therefore, at high energies and $n > \bar{n}_j(s)$ the jets number must be minimal (with exponential accuracy). This means that at $n \to \infty$ the processes of hadrons creation have a tendency to be Markovian (with sharp increase of transverse momentum $< k >$) and only in the last stage the (first order) phase transition (colored plasma) $\to$ (hadrons) may be seen.

One can say that in asymptotics over n we consider the "inflational" channel of thermalization which is so fast* that the usual confinement forces are "freezed" and do not play important role in final colored plasma creation.

3. S-MATRIX INTERPRETATION OF REAL-TIME FINITE-TEMPERATURE FIELD THEORIES

3.1. Vacuum Boundary Conditions. The starting point of our calculations is n- into m-particles transition amplitude $a_{n,m}$, the derivation of which is a well-known procedure in the perturbation theory framework. For this purpose the $(n+m)$-point Green function $G_{n,m}$ is introduced [42]. To calculate the nontrivial elements of S matrix one must put the external particles on the mass shell. Formally this procedure means amputation of the external legs of $G^c_{n,m}$ and further multiplication on the free particles wave functions. In result the amplitude of m- into n-particles transition $a_{n,m}$ in the momentum representation has the form:

$$a_{n,m}((q)_n;(p)_m) = (-i)^{n+m} \prod_{k=1}^{m} \hat{\phi}(q_k) \prod_{k=1}^{n} \hat{\phi}^*(p_k) Z(\phi). \tag{3.1}$$

Here we introduce the "annihilation" operator

$$\hat{\phi}(q) = \int dx \mathrm{e}^{-iqx} \hat{\phi}(x), \quad \hat{\phi}(x) = \frac{\delta}{\delta\phi(x)}, \tag{3.2}$$

*The partons life time with virtuality $|q|$ is $\sim 1/|q|$ and the time needed for hadrons of mass m_h formation is $\sim 1/m_h$. Therefore the partons have a time to decay before hadrons formation if $|q| >> m_h$. But this situation is rear since the thermal motion in the initial stage of process is high.

$\hat{\phi}^*(p_k)$ is the "creation" operator and q_k and p_k are the momenta of in- and out-going particles. In (3.1)

$$Z(\phi) = \int D\Phi \mathrm{e}^{iS(\Phi) - iV(\Phi+\phi)}$$

is the generating functional. The total action was divided into two parts, where $S(\Phi)$ is the free part and $V(\Phi, \phi)$ describes the interactions. At the very end one should put the auxiliary field $\phi = 0$.

To provide the convergence of the integral (3.1) over scalar field Φ the action $S(\Phi)$ must contain positive imaginary part. Usually for this purpose Feynman's $i\varepsilon$-prescription is used. It is better for us to shift infinitesimally time contour to the upper half plane [10, 43], i.e., to the Mills contour

$$C_+ : t \to t + i\varepsilon, \quad \varepsilon > 0$$

and after all calculations to return the time contour on the real axis, $\varepsilon \to +0$.

In Eq. (3.1) the integration is performed over all field configurations with standard vacuum boundary condition:

$$\int d^4x \partial_\mu (\Phi \partial^\mu \Phi) = \int_{\sigma_\infty} d\sigma_\mu \Phi \partial^\mu \Phi = 0,$$

which assumes zero contribution from the surface term.

Supposing that the particles number and momenta are insufficient for us we introduce the probability

$$r(P) = \sum_{n,m} \frac{1}{n!m!} \int d\omega_n(q) d\omega_m(p) \delta^{(4)}(P - \sum_{k=1}^{n} q_k) \delta^{(4)}(P - \sum_{k=1}^{n} p_k) |a_{n,m}|^2, \tag{3.3}$$

where

$$d\omega_n(q) = \prod_{k=1}^{n} d\omega(q_k) = \prod_{k=1}^{n} \frac{d^3 q_k}{(2\pi)^3 2\epsilon(q_k)}, \quad \epsilon = (q^2 + m_h^2)^{1/2},$$

is the Lorentz-invariant phase space element. We assume that the energy-momentum conservation δ-function was extracted from the amplitude. It was divided into two parts:

$$\delta^{(4)}(\sum q_k - \sum p_k) = \int d^4P \delta^{(4)}(P - \sum q_k) \delta^{(4)}(P - \sum p_k). \tag{3.4}$$

It is not too hard to see that, up to phase space volume,

$$r = \int d^4P r(P)$$

is the imaginary part of amplitude $< vac|vac >$. Therefore, computing $r(P)$ the standard renormalization procedure can be applied and the new divergences will not arise in our formalism.

The Fourier transformation of δ-functions in (3.3) allows one to write $r(P)$ in the form:

$$r(P) = \int \frac{d^4\alpha_1}{(2\pi)^4}\frac{d^4\alpha_2}{(2\pi)^4}\mathrm{e}^{iP(\alpha_1+\alpha_2)}\rho(\alpha_1,\alpha_2),$$

where

$$\rho(\alpha_1,\alpha_2) = \sum_{n,m}\frac{1}{n!m!}\int\prod_{k=1}^{n}\{d\omega(q_k)\mathrm{e}^{-i\alpha_1 q_k}\}\prod_{k=1}^{m}\{d\omega(p_k)\mathrm{e}^{-i\alpha_2 p_k}\}|a_{n,m}|^2. \tag{3.5}$$

Introduction of the "Fourier-transformed" probability $\rho(\alpha_1,\alpha_2)$ means only that the phase-space volume is not fixed exactly, i.e., it is proposed that 4-vector P is fixed with some accuracy if α_i are fixed. The energy and momentum in our approach are still locally conserved quantities since the amplitude a_{nm} is translational invariant. So, we can perform the transformation:

$$\alpha_1\sum q_k = (\alpha_1-\sigma_1)\sum q_k + \sigma_1\sum q_k \to (\alpha_1-\sigma_1)\sum q_k + \sigma_1 P$$

since 4-momenta are conserved. The choice of σ_1 fixes the reference frame. This degree of freedom of the theory was considered in [44,45].

Inserting (3.1) into (3.5) we find that

$$\rho(\alpha_1,\alpha_2) = \exp\{i\int dxdx'(\hat{\phi}_+(x)D_{+-}(x-x',\alpha_2)\hat{\phi}_-(x') - \\ -\hat{\phi}_-(x)D_{-+}(x-x',\alpha_1)\hat{\phi}_+(x'))\}Z(\phi_+)Z^*(\phi_-), \tag{3.6}$$

where D_{+-} and D_{-+} are the positive and negative frequency correlation functions,

$$D_{+-}(x-x',\alpha) = -i\int d\omega(q)\mathrm{e}^{iq(x-x'-\alpha)}$$

describes the process of particles creation at the time moment x_0 and its absorption at x_0', $x_0 > x_0'$, and α is the center of mass () 4-coordinate. Function

$$D_{-+}(x-x',\alpha) = i\int d\omega(q)\mathrm{e}^{-iq(x-x'+\alpha)}$$

describes the opposite process, $x_0 < x_0'$. These functions obey the homogeneous equations:

$$(\partial^2 + m_h^2)_x G_{+-} = (\partial^2 + m_h^2)_x G_{-+} = 0$$

since the propagation of mass-shell particles is described.

We suppose that $Z(\phi)$ may be computed perturbatively. For this purpose following transformation will be used:

$$\begin{aligned} \mathrm{e}^{-iV(\phi)} = \mathrm{e}^{-i\int dx\hat{j}(x)\hat{\phi}'(x)}\mathrm{e}^{i\int dxj(x)\phi(x)}\mathrm{e}^{-iV(\phi')} = \\ = \mathrm{e}^{\int dx\phi(x)\hat{\phi}'(x)}\mathrm{e}^{-iV(\phi')} = \\ = \mathrm{e}^{-iV(-i\hat{j})}\mathrm{e}^{i\int dxj(x)\phi(x)}, \end{aligned} \tag{3.7}$$

where $\hat{\phi}$ was defined in (3.2). At the end of calculations the auxiliary variables j, ϕ' should be taken equal to zero. Using the first equality in (3.7) we find that

$$Z(\phi) = \mathrm{e}^{-i\int dx\hat{j}(x)\hat{\Phi}(x)}\mathrm{e}^{-iV(\Phi+\phi)}\mathrm{e}^{-\frac{i}{2}\int dxdx'j(x)D_{++}(x-x')j(x')}, \tag{3.8}$$

where D_{++} is the causal Green function:

$$(\partial^2 + m_h^2)_x G_{++}(x-y) = \delta(x-y).$$

Inserting (3.8) into (3.6) after simple manipulations with differential operators, see (3.7), we find the expression:

$$\begin{aligned} \rho(\alpha_1,\alpha_2) = \mathrm{e}^{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)} \times \\ \times \exp\{\frac{i}{2}\int dxdx'(j_+(x)D_{+-}(x-x',\alpha_1)j_-(x') - \\ j_-(x)D_{-+}(x-x',\alpha_2)j_+(x') - \\ -j_+(x)D_{++}(x-x')j_+(x') + j_-(x)D_{--}(x-x')j_-(x'))\}, \end{aligned} \tag{3.9}$$

where

$$D_{--} = (D_{++})^*$$

is the anticausal Green function.

