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LIGHT-FRONT FORMALISM IN THE QUASI-POTENTIAL APPROACH IN QUANTUM FIELD THEORY

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The review of the light-front formulation of the quasi-potential approach in quantum field theory for bound state and scattering problems is given.

Обзор посвящен применению квазипотенциальной формулировки теории поля на нуль-плоскости для описания процессов рассеяния с участием связанных состояний.

1. INTRODUCTION

The most general information about two- and many-body systems in quantum field theory is contained in the corresponding many-time Green functions, which are related to each other by function equations. In some conditions from these equations one can obtain equations for two-body bound state and scattering problems (Bethe–Salpeter-type equations [1]). The dependence of the Bethe–Salpeter amplitude (wave function) on the relative time of two particles leads to the fact that it contains the information on bound state and on the states, which have nothing to do with bound states as well. «Electron today and proton tomorrow» do not form the bound state — hydrogen atom. Similar difficulties arise in the case of many-body systems.

A regular method for excluding the relative time, based on the two-time Green functions has been developed in Ref. 2, where relativistic three-dimensional equations for bound state and scattering problems were derived. These equations are known as quasi-potential equations, because of their similarity with the corresponding equations of quantum mechanics. For quasi-potential wave functions the boundary conditions corresponding to bound state and scattering problems can be imposed. Relativistically covariant form of these equations for two- and many-body systems is given in Ref. 3. Similar equation for two particles in the Hamiltonian formulation of quantum field theory has been derived in Ref. 4.

With the development of quark models and the study of structure of particles and nuclei at high momentum transfer it turned out to be convenient the light-front form of quasi-potential equations [5].

In this approach the relativistic composite system with the total 4-momentum P is described by means of the quasi-potential wave function $\Phi_P([x^{(i)}, \mathbf{p}_\perp^{(i)}])$, where the «longitudinal motion» of constituents is parametrized by means of the scale-invariant variables

$$x^{(i)} = \frac{p_0^{(i)} + p_3^{(i)}}{P_0 + P_3},$$

where $p_\mu^{(i)}(\mu$ =0,1,2,3 is the Lorentz index) and P_μ are the individual 4-momentum of the i-th constituent and the total 4-momentum of the system, respectively. Variables $x^{(i)}$ are ratios of the light-front variables. In terms of these variables the wave function of the composite system reflects, in particular, the dependence of the internal motion of constituents on the total momentum of the system. Square brackets in the argument of the wave function Φ_p denote the set of the variables $x^{(i)}$ and $\mathbf{p}_\perp^{(i)}$ which satisfy the conditions

$$\sum_{i=1}^{N} x^{(i)} = 1; \;\; 0 < x^{(i)} < 1; \;\; \sum_{i=1}^{N} \mathbf{p}_\perp^{(i)} = \mathbf{P}_\perp.$$

The review is organized as follows:

Section 2 is devoted to the formulation of the light-front formalism for composite systems. Equations for bound states and scattering problems are given. It is shown how equations of this approach are related to or differ from the equation obtained in the framework of the old-fashioned perturbation theory in the infinite momentum frame. Spectral and projective properties of the «two-time» Green functions are studied.

Section 3 deals with the method of constructing of relativistic elastic form factors and scattering amplitudes of composite systems in the light-front formalism. A general expression for the matrix element of the current of composite system in terms of relativistic wave functions and the generalized vertex operator $\tilde{\Gamma}_\mu$ is given. The electromagnetic form factor for a system, consisting of two or arbitrary number of constituents is presented in the impulse approximation.

Problems of the interaction of relativistic composite systems are also discussed in this section. The scattering amplitude is expressed in a general form, using relativistic wave functions and the transition operator. The constituent interchange mechanism is considered.

Section 4 is devoted to the study of deep inelastic form factors of composite systems. Like the case of elastic form factors, a general expression for the deep inelastic tensor $W_{\mu\nu}$, in terms of the relativistic wave functions are the generalized two-photon vertex $\tilde{\Gamma}_{\mu\nu}$ is given. The explicit form of the structure functions W_1

and νW_2 in the lowest order in the electromagnetic interaction is presented. It is shown that if the transverse motion of quarks is taken into account, the Bjorken scaling is violated and the structure functions become the square of the momentum transfer dependent.

In Section 5 inclusive hadron-hadron processes are considered. General representations for the inclusive cross sections in terms of the light-front wave functions are given. Approximations are treated which lead to the quark-parton description of these processes.

Note, that the review is based mainly on the results obtained in the Dubna school. Other forms of light-front dynamics and appropriate lists of references can be found in a number of original and review papers (see, e.g., [6–15]).

2. LIGHT-FRONT FORMULATION OF BOUND STATE AND SCATTERING PROBLEMS

Light-front variables have been introduced by Dirac [16] with the aim to construct the quantum theory with commutation relations on the light-front hyperplane (instead of traditionally used $t = 0$ hyperplane). In this section, following Ref. 2 equations for bound state and scattering problems in light-front variables are derived.

2.1. Equation for the Two-Body Bound State Wave Function. Consider the Bethe–Salpeter amplitude (wave function)

$$\chi_{P,\alpha} = \langle 0|T(\phi_1(x_1)\phi_2(x_2))|P,\alpha\rangle = \mathrm{e}^{-iPX}\chi_{P,\alpha}(x). \tag{2.1}$$

Here $|P,\alpha\rangle$ is the state vector with total 4-momentum P and quantum numbers α, $X = (x_1 + x_2)/2$ is the centre of mass coordinate, $P = p_1 + p_2$. Define the relative coordinate and momentum

$$x = x_1 - x_2, \quad p = \frac{p_1 - p_2}{2}, \tag{2.2}$$

and introduce the light-front variables

$$x_\pm = \frac{x_0 \pm x_3}{2}, \quad p_\pm = p_0 \pm p_3, \quad P_\pm = P_0 \pm P_3. \tag{2.3}$$

Introduce then the Fourier transform $\chi_{P,\alpha}(p) = \chi_{P,\alpha}(p_-, p_+, \mathbf{p}_\perp)$ of the Bethe–Salpeter amplitude

$$\chi_{P,\alpha}(p) = \chi_{P,\alpha}(p_-, p_+, \mathbf{p}_\perp) = \int d^4p\mathrm{e}^{-ipx}\chi_{P,\alpha}(p) =$$

$$= \frac{1}{2}\int dp_+dp_-d\mathbf{p}_\perp \mathrm{e}^{-i(p_+x_- + p_-x_+ - \mathbf{p}_\perp\mathbf{x}_\perp)}\chi_{P,\alpha}(p) \tag{2.4}$$

and define the light-front quasi-potential wave function [2,5]:

$$\Psi_{P,\alpha}(p_+, \mathbf{p}_1) = \int\limits_{-\infty}^{\infty} dp_- \chi_{P,\alpha}(p_-, p_+, \mathbf{p}_\perp). \tag{2.5}$$

It can be shown that the function $\Phi_{P,\alpha}(p_+, \mathbf{p}_\perp)$ depends on the values of the Bethe–Salpeter amplitude on the light-front hyperplane $x_0 + x_3 = 0$. In fact, using the definition (2.5) and the Fourier transformation (2.4) we get:

$$\Psi_{P,\alpha}(p_+, \mathbf{p}_\perp) =$$

$$= \frac{2}{(2\pi)^3} \int dx_+ dx_- d\mathbf{x}_\perp \delta(x_+) \mathrm{e}^{-i(p_+x_- + p_-x_+ - \mathbf{p}_\perp \mathbf{x}_\perp)} \chi_{P,\alpha}(x_+, x_-, \mathbf{x}_\perp). \tag{2.6}$$

Consider now the two-particle Green function

$$G(x_1, x_2; x_1', x_2') = G(X - X'; x, x') =$$

$$\langle 0 | T(\phi_1(x_1)\phi_2(x_2)\phi_1^+(x_1')\phi_2^+(x_2')) | 0 \rangle = \tag{2.7}$$

$$\frac{1}{(2\pi)^3} \int dP dp dp' \mathrm{e}^{-iP(X-X') - i(px - p'x')} G(P; p, p').$$

Here the total and relative 4-momenta and 4-coordinates in the initial and final states are introduced

$$P = p_1 + p_2, \quad , p = \frac{p_1 - p_2}{2}, \;\; X = \frac{x_1 + x_2}{2}, \;\; x = x_1 - x_2. \tag{2.8}$$

$$P = p_1' + p_2', \;\; p' = \frac{p_1' - p_2'}{2}, \;\; X' = \frac{x_1' + x_2'}{2}, \;\; x' = x_1' - x_2'. \tag{2.9}$$

Define the Fourier transform of the «two-time» quasi-potential Green function

$$\tilde{G}(P; p_+, \mathbf{p}_\perp; {p'}_+, {\mathbf{p}'}_\perp) = \int\limits_{-\infty}^{\infty} dp_- d{p'}_- G(P; p, p'). \tag{2.10}$$

For free particles we have

$$G^{(0)}(P; p, p') = \frac{-\delta^{(4)}(p - p')}{\left[\left(\frac{P}{2} + p\right)^2 - m_1^2 + i\epsilon\right]\left[\left(\frac{P}{2} - p\right)^2 - m_2^2 + i\epsilon\right]}. \tag{2.11}$$

Performing the integration according to the definition (2.10), we obtain

$$\tilde{G}^{(0)}(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp)=$$

$$\frac{4\pi i\delta(p_+-p'_+)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}'_\perp)\theta(x)\theta(1-x)}{P_+x(1-x)\left[P^2+\mathbf{P}_\perp^2-\frac{(\mathbf{P}/2+\mathbf{p})_\perp^2+m_1^2}{x}-\frac{(\mathbf{P}/2+\mathbf{p})_\perp^2-m_2^2}{1-x}\right]}= \tag{2.12}$$

$$\tilde{G}^{(0)}(P;p_+,\mathbf{p}_\perp)\delta(p_+-p'_+)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}'_\perp).$$

In this expression the variable x is introduced in the following way

$$x=\frac{1}{2}+\frac{p_+}{P_+}. \tag{2.13}$$

It is obvious that when the variable x varies in the limits

$$0<x<1, \tag{2.14}$$

the variable p_+ varies in the interval $(-P_+/2,\ P_+/2)$.

Define now the inverse operator by the relation

$$\int\limits_{-P_+/2}^{P_+/2} dp''_+\int d\mathbf{p}''_\perp \tilde{G}^{-1}(P;p_+,\mathbf{p}_\perp;p''_+,\mathbf{p}''_\perp)\times \tag{2.15}$$

$$\times\tilde{G}(P;p''_+,\mathbf{p}''_\perp;p'_+,\mathbf{p}'_\perp)=\delta(p_+-p'_+)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}'_\perp).$$

Introduce the interaction kernel V (quasi-potential) [2]:

$$\tilde{G}^{-1}(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp)=\tilde{G}^{(0)-1}(P;p_+,\mathbf{p}_\perp)\times$$

$$\times\delta(p_+-p'_+)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}'_\perp)-\frac{1}{4\pi i}V(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp). \tag{2.16}$$

After simple transformations the equation for the quasi-potential wave function

$$\Phi_{P,\alpha}(x,\mathbf{p}_\perp)=P_+x(1-x)\Psi_{P,\alpha}(p_+,\mathbf{p}_\perp) \tag{2.17}$$

takes the form [5]:

$$\left[P^2-\frac{(\mathbf{p}_\perp+(1/2-x)\mathbf{P}_\perp)^2+m_1^2}{x}-\frac{(\mathbf{p}_\perp+(1/2-x)\mathbf{P}_\perp)^2+m_2^2}{1-x}\right]\times$$

$$\times\Phi_{P,\alpha}(x,\mathbf{p}_\perp) = \int\limits_0^1 \frac{dx'}{x'(1-x')} \int d\mathbf{p}'_\perp V(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp)\Phi_{P,\alpha}(x',\mathbf{p}'_\perp). \tag{2.18}$$

The equation obtained gives the wave function of a bound state in an arbitrary Lorentz reference frame. Comparing it with the equation in the frame where $\mathbf{P}_\perp = 0$ we get the transformation property for the wave function from the arbitrary frame to the frame, in which the total transverse momentum of two-particle bound state is equal to zero:

$$\Phi_P(x,\mathbf{p}_\perp) = \Phi_{\mathbf{P}_\perp=0}(x,\mathbf{p}_\perp + (1/2-x)\mathbf{P}_\perp). \tag{2.19}$$

The case of spin particles is considered in Ref. 17.

2.2. Equation for the Scattering Amplitude and Relation to the Equation in the Infinite Momentum Frame. Derive now the equation for the two-body scattering amplitude. Definition of the scattering amplitude $T\ (P;p,p')$ in the 4-dimensional covariant Bethe–Salpeter formalism looks as follows:

$$G(P;p,p') = G^{(0)}(P;p,p') +$$

$$+\int d^4p''d^4p'''G^{(0)}(P;p,p'')T(P;p'',p''')G^{(0)}(P;p''',p') = \tag{2.20}$$

$$G^{(0)}(P;p)\delta^{(4)}(p-p') + G^{(0)}(P;p)T(P;p,p')G^{(0)}(P;p').$$

Define the quantity $\tilde{T}(P;p_+,\mathbf{p}_\perp;\mathbf{p}'_+,\mathbf{p}'_\perp)$ by the similar expression [2]:

$$\tilde{G}(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp) = \tilde{G}^{(0)}(P;p_+,\mathbf{p}_\perp)\delta(p_+-p'_+)\delta^{(2)}(\mathbf{p}_\perp-\mathbf{p}'_\perp) +$$

$$\tilde{G}^{(0)}(P;p_+,\mathbf{p}_\perp)\tilde{T}(P;p_+,\mathbf{p}_\perp;p'_+,\mathbf{p}'_\perp)\tilde{G}^{(0)}(P;p'_+,\mathbf{p}'_\perp). \tag{2.21}$$

Integrating (2.20) according to (2.10) we get:

$$\tilde{G} = \tilde{G}^{(0)} + \widetilde{G^{(0)}TG^{(0)}}. \tag{2.22}$$

Comapring formulae (2.22) and (2.21) we obtain:

$$\tilde{T} = \tilde{G}^{(0)-1}\cdot \widetilde{G^{(0)}TG^{(0)}} \cdot \tilde{G}^{(0)-1}. \tag{2.23}$$

It can be shown that on the mass shell the following equality holds

$$\tilde{T} = T. \tag{2.24}$$

Derive now the equation for the amplitude $\tilde{T}$. Using the definition (2.16) we get the equation for the Fourier transform of the «two-time» Green function

$$\tilde{G} = \tilde{G}^{(0)} + \tilde{G}^{(0)} V \tilde{G}. \tag{2.25}$$

In (2.25) the multiplication is understood as a three-dimensional intergation over the corresponding variables x and $\mathbf{p}_\perp$. Comparing (2.25) and (2.21) one can see that

$$\tilde{T}\tilde{G}^{(0)} = V\tilde{G} \tag{2.26}$$

from which the equation for scattering amplitude $\tilde{T}$ follows:

$$\tilde{T} = V + V\tilde{G}^{(0)}\tilde{T}. \tag{2.27}$$

In the frame, where total transverse momentum is zero $\mathbf{P}_\perp = 0$, the explicit form of the equation (2.27) looks as follows:

$$\tilde{T}(P; x, \mathbf{p}_\perp; x', \mathbf{p}'_\perp) = V(P; x, \mathbf{p}_\perp; x', \mathbf{p}'_\perp) +$$

$$\int_0^1 \frac{dx''}{x''(1-x'')} \int d\mathbf{p}''_\perp \frac{V(P; x, \mathbf{p}_\perp; x'', \mathbf{p}''_\perp)\tilde{T}(P; x'', \mathbf{p}''_\perp x', \mathbf{p}'_\perp)}{\left[\frac{m_1^2 + \mathbf{p}''^{\,2}_\perp}{x''} + \frac{m_2^2 + \mathbf{p}''^{\,2}_\perp}{1-x''} - P^2 - i\epsilon\right]}. \tag{2.28}$$

In a number of papers (see, e.g., [18–21]) composite systems have been described on the basis of the so-called old-fashioned perturbation theory in the infinite momentum frame, which has been used by Weinberg [22] in the relativistic quantum field theory. Equation (2.28) reproduces in the lowest order of perturbation theory the equation from [22] and at the same time contains the regular method of constructing the interaction kernel in the higher orders of perturbation theory. We will not discuss this point here, but recall that as in the canonical three-dimensional approach [2] there exist two methods of constructing of the interaction kernel (by means of the «two-time» Green function and by means of the scattering amplitude on the mass-shell).

The method of the constructing of the interaction kernel in lowest and high orders in perturbation theory can be used, for instance, for the relativistic generalization of one- or multi-boson exchange potentials to describe the nuclear forces. For the review of quark aspects of nuclear forces see, e.g., [23].