Considering the system with large number of particles we can simplify calculations choosing the CM frame $P = (P_0 = E, \vec{0})$. It is useful also [16, 20] to rotate the contours of integration over $\alpha_{0,k}$: $\alpha_{0,k} = -i\beta_k, \mathrm{Im}\beta_k = 0, k = 1, 2$. In result, omitting unnecessary constant, we will consider $\rho = \rho(\beta_1, \beta_2)$.

External particles play the double role in the S-matrix approach: their interactions create and annihilate the interacting fields system and, on the other hand, they are probes through which the measurement of the system is performed. Since β_k are the conjugate to the particles energies quantities we will interpret them as the inverse temperatures in the initial (β_1) and final (β_2) states of interacting fields. But there is the question: are constants β_k really the "good" parameters to describe the system.

The integrals over β_k:

$$r(E) = \int \frac{d\beta_1}{2\pi i}\frac{d\beta_2}{2\pi i}\mathrm{e}^{(\beta_1+\beta_2)E}\mathrm{e}^{-F(\beta_1,\beta_2)}, \tag{3.10}$$

where

$$F(\beta_1, \beta_2) = -\ln \rho(\beta_1, \beta_2),$$

can be computed by the stationary phase method. This assumes that the total energy E is a fixed quantity. The solutions of the equations (of state):

$$E = \frac{\partial F(\beta_1, \beta_2)}{\partial \beta_k}, \quad k = 1, 2, \tag{3.11}$$

give the mostly probable values of β_k at a given E. Eqs. (3.11) always have the real solutions and, because of energy conservation law, both Eqs. (3.11) have the same solution with the property [16]:

$$\beta_k = \beta(E), \quad \beta > 0.$$

Assuming that β is the "good" parameter, i.e., the fluctuations of β_k are Gaussian we can interpret $F(\beta_1, \beta_2)$ as the free energy and $1/\beta_k$ as the temperatures. Such definition of thermodynamical parameters is in a spirit of microcanonical description. We will return to this question in Sec.4.

The structure of generating functional (3.9) is the same as the generating functional of Niemi-Semenoff [12]. The difference is only in the definition of Green functions which follows from the choice of boundary condition (2.6). The Green functions $D_{ij}, i, j = +, -$ were defined on the time contours $C_\pm$ in the complex time plane ($C_- = C_+^*$). This definition of the time contours coincides with Keldysh' time contour [14]. The expression (3.9) can be written in the compact form if the matrix notations are used. Note also a doubling of the degrees of freedom. This doubling is unavoidable since Green functions D_{ij} are singular on the light cone.

3.2. Closed-Path Boundary Conditions. The generating functional $\rho(\alpha_1, \alpha_2)$ has important factorized structure, see (3.6):

$$\rho(\alpha_1, \alpha_2) = \mathrm{e}^{\hat{N}(\alpha_1, \alpha_2; \phi)} \rho_0(\phi_\pm),$$

where the operator

$$\hat{N}(\alpha_1, \alpha_2; \phi) = \int dx dx' (\hat{\phi}_+(x) D_{+-}(x - x', \alpha_2) \hat{\phi}_-(x') - \\ - \hat{\phi}_-(x) D_{-+}(x - x', \alpha_1) \hat{\phi}_+(x'))$$

acts on the generating functional

$$\rho_0(\phi_\pm) = Z(\phi_+) Z^*(\phi_-) = \\ = \int D\Phi_+ D\Phi_- \mathrm{e}^{iS(\Phi_+) - iS(\Phi_-) - iV(\Phi_+ + \phi_+) + iV(\Phi_- + \phi_-)}, \tag{3.12}$$

of measurables. All "thermodynamical" information was contained in the operator $\hat{N}(\alpha_1, \alpha_2; \phi)$ and interactions are hidden in $\rho_0(\phi_\pm)$. One can say that action of the operator $\hat{N}$ maps the system of interacting fields on the measurable states. Last ones are "labeled" by α_1 and α_2. Just this property allows one to say that we are dealing with "mechanical" fluctuations only. To regulate the particles number we can introduce into $\hat{N}$ the dependence from "activities" z_1 and z_2 for initial and final states, separately.

The independent fields ϕ_+, ϕ_- and Φ_+, Φ_- were defined on the time contours C_+, C_-. By definition, path integral (3.12) describes the closed path motion in the space of fields Φ. We want to use this fact and introduce a more general boundary condition which also guarantees cancellation of surface terms in the perturbation framework. We will introduce the equality:

$$\int_{\sigma_\infty} d\sigma_\mu \Phi_+ \partial^\mu \Phi_+ = \int_{\sigma_\infty} d\sigma_\mu \Phi_- \partial^\mu \Phi_-. \tag{3.13}$$

The solution of Eq. (3.13) requires that the fields Φ_+ and Φ_- (and their first derivatives $\partial_\mu \Phi_\pm$) coincide on the boundary hypersurface σ_∞:

$$\Phi_\pm(\sigma_\infty) = \Phi(\sigma_\infty),$$

where, by definition, $\Phi(\sigma_\infty)$ is the arbitrary, "turning-point", field.

The existence of nontrivial field $\Phi(\sigma_\infty)$, in absence of surface terms, has influence only on the structure of Green functions

$$\begin{aligned} G_{++} &= < T\Phi_+\Phi_+ >, \quad G_{+-} = < \Phi_+\Phi_- >, \\ G_{-+} &= < \Phi_-\Phi_+ >, \quad G_{--} = < \tilde{T}\Phi_-\Phi_- >, \end{aligned} \tag{3.14}$$

where $\tilde{T}$ is the antitemporal time ordering operator. These Green functions must obey the equations

$$\begin{aligned} (\partial^2 + m^2)_x G_{+-}(x-y) = (\partial^2 + m^2)_x G_{-+}(x-y) = 0, \\ (\partial^2 + m^2)_x G_{++}(x-y) = (\partial^2 + m^2)^*_x G_{--}(x-y) = \delta(x-y), \end{aligned} \tag{3.15}$$

and the general solution of these equations

$$\begin{aligned} G_{ii} &= D_{ii} + g_{ii}, \\ G_{ij} &= g_{ij}, \quad i \neq j \end{aligned} \tag{3.16}$$

contains the undefined terms g_{ij} which must obey the homogeneous equations:

$$(\partial^2 + m^2)_x g_{ij}(x-y) = 0, \quad i,j = +,-. \tag{3.17}$$

The general solution of these equations (they are distinguished by the choice of the time contours $C_{\pm}$)

$$g_{ij}(x-x') = \int d\omega(q) \mathrm{e}^{iq(x-x')} n_{ij}(q) \tag{3.18}$$

are defined by the functions n_{ij}. Last ones are the functionals of 'turning-point" field $\Phi(\sigma_\infty)$: if $\Phi(\sigma_\infty) = 0$, we must have $n_{ij} = 0$ and we will come back to the theory of previous section.

Our aim is to define n_{ij}. We can suppose that

$$n_{ij} \sim < \Phi(\sigma_\infty) \cdots \Phi(\sigma_\infty) > .$$

The simplest supposition gives:

$$n_{ij} \sim < \Phi_i \Phi_j > \sim < \Phi^2(\sigma_\infty) > . \tag{3.19}$$

We will find the exact definition of n_{ij} starting from the S-matrix interpretation of the theory.

We should suppose there are only free, mass-shell particles that are on the infinitely far hypersurface σ_∞. Formally this follows from (3.16)—(3.18) and is natural in the S-matrix framework [7]. In other respects the choice of the boundary condition is arbitrary.

Therefore, our aim is the description of evolution of the system in a background field of mass-shell particles. We will assume that there are no any special correlations among background particles and will take into account only the energy-momentum conservation laws constraints. Quantitatively this means that multiplicity distribution of background particles is Poison-like, i.e., is determined by the mean multiplicity only. This is in spirit of definition of n_{ij} in Eqs. (3.18), (3.19).

Our derivation is the same as in [45]. Here we restrict ourselves mentioning only the main quantitative points.

In the vacuum case of Sec.3.1 the process of particles creation and their further absorption was described. In the presence of the background particles this time-ordered picture is wiped out: there appears the possibility of particles absorption before their creation.

The particles creation and absorption was described by the product of operator exponent (3.6). One can derive (see also [45]) the generalizations of (3.6): the presence of the background particles will lead to the same structure:

$$\rho_{cp} = \mathrm{e}^{i\hat{N}(\phi_i^* \phi_j)} \rho_0(\phi_\pm),$$

where $\rho_0(\phi_\pm)$ is the same generating functional, see (3.12). But the operator $\hat{N}(\phi_i^* \phi_j), i, j = +, -$, should be changed wanting to take into account the external particles environment.

The operator $\hat{\phi}_i^*(q)$ was interpreted as the creation and $\hat{\phi}_i(q)$ as the annihilation operator, see definition (3.1). Correspondingly the product $\hat{\phi}_i^*(q)\hat{\phi}_j(q)$ acts as the activity operator. So, in the expansion of $\hat{N}(\phi_i^*\phi_j)$ we can leave only first nontrivial term:

$$\hat{N}(\phi_i^*\phi_j) = \int d\omega(q)\hat{\phi}_i^*(q)n_{ij}\hat{\phi}_j(q), \tag{3.20}$$

since no special correlation among background particles should be expected. If the external (nondynamical) correlations are present, then the higher powers of $\hat{\phi}_i^*\hat{\phi}_j$ will appear in expansion (3.20) [29]. Following the interpretation of $\hat{\phi}_i^*\hat{\phi}_j$ we conclude that n_{ij} is the mean multiplicity of background particles. In (3.20) the normalization condition $N(0) = 0$ was used and summation over all i, j was assumed. (In the vacuum case only the combination $i \neq j$ was present.)

Computing ρ_{cp} we must conserve the translational invariance of amplitudes and extract the energy-momentum conservation δ-functions. We must adjust to each vertex of in-going particle in $a_{n,m}$ the factor $\mathrm{e}^{-i\alpha_1 q/2}$ and for each out-going particle $\mathrm{e}^{-i\alpha_2 q/2}$ one, after Fourier transformation, of these δ-functions.

So, the product $\mathrm{e}^{-i\alpha_k q/2}\mathrm{e}^{-i\alpha_j q/2}$ can be interpreted as the probability factor of the one-particle $(creation + annihilation)$ process. The n-particles $(creation + annihilation)$ process' probability is the simple product of these factors if there are no special correlations among background particles. This interpretation is evident in the CM frame $\alpha_k = (-i\beta_k, \vec{0})$.

After these preliminaries it is not difficult to find that in the CM frame we have:

$$n_{++}(q_0) = n_{--}(q_0) == \frac{1}{\mathrm{e}^{\frac{\beta_1+\beta_2}{2}|q_0|} - 1} \equiv \tilde{n}(|q_0|\frac{\beta_1+\beta_2}{2}). \tag{3.21}$$

Computing n_{ij} for $i \neq j$ we must take into account that we have one additional particle:

$$n_{+-}(q_0) == \Theta(q_0)(1 + \tilde{n}(q_0\beta_1)) + \Theta(-q_0)\tilde{n}(-q_0\beta_1) \tag{3.22}$$

and

$$n_{-+}(q_0) = \Theta(q_0)\tilde{n}(q_0\beta_2) + \Theta(-q_0)(1 + \tilde{n}(-q_0\beta_2)). \tag{3.23}$$

Using (3.21), (3.22) and (3.23), and the definition (3.16) we find the Green functions (the matrix Green functions in the real-time finite-temperature field theories were introduced firstly in [46]):

$$G_{i,j}(x - x', (\beta)) = \int \frac{d^4q}{(2\pi)^4}\mathrm{e}^{iq(x-x')}\tilde{G}_{ij}(q, (\beta))$$

where

$$i\tilde{G}_ij(q, (\beta)) = \begin{pmatrix} \frac{i}{q^2-m^2+i\epsilon} & 0 \\ 0 & -\frac{i}{q^2-m^2-i\epsilon} \end{pmatrix} +$$

$$+2\pi\delta(q^2-m^2)\begin{pmatrix} \tilde{n}(\frac{\beta_1+\beta_2}{2}|q_0|) & \tilde{n}(\beta_2|q_0|)a_+(\beta_2) \\ \tilde{n}(\beta_1|q_0|)a_-(\beta_1) & \tilde{n}(\frac{\beta_1+\beta_2}{2}|q_0|) \end{pmatrix} \tag{3.24}$$

and

$$a_\pm(\beta) = -\mathrm{e}^{\frac{\beta}{2}(|q_0|\pm q_0)}.$$

The corresponding generating functional has the standard form:

$$\rho_{cp}(j_\pm) = \exp\{-iV(-i\hat{j}_+) + iV(-i\hat{j}_-)\} \times$$
$$\times \exp\{\frac{i}{2}\int dxdx' j_i(x)G_{ij}(x-x',(\beta))j_j(x')\}, \tag{3.25}$$

where the summation over repeated indices is assumed.

Inserting (3.25) into the equation of state (3.11) we can find that $\beta_1 = \beta_2 = \beta(E)$. If $\beta(E)$ is a "good" parameter, then $G_{ij}(x-x';\beta)$ coincide with the Green functions of the real-time finite-temperature field theory and the KMS boundary condition:

$$G_{+-}(t-t') = G_{-+}(t-t'-i\beta), \quad G_{-+}(t-t') = G_{+-}(t-t'+i\beta), \tag{3.26}$$

is restored. Eq. (3.26) can be deduced from (3.24) by the direct calculations.

4. APPLICABILITY OF FINITE-TEMPERATURE DESCRIPTION

4.1. The Schwinger-Keldysh Formalism. There are various approaches to build the real-time finite-temperature field theories of Schwinger-Keldysh type (e.g., [10]). All of them use various tricks for analytical continuation of imaginary-time Matsubara formalism to the real time [47]. The basis of the approaches is introduction of Matsubara field operator

$$\Phi_M(\mathbf{x},\beta) = \mathrm{e}^{\beta H}\Phi_S(\mathbf{x})\mathrm{e}^{-\beta H}, \tag{4.1}$$

where $\Phi_S(\mathbf{x})$ is the interaction-picture operator, instead of Heisenberg operator

$$\Phi(\mathbf{x},t) = \mathrm{e}^{itH}\Phi_S(\mathbf{x})\mathrm{e}^{-itH}.$$

This introduces the averaging over Gibbs ensemble instead of averaging over zero-temperature vacuum states.

If the interaction is switched on adiabatically at the instant t_i and is switched off at t_f, then there is the unitary transformation:

$$\Phi(x) = U(t_i,t_f)U(t_i,t)\Phi_S(x)U(t,t_i).$$

Introducing the complex Mills time contours [43] to connect t_i to t, t to t_f and t_f to t_i we form "closed-time" contour C (the end-points of the contours C_+ and C_- are $joint$ together). This allows one to write last equality in the compact form:

$$\Phi(x) = T_C\{\Phi(x)\mathrm{e}^{i\int_C d^4x' L_{int}(x')}\}_S,$$

where T_C is the time-ordering on the contour C operator.

The corresponding expression for the generating functional $Z(j)$ of correlation (Green) functions has the form:

$$Z(j) = R(0) < T_C\mathrm{e}^{i\int_C d^4x\{L_{int}(x)+j(x)\Phi(x)\}_S} >,$$

where $<>$ means averaging over initial state.