We note, however, that three exists one substantional difference between the equation derived here and equation of Ref. 22. In the light-front approach the equation is written in an arbitrary Lorentz frame and «longitudinal motion» of constituents is parametrized in terms of scale invariant and Lorentz invariant (under the transformations of reference frames along the z-axis) variable $x = (P/2 + p)_+/P_+$. In the infinite momentum frame «longitudinal motion» is parametrized in terms of the variable $x = (P/2 + p)_3/P_3$, which is not Lorentz invariant.

2.3. Equation for the Many-Body Bound State Wave Function. Formalism developed can be extended to the case of N relativistic interacting particles. The way of this extention can be seen if instead of the variable x, defined by the relative momentum, two variables $x^{(1)}$ and $x^{(2)}$, defined by the individual momenta of particles

$$x^{(i)} = \frac{p_+^{(i)}}{P_+}, \quad i = 1, 2 \tag{2.29}$$

are used. The variables $x^{(i)}$ vary in the interval $0 < x^{(i)} < 1$.

Define the Fourier transform of the many-body Bethe–Salpeter amplitude (wave function)

$$\chi_{P,\alpha}([x_\mu^{(i)}]) = \langle 0|T(\phi_1(x_\mu^{(1)})\phi_2(x_\mu^{(2)})...\phi_N(x_\mu^{(N)})|P, \alpha\rangle$$

by the following relation

$$\delta^{(4)}\left(P - \sum_{i=1}^{N} p^{(i)}\right)\chi_{P,\alpha}([p^{(i)}]) = \int \prod_{i=1}^{N} d^4x^{(i)} \exp\left[i\sum_{i=1}^{N} p^{(i)}x^{(i)}\right]\chi_{P,\alpha}([x_\mu^{(i)}]), \tag{2.30}$$

where

$$[p^{(i)}] = p^{(1)}, ..., p^{(N)}; \ \ [x_\mu^{(i)}] = x_1^{(1)}, ..., x^{(N)}.$$

Here we have ascribed the Lorentz index μ to the 4-coordinates $x_\mu^{(i)}$ in order to distinguish them from the scale-invariant variables $x^{(i)}$, which will be introduced later.

Introduce the light, front variables

$$P_\pm = P_0 \pm P_3; , \ \ p_\pm^{(i)} = p_0^{(i)} \pm p_3^{(i)}; , \ \ x_\pm^{(i)} = \frac{x_0^{(i)} \pm x_3^{(i)}}{2} \tag{2.31}$$

and integrate (2.30) over $\prod_{i=1}^{N} dp_-^{(i)}$. We obtain

$$2\delta\left(P_+ - \sum_{i=1}^{N} p_+^{(i)}\right)\delta^{(2)}\left(\mathbf{P}_\perp - \sum_{i=1}^{N}\mathbf{p}_\perp^{(i)}\right)\Psi_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) =$$

$$= (2\pi)^N \int \prod_{i=1}^{N} d^4x^{(i)}\delta(x_+^{(i)}) \exp\left[i\sum_{i=1}^{N}(p_+^{(i)}x_-^{(i)} - \mathbf{p}_\perp^{(i)}\mathbf{x}_\perp^{(i)})\right]\chi_{P,\alpha}([x_\mu^{(i)}]). \tag{2.32}$$

The function $\Psi_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])$ is related to the Bethe–Salpeter amplitude in the following way:

$$\Psi_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) = \int\limits_{-\infty}^{\infty} \prod_{i=1}^{N} dp_-^{(i)} \delta\left(P_- - \sum_{i=1}^{N} p_-^{(i)}\right) \chi_{P,\alpha}([p^{(i)}]). \tag{2.33}$$

Introduce now the Fourier transform of the «two-time» Green function

$$\tilde{G}(P; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \int\limits_{-\infty}^{\infty} \prod_{i=1}^{N} dp_-^{(i)} dp_-^{(i)'} \delta\left(P_- - \sum_{i=1}^{N} p_-^{(i)}\right) \delta\left(P_- - \sum_{i=1}^{N} p_-^{(i)'}\right) G(P; [p^{(i)}]; [p^{(i)'}]). \tag{2.34}$$

The function $G(P; [p^{(i)}]; [p^{(i)'}])$ is defined by the Fourier transformation

$$G([x_\mu^{(i)}]; [x_\mu^{(i)'}]) = \langle 0 | T(\phi_1(x_\mu^{(i)}) ... \phi_N(x_\mu^{(N)}) \phi_1^+(x_\mu^{(1)'}) ... \phi_N^+(x_\mu^{(N)'})) | 0 \rangle =$$

$$(2\pi)^{-4N} \int \prod_{i=1}^{N} d^4p^{(i)} d^4p^{(i)'} \exp\left[-i \sum_{i=1}^{N} (p^{(i)} x^{(i)} - p^{(i)'} x^{(i)'})\right] \times \tag{2.35}$$

$$\times G(P; [p^{(i)}]; [p^{(i)'}]).$$

For the case of free particles we have

$$G^{(0)}(P; [p^{(i)}]; [p^{(i)'}]) = \frac{i^N \prod\limits_{i=1}^{N} \delta^{(4)}(p^{(i)} - p^{(i)'})}{\prod\limits_{i=1}^{N} (p^{(i)2} - m^{(i)2} + i\epsilon)}. \tag{2.36}$$

Integrating both sides of (2.36) according to the definition (2.34) and omitting the δ-function corresponding to the total 4-momentum consevation, we get

$$\tilde{G}^{(0)}(P; [p_+^{(i)}, p_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$\frac{(2i)^N (2\pi i)^{N-1} \prod\limits_{i=1}^{N} \delta(p_+^{(i)} - p_+^{(i)'}) \delta^{(2)}(\mathbf{p}_\perp^{(i)} - \mathbf{p}_\perp^{(i)'}) \prod\limits_{i=1}^{N} \theta(x^{(i)}) \theta(1 - x^{(i)})}{P_+^{N-1} \prod\limits_{i=1}^{N} x^{(i)} \left[P^2 - \sum\limits_{i=1}^{N} \frac{(\mathbf{p}_\perp^{(i)} - x^{(i)} \mathbf{P}_\perp)^2 + m^{(i)2}}{x^{(i)}}\right]} = \tag{2.37}$$

$$\tilde{G}^{(0)}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}])\prod_{i=1}^{N}\delta(p_+^{(i)}-p_+^{(i)'})\delta^{(2)}(\mathbf{p}_\perp^{(i)},\mathbf{p}_\perp^{(i)'}]).$$

The variables $x^{(i)}$ are defined in the following way:

$$x^{(i)}=\frac{p_+^{(i)}}{P_+},\ \ i=1,2,...,N. \tag{2.38}$$

Thus, the function $\tilde{G}^{(0)}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}])$ is defined under the conditions:

$$\sum_{i=1}^{N}x^{(i)}=1;,\ \ 0<x^{(i)}<1;,\ \ \sum_{i=1}^{N}\mathbf{p}_\perp^{(i)}=\mathbf{P}_\perp. \tag{2.39}$$

Introduce now the inverse operator $\tilde{G}^{-1}$ by means of the relation

$$\int_0^{P_+}\prod_{i=1}^{N}dp_+^{(i)''}\int\prod_{i=1}^{N}d\mathbf{p}_\perp^{(i)''}\tilde{G}^{-1}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)''},\mathbf{p}_\perp^{(i)''}])\times$$

$$\times\tilde{G}(P;[p_+^{(i)''},\mathbf{p}_\perp^{(i)''}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}])=\prod_{i=1}^{N}\delta(p_+^{(i)}-p_+^{(i)'})\delta^{(2)}(\mathbf{p}_\perp^{(i)},\mathbf{p}_\perp^{(i)'}]) \tag{2.40}$$

and define the interaction kernel V:

$$\tilde{G}^{-1}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}])=\tilde{G}^{(0)-1}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}])-$$

$$-\frac{\delta\left(P_+-\sum\limits_{i=1}^{N}p_+^{(i)}\right)\delta^{(2)}\left(\mathbf{P}_\perp-\sum\limits_{i=1}^{N}\mathbf{p}_\perp^{(i)}\right)}{(2i)^N(2\pi i)^{N-1}}V(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]). \tag{2.41}$$

The equation for the wave function

$$\Phi_{P,\alpha}([x^{(i)},\mathbf{p}_\perp^{(i)}])=P_+^{N-1}\prod_{i=1}^{N}x^{(i)}\Psi_{P,\alpha}([p_+^{(i)},\mathbf{p}_\perp^{(i)}]) \tag{2.42}$$

looks as follows [24]:

$$\left[P^2-\sum_{i=1}^{N}\frac{(\mathbf{p}_\perp^{(i)}-x^{(i)}\mathbf{P}_\perp)^2+m^{(i)^2}}{x^{(i)}}\right]\Phi_{P,\alpha}([x^{(i)},\mathbf{p}_\perp^{(i)}])=$$

$$= \int\limits_0^1 \prod_{i=1}^N \frac{dx^{(i)'}}{x^{(i)'}} \delta\left(1 - \sum_{i=1}^N x^{(i)'}\right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)'} \delta^{(2)}\left(\mathbf{P}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)'}\right) \times \quad (2.43)$$

$$\times V(P; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) \Phi_{P,\alpha}([x^{(i)'}, \mathbf{p}_\perp^{(i)'}]).$$

The formalism developed can be used for the treatment of a wide class of elementary particle and nuclear physics problems.

2.4. Spectral and Projective Properties of the Two-Time Green Functions. The Green function for N interacting particles in the light-front quantum field theory is defined as a vacuum expectation value of the «chronologically» ordered Heisenberg field operators [25]:

$$G([x_\mu^{(i)}]; [x_\mu^{(i)'}]) = \langle 0|T_+(\psi_1(x_\mu^{(1)}) ... \psi_N(x_\mu^{(N)}) \bar{\psi}_N(x_\mu^{(N)'}) ... \bar{\psi}_1(x_\mu^{(1)'}))|0\rangle. \quad (2.44)$$

Define now the «two-time» Green function:

$$\tilde{G}(X_+; [x_-^{(i)}, \mathbf{x}_\perp^{(i)}]; X'_+; [x_-^{(i)'}, \mathbf{x}_\perp^{(i)'}]) = G([x_\mu^{(i)}]; [x_\mu^{(i)'}]) \Bigg|_{\substack{x_+^{(1)} = ... = x_+^{(N)} = X_+ \\ x_+^{(1)'} = ... = x_+^{(N)'} = X'_+}} . \quad (2.45)$$

It is convenient to introduce the operators

$$A([x_\mu^{(i)}]) = \psi_1(x_\mu^{(1)}) ... \psi_N(x_\mu^{(N)}) \Big|_{x_+^{(1)} = ... = x_+^{(N)} = X_+}$$

$$\bar{A}([x_\mu^{(i)'}]) = \bar{\psi}_N(x_\mu^{(N)'}) ... \psi_1(x_\mu^{(1)'}) \Big|_{x_+^{(1)'} = ... = x_+^{(N)'} = X'_+} \quad (2.46)$$

and rewrite the «two-time» Green function as follows:

$$\tilde{G}([x_\mu^{(i)}]; [x_\mu^{(i)'}]) = \langle 0|T_+(A([x_\mu^{(i)}]) \bar{A}([x_\mu^{(i)'}]))|0\rangle =$$

$$= \theta(X_+ - X'_+) \langle 0|A([x_\mu^{(i)}]) \bar{A}([x_\mu^{(i)'}])|0\rangle \pm \quad (2.47)$$

$$\pm \theta(X'_+ - X_+) \langle 0|\bar{A}([x_\mu^{(i)'}]) A([x_\mu^{(i)}])|0\rangle.$$

The signs $\pm$ are chosen depending on the number of fermion field operators in $A([x_\mu^{(i)}])$.

In what follows we will obtain the spectral representation for the Green function (2.47). Using the expansion in the complete set of physical states $|n\rangle$, translation invariance property and the Fourier representation for θ function the expression (2.47) can be rewritten as:

$$\tilde{G}([x_\mu^{(i)}];[x_\mu^{(i)'}]) = \tilde{G}(X_+ - X'_+;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) =$$

$$= \int\limits_{-\infty}^{\infty} dP_- \exp^{ip_-(X_+ - X'_+)} \int\limits_0^{\infty} dz \times \tag{2.48}$$

$$\times \left[\frac{\sigma_1(z;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}])}{P_- - z + i\epsilon} \mp \frac{\sigma_2(z;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}])}{P_- + z - i\epsilon} \right].$$

Spectral functions σ_1 and σ_2 are expressed via the three-dimensional light-front wave functions:

$$\sigma_1(z;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) =$$

$$= \frac{i}{2\pi} \sum_m \delta(z - P_-^{(m)}) \Psi_{om}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) \bar{\Psi}_{om}([x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]), \tag{2.49}$$

$$\sigma_2(z;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) =$$

$$= \frac{i}{2\pi} \sum_m \delta(z - P_-^{(m)}) \Psi_{mo}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) \bar{\Psi}_{mo}([x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]), \tag{2.50}$$

$$\Psi_{om}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) = \langle 0|A([0,x_-^{(i)},\mathbf{x}_\perp^{(i)}])|m\rangle =$$

$$= \langle 0|\psi_1(0,x_-^{(1)},\mathbf{x}_\perp^{(1)})...\psi_N(0,x_-^{(N)},\mathbf{x}_\perp^{(N)})|m\rangle, \tag{2.51}$$

$$\bar{\Psi}_{om}([x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) = \langle m|\bar{A}([0,x_-^{(i)'},\mathbf{x}_\perp^{(i)'}])|0\rangle, \tag{2.52}$$

$$\Psi_{mo}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) = \langle m|A([0,x_-^{(i)},\mathbf{x}_\perp^{(i)}])|0\rangle, \tag{2.53}$$

$$\bar{\Psi}_{mo}([x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) = \langle 0|\bar{A}([0,x_-^{(i)'},\mathbf{x}_\perp^{(i)'}])|m\rangle. \tag{2.54}$$

Summation in Eqs. (2.49), (2.50) is understood as the integration over 4-momentum $P^{(m)}(P_+^{(m)} > 0; P_-^{(m)} > 0)$ under the condition $P^{(m)^2} > 0$ and the summation over other quantum numbers on which the given physical state $|m\rangle$ can be dependent.

Define the Fourier transforms of the spectral functions:

$$\sigma_{1,2}(z;[x_-^{(i)},\mathbf{x}_\perp^{(i)}];[x_-^{(i)'},\mathbf{x}_\perp^{(i)'}]) = \frac{1}{(2\pi)^{4N}}\int\prod_{i=1}^{N} dp_+^{(i)} d\mathbf{p}_\perp^{(i)} dp_+^{(i)'} d\mathbf{p}_\perp^{(i)'}\times$$

$$\times\exp\left\{-i\sum_{i=1}^{N}[(p_+^{(i)}x_-^{(i)}-\mathbf{p}_\perp^{(i)}\mathbf{x}_\perp^{(i)})-(p_+^{(i)'}x_-^{(i)'}-\mathbf{p}_\perp^{(i)'}\mathbf{x}_\perp^{(i)'})]\right\}\times \tag{2.55}$$

$$\times\sigma_{1,2}(z;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]).$$

From this definition and Eqs.(2.51–(2.54) for wave functions one obtains:

$$\sigma_1(z;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]) =$$

$$= \frac{i}{(2\pi)^{1-4N}}\sum_m \delta(z-P_-^{(m)})\Psi_{om}([p_+^{(i)},\mathbf{p}_\perp^{(i)}])\bar{\Psi}_{om}([p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]), \tag{2.56}$$

$$\sigma_2(z;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]) =$$

$$= \frac{i}{(2\pi)^{1-4N}}\sum_m \delta(z-P_-^{(m)})\Psi_{mo}([p_+^{(i)},\mathbf{p}_\perp^{(i)}])\bar{\Psi}_{mo}([p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]), \tag{2.57}$$

where

$$\Psi_{mo}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) =$$

$$= \int\prod_{i=1}^{N} dp_+^{(i)} d\mathbf{p}_\perp^{(i)} \exp\left[-i\sum_{i=1}^{N}[(p_+^{(i)}x_-^{(i)}-\mathbf{p}_\perp^{(i)}\mathbf{x}_\perp^{(i)})\right]\Psi_{mo}([p_+^{(i)},\mathbf{p}_\perp^{(i)}]), \tag{2.58}$$

$$\Psi_{om}([x_-^{(i)},\mathbf{x}_\perp^{(i)}]) =$$

$$= \int\prod_{i=1}^{N} dp_+^{(i)} d\mathbf{p}_\perp^{(i)} \exp\left[-i\sum_{i=1}^{N}[(p_+^{(i)}x_-^{(i)}-\mathbf{p}_\perp^{(i)}\mathbf{x}_\perp^{(i)})\right]\Psi_{om}([p_+^{(i)},\mathbf{p}_\perp^{(i)}]). \tag{2.59}$$

We will show now that the functions $\sigma_{1,2}(z;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}])$ possess the following properties:

$$\sigma_1(z;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}]) = 0 \tag{2.60}$$

if even one of the variables $p_+^{(i)}, p_+^{(i)'} < 0$ and

$$\sigma_2(z; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) = 0 \tag{2.61}$$

if even one of the variables $p_+^{(i)}, p_+^{(i)'} > 0$.