If the initial correlations have little effect, we can perform averaging over Gibbs ensemble. This is the main assumption of formalism: the generating functional of the Green functions $Z(j)$ has the form in this case:

$$Z(j) = \int D\Phi' < \Phi'; t_i|\mathrm{e}^{-\beta H}T_C\mathrm{e}^{i\int_C d^4x j(x)\Phi(x)}|\Phi'; t_i >$$

with $\Phi' = \Phi'(\mathbf{x})$. In accordance with (4.1) we have:

$$< \Phi'; t_i|\mathrm{e}^{-\beta H} =< \Phi'; t_i - i\beta|$$

and, in result,

$$Z(j) = \int D\Phi'\mathrm{e}^{i\int_{C_\beta} d^4x\{L(x)+j(x)\Phi(x)}, \tag{4.2}$$

where path integration is performed with KMS periodic boundary condition:

$$\Phi(t_i) = \Phi(t_i - i\beta).$$

In (4.2) the contour C_β connects t_i to t_f, t_f to t_i and t_i to $t_i - i\beta$. Therefore it contains imaginary-time Matsubara part t_i to $t_i - i\beta$. More symmetrical formulation uses following realization: t_i to t_f, t_f to $t_f - i\beta/2$, $t_f - i\beta/2$ to $t_i - i\beta/2$ and $t_i - i\beta/2$ to $t_i - i\beta$ (e.g., [12]). This case also contains the imaginary-time parts of time contour. Therefore, Eq. (4.2) presents the analytical continuation of Matsubara generating functional to real times.

One can note that if this analytical continuation is possible in $Z(j)$, then representation (4.2) gives good recipe of regularization of frequency integrals in the Matsubara perturbation theory, e.g., [10], but nothing new for our problem since the Matsubara formalism is a formalism for equilibrium states only.

Taking $t_i = -\infty$ and $t_f = +\infty$ and calculating integral (4.2) perturbatively we find coincidence of $Z(j)$ and $\rho(\beta)$ from (3.25) with Green functions defined

in (3.24) if $\beta_1 = \beta_2$. This "factorization" of contributions from contours C_+ and C_- in the integral (4.2) follows from the Riemann–Lebesque lemma [48] which is applicable in the perturbation framework [12, 43]. Note absence of Matsubara parts of contour, which prevents the factorization, in the derived "S-matrix generating functional" (3.25) by definition (importance of this circumstances is discussed in Sec.7).

4.2. Range of the "Hydrodynamical" Approximation. Let us return now to Eq. (3.5). To find the physical meaning $\beta_{1(2)}$ we must show the way as they can be measured. If there is nonequilibrium flow it is hard to invent a thermometer (or thermodynamical calorimeter) which measures locally in space-time the temperatures of this dissipative processes. But there was described another way — to define the temperatures through equations of state. This is possible in the accelerator experiments where the total energy E is fixed. So, we will define $\beta_{1(2)}$ through equations of state (3.11), i.e., considering $1/\beta_{1(2)}$ as the mean energy of particles in the initial (final) state. But even knowing solutions of these equations one cannot find $\rho(E, z)$ correctly if the assumption that $\beta_{1(2)}$ are "good" quantities is not added, i.e., that the fluctuations near solutions of Eqs. (3.11) are small (Gaussian).

This assumption is the main problem toward nonequilibrium thermodynamics. The problem in our terms looks as follows: the expansion near $\beta_{1(2)}(E)$ gives asymptotic series over

$$\int \mathbf{D}_s \sim \int \prod \{d\omega(k_i) dr_i\} < \varepsilon(k_1)\varepsilon(k_2)\cdots > |_{(r_1, r_2, \ldots)},$$

where $<>_{()}$ means averaging over fields drown on fixed points of phase space $(k, r)_i$. In other words, the fluctuations near $\beta_{1(2)}(E)$ are defined by the value of inclusive spectra familiar in particle physics. Therefore, $\beta_{1(2)}(E)$ are "good" quantities if this inclusive spectra are small. But this is too strong assumption. More careful analysis shows that it is enough to have the factorization properties [49]:

$$\int \prod \{d\omega(k_i) dr_i\} < \varepsilon(k_1)\varepsilon(k_2)\cdots > |_{(r_1, r_2, \ldots)} - \prod \int d\Omega(k_i) dr_i < \varepsilon(k_i) > |_{(r_i)} \sim 0. \tag{4.3}$$

It must be noted that this is the unique solution of the problem since the expansion near $\beta_{1(2)}(E)$ $unavoidably$ leads to asymptotic series with zero radii of convergence.

One can hope to avoid this problem working permanently in the energy-momentum representation, i.e., without introduction of temperatures. Of course this is possible in particle physics, but if $\beta_{1(2)}(E)$ is not the "good" parameter

this means that all correlations between created particles are sufficient, i.e., only the energy-momentum representation did not solve the problem.

At the end, discussed factorization property of $\mathbf{D}_s$, $s > 1$, is the well-known Bogoliubov condition of "shortened" description of nonequilibrium thermodynamical systems with s-particle distribution functions $\mathbf{D}_s$, $s > 1$, expressed in terms of $\mathbf{D}_1$. It is the condition for the "hydrodynamical" descriptions applicability since it assumes that the constant $\beta_1(E) = \beta_2(E)$ is a "good" parameter for description of whole system.

Considering a problem with nonzero nonequilibrium flaw it is hard to expect that $\beta_{1(2)}(E)$ is a good parameter, i.e., that the factorization conditions are held. Nevertheless, as was mentioned above, there is a possibility to have the *mean values* of correlators sufficiently small in restricted ranges of phase space. It is the so-called "kinetic" phase of the process: when the memory of initial state disappeares, the "fast" fluctuations are averaged over and we can consider the long-range fluctuations only.

5. LOCAL EQUILIBRIUM HYPOTHESIS

Let us return now to description of experimental situation in the high multiplicity experiments. Having at energies of modern accelerators thousands of particles in a final state it is a difficult problem even to count such big numbers. So, the number of particles n cannot be considered as a trigger. Moreover, it seems natural that it is not important whether we have hundred thousand of particles or hundred thousand plus one. To do first step toward CQGP it is enough to be sure that in experiment the transition of "hot" initial state into "cold" final one is examined. For this purpose the ordinary calorimeters can be used [50].

So, we must assume that the energies of created particles $\varepsilon_i \leq \varepsilon_0$, where ε_0 is fixed by experiment. Then using energy conservation law at given ε_0 the number of created particles is bounded from below: $n > \sqrt{s}/\varepsilon_0 \equiv n_{\min}$. With this constraint the integral cross section

$$\sigma_{\varepsilon_0}(s) = \sum_{n=n_{\min}} \sigma_n(s)$$

is measured. Choosing $n_{\min} >> \bar{n}$, i.e., $\varepsilon_0 << \sqrt{s}/\bar{n}(s)$, we get into high multiplicity region. There is also a theoretical possibility of restoring the quantity $\sim \sigma_n$ calculating the difference $\sigma_{\varepsilon_0}(s) - \sigma_{\varepsilon_0+\delta\varepsilon_0}(s)$ [50].

It is not necessary to measure energy of each particle to have $n_{\min} >> \bar{n}$. Indeed, let $\tilde{\varepsilon}_i$ is the energy of i-th group of particles, $\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2 + ... + \tilde{\varepsilon}_k = \sqrt{s}$ and let $\tilde{n}_i$ is the number of particles in the group, $\tilde{n}_1 + \tilde{n}_2 + ... + \tilde{n}_k = n^*$. Then,

*It is assumed that the number of calorimeter cells $K \geq k$.

if $\tilde{\varepsilon}_i < \varepsilon_0$, $i = 1, 2, ..., k$, we have inequality: $k > n_{\min}$. Therefore, we get into high multiplicities domain since $n \geq k$, if $\varepsilon_0 << \sqrt{s}/\bar{n}(s)$. We can use the calorimeter demanding that the induced in each cell energy $\tilde{\varepsilon}_i < \varepsilon_0$.

The preparation of such experiment is not hopeless task and it may be sufficiently informative. This formulation of experiment we will put in the basis of the theory. Theoretically, we should shrink the 4-dimension of calorimeter cells up to zero since we do not know *ad hoc* the cells dimension. Then the cells index i is transformed into the position of particle r. So we come to contradiction with quantum uncertainty principle. This forces one to use the Wigner functions formalism and the first question which must be solved is to find a way how this formalism can be adopted for description of our experiment (there is also interesting ideas concerning applicability of Wigner functions in [51]).