Let us show first the validity of (2.60). Consider for this purpose the Fourier transform of the light-front wave function

$$\Psi_{om}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) = \frac{1}{(2\pi)^{3N}} \int \prod_{i=1}^{N} dx_-^{(i)} d\mathbf{x}_\perp^{(i)} \exp\left[i \sum_{i=1}^{N} [(p_+^{(i)} x_-^{(i)} - \mathbf{p}_\perp^{(i)} \mathbf{x}_\perp^{(i)}) \right] \times$$

$$\times \langle 0 | \psi_1(0, x_-^{(1)}, \mathbf{x}_\perp^{(1)}) ... \psi_N(0, x_-^{(N)}, \mathbf{x}_\perp^{(N)}) | m \rangle =$$

$$= \frac{1}{(2\pi)^{3N-3}} \sum_{m_1} \delta(p_+^{(1)} - p_+^{(m_1)}) \int \prod_{i=2}^{N} dx_-^{(i)} d\mathbf{x}_\perp^{(i)} \exp\left[i \sum_{i=2}^{N} (p_+^{(i)} x_-^{(i)} - \mathbf{p}_\perp^{(i)} \mathbf{x}_\perp^{(i)}) \right] \times$$

$$\times \langle 0 | \psi_1(0) | m_1 \rangle \langle m_1 | \psi_2(x_-^{(2)}, \mathbf{x}_\perp^{(2)}) ... \psi_N(x_-^{(N)}, \mathbf{x}_\perp^{(N)}) | m \rangle. \tag{2.62}$$

Taking into account that $p_+^{(m_1)} > 0$, it is evident that $\Psi_{om}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])$, if $p_+^{(1)} < 0$. In order to show the validity of this statement for arbitrary $p_+^{(i)}$ we will use the light-front commutation properties of the fields $\psi_i(x_\mu^{(i)})$ and locate on the first place arbitrary operator $\psi_i(x_\mu^{(i)})$:

$$\Psi_{om}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) = \frac{1}{(2\pi)^{3N-3}} \sum_{m_1} \delta(p_+^{(i)} - p_+^{(m_1)}) \times$$

$$\times \int \prod_{\substack{j \neq i \\ j=1}}^{N} dx_-^{(j)} d\mathbf{x}_\perp^{(j)} \times \exp\left[i \sum_{\substack{j \neq i \\ j=1}}^{N} [(p_+^{(j)} x_-^{(j)} - \mathbf{p}_\perp^{(j)} \mathbf{x}_\perp^{(j)}) \right] \times \tag{2.63}$$

$$\times \langle 0 | \psi_1(0) | m_1 \rangle \langle m_1 | \psi_1(x_-^{(1)}, \mathbf{x}_\perp^{(1)}) ... \psi_{i-1}(x_-^{(i-1)}, \mathbf{x}_\perp^{(i-1)}) \psi_{i+1}(x_-^{(i+1)}, \mathbf{x}_\perp^{(i+1)}) | m \rangle.$$

Taking into account that for physical states $p_+^{(m_1)} \geq 0$, we see that

$$\Psi_{om}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) = 0 \tag{2.64}$$

if even one of $p_+^{(i)} < 0$.

In a similar way one can show that

$$\Psi_{mo}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) = 0 \tag{2.65}$$

if even one of $p_+^{(i)} > 0$.

Taking into account (2.64) and (2.65) one can see the validity of Eqs. (2.60) and (2.61).

Define now the Fourier transform of the «two-time» Green function:

$$\tilde{G}(X_+ - X'_+; [x_-^{(i)}, \mathbf{x}_\perp^{(i)}]; [x_-^{(i)'}, \mathbf{x}_\perp^{(i)'}]) = \frac{1}{(2\pi)^{4N}} \int \prod_{i=1}^{N} dp_+^{(i)} d\mathbf{p}_\perp^{(i)} dp_+^{(i)'} d\mathbf{p}_\perp^{(i)'} \times$$

$$\times \exp\left\{ -iP_-(X_+ - X'_+) - i \sum_{i=1}^{N} [(p_+^{(i)} x_-^{(i)} - \mathbf{p}_\perp^{(i)} \mathbf{x}_\perp^{(i)}) - \right. \qquad (2.66)$$

$$\left. - (p_+^{(i)'} x_-^{(i)'} - \mathbf{p}_\perp^{(i)'} \mathbf{x}_\perp^{(i)'})] \right\} \tilde{G}(P_-; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]).$$

Inserting (2.55) and (2.66) into (2.48) one obtains:

$$\tilde{G}(P_-; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \int dz \left[\frac{\sigma_1(z; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{P_- - z + i\epsilon} \mp \frac{\sigma_2(z; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{P_- + z - i\epsilon} \right]. \qquad (2.67)$$

Here $\sigma_1 = 0$ if even one of $p_+^{(i)}$ or $p_+^{(i)'}$ is less than zero, $\sigma_2 = 0$ of even one of $p_+^{(i)}$ or $p_+^{(i)'}$ is bigger than zero.

Spectral representation (2.67) is an analogue of the spectral representation of the «two-time» Green function [3] with respect to total energy. Here, however, an essential difference between the upper and lower parts of the light cone is realized which is characteristic for the light-front quantum field theory. «Retarded» part of the Green function (first term) determines completely the behaviour of the Green function for positive $p_+^{(i)}$, $p_+^{(i)'}$, «advanced» part (second term) determines the behaviour of the Green function for negative $p_+^{(i)}$, $p_+^{(i)'}$.

Taking into account the definition of spectral densities and translation invariance property of wave functions one can obtain the spectral representation of the «two-time» Green function with respect to P^2 [25]:

$$\tilde{G}(P^2, [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) = \int_0^\infty ds \frac{\sigma(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{P^2 - s + i\epsilon}, \qquad (2.68)$$

where

$$\sigma(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \sigma_1(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) \prod_{i=1}^{N} \theta(p_+^{(i)})\theta(p_+^{(i)'}) \mp \tag{2.69}$$

$$\mp\sigma_2(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) = \prod_{i=1}^{N} \theta(-p_+^{(i)})\theta(-p_+^{(i)'}),$$

$$\sigma_1(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= i(2\pi)^{4N-1} \sum_m P_+ \delta(s - P^{(m)^2}) \Psi_{om}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) \bar{\Psi}_{om}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]), \tag{2.70}$$

$$\sigma_2(s; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= i(2\pi)^{4N-1} \sum_m P_+ \delta(s - P^{(m)^2}) \Psi_{mo}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]) \bar{\Psi}_{mo}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]). \tag{2.71}$$

Taking into account the momentum conservation

$$\sum_{i=1}^{N} p_+^{(i)} = \sum_{i=1}^{N} p_+^{(i)'}, \quad \sum_{i=1}^{N} \mathbf{p}_\perp^{(i)} = \sum_{i=1}^{N} \mathbf{p}_\perp^{(i)'} \tag{2.72}$$

one can rewrite the «two-time» Green function as follows:

$$\tilde{G}(P^2, [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \delta\left(\sum_{i=1}^{N} p_+^{(i)} - \sum_{i=1}^{N} p_+^{(i)'}\right) \delta^{(2)}\left(\sum_{i=1}^{N} \mathbf{p}_\perp^{(i)} - \sum_{i=1}^{N} \mathbf{p}_\perp^{(i)'}\right) \times \tag{2.73}$$

$$\times \tilde{G}(P; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])_{i=1,\ldots,N-1}.$$

Let us show now that the Green function (2.73) depends on its variables in a special manner:

$$(P^+)^{2N-2} \tilde{G}(P, [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) \equiv$$

$$\equiv S_P \tilde{G}(P; [x^{(i)}, \mathbf{p}_\perp^{(i)} - x^{(i)}\mathbf{P}_\perp]; [x^{(i)'}, \mathbf{p}_\perp^{(i)'} - x^{(i)'}\mathbf{P}_\perp]) S_P^{-1}. \tag{2.74}$$

S_P and S_P^{-1} are the known transformation matrices acting on spin indices. For scalar particles $S_P = 1$.

The fact that the Green function depends only on the scaling variables $x^{(i)}$ and $x^{(i)'}$ is the consequence of the invariance of the Green function under the rotations in the (x_0, x_3)-plane:

$$G([x_\mu^{(i)}];[x_\mu^{(i)'}]) =$$

$$= S_\lambda G([\lambda x_+^{(i)}, \lambda^{-1} x_-^{(i)}, \mathbf{x}_\perp^{(i)}];[\lambda x_+^{(i)'}, \lambda^{-1} x_-^{(i)'}, \mathbf{x}_\perp^{(i)'}]) S_\lambda^{-1}. \quad (2.75)$$

The matrix S_λ acts on the spin indices. Remind that an arbitrary 4-vector $A(A_+, A_-, \mathbf{A}_\perp)$ is transformed according to:

$$A_+ \to \lambda A_+,\;\; A_- \to \lambda^{-1} A_-,\;\; \mathbf{A}_\perp \to \mathbf{A}_\perp \quad (2.76)$$

under the rotations in the (x_0, x_3)-plane.

The property (2.75) is preserved for «two-time» Green function. As a result the Fourier transform is a homogeneous function of the variables $P_+, p_+^{(i)}, p_+^{(i)'}$:

$$\tilde{G}(P_-, P_+, \mathbf{P}_\perp;[p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \lambda^{2N-2} S_\lambda \tilde{G}(P^2;\lambda P_+, \mathbf{P}_\perp;[\lambda p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[\lambda p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) S_\lambda^{-1}. \quad (2.77)$$

From (2.77) it follows that $\tilde{G}$ depends only on the scaling variables $x^{(i)}$ and $x^{(i)'}$.

Consider the Lorentz transformation which is given by the 2-vector $\mathbf{u}_\perp$:

$$A_+ \to \lambda A_+;\;\; A_- \to A_- + \mathbf{u}_\perp \mathbf{A}_\perp + \frac{1}{2} A^+ \mathbf{u}_\perp^2; \mathbf{A}_\perp \to \mathbf{A}_\perp + A_+ \mathbf{u}_\perp. \quad (2.78)$$

The Green function is invariant under these transformations. For the Fourier transform of the «two-time» Green function it follows that:

$$\tilde{G}(P^2, \mathbf{P}_\perp;[x^{(i)}, \mathbf{p}_\perp^{(i)}];[x^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= S_{\mathbf{u}_\perp} \tilde{G}(P^2, \mathbf{P}_\perp + P_+ \mathbf{u}_\perp;[x^{(i)}, \mathbf{p}_\perp^{(i)} + p_+^{(i)} \mathbf{u}_\perp];\times$$

$$\times[x^{(i)'}, \mathbf{p}_\perp^{(i)'} + p_+^{(i)'} \mathbf{u}_\perp]) S_{\mathbf{u}_\perp}^{-1}. \quad (2.79)$$

Choosing $\mathbf{u}_\perp = \frac{\mathbf{P}_\perp}{P_+}$ one obtains the formula (2.74).

3. RELATIVISTIC ELASTIC FORM FACTORS AND SCATTERING AMPLITUDES FOR COMPOSITE SYSTEMS

3.1. Formulation of the Method. The reaction of a composite systems on a weak external perturbation corresponding to the local field $A(x)$ is described in quantum field theory by the expression [26,27]:

$$\langle P,\alpha|\frac{\delta S}{\delta A(k)}|P',\beta\rangle\,|_{A=0} = (2\pi)^4\delta^{(4)}(P-P'-k)\langle P,\alpha|J(0)|P',\beta\rangle. \tag{3.1}$$

Here $J(x)$ is the local current of the system

$$J(x) = i\frac{\delta S}{\delta A(x)}S^+, \tag{3.2}$$

$|P,\alpha\rangle$ and $|P',\beta\rangle$ are the state vectors of composite particles with momenta P and P' and the sets of additional quantum numbers α and β, respectively, normalized in a relativistically invariant manner

$$\langle P,\alpha|P',\beta\rangle = 2P_0(2\pi)^3\delta^{(3)}(\mathbf{P}-\mathbf{P}'). \tag{3.3}$$

Below we suggest a method of constructing relativistically covariant form factors of composite systems in terms of light-front wave functions.

Consider first the case of two-particle system. Introduce the quantity R defined by the vacuum expectation value of the chronologically ordered product of Heisenberg field operators of scalar particles $\phi_i(x_i)$ and the same local current $J(x)$:

$$R(x_1,x_2,x_1',x_2') = \langle 0|T(\phi_1(x_1)\phi_2(x_2)J(0)\phi_1^+(x_1')\phi_2^+(x_2')|0\rangle =$$

$$= (2\pi)^{-16}\int d^4p_1d^4p_2d^4p_1'd^4p_2'\,\times \tag{3.4}$$

$$\times\exp\left[-i\sum_{j=1}^{2}(p_jx_j-p_j'x_j')\right]R(p_1,p_2;p_1',p_2').$$

Introducing, as above, relative 4-coordinates and 4-momenta

$$X=\frac{x_1+x_2}{2},\;\; x=x_1-x_2;\;\; X'=\frac{x_1'+x_2'}{2},\;\; x'=x_1'-x_2';$$

$$P=p_1+p_2;\;\; p=\frac{p_1-p_2}{2};\;\; P'=p_1'+p_2';\;\; p'=\frac{p_1'-p_2'}{2}, \tag{3.5}$$

we rewrite expression (3.4) in the form

$$R(X, x : X', x') = (2\pi)^{-16} \int d^4P d^4p d^4P' d^4p' \times$$

$$\times \exp[-i(PX - P'X' + px - p'x')] R(P, p; P', p'). \quad (3.6)$$

As is known [28], the quantity R can be presented in the form

$$R = G\Gamma G \quad (3.7)$$

or in the detailed form

$$R(X, x; X', x') = \int d^4X'' d^4x'' d^4X''' d^4x''' G(X - X''; x, x'') \times$$

$$\times \Gamma(X'', x''; X''', x''') G(X''' - X'; x''', x'). \quad (3.8)$$

In the momentum space we get

$$R(P, p; P', p') = \int d^4p'' d^4p''' G(P, p, p'') \Gamma(P, p''; P', p''') G(P'; p''', p'). \quad (3.9)$$

Here G is two-particle Green function of fields $\phi_i(x_i)$ and the vertex function Γ is the sum of all two-particle irreducible diagrams for the 5-poin Green function (3.6) (see Fig.1).

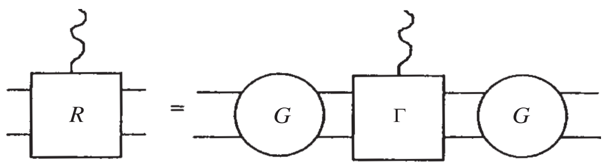

Fig. 1.