5.1. Vacuum Boundary Condition. We start consideration from the assumption that the temperature fluctuations are large scale. In a cell, the dimension of which is much smaller than the fluctuation scale of temperature, we can assume that the temperature is a "good" parameter. (The "good" parameter means that the corresponding fluctuations are Gaussian.)

Let us surround the interaction region, i.e., the system under consideration, by N cells with known space-time position and let us propose that we can measure the energy and momentum of groups of in- and out-going particles in each cell. The 4-dimension of cells cannot be arbitrary small in this case because of the quantum uncertainty principle.

To describe this situation we decompose δ-functions in (3.4) on the product of $(N+1)$ δ-functions:

$$\delta^{(4)}(P - \sum_{k=1}^{n} q_k) = \int \prod_{\nu=1}^{N} \{dQ_\nu \delta(Q_\nu - \sum_{k=1}^{n_\nu} q_{k,\nu})\} \delta^{(4)}(P - \sum_{\nu=1}^{N} Q_\nu),$$

where $q_{k,\nu}$ are the momentum of k-th in-going particle in the ν-th cell and Q_ν is the total 4-momenta of n_ν in-going particles in this cell, $\nu = 1, 2, ..., N$. The same decomposition will be used for the second δ-function in (3.4). We must take into account the multinomial character of particles decomposition on N groups. This will give the coefficient:

$$\frac{n!}{n_1! \cdots n_N!} \delta_K(n - \sum_{\nu=1}^{N} n_\nu) \frac{m!}{m_1! \cdots m_N!} \delta_K(m - \sum_{\nu=1}^{N} m_\nu),$$

where δ_K is Kronecker's δ-function.

In result, the quantity

$$r((Q)_N, (P)_N) = \sum_{(n.m)} \int |a_{(n,m)}|^2 \times$$

$$\times \prod_{\nu=1}^{N} \{ \prod_{k=1}^{n_\nu} \frac{d\omega(q_{k,\nu})}{n_\nu!} \delta^{(4)}(Q_\nu - \sum_{k=1}^{n_\nu} q_{k,\nu}) \prod_{k=1}^{m_\nu} \frac{d\omega(p_{k,\nu})}{m_\nu!} \delta^{(4)}(P_\nu - \sum_{k=1}^{m_\nu} p_{k,\nu}) \} \quad (5.1)$$

describes a probability to measure in the ν-th cell the fluxes of in-going particles with total 4-momentum Q_ν and of out-going particles with the total 4-momentum P_ν. The sequence of this two measurements is not fixed.

The Fourier transformation of δ-functions in (5.1) gives:

$$r((Q)_N, (P)_N) = \int \prod_{k=1}^{N} \frac{d^4\alpha_{1,\nu}}{(2\pi)^4} \frac{d^4\alpha_{2,\nu}}{(2\pi)^4} \mathrm{e}^{i \sum_{\nu=1}^{N} (Q_\nu \alpha_{1,\nu} + P_\nu \alpha_{2,\nu})} \rho((\alpha_1)_N, (\alpha_2)_N),$$

where

$$\rho((\alpha_1)_N, (\alpha_2)_N) = \rho(\alpha_{1,1}, \alpha_{1,2} ..., \alpha_{1,N}; \alpha_{2,1}, \alpha_{2,2}, ..., \alpha_{2,N})$$

has the form:

$$\rho((\alpha_1)_N, (\alpha_2)_N) = \int \prod_{\nu=1}^{N} \{ \prod_{k=1}^{n_\nu} \frac{d\omega(q_{k,\nu})}{n_\nu!} \mathrm{e}^{-i\alpha_{1,\nu} q_{k,\nu}} \times$$
$$\times \prod_{k=1}^{m_\nu} \frac{d\omega(p_{k,\nu})}{m_\nu!} \mathrm{e}^{-i\alpha_{2,\nu} p_{k.\nu}} \} |a_{(n,m)}|^2. \quad (5.2)$$

Inserting (3.1) into (5.2) we find:

$$\rho((\alpha_-)_N, (\alpha_+)_N) = \exp\{ i \sum_{\nu=1}^{N} \int dx dx' [\hat{\phi}_+(x) D_{+-}(x - x'; \alpha_{2,\nu}) \hat{\phi}_-(x') -$$
$$- \hat{\phi}_-(x) D_{-+}(x - x'; \alpha_{1,\nu}) \hat{\phi}_+(x')] \} Z(\phi_+) Z^*(\phi_-), \quad (5.3)$$

where ϕ_- is defined on the complex conjugate contour $C_- : t \to t - i\varepsilon$ and $D_{+-}(x - x'; \alpha)$, $D_{-+}(x - x'; \alpha)$ are the positive and negative frequency correlation functions correspondingly.

We must integrate over sets $(Q)_N$ and $(P)_N$ if the distribution of fluxes momenta over cells is not fixed. In result,

$$r(P) = \int D^4\alpha_1(P) d^4\alpha_2(P) \rho((\alpha_1)_N, (\alpha_2)_N), \quad (5.4)$$

where the differential measure

$$D^4\alpha(P) = \prod_{\nu=1}^{N} \frac{d^4\alpha_\nu}{(2\pi)^4} K(P, (\alpha)_N)$$

takes into account the energy-momentum conservation laws:

$$K(P,(\alpha)_N)=\int\prod_{\nu=1}^{N}d^4Q_\nu \mathrm{e}^{i\sum_{\nu=1}^{N}\alpha_\nu Q_\nu}\delta^{(4)}(P-\sum_{\nu=1}^{N}Q_\nu).$$

The explicit integration gives that

$$K(P,(\alpha)_N)\sim\prod_{\nu=1}^{N}\delta^{(3)}(\alpha-\alpha_\nu),$$

where $\vec{\alpha}$ is the center of mass (CM) 3-vector.

To simplify the consideration let us choose the CM frame and put $\alpha=(-i\beta,\vec{0})$. In result,

$$K(E,(\beta)_N)=\int_0^\infty\prod_{\nu=1}^{N}dE_\nu \mathrm{e}^{\sum_{\nu=1}^{N}\beta_\nu E_\nu}\delta(E-\sum_{\nu=1}^{N}E_\nu).$$

Correspondingly, in the CM frame,

$$r(E)=\int D\beta_1(E)D\beta_2(E)\rho((\beta_1)_N,(\beta_2)_N),$$

where

$$D\beta(E)=\prod_{\nu=1}^{N}\frac{d\beta_\nu}{2\pi i}K(E,(\beta)_N)$$

and $\rho((\beta)_N)$ was defined in (5.3) with $\alpha_{k,\nu}=(-i\beta_{k,\nu},\vec{0})$, $Re\beta_{k,\nu}>0$, $k=1,2$.

We will calculate integrals over β_k using the stationary phase method. The equations for mostly probable values of β_k:

$$-\frac{1}{K(E,(\beta_k)_N)}\frac{\partial}{\partial\beta_{k,\nu}}K(E,(\beta_k)_N)=\frac{1}{\rho((\beta_1)_N)}\frac{\partial}{\partial\beta_{k,\nu}}\rho((\beta)_N),\quad k=1,2, \tag{5.5}$$

always have the unique positive solutions $\tilde{\beta}_{k,\nu}(E)$. We propose that the fluctuations of β_k near $\tilde{\beta}_k$ are small, i.e., are Gaussian. This is the basis of the local-equilibrium hypothesis [32]. In this case $1/\tilde{\beta}_{1,\nu}$ is the temperature in the initial state in the measurement cell ν and $1/\tilde{\beta}_{2,\nu}$ is the temperature of the final state in the ν-th measurement cell.

The last formulation (5.4) implies that the 4-momenta $(Q)_N$ and $(P)_N$ cannot be measured. It is possible to consider another formulation also. For instance,

we can suppose that the initial set $(Q)_N$ is fixed (measured) but $(P)_N$ is not. In this case we will have mixed experiment: $\tilde{\beta}_{1,\nu}$ is defined by the equation:

$$E_\nu = -\frac{1}{\rho}\frac{\partial}{\partial\beta_{1,\nu}}\rho$$

and $\tilde{\beta}_{2,\nu}$ is defined by second equation in (5.5).