Passing to the «two-time» description in terms of light-front variables we define the quantity

$$\tilde{R}(P; p_+, \mathbf{p}_\perp; P'; p'_+, \mathbf{p}'_\perp) = \int_{-\infty}^{+\infty} dp_- dp'_- R(P, p; P', p'). \quad (3.10)$$

The quantity $\tilde{R}$ can be presented in the form

$$\tilde{R} = \tilde{G}\tilde{\Gamma}\tilde{G} \quad (3.11)$$

or in the detailed form

$$\tilde{R}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp) = \int\limits_{-P_+/2}^{P_+/2} dp''_+ \int d\mathbf{p}''_\perp \int\limits_{-P'_+/2}^{P'_+/2} dp'''_+ \int d\mathbf{p}'''_\perp \times$$

$$\times \tilde{G}(P;p_+,\mathbf{p}_\perp;p''_+,\mathbf{p}''_\perp)\tilde{\Gamma}(P;p''_+,\mathbf{p}''_\perp;p'''_+,\mathbf{p}'''_\perp) \times \tag{3.12}$$

$$\times \tilde{G}(P';p'''_+,\mathbf{p}'''_\perp;p'_+,\mathbf{p}'_\perp).$$

Here $\tilde{\Gamma}$ is the vertex integral operator. Let us show that the quantity $\tilde{\Gamma}$ defines the form factor of composite system. Starting from the spectral properties [29] of the 5-point Green function (3.10), it is possible to show that the quantity $\tilde{R}$ has the pole singularities near the points corresponding to the masses M_α and M_β of composite system:

$$\tilde{R}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp) \cong \atop P^2 \to M_\alpha^2; P'^2 \to M_\beta^2$$

$$\cong [i(2\pi)^4]^2 \frac{\Psi_{P,\alpha}(p_+,\mathbf{p}_\perp)\langle P,\alpha|J(0)|P',\beta\rangle \Psi^+_{P',\beta}(p'_+,\mathbf{p}'_\perp)}{(P^2-M_\alpha^2);(P'^2-M_\beta^2)}. \tag{3.13}$$

On the other hand, taking into account the pole singularities of the two-particle «two-time» Green function

$$\tilde{G}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp) \cong \atop P^2 \to M_\alpha^2 \tag{3.14}$$

$$\cong i(2\pi)^4 \frac{\Psi_{P,\alpha}(p_+,\mathbf{p}_\perp)\Psi^+_{P,\alpha}(p'_+,\mathbf{p}'_\perp)}{P^2-M_\alpha^2}.$$

We find from (3.12):

$$\tilde{R}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp) \cong$$

$$\cong [i(2\pi)^4]^2 \frac{\Psi_{P,\alpha}(P_+,\mathbf{P}_\perp)\Psi^+_{P',\beta}(P'_+,\mathbf{P}'_\perp)}{(P^2-M_\alpha^2);(Q^2-M_\beta^2)} \tag{3.15}$$

$$\int\limits_{-P_+/2}^{P_+/2} dp''_+ \int d\mathbf{p}''_\perp \int\limits_{-P'_+/2}^{P'_+/2} dp'''_+ \int d\mathbf{p}'''_\perp$$

$$\times \Psi^+_{P,\alpha}(p''_+,\mathbf{p}''_\perp)\tilde{\Gamma}_{\alpha\beta}(P;p''_+,\mathbf{p}''_\perp;P';p'''_+,\mathbf{p}'''_\perp)\Psi_{P',\beta}(p'''_+,\mathbf{p}'''_\perp),$$

where

$$\tilde{\Gamma}_{\alpha\beta}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp)=\tilde{\Gamma}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp)\Big|_{\substack{P^2=M_\alpha^2\\ P'^2=M_\beta^2}}. \qquad (3.16)$$

Comparing equations (3.13) and (3.15) we get the following expression for the matrix element of the local current J:

$$\langle P,\alpha|J(0)|P',\beta\rangle=\int\limits_{-P_+/2}^{P_+/2}dp_+\int d\mathbf{p}_\perp\int\limits_{-P'_+/2}^{P'_+/2}dp'_+\int d\mathbf{p}'_\perp\times$$

$$\times\Psi^+_{P,\alpha}(p_+,\mathbf{p}_\perp)\tilde{\Gamma}_{\alpha\beta}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp)\Psi_{P',\beta}(p'_+,\mathbf{p}'_\perp). \qquad (3.17)$$

Equations (3.15) and (3.17) give an exact expression for vertex operator of the composite system

$$\tilde{\Gamma}_{\alpha\beta}(P;p_+,\mathbf{p}_\perp;P';p'_+,\mathbf{p}'_\perp)=$$

$$=\lim_{\substack{P^2\to M_\alpha^2\\ P'^2\to M_\beta^2}}\int\limits_{-P_+/2}^{P_+/2}dp''_+\int d\mathbf{p}''_\perp\int\limits_{-P'_+/2}^{P'_+/2}dp'''_+\int d\mathbf{p}'''_\perp\times$$

$$\times\tilde{G}^{-1}(P;p_+,\mathbf{p}_\perp;p''_+,\mathbf{p}''_\perp)[\widetilde{G\Gamma G}](P;p''_+,\mathbf{p}''_\perp;P';p'''_+,\mathbf{p}'''_\perp)\times \qquad (3.18)$$

$$\times\tilde{G}^{-1}(P';p'''_+,\mathbf{p}'''_\perp;p'_+,\mathbf{p}'_\perp).$$

in terms of 4- and 5-point Green functions G and Γ. Using the perturbation theory methods for these functions one can construct the coupling constant expansion for the vertex function of composite system.

3.2. Elastic Form Factor in the Impulse Approximation. To demonstrate this method we consider the so-called impulse approximation for the vertex operator $\tilde{\Gamma}$, which corresponds to the limit of «weakly bound» (noninteracting) particles (see Fig.2).

Fig. 2.

For the vertex operator corresponding to the conserved vector current we find

$$\tilde{\Gamma}_\mu=\tilde{\Gamma}^{(0)}_{1\mu}+\tilde{\Gamma}^{(0)}_{2\mu}, \qquad (3.19)$$

$$\tilde{\Gamma}_{i\mu}=[\tilde{G}^{(0)}]^{-1}[G^{(0)}\widetilde{\Gamma^0_{i\mu}}G^{(0)}][\tilde{G}^{(0)}]^{-1}, \qquad (3.20)$$

where

$$\tilde{\Gamma}_{i\mu}^{(0)} = (2\pi)^4 e_i (p_i + p'_i)_\mu \delta^{(4)}(p_j - p'_j)[G_j^{(0)}(p_j)]^{-1}\,|_{i\neq j}\ . \qquad (3.21)$$

Here

$$G^{(0)}(p_1, p_2) = G_1^{(0)}(p_1) G_2^{(0)}(p_2) = i^2 \prod_{i=1}^{2} (p_i^2 - m_i^2)^{-1}, \qquad (3.22)$$

$$G^{(0)}(p'_1, p'_2) = G_1^{(0)}(p'_1) G_2^{(0)}(p'_2) = i^2 \prod_{i=1}^{2} ({p'}_i^2 - m_i^2)^{-1} \qquad (3.23)$$

are two-particle Green functions for free particles with masses m_i and charges e_i. Then for the invariant form factor of the composite system defined by the relation

$$\langle P, \alpha | J_\mu(0) | P', \beta\rangle = (P + P')_\mu F(\Delta^2);\ \ \Delta = P - P' \qquad (3.24)$$

in the reference frame, in which

$$P_+ = P'_+,\ \ (P - P')^2 = \Delta^2 = -\boldsymbol{\Delta}_\perp^2 = -(\mathbf{P}_\perp - {\mathbf{P}'}_\perp)^2 \qquad (3.25)$$

we have

$$F(\boldsymbol{\Delta}_\perp^2) = \frac{e_i (2\pi)^2}{2} \int_0^1 \frac{dx}{x(1-x)} \int d\mathbf{p}_\perp \Phi_{\mathbf{P}_\perp = 0}(x, \mathbf{p}_\perp + (1-x)\boldsymbol{\Delta}_\perp)$$

$$\Phi_{\mathbf{P}_\perp = 0}(x, \mathbf{p}_\perp) + \text{similar term with } \mathrm{e}_2. \qquad (3.26)$$

Note that construction of relativistic form factors of composite systems in other versions of relativistic description of bound states is considered, e.g., in Refs. 29–31.

3.3. Relativistic Form Factor for the Many-Body System. Let's construct now the form factor for the relativistic many-body system in terms of the many-body light-front wave functions $\Phi_P([x^{(i)}, \mathbf{p}_\perp^{(i)}])$. Consider, as in the case of two constituents, the quantity R, which is defined by the vacuum expectation value of the chronologically ordered product of the Heisenberg field operators $\phi_i(x_\mu^{(i)})$ and the local current $J(x)$

$$R([x_\mu^{(i)}]; [x_\mu^{(i)'}]) =$$

$$= \langle 0 | T(\phi_1(x_\mu^{(1)}) \ldots \phi_N(x_\mu^{(N)}) J(0) \phi_1^+(x_\mu^{(1)'}) \ldots \phi_N^+(x_\mu^{(N)'})) | 0\rangle = \qquad (3.27)$$

$$= (2\pi)^{-4N} \int \prod_{i=1}^{N} d^4p^{(i)} d^4p^{(i)'} \exp\left[-i \sum_{i=1}^{N} (p^{(i)}x^{(i)} - p^{(i)'}x^{(i)'})\right] R([p^{(i)}];[p^{(i)'}]).$$

The quantity R can be presented in the form

$$R = G\Gamma G. \tag{3.28}$$

Multiplication in (3.28) has to be understood as an integration over the 4-coordinates of particles. G is the many-body Green function of the fields $\phi_i(x_\mu^{(i)})$ and the vertex function Γ is defined by the sum of the irreducible diagrams of the $(2N+1)$-point function (3.27).

Proceeding now to the light-front description we introduce the quantity $\tilde{R}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])$ by the relation

$$\tilde{R}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) =$$

$$= \int_{-\infty}^{\infty} \prod_{i=1}^{N} dp_-^{(i)} d^4p_-^{(i)'} \delta\left(P_- - \sum_{i=1}^{N} p_-^{(i)}\right) \delta\left({P'}_- - \sum_{i=1}^{N} p_-^{(i)'}\right) R([p^{(i)}];[p^{(i)'}]) \tag{3.29}$$

and write it in the form

$$\tilde{R} = \tilde{G}\tilde{\Gamma}\tilde{G}. \tag{3.30}$$

Multiplication in Eq. (3.30) has to be understood in the operator sense:

$$\tilde{A}\tilde{B} = \int_0^{P_+} \prod_{i=1}^{N} dp_+^{(i)''} \delta\left(P_+ - \sum_{i=1}^{N} p_+^{(i)''}\right) \int \prod_{i=1}^{N} d\mathbf{p}_\perp^{(i)'} \delta^{(\alpha)}\left(\mathbf{P}_\perp - \sum_{i=1}^{N} \mathbf{p}_\perp^{(i)''}\right) \times$$

$$\tilde{A}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[{p''}_+, \mathbf{p}''_\perp])\tilde{B}([{p''}_+, \mathbf{p}''];[p'_+, p']). \tag{3.31}$$

From the spectral properties of the function $\tilde{G}$ it follows that $\tilde{R}$ possesses the double pole singularities

$$\tilde{R}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}];[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) \cong$$

$$\cong [i(2\pi)^4]^2 \frac{\Psi_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])\langle P,\alpha|J(0)|P',\beta\rangle \Psi^+_{P',\beta}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{(P^2 - M_\alpha^2);({P'}^2 - M_\beta^2)} \tag{3.32}$$

in the vicinity of the points, where N-particle system forms the bound states with masses M_α and M_β and sets of other quantum numbers α and β, respectively.

On the other hand, knowing the pole singularities of the Green function one can reduce the Eq. (3.32) to the form

$$\tilde{R}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]) \cong$$

$$\cong [i(2\pi)^4]^2 \frac{\Psi_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])\Psi^+_{P',\beta}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{(P^2 - M_\alpha^2); (P'^2 - M_\beta^2)} \times$$

$$\times \int_0^{P_+} \prod_{i=1}^N dp_+^{(i)''} \delta\left(P_+ - \sum_{i=1}^N p_+^{(i)''}\right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)''} \delta^{(2)}\left(\mathbf{P}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)''}\right) \times \tag{3.33}$$

$$\times \int_0^{P'_+} \prod_{i=1}^N dp_+^{(i)'''} \delta\left(P'_+ - \sum_{i=1}^N p_+^{(i)'''}\right) \int \prod_{i=1}^N d\mathbf{p}_+^{(i)'''} \delta^{(2)}\left(\mathbf{P}'_\perp - \sum_{i=1}^N \mathbf{p}_+^{(i)'''}\right) \times$$

$$\times \Psi^+_{P,\alpha}([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}])\tilde{\Gamma}_{\alpha\beta}([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}]; [p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}])\Psi^+_{P',\beta}([p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}]).$$

Comparing (3.32) with (3.33) we get the following expression for the matrix element of the bound state current:

$$\langle P, \alpha | J(0) | P', \beta \rangle =$$

$$\int_0^{P_+} \prod_{i=1}^N dp_+^{(i)} \delta\left(P_+ - \sum_{i=1}^N p_+^{(i)}\right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)} \delta^{(2)}\left(\mathbf{P}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)}\right) \times \tag{3.34}$$

$$\int_0^{P'_+} \prod_{i=1}^N dp_+^{(i)'} \delta\left(P'_+ - \sum_{i=1}^N p_+^{(i)'}\right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)'} \delta^{(2)}\left(\mathbf{P}'_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)'}\right) \times$$

$$\times \Psi^+_{P,\alpha}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])\tilde{\Gamma}_{\alpha\beta}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])\Psi_{P',\beta}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]).$$

The vertex operator $\Gamma_{\alpha\beta}$ can be constructed, using, for instance, perturbation theory methods of quantum field theory. Phenomenological vertex operators can also be used. Here we consider the so-called «impulse approximation». In this case we obtain:

$$\langle P, \alpha | J_\mu(0) | P', \beta \rangle = \sum_{k=1}^N \langle P, \alpha | J_\mu(0) | P', \beta \rangle_k \tag{3.35}$$

where, for instance

$$\langle P,\alpha|J_+(0)|P',\beta\rangle_k = (P_+ + P'_+)F_k(-\mathbf{\Delta}_\perp^2) =$$

$$= \frac{-(2\pi)^4 e_k (P_+ + P'_+)}{(2i)^{N+1}(2\pi i)^{N-1}} \int\limits_0^1 \prod_{i=1}^N \frac{dx^{(i)}}{x^{(i)}} \times \tag{3.36}$$

$$\times\delta\left(1-\sum_{i=1}^N x^{(i)}\right)\int\prod_{i=1}^N d\mathbf{p}_\perp^{(i)}\delta^{(2)}\left(\sum_{i=1}^N \mathbf{p}_\perp^{(i)}\right)\times$$

$$\times\Phi^+_{\mathbf{P}_\perp=0,\alpha}([x^{(i)},\mathbf{p}_\perp^{(i)} - x^{(i)}\mathbf{\Delta}_\perp]_{i\neq k}, x^{(k)}, \mathbf{p}_\perp^{(k)} +$$

$$+(1-x^{(k)})\mathbf{\Delta}_\perp)\Phi_{\mathbf{P}_\perp=0,\beta}([x^{(i)},\mathbf{p}_\perp^{(i)}]).$$

Taking into account the normalization condition for the wave functions for $\mathbf{\Delta}_\perp = 0$ we get:

$$F(\Delta^2=0) = \sum_{k=1}^N e_k. \tag{3.37}$$

Thus, the form factor at zero momentum transfer is normalized to the total charge of the system. Note, that problems of normalization of three-dimensional relativistic wave function have been considered in Ref. 32.

3.4. Scattering of Relativistic Composite Systems. Experimental study of high energy processes during the last decades revealed a number of scaling properties of observable quantities. Many of these properties can be understood on the basis of the composite quark parton structure of elementary particles. In particular, the asymptotic scaling property of differential cross section of hadron-hadron scattering

$$\frac{do}{dt}\Big|_{\substack{s\to\infty\\ |t/s|=\text{const.}}} \sim \frac{1}{s^N} f(\cos\theta_s), \tag{3.38}$$

where N is integer number, can be explained in the framework of dimensional analysis and the assumption on three-quark structure of baryons and quark antiquark structure of mesons (quark counting rules) [33,34].

In connection with the development of composite models of elementary particles a problem of the description of their interactions becomes of special interest. Study of interactions of relativistic composite systems is important also in connection with current and future experiments with beams of relativistic nuclei. Here we outline a method for the treatment of problems of that kind [35]. Below we present a description of the scattering of two composite particles. It will be shown that some simple assumption on the hadron interactions in the scattering process allows one to reproduce the results of quark counting rules.