Considering limit $N \to \infty$ the dimension of cells tends to zero. In this case we are forced by quantum uncertainty principle to propose that the 4-momenta sets (Q) and (P) are not fixed. This formulation becomes pure thermodynamical: we must assume that (β_1) and (β_2) are measurable quantities. For instance, we can fix (β_1) and try to find (β_2) as the function of total energy E and the functional of (β_1). In this case Eqs. (5.5) become the functional equations.

In the considered microcanonical description the finiteness of temperature does not touch the quantization mechanism. Really, one can see from (5.3) that all thermodynamical information is confined in the operator exponent

$$\mathrm{e}^{\hat{N}(\phi_i^*\phi_j)} = \prod_\nu \prod_{i\neq j} \mathrm{e}^{i\int \hat{\phi}_i D_{ij} \hat{\phi}_j}$$

the expansion of which describes the environment, and the "mechanical" perturbations are described by the amplitude $Z(\phi)$. This factorization was achieved by introduction of auxiliary field ϕ and is independent of the choice of boundary conditions, i.e., of the choice of considered systems environment.

5.2. Wigner Functions Formalism. We will use the Wigner functions formalism in the Carrusers-Zachariasen formulation [25]. For the sake of generality the m into n particles transition will be considered. This will allow one to include into consideration the heavy ion-ion collisions.

In the previous section the generating functional $\rho((\beta)_N)$ was calculated by means of dividing the "measuring device" (calorimeter) on the N cells. It was assumed that the dimension of device cells tends to zero ($N \to \infty$). Now we will specify the cells coordinates using Wigner's description.

Let us introduce the distribution function F_n which defines the probability to find n particles with definite momentum and with arbitrary coordinates. This probabilities (cross sections) are usually measured in particle physics. The corresponding Fourier-transformed generating functional can be deduced from (5.3):

$$F(z,(\beta_+)_N,(\beta_-)_N) = \prod_{\nu=1}^{N}\prod_{i\neq j} \mathrm{e}^{\int d\omega(q)\hat{\phi}_i^*(q)\mathrm{e}^{-\beta_{j,\nu}\epsilon(q)}\hat{\phi}_j(q)z_{ij}^\nu(q)} \times$$
$$\times Z(\phi_+)Z^*(\phi_-). \qquad (5.6)$$

The variation of F over $z_{ij}^\nu(q)$ generates corresponding distribution functions. One can interpret $z_{ij}^\nu(q)$ as the local activity: the logarithm of $z_{ij}^\nu(q)$ is conjugate

to the particles number in the cell ν with momentum q for the initial ($ij = 21$) or final ($ij = 12$) states. Note that $z^{\nu}_{ij}(q)\hat{\phi}^{*}_{i}(q)\hat{\phi}_{j}(q)$ can be considered as the operator of activity.

The Boltzman factor $\mathrm{e}^{-\beta_{i,\nu}\epsilon(q)}$ can be interpreted as the probability to find a particle with the energy $\epsilon(q)$ in the final state ($i = 2$) and in the initial state ($i = 1$). The total probability, i.e., the process of creation and further absorption of n particles, is defined by multiplication of this factors.

The generating functional (5.6) is normalized as follows:

$$F(z = 1, (\beta)) = R((\beta)), \tag{5.7}$$

$$F(z = 0, (\beta)) = |Z(0)|^2 = \rho_0(\phi_{\pm})|_{\phi_{\pm}=0},$$

where

$$\rho_0(\phi_{\pm}) = Z(\phi_{+})Z^{*}(\phi_{-})$$

is the "probability" of the vacuum into vacuum transition in presence of auxiliary fields $\phi_{\pm}$. The one-particle distribution function

$$F_1((\beta_1)_N, (\beta_2)_N; q) = \frac{\delta}{\delta z^{\nu}_{ij}(q)} F|_{z=0} =$$

$$= \{\hat{\phi}^{*}_{i}(q)\mathrm{e}^{-\beta^{\nu}_{i}\epsilon(q)/2}\}\{\hat{\phi}_{j}(q)\mathrm{e}^{-\beta^{\nu}_{i}\epsilon(q)/2}\}\rho_0(\phi_{\pm}) \tag{5.8}$$

describes the probability to find one particle in the vacuum.

Using definition

$$F_1((\beta_1)_N, (\beta_2)_N; q) = \int dx dx' \mathrm{e}^{iq(x-x')}\mathrm{e}^{-\beta_{i,\nu}\epsilon(q)}\}\hat{\phi}_{i}(x)\hat{\phi}_{j}(x')\rho_0(\phi_{\pm}) =$$

$$= \int dr\{dy\mathrm{e}^{iqy}\mathrm{e}^{-\beta_{i,\nu}\epsilon(q)}\}\hat{\phi}_{i}(r + y/2)\hat{\phi}_{j}(r - y/2)\rho_0(\phi_{\pm})\}, \tag{5.9}$$

we introduce the one-particle Wigner function W_1 [25]:

$$F_1((\beta_1)_N, (\beta_2)_N; q) = \int dr W_1((\beta_1)_N, (\beta_2)_N; r, q).$$

So,

$$W_1((\beta_1)_N, (\beta_2)_N; r, q) = \int dy\mathrm{e}^{iqy}\mathrm{e}^{-\beta_{i,\nu}\varepsilon(q)}\hat{\phi}_{i}(r + y/2)\hat{\phi}_{j}(r - y/2)\rho_0(\phi_{\pm}).$$

This distribution function describes the probability to find in the vacuum the particle with momentum q at the point r in the cell ν.

Since the choice of the device coordinates is in our hands it is natural to adjust the cell coordinate to the coordinate of measurement r:

$$W_1((\beta_1)_N, (\beta_2)_N; r, q) = \int dy\mathrm{e}^{iqy}\mathrm{e}^{-\beta_{i}(r)\epsilon(q)}\}\hat{\phi}_{i}(r + y/2)\hat{\phi}_{j}(r - y/2)\rho_0(\phi_{\pm}).$$

This choice of the device coordinates leads to the following generating functional:

$$F(z,\beta) = \exp\{i \int dydr[\hat{\phi}_+(r+y/2)D_{+-}(y;\beta_2(r),z)\hat{\phi}_-(r-y/2) - \\ -\hat{\phi}_-(r+y/2)D_{-+}(y;\beta_1(r),z)\hat{\phi}_+(r-y/2)]\}\rho_0(\phi_\pm), \quad (5.10)$$

where

$$D_{+-}(y;\beta(r),z) = -i\int d\omega(q) z_{+-}(r,q)\mathrm{e}^{iqy}\mathrm{e}^{-\beta(r)\epsilon(q)},$$

$$D_{-+}(y;\beta(r),z) = i\int d\omega(q) z_{-+}(r,q)\mathrm{e}^{-iqy}\mathrm{e}^{-\beta(r)\epsilon(q)}$$

are the modified positive and negative correlation functions.

The inclusive, partial, distribution functions are familiar in the particle physics. These functions describe the distributions in presence of arbitrary number of other particles. For instance, one-particle partial distribution function

$$P_{ij}(r,q;(\beta)) = \frac{\delta}{\delta z_{ij}(r,q)} F(z,(\beta))|_{z=1} = \\ = \frac{\mathrm{e}^{-\beta_i(r)\epsilon(q)}}{(2\pi)^3\epsilon(q)}\int dy \mathrm{e}^{iqy}\hat{\phi}_i(r+y/2)\hat{\phi}_j(r-y/2)\rho(\phi_\pm,(\beta)), \quad (5.11)$$

where Eq. (5.7) was used.

The mean multiplicity $n_{ij}(r,q)$ of particles in the infinitesimal cell Y with momentum q is

$$n_{ij}(r,q) = \int dq \frac{\delta}{\delta z_{ij}(r,q)} \ln F(z,(\beta))|_{z=1}.$$

If the interactions among fields are switched out, we can find that (omitting indexes):

$$n(r,q_0) = \frac{1}{\mathrm{e}^{\beta(r)q_0}-1}, \quad q_0 = \epsilon(q) > 0.$$

This is the mean multiplicity of black-body radiation.

5.3. Closed Path Boundary Conditions. The developed formalism allows one to introduce more general "closed-path" boundary conditions. Presence of external black-body radiation flow will reorganize the differential operator $\exp\{\hat{N}(\phi_i^*\phi_j)\}$ only and new generating functional ρ_{cp} has the form:

$$\rho_{cp}(\alpha_1,\alpha_2) = \mathrm{e}^{\hat{N}(\phi_i^*\phi_j)}\rho_0(\phi_\pm).$$

The calculation of operator $\hat{N}(\phi_i^*\phi_j)$ is strictly the same as in Sec.3. Introducing the cells we will find that

$$\hat{N}(\phi_i^*\phi_j) = \int drdy\hat{\phi}_i(r+y/2)\tilde{n}_{ij}(r,y)\hat{\phi}_j(r-y/2),$$

where the occupation number $\tilde{n}_{ij}$ carries the cells index r:

$$\tilde{n}_{ij}(r,y)=\int d\omega(q)\mathrm{e}^{iqy}n_{ij}(r,q)$$

and ($q_0=\epsilon(q)$)

$$n_{++}(r,q_0)=n_{--}(r,q_0)=\tilde{n}(r,(\beta_1+\beta_2)|q_0|/2)=\frac{1}{\mathrm{e}^{(\beta_1+\beta_2)(r)|q_0|/2}-1},$$

$$n_{+-}(r,q_0)=\Theta(q_0)(1+\tilde{n}(r,\beta_2 q_0))+\Theta(-q_0)\tilde{n}(r,-\beta_1 q_0),$$

$$n_{-+}(r,q_0)=n_{+-}(r,-q_0).$$

For simplicity the CM system was used.

Calculating ρ_0 perturbatively we will find that

$$\rho_{cp}(\beta)=\exp\{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)\}\times$$
$$\exp\{i\int drdy[\hat{j}_i(r+y/2)G_{ij}(y,(\beta(r))\hat{j}_j(r-y/2)\} \tag{5.12}$$

where, using the matrix notations,

$$iG(q,(\beta(r)))=\begin{pmatrix}\frac{i}{q^2-m^2+i\varepsilon} & 0\\ 0 & -\frac{i}{q^2-m^2-i\varepsilon}\end{pmatrix}+$$
$$+2\pi\delta(q^2-m^2)\begin{pmatrix}n(\frac{(\beta_1+\beta_2)(r)}{2}|q_0|) & n(\beta_1(r)|q_0|)a_+(\beta_1)\\ n(\beta_2(r)|q_0|)a_-(\beta_2) & n(\frac{(\beta_1+\beta_2)(r)}{2}|q_0|)\end{pmatrix}, \tag{5.13}$$

and

$$a_\pm(\beta)=-\mathrm{e}^{\beta(|q_0|\pm q_0)/2}. \tag{5.14}$$

Formally these Green functions obey the standard equations in the y space:

$$(\partial^2-m^2)_y G_{ii}=\delta(y),$$

$$(\partial^2-m^2)_y G_{ij}=0,\ \ i\neq j$$

since $\Phi(\sigma_\infty)\neq 0$ reflects the mass-shell particles. But the boundary conditions for these equations are not evident.

It should be underlined that in our consideration r is the coordinate of $measurement$, i.e., r is as the calorimeter cells coordinate and there is no necessity to divide the interaction region of QGP on domains (cells). This means that L must be smaller than the typical range of fluctuations of QGP. But, on the other hand, L cannot be arbitrary small since this will lead to assumption of $local$ factorization property of correlators, i.e., to absence of interactions.

So, changing $\beta \to \beta(r)$ we should assume that $\beta_{1(2)}(r)$ and $z_{+-(-+)}(r,k)$ are constants on interval L. This prescription adopts Wigner functions formalism for the case of high multiplicities. It describes the temperature fluctuations larger than L and averages the fluctuations smaller than L leading to absence, in average, of "non-Gaussian" fluctuations.

It is the typical "calorimetric" measurement since in a dominant number of calorimeter cells the measured mean values of energy, with exponential accuracy, are the "good" parameters $\sim 1/\beta_2(r,E)$. We will assume that the dimension of calorimeter cells $L << L_{cr}$, where L_{cr} is the dimension of characteristic fluctuations at given n. In deep asymptotic over n we must have $L_{cr} \to \infty$. This consideration shows that the offered experiment with calorimeter as the measuring device of particles energies is sufficiently informative in the high multiplicities domain.

6. NONSTATIONARY STATISTICAL OPERATOR

One cannot expect the evident connection between the above considered S-matrix (microcanonical) and Zubarev's [32] approaches. The reason is introduction into Zubarev's formalism of an interaction with a heat bath, external to system under consideration. This interaction is crucial for definition of NSL for explanation of the trend to maximal-entropy state, starting evolution from local-equilibrium state*.

Therefore, in Zubarev's theory the local-equilibrium state was chosen as the boundary condition. It is assumed that in the suitably defined cells of the $system$ at a given temperature distribution $T(\vec{x},t) = 1/\beta(\vec{x},t)$, where $(\vec{x},t)$ is the index of the cell, the entropy is maximal. The corresponding nonequilibrium statistical operator

$$\rho_z \sim \mathrm{e}^{-\int d^3x \beta(\vec{x},t) T_{00}} \tag{6.1}$$

describes evolution of a system in the time scale t. Here $T_{\mu\nu}$ is the energy-momentum tensor. It is assumed that the system "follows' to $\beta(\vec{x},t)$ evolution and the local temperature $T(\vec{x},t)$ is defined as the external parameter which is the regulator of systems dynamics. For this purpose the special $i\varepsilon$-prescription was introduced (it was not shown in (6.1)) [32]. It brings the interaction with heat bath.

The KMS periodic boundary condition cannot be applied for nonstationary temperature distribution and by this reason the decomposition:

$$\beta(\vec{x},t) = \beta_0 + \beta_1(\vec{x},t) \tag{6.2}$$

*This condition is not necessary in the S-matrix formalism since it is "dynamical" by its nature, i.e., includes the notion of initial- and final-states as the boundary conditions.

was offered in the paper [30]. Here β_0 is the constant, and the inequality

$$\beta_0 >> |\beta_1(\vec{x},t)|$$

is assumed. Then,

$$\rho_z \sim \mathrm{e}^{-\beta_0(H_0+V+B)}, \tag{6.3}$$

where H_0 is the free part of the Hamiltonian, V describes the interactions, and the linear over β_1/β_0 term B is connected with the deviation of temperature from the "equilibrium" value $1/\beta_0$. Presence of B perturbations creates the "thermal" flows in the system to explain increasing entropy. Considering V and B as the perturbations one can calculate the observables averaging over equilibrium states, i.e., adopting the KMS boundary condition. Using standard terminology one can consider V as the "mechanical" and B as the "thermal" perturbations.

The quantization problem of operator (6.3) is connected with definition of the space-time sequence of mechanical (V) and thermal (B) excitations. It is necessary since the mechanical excitations give the influence on the thermal ones and vice versa. It was assumed in [30] that V and B are commuting operators, i.e., the sequence of V- and B perturbations is not sufficient. The corresponding generating functional has the form [30]:

$$Z(j) = \exp\{-i\int_{C_\beta} d^4x(V(-i\hat{j}(x)) + \frac{\beta_1(\mathbf{x},\tau)}{\beta_0}T_{00}[-i\hat{j}(x)] -$$

$$\int_{-\infty}^{0} dt_1 \frac{\beta_1(\mathbf{x},\tau+t_1)}{\beta_0}T_{00}[-i\hat{j}(\mathbf{x},x_0t_1)])\}Tr(\mathrm{e}^{-\beta_0 H_0}T_C\mathrm{e}^{i\int_C d^4yj(y)\Phi(y)}),$$

where the time contour C_β was described in Sec.4.1, and τ is the measurement time.

It is evident that this solution leads to the renormalization by the interactions with the external field $\beta(\vec{x},t)$ even without interactions among fundamental fields Φ. The source of this renormalizations is the kinetic term in the energy-momentum tensor T_{00}, i.e., follows from "thermal" interactions with external heat bath. Note absence of this renormalizations in the S-matrix formalism, see, for instance (3.25), where the interactions are generated by V perturbations only.