Consider the eight-point Green function G:

$$G(x_1,x_2,x_3,x_4;x'_1,x'_2,x'_3,x'_4) =$$
$$= \langle 0|T(\phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\phi_1^+(x'_1)\phi_2^+(x'_2)\phi_3^+(x'_3)\phi_4^+(x'_4))|0\rangle =$$
$$= [(2\pi)^4]^{-8}\int\prod_{i=1}^{4} d^4p_i d^4p'_i \exp\left[-i\sum_{i=1}^{4}(p_ix_i - p'_ix'_i)\right]\times$$
$$\times G(p_1,p_2,p_3,p_4;p'_1,p'_2,p'_3,p'_4) =$$

$$= [(2\pi)^4]^{-8}\int d^4P^{(12)}d^4p^{(12)}d^4P^{(34)}d^4p^{(34)}d^4P'^{(12)}d^4p'^{(12)}d^4P'^{(34)}d^4p'^{(34)}\times \tag{3.39}$$

$$\times\exp[-i(P^{(12)}X^{(12)}+P^{(34)}X^{(34)}+p^{(12)}x^{(12)}+p^{(34)}x^{(34)}-$$
$$-P'^{(12)}X'^{(12)}+P'^{(34)}X'^{(34)}+p'^{(12)}x'^{(12)}+p'^{(34)}x'^{(34)})]\times$$
$$\times G(P^{(12)},p^{(12)},P^{(34)},p^{(34)};P'^{(12)},p'^{(12)},P'^{(34)},p'^{(34)}).$$

In (3.39) the momenta $P^{(12)},p^{(12)},P^{(34)},p^{(34)};P'^{(12)},p'^{(12)},P'^{(34)},p'^{(34)}$ are introduced according to the following equations

$$P^{(12)} = p_1+p_2, p^{(12)} = \frac{p_1-p_2}{2}, P^{(34)} = p_3+p_4, p^{(34)} = \frac{p_3-p_4}{2},$$

$$P'^{(12)} = p'_1+p'_2, p'^{(12)} = \frac{p'_1-p'_2}{2}, P'^{(34)} = p'_3+p'_4, p'^{(34)} = \frac{p'_3-p'_4}{2}. \tag{3.40}$$

Passing now to the «two-time» description, we introduce the light-front variables and define the quantity

$$\tilde{G}(P^{(12)},p_+^{(12)},\mathbf{p}_\perp^{(12)};P^{(34)},p_+^{(34)},\mathbf{p}_\perp^{(34)};P'^{(12)},p'^{(12)}_+,\mathbf{p}'^{(12)}_\perp;P'^{(34)},p'^{(34)}_+,\mathbf{p}'^{(34)}_\perp) =$$

$$= \int\limits_{-\infty}^{\infty} dp_-^{(12)}dp_-^{(34)}dp'^{(12)}_-dp'^{(34)}_- \times \tag{3.41}$$

$$\times G(P^{(12)},p^{(12)},P^{(34)},p^{(34)};P'^{(12)},p'^{(12)},P'^{(34)},p'^{(34)}).$$

Introduce now the quantity M by the relation:

$$G(P^{(12)},p^{(12)},P^{(34)},p^{(34)};P'^{(12)},p'^{(12)},P'^{(34)},p'^{(34)}) =$$

$$= \int d^4p^{(12)''}d^4p^{(34)''}d^4p^{(12)'''}d^4p^{(34)'''}\times$$

$$\times G_{12}(P^{(12)},p^{(12)},p^{(12)''})G_{34}(P^{(34)},p^{(34)},p^{(34)''})\times \tag{3.42}$$

$$\times M(P^{(12)},p^{(12)''};P^{(34)},p^{(34)''};P'^{(12)},P^{(12)'''};P^{(34)},P^{(34)'''})\times$$

$$\times G_{12}(P'^{(12)},p^{(12)'''},p^{(12)'})G_{34}(P'^{(34)},p^{(34)'''},p^{(34)'}) \equiv (G_{12}G_{34})M(G_{12}G_{34}).$$

The quantity $\tilde{G}$ can be presented in the form (here and in what follows we omit the arguments which are related to the relative momenta and this will not cause any misunderstanding):

$$\tilde{G}(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)}) =$$

$$= \tilde{G}_{12}(P^{(12)})\tilde{G}_{34}(P^{(34)})\tilde{M}(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)})\tilde{G}_{12}(P'^{(12)})\tilde{G}_{34}(P'^{(34)}). \tag{3.43}$$

The multiplication in the (3.43) has to be understood in the following sense

$$\tilde{A}\tilde{B} = \int\limits_{-P_+^{(12)}/2}^{P_+^{(12)}/2} dp_+^{(12)} \int\limits_{-P_+^{(34)}/2}^{P_+^{(34)}/2} dp_+^{(34)} \int d\mathbf{p}_\perp^{(12)} \int d\mathbf{p}_\perp^{(34)} \times$$

$$\times \tilde{A}(...,p_+^{(12)},\mathbf{p}_\perp^{(12)};p_+^{(34)},\mathbf{p}_\perp^{(34)})\tilde{B}(p_+^{(12)},\mathbf{p}_\perp^{(12)};p_+^{(34)},\mathbf{p}_\perp^{(34)},...) \tag{3.44}$$

and dots correspond to the set of other arguments the operators $\tilde{A}$ and $\tilde{B}$ can depend on.

Knowing the pole singularities of the two-particle Green functions $\tilde{G}_{12}$, $\tilde{G}_{34}$ one can show that in the vicinity of these poles the function $\tilde{G}$ looks as follows:

$$\tilde{G}(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)}) \cong$$

$$\cong [i(2\pi)^4]^4 \frac{\Psi_{12}(P^{(12)})\Psi_{34}(P^{(34)})\Psi_{12}^+(P'^{(12)})\Psi_{34}^+(P'^{(34)})}{(P^{(12)^2}-M_{12}^2)(P^{(34)^2}-M_{34}^2)(P'^{(12)^2}-M_{12}^2)(P'^{(34)^2}-M_{34}^2)} \times \tag{3.45}$$

$$\times\Psi^{+}_{12}(P^{(12)})\Psi^{+}_{34}(P^{(34)})\tilde{M}(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)})\Psi_{12}(P'^{(12)})\Psi^{+}_{34}(P'^{(34)}).$$

Here

$$\tilde{M}_{1234}=\lim_{\substack{P^{(12)2}\to M^2_{12},P^{(34)2}\to M^2_{34}\\ P'^{(12)2}\to M^2_{12},P'^{(34)2}\to M^2_{34}}}\tilde{G}^{-1}_{12}(P^{(12)})\tilde{G}^{-1}_{34}(P^{(34)})\cdot$$

$$\cdot G_{12}G_{34}\widetilde{M}G_{12}G_{34}(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)})\cdot\tilde{G}^{-1}_{12}(P'^{(12)})\tilde{G}^{-1}_{34}(P'^{(34)}), \tag{3.46}$$

M^2_{12}, M^2_{34}, M'^2_{12}, M'^2_{34} are the masses of corresponding states.

From the equations (3.43) and (3.45) we get the following expression for the scattering amplitude

$$T(P^{(12)},P^{(34)},P'^{(12)},P'^{(34)})=\Psi^{+}_{12}(P^{(12)})\Psi^{+}_{34}(P^{(34)})\times$$

$$\times\tilde{M}(P^{(12)},P^{(12)},P'^{(12)},P'^{(12)})\Psi_{12}(P'^{(12)})\Psi^{+}_{34}(P'^{(34)}). \tag{3.47}$$

Eq. (3.47) gives a general expression for the scattering amplitude in the case of scattering of composite particles. The detailed form of the scattering amplitude depends on the interaction mechanism in the intermediate state and on a specific form of the wave functions of the scattered objects.

3.5. Constituent Interchange Mechanism. Considering the constituent interchange mechanism (Fig.3) one gets the following expression for the scattering amplitude

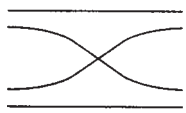

Fig. 3.

$$T=\frac{-1}{2(2\pi)^3}\int\limits_0^1\frac{dx}{x^2(1-x)^2}\int d\mathbf{p}_\perp\Phi^{+(12)}_{\mathbf{P}_\perp=0}(x,\mathbf{p}_\perp-x\mathbf{\Delta}^{(u)}_\perp+$$

$$+(1-x)\mathbf{\Delta}^{(t)}_\perp)\Phi^{+(34)}_{\mathbf{P}_\perp=\mathbf{0}}(x,\mathbf{p}_\perp)\times$$

$$\times[M^2_{12}+M^2_{34}-S(x,\mathbf{p}_\perp+x\mathbf{\Delta}^{(t)}_\perp-(1-x)\mathbf{\Delta}^{(u)}_\perp)-S(x,\mathbf{p}_\perp)]\times \tag{3.48}$$

$$\times\Phi^{(12)}_{\mathbf{P}_\perp=0}(x,\mathbf{p}_\perp-x\mathbf{\Delta}^{(u)}_\perp)\Phi^{(34)}_{\mathbf{P}_\perp=\mathbf{0}}(x,\mathbf{p}_\perp+(1-x)\mathbf{\Delta}^{(t)}_\perp),$$

where $\mathbf{\Delta}^{(t)}_\perp=-t$, $\mathbf{\Delta}^{(u)}_\perp=-u$.

Here the notation has been introduced

$$S(x,\mathbf{p}_\perp)=\frac{m_1^2+\mathbf{p}^2_\perp}{1-x}+\frac{m_2^2+\mathbf{p}^2_\perp}{x} \tag{3.49}$$

and the following properties of wave functions

$$\Phi(x, \mathbf{p}_\perp) = \Phi(x, -\mathbf{p}_\perp), \Phi(x, \mathbf{p}_\perp) = \Phi(1 - x, \mathbf{p}_\perp) \tag{3.50}$$

have been used.

Let us choose now the wave function of the composite particles in the form

$$\Phi_N(x, \mathbf{p}_\perp) = \frac{\phi_N(x)}{[S(x, \mathbf{p}_\perp)]^N}, N = A, B, C, D \tag{3.51}$$

A, B and C, D denote the hadrons before and after the scattering and corresponding powers, respectively.

Inserting the wave functions (3.51) into the Eq. (3.48) for the scattering amplitude one gets in the asymptotic region

$$T \underset{\substack{s\to\infty \\ |t|\to\infty}}{\sim} \frac{1}{s A + C + D - 1} \left(\frac{1+z}{2}\right)^{-C} \left(\frac{1-z}{2}\right)^{-D} f(z), \tag{3.52}$$

where

$$f(z) = \int_0^1 \frac{dx \phi_A^+(x) \tilde{\phi}_B^+(x) \phi_C(x) \phi_D(x)}{\left[(1-x)^2 \frac{1-z}{2} + x^2 \frac{1+z}{2}\right]^A} \left[(1-x)^2 \frac{1+z}{2} + x^2 \frac{1-z}{2}\right] \times$$

$$\times [x(1-x)]^{A+B+C+D-3} x^{-2C} (1-x)^{-2D}, \tag{3.53}$$

$$\tilde{\phi}_B^+(x) = \frac{-1}{(2\pi)^3} \int d\mathbf{p}_\perp \Phi_B^+(x, \mathbf{p}_\perp) [x(1-x)]^{-B}$$

$z = \cos \vartheta s$, where ϑs is the scattering angle in the c.m.s.

$$-t \cong \frac{s}{2}(1-z), \;\; -u \cong \frac{s}{2}(1+z).$$

Eq.(3.53) is in close connection with the results of quark counting rules [33,34,36].

4. DEEP INELASTIC FORM FACTORS OF COMPOSITE SYSTEMS

The great interest to deep inelastic interaction processes is caused by the possibility of studying the internal structure of hadrons and nuclei experimentally and checking different theoretical models based on the assumptions about composite nature of strongly interacting particles. The main part of experimentally observed properties of these processes (in particular, the scale properties of structure functions) have been explained in the framework of composite quark-parton models of hadrons, in which hadron is considered as a bound state of some parallelly moving pointlike constituents. Interaction between constituents and their transverse motion inside hadron is neglected [37,38].

More precise measurements in wider range of kinematic variables have led to the observation of deviations from exact scale invariance in the behaviour of structure functions [39–41]. Attempts were made to explain these deviations on the kinematical (search for new scale-invariant variables [42]) and dynamical (taking into account chromodynamical corrections [43]) basis. Quark-parton picture of the deep inelastic scattering in the quasi-potential approach can be found in Ref. 44.

Here we incorporate the transverse motion of constituents in the composite system, which leads to the violation of Bjorken scaling of structure functions.

4.1. Construction of the Tensor $W_{\mu\nu}$. Consider the quantity $R_{\mu\nu}$, which is defined by the vacuum expectation value of the chronologically ordered product of the Heisenberg field operators $\phi_i(x_\mu^{(i)})$ and local currents J_μ and J_ν:

$$R_{\mu\nu}([x_\mu^{(i)}];[x_\mu^{(i)'}];z) =$$

$$= \langle 0|T(\phi_1(x_\mu^{(1)})...\phi_N(x_\mu^{(N)})J_\mu(z)J_\nu(0)\phi_1^+(x_\mu^{(1)'})...\phi_N^+(x_\mu^{(N)'}))|0\rangle =$$

$$= (2\pi)^{-3N}\int\prod_{i=1}^{N} d^4p^{(i)}d^4p^{(i)'}\exp\left[-i\sum_{i=1}^{N}(p^{(i)}x^{(i)}-p^{(i)'}x^{(i)'})\right]\times \qquad (4.1)$$

$$\times R_{\mu\nu}([p_\mu^{(i)}];[p_\mu^{(i)'}];z).$$

Here $[x_\mu^{(i)}], [x_\mu^{(i)'}], [p_\mu^{(i)}], [p_\mu^{(i)'}]$ are the sets of corresponding 4-vectors. The quantity $R_{\mu\nu}$ can be presented as (see Fig.4)

$$R_{\mu\nu} = G\Gamma_{\mu\nu}G,$$

where G is the N-particle Green function of fields $\phi_i(x_\mu^{(i)})$ and «two-photon» vertex function $\Gamma_{\mu\nu}$ is defined by the sum of the irreducible diagrams with $2N+2$ points (legs).

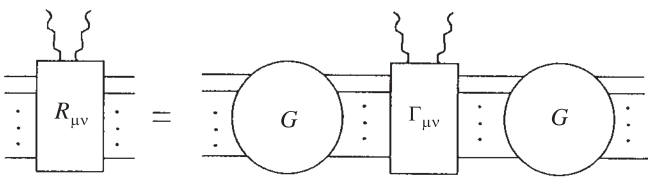

Fig. 4.

Introduce now the three-dimensional quantity $\tilde{R}_{\mu\nu}$ equating all $x_+^{(i)} = x_+$ and $x_+^{(i)'} = x'_+$ in (4.1)

$$\tilde{R}_{\mu\nu}(x_+, [x_-^{(i)}, \mathbf{x}_\perp^{(i)}]; x'_+, [x_-^{(i)'}, \mathbf{x}_\perp^{(i)'}]; z) =$$

$$= \langle 0|T(\phi_1(x_+, x_-^{(i)}, \mathbf{x}_\perp^{(i)})...\phi_N(x_+, x_-^{(N)}, \mathbf{x}_\perp^{(N)})J_\mu(z)J_\nu(0)\times$$

$$\times\phi_1^+(x'_+, x_-^{(i)'}, \mathbf{x}_\perp^{(i)'})...\phi_N^+(x'_+, x_-^{(N)'}, \mathbf{x}_\perp^{(N)'}))|0\rangle = \tag{4.2}$$

$$= (2\pi)^{-3N}\int\prod_{i=1}^{N}(dp_+^{(i)}, d\mathbf{p}_\perp^{(i)})(dp_+^{(i)'}, d\mathbf{p}_\perp^{(i)'})\tilde{R}_{\mu\nu}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]; z)\times$$

$$\times\exp\left[-i(x_+P_- - x'_+P'_-) - i\sum_{i=1}^{N}(p_+^{(i)}x_-^{(i)} - p_+^{(i)}x_-^{(i)}) + i\sum_{i=1}^{N}(p_+^{(i)'}x_-^{(i)'} - \mathbf{p}_\perp^{(i)'}\mathbf{x}_\perp^{(i)'})\right].$$

Fourier transforms of $R_{\mu\nu}$ and $\tilde{R}_{\mu\nu}$ are related to each other in the following way:

$$\tilde{R}_{\mu\nu}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]; z) = \int_{-\infty}^{\infty}\prod_{i=1}^{N}dp_-^{(i)}dp_-^{(i)'}\delta\times$$

$$\times\left(P_- - \sum_{i=1}^{N}p_-^{(i)}\right)\delta\left(P'_- - \sum_{i=1}^{N}p_-^{(i)'}\right)R_{\mu\nu}([p^{(i)}]; [p^{(i)'}]; z). \tag{4.3}$$

Single out now the contribution of N-particle bound states in the matrix element (4.3) expressing the T-product via θ-functions and using the compliteness of physical states:

$$\tilde{R}_{\mu\nu}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]; z) \cong$$

$$\cong [i(2\pi)^4]^2\frac{\Psi_P([p_+^{(i)}, \mathbf{p}_\perp^{(i)}])\langle P,\alpha|T(J_\mu(z)J_\nu(0))|P',\beta\rangle\Psi_{P'}^+([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}])}{(P^2 - M_\alpha^2)(P'^2 - M_\beta^2)}. \tag{4.4}$$

Finally, we get the following expression for the matrix element of T-product of currents:

$$\langle P,\alpha|T(J_\mu(z)J_\nu(0))|P',\beta\rangle =$$

$$= \int_0^{P_+}\prod_{i=1}^{N}dp_+^{(i)''}\delta\left(P_+ - \sum_{i=1}^{N}p_+^{(i)''}\right)\int\prod_{i=1}^{N}d\mathbf{p}_\perp^{(i)''}\delta^{(2)}\left(\mathbf{P}_\perp - \sum_{i=1}^{N}\mathbf{p}_\perp^{(i)''}\right)\times \tag{4.5}$$

$$= \int\limits_0^{p'_+} \prod_{i=1}^N dp_+^{(i)'''} \delta \left(P'_+ - \sum^N p_-^{(i)'''} \right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)'''} \delta^{(2)} \left(\mathbf{P'}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)'''} \right) \times$$

$$\times \Psi_P^+([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}]) \tilde{\Gamma}_{\mu\nu}([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}]; [p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}]; z) \Psi_{P'}([p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}]).$$

Fourier transform of this matrix element defined the amplitude of virtual Compton scattering of photon with space-like 4-momentum q_μ on the hadron with 4-momentum P_μ:

$$T_{\mu\nu}(P, q) = i \int d^4 z e^{lqz} \langle P, \alpha | T(J_\mu(z) J_\nu(0)) | P, \alpha \rangle =$$

$$= \int\limits_0^{P_+} \prod_{i=1}^N dp_+^{(i)''} \left(P_+ - \sum^N p_+^{(i)''} \right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)''} \delta^{(2)} \left(\mathbf{P}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)''} \right) \times \quad (4.6)$$

$$= \int\limits_0^{P'_+} \prod_{i=1}^N dp_+^{(i)'''} \delta \left(P'_+ - \sum^N p_+^{(i)'''} \right) \int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)'''} \delta^{(2)} \left(\mathbf{P'}_\perp - \sum_{i=1}^N \mathbf{p}_\perp^{(i)'''} \right) \times$$

$$\times \Psi_P^+([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}]) \int d^4 z e^{lqz} \tilde{\Gamma}_{\mu\nu}([p_+^{(i)''}, \mathbf{p}_\perp^{(i)''}]; [p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}]; z) \times$$

$$\times \Psi_{P'}([p_+^{(i)'''}, \mathbf{p}_\perp^{(i)'''}]).$$

According to optical theorem the tensor $W_{\mu\nu}$, which defines the hadronic part of deep inelastic lepton-hadron scattering cross section is related to the imaginary part of the amplitude of the zero angle virtual Compton scattering in the following way:

$$W_{\mu\nu}(P, q) = \sum_\alpha \int d^4 z e^{iqz} \langle P, \alpha | T(J_\mu(z) J_\nu(0)) | P, \alpha \rangle = \frac{1}{2\pi} \mathrm{Im} T_{\mu\nu}(P, q). \quad (4.7)$$

Taking into account the current conservation, the tensor $W_{\mu\nu}$ can be expressed via two invariant structure functions W_1, and W_2:

$$W_{\mu\nu}(P, q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1(q^2, \nu) +$$

$$+ \frac{1}{M^2} \left(P_\mu - \frac{Pq}{q^2} q_\mu \right) \left(P_\nu - \frac{Pq}{q^2} q_\nu \right) W_2(q^2, \nu),$$

where $M\nu = Pq$, M is the hadron mass.

Thus, using Eqs. (4.6)–(4.8) one can express the structure functions of deep inelastic lepton-hadron scattering in terms of the light-front many-body wave functions, describing the internal motion of partons inside hadron, and the «two-photon» vertex function:

$$\tilde{\Gamma}_{\mu\nu}([p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}];q)=\int d^4ze^{lqz}\tilde{\Gamma}_{\mu\nu}([p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)'},\mathbf{p}_\perp^{(i)'}];z)=$$

$$=\int d^4ze^{lqz}\int\limits_0^{P_+}\prod_{i=1}^{N}dp_+^{(i)''}\delta\left(P_+-\sum^{N}p_+^{(i)''}\right)\int\prod_{i=1}^{N}d\mathbf{p}_\perp^{(i)''}\delta^{(2)}\times \qquad (4.8)$$

$$\times\left(\mathbf{P}_\perp-\sum_{i=1}^{N}\mathbf{p}_\perp^{(i)''}\right)\times$$

$$=\int\limits_0^{P'_+}\prod_{i=1}^{N}dp_+^{(i)'''}\delta\left(P'_+-\sum^{N}p_-^{(i)'''}\right)\int\prod_{i=1}^{N}d\mathbf{p}_\perp^{(i)'''}\delta^{(2)}\left(\mathbf{P'}_\perp-\sum_{i=1}^{N}\mathbf{p}_\perp^{(i)'''}\right)\times$$

$$\times\tilde{G}^{-1}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)''},\mathbf{p}_\perp^{(i)''}])$$

$$\times[G\widetilde{\Gamma_{\mu\nu}}G]([p_+^{(i)''},\mathbf{p}_\perp^{(i)''}][p_+^{(i)'''},\mathbf{p}_\perp^{(i)'''}])\times$$

$$\times\tilde{G}^{-1}(P';[p_+^{(i)'''},\mathbf{p}_\perp^{(i)'''}][p_+^{(i)'},\mathbf{p}_\perp^{(i)'}])$$

4.2. Lowest Order in the Electromagnetic Interaction. The «two-photon» vertex operator $\tilde{\Gamma}_{\mu\nu}$ can be constructed using methods of perturbation theory and expanding the functions $\tilde{G}^{-1}$ and $\tilde{R}_{\mu\nu}=G\widetilde{\Gamma_{\mu\nu}}G$ in the series in coupling constant. In the lowest order two types of diagrams, shown in Fig.5 contribute to $\tilde{\Gamma}^{(0)}_{\mu\nu}$.

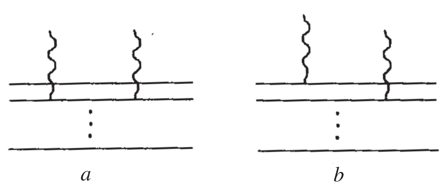

Fig. 5.

Assuming that the partons constituting hadron are on mass shell and neglecting small terms of the order of $P_--\sum\limits_{j=1}^{N}(\mathbf{p}_\perp^{(j)2}+m^{(j)2})/p_+^{(j)}$, we obtain that diagrams of type b) do not contribute to $\tilde{\Gamma}^{(0)}_{\mu\nu}$. Summing the contribution of all diagrams of type a), inserting into the expression (4.6) for virtual Compton scattering amplitude and extracting the imaginary part we obtain for the structure functions W_1 and W_2:

$$W_1(q^2,\nu) = \frac{1}{8(4\pi)^{N-1}} \int\limits_0^1 \prod_{i=1}^N \frac{dx^{(i)}}{x^{(i)}} \delta\left(1-\sum_{i=1}^N x^{(i)}\right) \times$$

$$\int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)} \delta^{(2)}\left(\mathbf{P}_\perp - \sum^N \mathbf{p}_\perp^{(i)}\right) |\Phi_P([x^{(i)},\mathbf{p}_\perp^{(i)})|^2 \times$$

$$\times \sum_{i=1}^N \frac{e_i^2}{x^{(i)}} \left[\frac{(M\nu+2\bar{p}^{(i)}P)^2}{M^2(1-\nu^2/q^2)} - (4m^{(i)2}-q^2)\right] \delta(q^2+2\bar{p}^{(i)}q), \qquad (4.9a)$$

$$\nu W_2(q^2,\nu) = \frac{1}{8(4\pi)^{N-1}} \frac{\nu}{(1-\nu^2/q^2)} \int\limits_0^1 \prod_{i=1}^N \frac{dx^{(i)}}{x^{(i)}} \delta\left(1-\sum_{i=1}^N x^{(i)}\right) \times$$

$$\int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)} \delta^{(2)}\left(\mathbf{P}_\perp - \sum^N \mathbf{p}_\perp^{(i)}\right) |\Phi_p([x^{(i)},\mathbf{p}_\perp^{(i)})|^2 \times$$

$$\times \sum_{i=1}^N \left\{\frac{e_i^2}{x^{(i)}} \left[\frac{3(M\nu+2\bar{p}^{(i)}P)^2}{M^2(1-\nu^2/q^2)} - (4m^{(i)2}-q^2)\right] \delta(q^2+2\bar{p}^{(i)}q)\right\}. \qquad (4.9b)$$

Here $\bar{p}^{(i)}$ is the momentum of parton on the mass shell $\bar{p}^{(i)} = = \left(\frac{\mathbf{p}_\perp^{(i)^2}+m^{(i)^2}}{p_+^{(i)}}, p_+^{(i)}, \mathbf{p}_\perp\right)$.

For further consideration we proceed to the frame, where the virtual photon and hadron are moving along the z axis:

$$P = (P_-, P_+, \mathbf{0}_\perp), \;\; q = (q_-, q_+, \mathbf{0}_\perp).$$

In this frame

$$2\bar{p}^{(i)}P = x^{(i)}M^2 + \frac{\mathbf{p}_\perp^{(i)^2} + m^{(i)^2}}{x^{(i)}}$$

and δ-function in (4.9) can be rewritten in the form:

$$\delta(q^2+2\bar{p}^{(i)}q) = \frac{1}{\xi}\delta\left[\frac{\mathbf{p}_\perp^{(i)^2}+m^{(i)^2}}{x^{(i)}} + \frac{Q^2(\xi-x^{(i)})}{\xi^2}\right]. \qquad (4.10)$$

Here we have introduced the variables $Q^2 = -q^2$ and

$$\xi = -\frac{q_+}{P_+} = \frac{Q^2}{M(\nu+\sqrt{\nu^2+Q^2})}. \qquad (4.11)$$

Scaling properties of structure functions with respect of variable ξ are discussed in several papers [50–53]. The Nachtman variable ξ is the generalization of the usual Bjorken variable x_B taking into account the hadron mass and is related to x_B by following relation:

$$\xi = -\frac{2x_B}{1+\sqrt{1+\frac{4Mx_B}{Q^2}}}. \tag{4.12}$$

If one neglects the masses and transverse momenta of partons ($m^{(i)2} << Q^2, \mathbf{p}_\perp^{(i)2} << Q^2$), the δ-function takes the form

$$\delta(2\bar{p}^{(i)}q - Q^2) = \frac{\xi}{Q^2}\delta(x^{(i)} - \xi).$$

Then the structure function W_1 vanishes and for the structure function νW_2 we obtain

$$\nu W_2(Q^2,\xi) = \frac{MQ^2}{2(4\pi)^{N-1}\xi}\frac{(Q^2/\xi^2 - M^2)}{(Q^2/\xi^2 + M^2)^2}\times$$
$$\int_0^1 \prod_{i=1}^N \frac{dx^{(i)}}{x^{(i)}}\delta\left(1 - \sum_{i=1}^N x^{(i)}\right)\times \tag{4.13}$$
$$\int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)}\delta^{(2)}\left(\sum_{i=1}^N \mathbf{p}_\perp^{(i)}\right)|\Phi_P([x^{(i)},\mathbf{p}_\perp^{(i)})|^2 \sum_{i=1}^N e_i^2\delta(x^{(i)} - \xi).$$

In the asymptotic limit ($\nu, Q^2 >> M^2$, x_B is fixed) the variable ξ coincides with the Bjorken variable x_B and we obtain that the structure function νW_2, is scale invariant with respect to the variable x_B:

$$\nu W_2(x_B) = \frac{Mx_B}{2(4\pi)^{N-1}}\int_0^1 \prod_{i=1}^N \frac{dx^{(i)}}{x^{(i)}}\delta\left(1 - \sum_{i=1}^N x^{(i)}\right)\times$$
$$\int \prod_{i=1}^N d\mathbf{p}_\perp^{(i)}\delta^{(2)}\left(\sum_{i=1}^N \mathbf{p}_\perp^{(i)}\right)|\Phi_P([x^{(i)},\mathbf{p}_\perp^{(i)}])|^2 \sum_{i=1}^N e_i^2\delta(x^{(i)} - x_B). \tag{4.14}$$

Assuming that the interaction kernel does not depend on the total energy and using the explicit expression for the Green function of N free particles we obtain the following sum rule:

$$\int_0^1 \frac{\nu W_2(x_B)}{Mx_B}dx_B = \sum_{i=1}^N e_i^2. \tag{4.15}$$

4.3. Model Parametrisation of the Wave Function. Consider now the case, when the hadron consists of two constituents. This case corresponds to meson, which consists of quark and antiquark. We will neglect contributions of gluons and quark-antiquark sea. Expressions for structure functions in the case $N = 2$ have the form:

$$W_1(Q^2,\xi) = \frac{e_1^2+e_2^2}{8\pi\xi}\int\limits_{\alpha}^{1}\frac{dx}{x(1-x)}\int d\mathbf{p}_\perp|\Phi_P(x,\mathbf{p}_\perp)|^2\times$$

$$\times\left[\frac{Q^2x(x-\xi)}{\xi^2}-m^2\right]\delta\left[\mathbf{p}_\perp^2+m^2-\frac{Q^2x(x-\xi)}{\xi^2}\right],$$

$$\nu W_2(Q^2,\xi) = \frac{e_1^2+e_2^2}{8\pi\xi}MQ^2\frac{Q^2/\xi^2-M^2}{(Q^2/\xi^2+M^2)^2}\int\limits_{a}^{1}\frac{dx}{x(1-x)}\int d\mathbf{p}_\perp|\Phi_P(x,\mathbf{p}_\perp)|^2\times$$

$$\times\left[\frac{6Q^2x(x-\xi)}{\xi^2}+Q^2-2m^2\right]\delta\left(\mathbf{p}_\perp^2+m^2-\frac{Q^2x(x-\xi)}{\xi^2}\right). \quad (4.16)$$

Here we assume that masses of constituents are equal to each other $m^{(1)} = m^{(2)} = m$.

In (4.16) the limit of integration over x is defined from the δ-function:

$$a = \frac{\xi}{2}\left(1+\sqrt{1+\frac{4m^2}{Q^2}}\right). \quad (4.17)$$

Neglecting masses and transverse momenta of quarks we obtain that the structure function W_1, vanishes and the structure function νW_2 takes the following form:

$$\nu W_2(Q^2,\xi) = \frac{(e_1^2+e_2^2)MQ^2}{8\pi\xi^2(1-\xi)}\frac{Q^2/\xi^2-M^2}{(Q^2/\xi^2+M^2)^2}\int d\mathbf{p}_\perp|\Phi_P(\xi,\mathbf{p}_\perp)|^2. \quad (4.18)$$

If we choose the following parametrization for wave function Φ_P

$$\Phi_P(x,\mathbf{p}_\perp) = C\left[\frac{\mathbf{p}_\perp^2+m^2}{x(1-x)}-\alpha\right]^{-n} \quad (4.19)$$

for the structure function νW_2 we get

$$\nu W_2(Q^2,\xi) = \frac{(e_1^2+e_2^2)MQ^2}{8}\frac{Q^2/\xi^2-M^2}{(Q^2/\xi^2+M^2)^2}\frac{|C|^2\xi^{2n-1}(1-\xi)^{2n-1}}{(2n-1)[m^2-\alpha\xi(1-\xi)]^{2n-1}}. \quad (4.20)$$

In the Bjorken limit ($Q^2 >> m^2$, $\xi \to x_B$) for $n = 1$ we obtain [45]:

$$\nu W_2(x_B) = \frac{e_1^2 + e_2^2}{8} M|C|^2 \frac{x_B(1 - x_B)}{m^2 - \alpha x_B(1 - x_B)}. \tag{4.21}$$

5. INCLUSIVE PROCESSES IN LIGHT-FRONT FORMALISM

Inclusive processes which have been proposed by Logunov and collaborators [46] are in effective tool to study the hadron structure at high energy. Consequences of a number of theoretical models have been formulated in this way. Majority of experiments in the relativistic nuclear physics (in particular, experiments on the cumulative production predicted by Baldin [47]) are also inclusive.

In this section expressions for inclusive cross sections in terms of the light-front quasi-potential wave functions are given [48].

5.1. Some Preliminary Relations. Let us construct the scattering amplitudes of the multiparticle production processes. «Two-time» N-particle Green function obeys the following equation:

$$\tilde{G}^{(N)} = \tilde{G}_0^{(N)} + \tilde{G}_0^{(N)} V^{(N)} \tilde{G}^{(N)}. \tag{5.1}$$

Here $V^{(N)}$ is the interaction kernel for N particles which can be constructed by the perturbation expansion.