In [53] the operators V and B are noncommuting ones and B perturbations were switched on after V perturbations. In this formulation the nondynamical renormalizations are also present but it is not unlikely that they are canceled at the very end of calculations [54].

This formulation with $\beta(\vec{x},t)$ as the external field reminds the old, firstly quantized, field theory in which matter is quantized but fields are not. It is well known that consistent quantum field theory requires the second quantization. Following to this analogy, if we want to take into account consistently the reciprocal

influence of V and B perturbations, the field $\beta(\vec{x},t)$ must be fundamental, i.e., must be quantized (and the assumption of paper [30] becomes true). But it is evidently the wrong idea in the canonical Gibbs formalism. So, as in the firstly quantized theory, the theory with operator (6.1) must have the restricted range of validity [32].

7. CONCLUSION

In our interpretation of the real-time finite-temperature field theory the statistics and the fields quantum dynamics were factorized: statistics is fixed by the operator $\exp\{\hat{N}(\phi_i^*\phi_j\}$ and a pure field-theoretical dynamics is described by $\rho_0(\phi_\pm) = Z(\phi_+)Z^*(\phi_-)$, where $Z(\phi_\pm)$ is the vacuum into vacuum transition amplitude in the presence of the external (auxiliary) fields $< vac|vac >_\phi$. We can say that the operator $\exp\{\hat{N}(\phi_i^*\phi_j\}$ maps the system of interacting fields on the state with definite thermodynamical parameters. We had concentrated our attention in this paper on the structure and origin of operator $\exp\{\hat{N}(\phi_i^*\phi_j\}$ only and do not discuss $\rho_0(\phi_\pm)$. But the developed formalism allows one to use following "S-matrix" properties which are new for thermodynamics to define ρ_0.

First of them is the absence of Matsubara imaginary parts of time contour in ρ_0 by definition: the approach is pure "real-time". This allows one to construct the formalism without referring to time asymptotic properties of correlation (Green) functions, and introduce the temperature description without using a notion of grand canonical ensemble constructing the environment of the system, i.e., the measuring device, "by hand".

Moreover, discussed factorization property has important consequence which would allow one to calculate expectation values with high accuracy. Let us consider the theoretical problem of the $\rho_0(\phi_\pm)$ calculation. To define the functional measure the orthonormalizability (i.e., the unitarity) condition may be used. It leads to the following representation [55]:

$$\rho_0(\phi) = \mathrm{e}^{-i\hat{K}(j,e)} \int DM(\Phi)\mathrm{e}^{-U(\Phi,e)}\mathrm{e}^{\int dx(v'(\Phi)+j)\phi}, \tag{7.1}$$

where the expansion over operator

$$\hat{K}(j,e) = 2Re \int dx \frac{\delta}{\delta j(x)} \frac{\delta}{\delta e(x)}$$

generates perturbation series and

$$U(\Phi,e) = V(\Phi+e) - V(\Phi-e) - 2Re \int dx e v'(\Phi)$$

weights quantum fluctuations. The most important term in (7.1) is the measure

$$DM(\Phi) = \prod_x d\Phi(x)\delta(\partial_\mu^2\Phi + m^2\Phi + v'(\Phi) - j),$$

where $v'(\Phi) \equiv \delta V(\Phi)/\delta\Phi(x)$. So, solving the equation

$$\partial_\mu^2\Phi + m^2\Phi + v'(\Phi) = j \tag{7.2}$$

we will find all contributions*.

At the very end of calculations one must put $e = j = 0$. Therefore, Eq. (7.2) can be solved expending it over j. This shows that (7.1) restores at $j = 0$ the usual stationary phase method. Indeed, it can be verified that (7.1) gives usual perturbation theory [55].

But Eq. (7.2) gives much more possibilities. Note that l.h.s. of this equation is the sum of known classical forces and the r.h.s. is the quantum force j. Eq. (7.2) establishes the local equilibrium between this forces. This solves the old standing problem of quantization with constraints: it can be done by field transformations in path integrals since Eq. (7.2) shows the way as j must be transformed when the l.h.s. is transformed. Presence of derivatives in (7.2) shows that the quantum force must be transformed in the tangent space of fields**.

The r.h.s. of Eq. (7.2) may contain also an additional force to describe the external influence on the system of interacting fields. This force was omitted in Eq. (7.2) assuming that a process of particles creation (and absorption) is switched on adiabatically.

As was mentioned above the action of operator $\mathrm{e}^{-\hat{N}(\beta,z;\phi)}$ on $\rho_0(\phi)$ maps interacting fields system on measurable states. Let us consider what this gives. Result of action has the form:

$$\rho(\beta, z) = \mathrm{e}^{-i\hat{K}(j,e)} \int DM(\Phi)\mathrm{e}^{-U(\Phi,e)}\mathrm{e}^{-N(\beta,z.;\Phi)},$$

where $N = N_1 + N_2$ and

$$N_{1(2)}(\beta, z; \Phi) = \int drd\omega(k)\mathrm{e}^{-\beta_{1(2)}(r)\varepsilon(k)} z_{+-(-+)}(k, r)|\Gamma(k, \Phi)|^2. \tag{7.3}$$

Here r is considered as the $index$ of calorimeter cell. This formulae needs more careful explanation. Instead of (7.3) we must consider

$$N_{1(2)}(\beta, z; \Phi) = \int drd\omega(k)\mathrm{e}^{-\beta_{1(2)}(r)\varepsilon(k)} z_{+-(-+)}(k, r)$$

*This means that the unitarity condition is necessary and sufficient for definition of path integral measure for $R_0(\phi_\pm)$ [56]

**This explains why the ordinary transformation of path integral is impossible, gives wrong result [57].

$$\int dq \delta_L(q) \Gamma(k+q,\Phi)\Gamma^*(k-q,\Phi). \tag{7.4}$$

where L is the scale, where $\beta_{1(2)}(r)$ and $z_{+-(-+)}(k,r)$ can be considered as the constants (L is the dimension of calorimeter cell). If $L \to \infty$, then $\delta_L(q)$ can be changed on usual δ-function $\delta(q)$ and, therefore, in this limit we will have (7.3). We had considered this limit supposing that the measurement is not in contradiction with quantum uncertainty principle.

So, deriving $N_{1(2)}(\beta, z; \Phi)$ we used the condition that r is the coordinate of size L cell. With this condition

$$\Gamma(k,\Phi) = \int dx \mathrm{e}^{ikx}(\partial_\mu^2 + m^2)\Phi \tag{7.5}$$

can be considered as the order parameter. Indeed, $\Gamma(k,\Phi)$ is the element of actions symmetry group since it is linear over field Φ and the generating functional $\rho(\beta, z)$ is trivial if $< |\Gamma(k,\Phi)|^2 >= 0$. In this case there is no creation of particles, i.e., there are no measurable asymptotic states (fields).

Indeed, it can be shown [58] that all quantum corrections to solitons contribution in the (1+1)-dimensional sine-Gordon model equal zero. This is in accordance with the result of [59] and with factorizability of solitons S matrix. Then it is easily seen computing integral in (7.5) by parts that $\Gamma(k, \Phi_s) = 0$, where Φ_s is the soliton solution. This result shows that hidden symmetry of sine-Gordon model cannot be broken and corresponding (polynomial) integrals of motion are conserved. The application of this idea for non-Abelian field theory should be fruitful.

ACKNOWLEDGMENT

I would like to thank my colleagues from the Institute of Physics (Tbilisi), Institute of Mathematics (Tbilisi), Institute of Nuclear Physics (St.-Petersburg), Joint Institute of Nuclear Research (Dubna), Institute of Experimental and Theoretical Physics (Moscow), Institute of Theoretical Physics (Kiev) and DAMPT (Cambridge), and especially I.Paziashvili and T.Bibilashvili, for fruitful discussions which gave me the chance of extracting the questions that were necessary to show up in this paper. I would like also to thank A.Sissakian for fruitful collaboration in the problem of high-multiplicity hadron reactions. It must be underlined the stimulating interest to this work by V.Kadyshevsky. The experimental side of the problem I learn from T.Lomtadze and T.Grigalashvili. I want to mention also the important information about Wigner functions applicability in the modern solid state physics from M.Tomak. The work was supported in part by the U.S. National Science Foundation and was granted in part by the Georgian Academy of Sciences.

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