If the total Hamiltonian processes one particles states with quantum numbers of $(N - k)$ and k particles of the initial state and $k_1, ...k_M$ particles of the final state the Green function possesses the poles with respect to the variables $P_A^2 = \left(\sum\limits_{i=1}^{k} p^{(i)}\right)^2$, $P_B^2 = \left(\sum\limits_{i=k+1}^{N} p^{(i)}\right)^2$, $P_1^2 = \left(\sum\limits_{i=1}^{k_1} p^{(i)'}\right)^2$, ..., $P_M^2 = \left(\sum\limits_{\substack{i=N-k_1-\\ -k_2 \ldots k_{M-1}}}^{N} p^{(i)}\right)^2$. In the vicinity of these poles it is of the form:

$$G(P; [p^{(i)}]; [p^{(i)'}]) \approx$$

$$\approx \frac{\chi_1(p^{(i)'}, ..., p^{(k_1)'})...\chi_M(p^{(N-k_1-...-k_M)'}, ..., p^{(N)'})}{\prod\limits_{i=1}^{M}(P_i^2 - M_i^2 + i\epsilon)} \times \tag{5.2}$$

$$\times \frac{T(P_1, ..., P_M; P_A, P_B)\bar{\chi}_A(p^{(1)}, ..., p^{(k)})\bar{\chi}_B(p^{(k+1)}, ..., p^{(N)})}{(P_A^2 - M_A^2 + i\epsilon)(P_B^2 - M_B^2 + i\epsilon)}.$$

T is the scattering amplitude corresponding to the process: $A+B \to 1+2+...+M$.

The «two-time» Green function has the following pole structure in the vicinity of corresponding poles:

$$G^{(N)}(P;[p_+^{(i)},\mathbf{p}_\perp^{(i)}];[p_+^{(i)\prime},\mathbf{p}_\perp^{(i)\prime}]) \cong [2(2\pi)^3]^{N-M-1}\times$$

$$\times\left[\frac{\Psi_1([p_+^{(i)\prime},\mathbf{p}_\perp^{(i)\prime}]...\bar{\Psi}_M([p_+^{(i)\prime},\mathbf{p}_\perp^{(i)\prime}])T\bar{\Psi}_A([p_+^{(i)},\mathbf{p}_\perp^{(i)}])\bar{\Psi}_B[p_+^{(i)},\mathbf{p}_\perp^{(i)}])}{x^{(i)\prime}...x^{(M)\prime}\left(P^2-\sum\limits_{i=1}^{M}\frac{\mathbf{P}_\perp^{(i)2}+M^{(i)2}}{x^{(i)}}\right)x^{(A)}...x^{(B)}\left(P^2-\sum\limits_{i=A,B}^{M}\frac{\mathbf{P}_\perp^{(i)2}+M^{(i)2}}{x^{(i)}}\right)}\right]. \tag{5.3}$$

Define transition operators $M_{\alpha\beta}^{(N)}$:

$$G^{(N)}=G_\alpha^{(N)}M_{\alpha,AB}^{(N)}G_{AB}^{(N)}+R,$$

where $G_\alpha^{(N)}$ is the Green functions which take into account interactions only inside the subsystems. α denotes the final states.

5.2. Production of Leading Hadrons with Large Transverse Momenta. Taking into account expression for Green functions one obtains the expression for the transition amplitude of the inclusive process $A+B\to C+X$ with the leading hadron C (assume that hadron C coincides with hadron A, or it is its excited state)

$$T(AB\to C)=[2(2\pi)^{N-M-1}]\times$$

$$\times\int\prod_{i=1}^{C}dx^{(i)\prime}d\mathbf{p}_\perp^{(i)\prime}C_+\delta\left(C_+-\sum_{i=1}^{C}p_+^{(i)\prime}\right)\delta^{(2)}\left(\mathbf{C}_\perp-\sum_{i=1}^{C}\mathbf{p}_\perp^{(i)\prime}\right)\times$$

$$\times\prod_{i=1}^{A+B}dx^{(i)}d\mathbf{p}_\perp^{(i)}A_+\delta\left(A_+-\sum_{i=1}^{A}p_+^{(i)}\right)\delta^{(2)}\left(\mathbf{A}_\perp-\sum_{i=1}^{A}\mathbf{p}_\perp^{(i)}\right)\times$$

$$B_+\delta\left(B_+-\sum_{i=1}^{B}p_+^{(i)}\right)\delta^{(2)}\left(\mathbf{B}_\perp-\sum_{i=1}^{B}\mathbf{p}_\perp^{(i)}\right)\times \tag{5.4}$$

$$\times\bar{\Psi}_C([p_+^{(i)\prime},\mathbf{p}_\perp^{(i)\prime}])M_{ABC}([p_+^{(i)\prime},\mathbf{p}_\perp^{(i)\prime}];[q_+^{(i)\prime},\mathbf{q}_\perp^{(i)\prime}];[p_+^{(i)},\mathbf{p}_\perp^{(i)}])\times$$

$$\times\bar{\Psi}_A([p_+^{(i)},\mathbf{p}_\perp^{(i)}])\bar{\Psi}_B([p_+^{(i)},\mathbf{p}_\perp^{(i)}]).$$

Here by A, B, C we denote hadrons A, B, C and the same time the number of elementary constituents inside them and their 4-momenta;

$$x^{(i)\prime} = \frac{p_+^{(i)\prime}}{C_+};\ x^{(i)} = \frac{p_+^{(i)}}{A_+};\ x^{(i)} = \frac{p_+^{(i)}}{B_+}$$

$q_+^{(i)}$, $\mathbf{q}_\perp^{(i)}$ are components of 4-momenta of the constituents of undetected hadrons.

Let us assume that the states A and C occur in the same sybsystem of M constituents and use pairwise approximation for the transition operator M_{ABC}. This is presented schematically in the Fig.6.

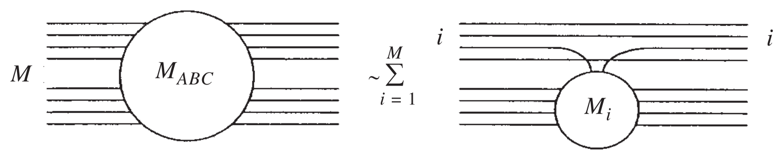

Fig. 6.

M_{iB} is the transition operator corresponding to the collision of the i-th constituent of hadron A with hadron B.

In this approximation the transition amplitude (5.4) takes the form:

$$T(AB \to C) = [2(2\pi)^{N-M-1}]\times$$

$$\times \sum_{i=1}^{C} \int \prod_{i=1}^{C} dx^{(i)} x^{(i)} d\mathbf{p}_\perp^{(i)} \delta\left(1 - \sum_{i=1}^{C} x^{(i)}\right) \delta^{(2)}\left(\sum_{i=1}^{C} \mathbf{p}_\perp^{(i)\prime}\right) \times$$

$$\times \bar{\Psi}_C(x^{(1)}, \mathbf{p}_\perp^{(1)} - x^{(1)}\mathbf{\Delta}_\perp; ...; x^{(i)}, (1 - x^{(i)})\mathbf{\Delta}_\perp; ...; x^{(c)}, \mathbf{p}_\perp^{(c)} - x^{(c)}\mathbf{\Delta}_\perp) \times \tag{5.5}$$

$$\times \bar{\Psi}_A([x^{(1)}, \mathbf{p}_\perp^{(1)}]) T_{iB}(B; [p_+^{(i)\prime}, \mathbf{p}_\perp^{(i)\prime}]; [p_+^{(i)\prime\prime}, \mathbf{p}_\perp^{(i)\prime\prime}]),$$

where: $x^{(i)} = p_+^{(i)}/A_+$,

$$p_+^{(i)\prime} = p_+^{(i)} + x^{(i)}/A_+;\ \mathbf{p}_\perp^{(i)\prime} = \mathbf{p}_\perp^{(i)} + x^{(i)}\mathbf{A}_\perp;\ \Delta = C - A$$

$$p_+^{(i)\prime\prime} = p_+^{(i)} + x^{(i)}/C_+;\ \mathbf{p}_\perp^{(i)\prime\prime} = \mathbf{p}_\perp^{(i)} + x^{(i)}\mathbf{A}_\perp; \tag{5.6}$$

T_{iB} is the inelastic amplitude of the scattering of i-th constituent of hadron A with hadron B.

For the following we recall the calculations of the electromagnetic form factors of composite systems. Rewrite the expression for the electromagnetic form factor in the impulse approximation in the form:

$$eF(t) = \sum_{n,i} e_i \int dx S_i^{(n)}(x, (1-x)\mathbf{\Delta}_\perp), \;\; t = (P - P')^2 = -\mathbf{\Delta}_\perp)^2,$$

where the quantity $S_i^{(n)}$ is defined as:

$$S_i^{(n)}(x, (1-x)\mathbf{\Delta}_\perp) = [2(2\pi)^3]^{n-1} x \int_0^1 \prod_{\substack{k=1 \\ k\neq i}}^{n} dx^{(k)} x^{(k)} \delta\left(1 - x - \sum_{\substack{k=1 \\ k\neq i}}^{n} x^{(k)}\right) \times$$

$$\times \int \prod_{k=1}^{n} d\mathbf{p}_\perp^{(k)} \delta\left(\sum_{k=1}^{n} \mathbf{p}_\perp^{(k)}\right) \bar{\Psi}^{(n)}([x^{(k)}, \mathbf{p}_\perp^{(k)} - x^{(k)}\mathbf{\Delta}_\perp]_{k\neq i}\, x, \mathbf{p}_\perp^{(i)} + (1-x)\mathbf{\Delta}_\perp) \times \quad (5.7)$$

$$\times \bar{\Psi}^{(n)}([x^{(k)}, \mathbf{p}_\perp^{(k)}]).$$

Taking into account the normalization condition for wave functions one can obtain the following normalization condition for $S_i^{(n)}$:

$$\sum_i \int_0^1 S_i^{(n)}(x, \mathbf{0}_\perp) dx = 1. \quad (5.8)$$

In the case when the interaction of the i-th constituent of hadron A with hadron B is effectively local [49] the transition amplitude $T(AB \to C)$ in the Eq. (5.5) takes the form:

$$T(AB \to C) = \sum_i f_i(\mathbf{\Delta}_\perp) T_{iB}, \quad (5.9)$$

where

$$f_i^{(\ell)}(\mathbf{\Delta}_\perp) = \int_0^1 \frac{dx}{x} S_i^{(\ell)}(x, (1-x)\mathbf{\Delta}_\perp)\phi(x).$$

The function $\phi(x)$ characterizes the local interaction vertex and in the case of the exchange of vectors particles (gluons), for instance, $\phi(x) = x$. In this case Eq. (5.9) takes the form:

$$T(AB \to C) = \sum_i F_i(\mathbf{\Delta}_\perp) T_{iB}, \quad (5.10)$$

where $\sum\limits_i F_i(\mathbf{\Delta}_\perp) = F_A(\mathbf{\Delta}_\perp)$ is the form factor of particle A, T_{iB} is the transition amplitude.

For the inclusive cross section one obtains:

$$E\frac{d\sigma(AB \to A)}{d\mathbf{p}} = \sum_i F_A^2(t) E \frac{d\sigma^{iB\to i+x}}{d\mathbf{p}}(s', t', u'). \qquad (5.11)$$

Here s', t', u' are the Mandelstam invariant variables for the subprocess.

5.3. Production of Hadron Systems with Large Transverse Momenta. Consider now the possibility when the transition operator takes into account interaction between some systems of constituents of hadrons A and B. Schematically this process is presented in Fig.7.

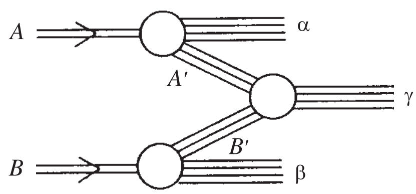

Fig. 7.

Let us assume that the wave function of the hadron A contains $\alpha + \ell$ constituents, the wave function of hadron B contains $\beta + \ell'$ constituents and $\gamma = \ell + \ell'$. In this approximation the transition amplitude takes the form:

$$T(A, B; [p_+^{(i)}, \mathbf{p}_\perp^{(i)}]; [p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]; [k_+^{(i)}, \mathbf{k}_\perp^{(i)}]) =$$

$$= [2(2\pi)]^{\alpha+\beta+\gamma-1} \int \prod_{i=1}^{\ell-1} dx^{(i)'} d\mathbf{k}_\perp^{(i)'} \prod_{i=\ell+1}^{\gamma-1} dx^{(i)'} d\mathbf{k}_\perp^{(i)'} \times$$

$$\times \prod_{i=1}^{\alpha} \frac{p_+^{(i)}}{A_+} \prod_{i=1}^{\beta} \frac{p_+^{(i)'}}{B_+} M_{A'B'}([k_+^{(i)}, \mathbf{k}_\perp^{(i)}]_{i=1,\ldots,\gamma}; [k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]_{i=1,\ldots,\gamma}) \times$$

$$\times \Psi_B^{(\beta+\ell')}([p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]_{i=1,\ldots,\beta}; [k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]_{i=\ell+1,\ldots,\gamma}) \Psi_A^{(\beta+\ell')}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]_{i=1,\ldots,\alpha}; [k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]_{i=1,\ldots,\ell})$$

$$x^{(i)'} = \begin{cases} k_+^{(i)'}/A_+, \ i = 1, ..., \ell \\ k_+^{(i)'}/B_+, \ i = \ell+1, ..., \gamma. \end{cases}$$

$M_{A'B'}$ is the transition operator of two interacting subsystems A' and B'. Let us connect the transition operator $M_{A'B'}$ with the transition operator $M_{A'B'\to C'D'}$ corresponding to inelastic «two-body» scattering (Fig.8). Here $G_{C'}$ and $G_{D'}$ are the Green functions of the subsystems C' and D'.

Fig. 8.

Using spectral properties of the Green functions $G_{C'}$ and $G_{D'}$ and restricting to the «one-particle» contributions one can represent the operator $M_{A'B'}$ in the form:

$$M_{A'B'}([\underset{i=1,\dots,\gamma}{k_+^{(i)},\mathbf{k}_\perp^{(i)}}];[\underset{i=1,\dots,\gamma}{k_+^{(i)'},\mathbf{k}_\perp^{(i)'}}]) = [2(2\pi)^3]^{\gamma-2}\prod_{i=1}^{\delta}\frac{k_+^{(i)}}{C_+}\prod_{i=\delta+1}^{\delta}\frac{k_+^{(i)}}{D_+}\times$$

$$\times\Psi_{C'}([\underset{i=1,\dots,\delta}{k_+^{(i)},\mathbf{k}_\perp^{(i)}}];)\Psi_{D'}([\underset{i=\delta+1,\dots,\gamma}{k_+^{(i)},\mathbf{k}_\perp^{(i)}}];)\prod_{i=1}^{\delta-1}dx^{(i)''}d\mathbf{k}_\perp^{(i)''}\prod_{i=\delta+1}^{\gamma-1}dx^{(i)''}d\mathbf{k}_\perp^{(i)''}\times$$

$$\times\Psi_{C'}([\underset{i=1,\dots,\delta}{k_+^{(i)''},\mathbf{k}_\perp^{(i)''}}];)\Psi_{D'}([\underset{i=\delta+1,\dots,\gamma}{k_+^{(i)''},\mathbf{k}_\perp^{(i)''}}];)M_{A'B'C'D'}([\underset{i=1,\dots,\gamma}{k_+^{(i)''},\mathbf{k}_\perp^{(i)''}}];[\underset{i=1,\dots,\gamma}{k_+^{(i)'},\mathbf{k}_\perp^{(i)'}}]),$$

where $x^{(i)''}=\begin{cases} k_+^{(i)''}/C_+,\ i=1,...,\delta \\ k_+^{(i)''}/D_+,\ i=\delta+1,...,\delta. \end{cases}$

This approximation corresponds to the pole contribution in the spectral representation for the Green functions of A', B', C', D' states and switching off the interactions with $(A-A')$ and $(B-B')$ subsystems. In the same approximation the wave functions Ψ_A and Ψ_B can be represented in the form:

$$\Psi_A^{(\alpha+\ell)}([\underset{i=1,\dots\alpha}{p_+^{(i)},\mathbf{p}_\perp^{(i)}}];[\underset{i=1,\dots,\ell}{k_+^{(i)'},\mathbf{k}_\perp^{(i)'}}])\sim$$

$$\sim(A_+/A'_+)^{\ell-1}\Psi_{AA'}([\underset{i=1,\dots\alpha}{p_+^{(i)},\mathbf{p}_\perp^{(i)}}]\Psi_{A\ell'}([\underset{i=1,\dots\ell}{k_+^{(i)'},\mathbf{k}_\perp^{(i)'}}],$$

where $\Psi_{A\ell'}([\underset{i=1,\dots\ell}{k_+^{(i)'},\mathbf{k}_\perp^{(i)'}}])$ is the usual light-front wave function of the system A', $\Psi_{AA'}([\underset{i=1,\dots\alpha}{p_+^{(i)},\mathbf{p}_\perp^{(i)}}])$ is defined as:

$$\Psi_{AA'}([\underset{i=1,\dots\alpha}{p_+^{(i)},\mathbf{p}_\perp^{(i)}}])\delta\left(A_+-A'_+-\sum_{i=1}^{\alpha}p_+^{(i)}\right)\delta^{(2)}\left(\mathbf{A}_\perp-\mathbf{A'}_\perp-\sum_{i=1}^{\alpha}\mathbf{p}_\perp^{(i)}\right)=$$

$$= \int \prod_{i=1}^{\alpha} dx_-^{(i)} d\mathbf{x}_\perp^{(i)} \exp\left[i \sum_{i=1}^{\alpha} (p_+^{(i)} x_-^{(i)} - \mathbf{p}_\perp^{(i)} \mathbf{x}_\perp^{(i)}) \right] \langle A' | \psi_1(x_\mu^{(1)} ... \psi_\alpha(x_\mu^{(\alpha)}) | A \rangle. \tag{5.12}$$

It is seen from the definition that $\Psi_{AA'}$ is the «one-time vertex function». Schematically the approximations made can be represented in the form (Fig.9).

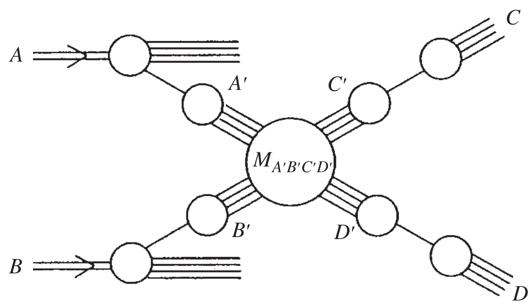

Fig. 9.

Corresponding expression for the scattering amplitude takes the form:

$$T(A, B; \underset{i=1,...\alpha}{[p_+^{(i)}, \mathbf{p}_\perp^{(i)}]}; \underset{i=1,...\beta}{[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]}; \underset{i=1,...\gamma}{[k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]}) =$$

$$= [2(2\pi)]^{\alpha+\beta+\gamma-2} [2(2\pi)]^{\gamma-2} \prod_{i=1}^{\delta} \frac{k_+^{(i)}}{C'_+} \prod_{i=\delta+1}^{\gamma} \frac{k_+^{(i)}}{D'_+} \times$$

$$\times \Psi_{C'}(\underset{i=1,...,\delta}{[k_+^{(i)}, \mathbf{k}_\perp^{(i)}]} \Psi_{D'} \underset{i=1,...,\gamma}{[k_+^{(i)}, \mathbf{k}_\perp^{(i)}]}) \times$$

$$\times \left\{ \int \prod_{i=1}^{\delta-1} dx^{(i)''} d\mathbf{k}_\perp^{(i)''} \prod_{i=1}^{\ell-1} dx^{(i)'} d\mathbf{k}_\perp^{(i)'} \prod_{i=1}^{\gamma-1} dx^{(i)'} d\mathbf{k}_\perp^{(i)'} \times \right.$$

$$\times \Psi_{C'}(\underset{i=1,...,\delta}{[k_+^{(i)''}, \mathbf{k}_\perp^{(i)''}]} \Psi_{D'} \underset{i=\delta+1,...,\gamma}{[k_+^{(i)''}, \mathbf{k}_\perp^{(i)''}]}) M_{A'B'C'D'}([k_+^{(i)''}, \mathbf{k}_\perp^{(i)''}]; [k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]) \times$$

$$\left. \times \Psi_{A\ell'} (\underset{i=1,...\ell}{[k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]} \Psi_{B'(\gamma-\ell)} \underset{i=\ell+1,...\gamma}{[k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]}) \right\} \Psi_{AA'}(\underset{i=1,...\alpha}{[p_+^{(i)}, \mathbf{k}_\perp^{(i)}]} \Psi_{BB'} \underset{i=\ell+1,...\gamma}{[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]}) \times$$

$$\times \prod_{i=1}^{\alpha} \frac{p_+^{(i)}}{A_+} \prod_{i=1}^{\beta} \frac{p_+^{(i)'}}{B_+}, \tag{5.13}$$

where

$$x^{(i)'} = \begin{cases} k_+^{(i)'}/A'_+, \ i = 1, ..., \ell \\ k_+^{(i)'}/B'_+, \ i = \ell + 1, ..., \gamma, \end{cases}$$

$$x^{(i)''} = \begin{cases} k_+^{(i)''}/C'_+, \ i = 1, ..., \delta \\ k_+^{(i)''}/D'_+, \ i = \delta + 1, ..., \gamma. \end{cases}$$

The exppression in paranthesis in Eq. (5.14) is by the definition the scattering amplitude of the process $A'B' \to C'D'$. Finally for the scattering amplitude we obtain:

$$T(A, B; \underset{i=1,\dots,\alpha}{[p_+^{(i)}, \mathbf{p}_\perp^{(i)}]}; \underset{i=1,\dots,\beta}{[p_+^{(i)'}, \mathbf{p}_\perp^{(i)'}]}; \underset{i=1,\dots,\gamma}{[k_+^{(i)}, \mathbf{k}_\perp^{(i)}]}) =$$

$$= [2(2\pi)^3]^{\alpha+\beta+\gamma-2} \prod_{i=1}^{\delta} \frac{k_+^{(i)}}{C'_+} \prod_{i=\delta+1}^{\gamma} \frac{k_+^{(i)}}{D'_+} \times$$

$$\times \Psi_{C'}(\underset{i=1,\dots,\delta}{[k_+^{(i)}, \mathbf{k}_\perp^{(i)}]} \Psi_{D'} \underset{i=\delta+1,\dots,\gamma}{[k_+^{(i)'}, \mathbf{k}_\perp^{(i)'}]}) \times$$

$$\times T(A'B' \to C'D') \prod_{i=1}^{\alpha} \frac{p_+^{(i)}}{A_+} \prod_{i=1}^{\beta} \frac{p_+^{(i)'}}{B_+} \times$$

$$\times \Psi_{AA'}(\underset{i=1,\dots,\alpha}{[p_+^{(i)}, \mathbf{p}_\perp^{(i)}]} \Psi_{BB'} \underset{i=1,\dots,\beta}{[p_+^{(i)}, \mathbf{p}_\perp^{(i)}]}). \tag{5.14}$$

Thus the transition amplitude is expressed through the scattering amplitude $T(A'B' \to C'D')$ of constituents which in particular, can be quarks. This mechanism corresponds to the production of jets.

Constructing the inclusive cross section of the process $AB \to c(C)$ we obtain:

$$E_C \frac{d\sigma(Ab \to c(C))}{d\mathbf{C}} = \frac{1}{4(2\pi)^2 s} \sum_{A'B'C'} \sum_{c(C)} \int dx d\mathbf{p}_\perp dx' d\mathbf{p}'_\perp \frac{dk_+}{k_+} (C_+ x x')^{-1} \times$$

$$\times \rho_A^{A'}(x, \mathbf{p}_\perp - \mathbf{A}_\perp, \alpha) \rho_B^{B'}(x', \mathbf{p}'_\perp - x' \mathbf{B'}_\perp, \beta) x'' \rho_C^{c^{(C)}}(z, \mathbf{k}_\perp - x'' \mathbf{C}_\perp, \gamma) \times$$

$$\times |T(A'B' \to C'D')|^2 \delta \left[\frac{A^2 - \alpha}{A_+} + \frac{B^2 - \beta}{B_+} + \frac{C^2 - \gamma}{C_+} + \right.$$

$$\left. + \frac{1}{k_+}(s' + t' + u' - c^2 - k^2 - {A'}^2 - {B'}^2) \right] d\alpha d\beta d\gamma. \tag{5.15}$$

Here $s' = xx's$; $t' = \frac{x}{x'}t$; $u' = \frac{x'}{x''}u$. Summation is performed over all possible intermediate states A', B', C'. The functions $\rho_A^{A'}$, $\rho_B^{B'}$, $\rho_C^{C'}$ are related to the squares of the corresponding wave functions.

Neglecting all the terms in the argument of δ-function as compared to $(s' + t' + u')$, and taking into account that transverse momenta in the functions ρ are limited, in the limit of high energies $(s \to \infty)$ and large momentum transvers $(t/s \to \text{const},\ u/s \to \text{const})$ one obtains:

$$E_C \frac{d\sigma(AB \to c(C))}{\mathbf{C}} = \frac{s}{\pi} \int dx dx' dx'' \frac{xx'}{x''^2} \rho_A^{A'}(x) \rho_B^{B'}(x') \rho_{C'}^{c(C)}(x') \times$$

$$\times \frac{d\sigma}{dt'}(s', t', u') \delta(s' + t' + u'). \tag{5.16}$$

In the case, when A', B', C', D' are quark states, corresponding quantities $\rho_A^{A'}$, $\rho_B^{B'}$ go over to the corresponding quark distribution functions and $\frac{d\sigma}{dt'}$ goes over to the quark-quark elastic cross section. Let us write finally the relation of quark distribution functions ρ_A to the light-front quasi-potential wave function:

$$\rho_A^{(N-1)}([p_i^{(+)}, \mathbf{p}_\perp^{(i)}]_{i=2,\ldots,N}) \prod_{i=2}^{N} dx^{(i)} d\mathbf{p}_\perp^{(i)} = [2(2\pi)^3]^{N-2} \left| \Psi_A^{(N-1)}([p_+^{(i)}, \mathbf{p}_\perp^{(i)}]_{i=2,\ldots,N}) \right|^2 \times$$

$$\times A_+ \prod_{i=2}^{N} x^{(i)} dx^{(i)} d\mathbf{p}_\perp^{(i)} \delta\left(A_+ - \sum_{i=2}^{N} p_+^{(i)}\right) \delta^{(2)}\left(\mathbf{A}_\perp - \sum_{i=2}^{N} \mathbf{p}_\perp^{(i)}\right). \tag{5.17}$$

REFERENCES

1. **Bethe H.A., Salpeter E.E.** — Phys. Rev., 1951, v.84, p.1232.
2. **Logunov A.A., Tavkhelidze A.N.** — Nuovo Cim., 1963, v.29, p.380.
3. **Matveev V.A., Muradyan R.M., Tavkhelidze A.N** — JINR Communications E2-3785, Dubna, 1967, P2-3900, Dubna, 1968.
4. **Kadyshevsky V.G.** — Nucl. Phys., 1968, v.B6, p.125.
5. **Garsevanishvili V.R., Kvinikhidze A.N., Matveev V.A., Tavkhelidze A.N, Faustov R.N.** — Sov. J. of Theor. and Math. Phys., 1975, v.23, p.310.
6. **Chang S.-J., Root R.G., Yan T.-M.** — Phys. Rev., 1973, v.D7, p.1133.
   Chang S.-J., Yan T.-M. — Phys. Rev., 1973, v.D7, p.1167.
   Yan T.-M. — Phys. Rev., 1973, v.D7, p.1760.
7. **Leutwyler H.** — Nucl. Phys., 1974, v.B76, p.413.
   Leutwyler H., Stern J. — Ann. Phys.(N.Y.), 1978, v.112, p.98.
8. **Berestetsky V.B., Terentyev M.V.** — Sov. J. Nucl. Phys., 1976, v.24, p.547.

9. **Harindranath A., Vary J.P.** — Phys. Rev., 1987, v.D36, p.1141; Phys. Rev., 1988, v.D37, p.1064.
10. **Karmanov V.A.** — Sov. J. of Particles and Nuclei, 1988, v.19, p.525.
11. **Brodsky S.J., Pauli H.-C.** — In: Recent Aspects of Quantum Fields. Eds. H.Mitter, H.Gausterer, Springer, Berlin, 1991.
Brodsky S.J., McCartor G., Pauli H.-C., Pinsky S.S. — Particle World, 1993, v.3, p.109.
12. **Bassetto A.** — Talk at the Meeting on the New Perturbative Methods and Quantization on the Light Cone. Les Houches, 24 Feb. – 4 Mar. 1997.
13. **Grangé P., Werner E.** — Talk at the Meeting on the New Perturbative Methods and Quantization on the Light Cone. Les Houches, 24 Feb. – 4 Mar. 1997.
14. **Wilson K.G., Walhout T.S., Harindranath A., Zhang W.-M., Perry S.J., Glazek St.D.** — Phys. Rev., 1994, v.D49, p.6720.
Brisudova M.M., Perry R.J., Wilson K.G. — Phys. Rev. Lett., 1997, v.78, p.1227.
15. **Carbonell J., Desplangues B., Karmanov V.A., Mathoit J.-F.,** to appear in Phys. Reports.
16. **Dirac P.A.M.** — Rev. Mod. Phys., 1949, v.21, p.392.
17. **Khelashvili A.A.** — JINR Communications P2-8750, Dubna, 1975.
18. **Susskind L.** — Phys. Rev., 1968, v.165, p.1535.
19. **Kogut J.B., Soper D.E.** — Phys. Rev., 1970, v.D1, p.2901.
20. **Kogut J.B., Susskind L.** — Phys. Reports, 1973, v.8, p.75.
21. **Gunion J.F., Brodsky S.T., Blankenbekler R.** — Phys. Rev., 1973, v.D8, p.287.
22. **Weinberg S.** — Phys. Rev., 1966, v.150, p.1313.
23. **Kopaleishvili T.I.** — In: Proc. of the VI Intern. Seminar on High Energy Physics Problems. JINR D1,2-81-728, Dubna, 1981.
24. **Garsevanishvili V.R., Matveev V.A.** — Sov. J. of Theor. and Math. Phys., 1975, v.24, p.3.
25. **Kvinikhidze A.N., Matveev V.A., Tavkhelidze A.N., Khelashvili A.A.** — Sov. J. of Theor. and Math. Phys., 1976, v.29, p.3.
26. **Bogolubov N.N., Shirkov D.V.** — Introduction to the Theory of Quantized Fields, Willy, New York, 1980.
27. **Tavkhelidze A.N.** — Talk at the Solvey Congress, 1967.
28. **Mandelshtam S.** — Proc. Roy Soc., 1955, v.A233, p.248.
29. **Faustov R.N.** — Ann. Phys.(N.Y.), 1973, v.78, p.176.
30. **Bogolubov N.N., Matveev V.A., Tavkhelidze A.N.** — In: Proc. of the Intern. Seminar on Elementary Particle Physics, Varna, Bulgaria, 1968, JINR, P2-4050, Dubna, 1968.
31. **Alabiso C., Schierholz G.** — Phys. Rev., 1974, v.D10, p.960.
32. **Faustov R.N., Khelashvili A.A.** — Sov. J. of Nucl. Phys., 1970, v.10, p.619.
33. **Matveev V.A., Muradyan R.M., Tavkhelidze A.N.** — Lett. Nuovo Cim., 1973, v.7, p.719.
34. **Brodsky S.J., Farrar G.** — Phys. Rev. Lett., 1973, v.31, p.1153.
35. **Garsevanishvili V.R., Kvinikhidze A.N., Matveev V.A., Tavkhelidze A.N., Faustov R.N.** — Sov. J. of Theor. and Math. Phys., 1975, v.25, p.37.
36. **Matveev V.A., Muradyan R.M., Tavkhelidze A.N** — JINR Communications E2-8048, Dubna, 1974.
37. **Feynman R.P.** — Phys. Rev. Lett., 1969, v.23, p.1415.

38. **Bjorken J.D., Paschos E.A.** — Phys. Rev., 1969, v.185, p.1975; Phys. Rev., 1970, v.D1, p.3151.

39. **Bodek A. et al.** — Phys. Rev. Lett., 1973, v.30, p.1087.

40. **Riordan E.M. et al.** — Phys. Rev. Lett., 1974, v.33, p.561.

41. **Gordon B.A. et al.** — Phys. Rev., 1979, v.D20, p.2645.

42. **Nachtmann O.** — Phys. Rev., 1973, v.B63, p.237.

43. **De Rujula A., Georgi H., Politzer H.D.** — Phys. Rev., 1974, v.D10, p.2141; Ann. Phys. (N.Y.), 1977, v.103, p.315.

44. **Krasnikov N.N., Tavkhelidze A.N., Chetyrkin K.G.** — In: Proc. of the Intern. Conf. «Neutrino 77», Nauka, Moscow, 1978.

45. **Atakishiev N.M., Mir-Kasimov R.M., Nagiev Sh.M.** — JINR Communications P2-80-635, Dubna, 1980.

46. **Logunov A.A., Mestvirishvili M.A., Nguen Van Hieu** — Phys. Lett., 1967, v.25B, p.612.

47. **Baldin A.M.** — Short Communications in Physics, 1971, v.1, p.35.

48. **Kvinikhidze A.N., Sissakian A.N., Slepchenko L.A., Tavkhelidze A.N.** — Sov. J. of Particles and Nuclei, 1977, v.8, p.478.

49. **Logunov A.A., Mestvirishvili M.A., Petrov V.A.** — IHEP, 74-66, Protvino, 1974.