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ABOUT FORCES, ACTING ON RADIATING CHARGE

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It is shown, that the force acting on a radiating charge is stipulated by two reasons — owing to exchange of a momentum between radiating charge and electromagnetic field of radiation, and also between a charge and field accompanying the charge.

It is well known that the charged particle moving with acceleration radiates, and as a result an additional force (apart from the external one, $\mathbf{F}_0$) — force of radiation reaction acts on it. In the present paper it is shown that this force (we shall call it as a self-action force or simply by self-action) is a sum of two parts: the first force is due to the exchange of the momentum between a particle and radiation fields, i. e., the fields, which go away to infinity. For the second force in the exchange of a momentum the fields, accompanying a charge participate as well. These fields do not go away to infinity, i. e., at infinity they have zero flux of energy (details see below).

We shall start with the momentum conservation law for a system of charge and electromagnetic field [1, 2]

$$\frac{d}{dt}\left(\mathbf{P}+\frac{1}{4\pi c}\int_V [\mathbf{EH}]\,dV\right)=\frac{1}{4\pi}\oint_S\left\{\mathbf{E}\,(\mathbf{nE})+\mathbf{H}\,(\mathbf{nH})-\frac{E^2+H^2}{2}\mathbf{n}\right\}dS, \tag{1}$$

where $\mathbf{P}$ is the particle momentum; $\mathbf{E}$ and $\mathbf{H}$ are the vectors for electromagnetic field; $\mathbf{n}$ is the normal to the surface S, enclosing volume V. In the right-hand side of formula (1) the external force $\mathbf{F}_0$ is omitted. From (1) we can see that, apart from external force, two forces act on the particle: force $\mathbf{f}_1$, expressed by a surface integral, and force $\mathbf{f}_2$, expressed by a volume integral.

As a surface S we shall take sphere of a large radius $R \to \infty$, with the centre at the point of instantaneous place of the charge, then $\mathbf{n} = \mathbf{R}/R$. For $\mathbf{E}$ and $\mathbf{H}$ we shall use the known expressions for the fields created by a charged particle moving with arbitrary velocity $\mathbf{v}\,(t)$ [2, 3]

$$\mathbf{H}=[\mathbf{n}E]\,,\quad \mathbf{E}\,(\mathbf{r},t)=\frac{e\,(\mathbf{n}-\boldsymbol{\beta})}{\gamma^2R^2x^3}+\frac{e}{cRx^3}\left[\mathbf{n}\left[\mathbf{n}-\boldsymbol{\beta},\dot{\boldsymbol{\beta}}\right]\right], \tag{2}$$

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where $c\boldsymbol{\beta} = \mathbf{v}$, $\gamma = \left(1 - \beta^2\right)^{-1/2}$, $x = 1 - \mathbf{n}\boldsymbol{\beta}$, $\dot{\boldsymbol{\beta}} \equiv d\boldsymbol{\beta}/dt$. Note, that all quantities in the right-hand side of equation (2) are taken at the moment $t' = t - R(t')/c$.

Calculating the force $\mathbf{f}_1$ we have to substitute in (1) the term with the lowest order of R^{-1} (the second term on the right in (2)), corresponding, to spherical electromagnetic fields going away to infinity, i. e., radiation fields. Then, taking into account the remark after formula (2), it is possible to write the force $\mathbf{f}_1$ in the form

$$\mathbf{f}_1 = -\oint_S \frac{E^2}{4\pi}\mathbf{n}dS = -\oint \mathbf{n}\frac{dI_n}{c}, \tag{3}$$

where dI_n is the energy, radiated per unit of time in the element of the solid angle $d\Omega$ in an arbitrary direction $\mathbf{n}$ [3]

$$dI_n = \frac{e^2}{4\pi c x^3}\left\{\dot{\beta}^2 + \frac{2}{x}\left(\mathbf{n}\dot{\boldsymbol{\beta}}\right)\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right) - \frac{\left(\mathbf{n}\dot{\boldsymbol{\beta}}\right)^2}{\gamma^2 x^2}\right\}d\Omega. \tag{4}$$

The formula (3) allows the following clear interpretation of the origin of the force $\mathbf{f}_1$: the radiation in a direction $\mathbf{n}$ per unit time carries away with itself momentum $\mathbf{n}dI_n/c$, and therefore, the charge acquires a momentum $-\mathbf{n}dI_n/c$. As the change of a momentum per unit time is equal to the acting force, then as a result of radiation in a direction $\mathbf{n}$ the force will act on the particle, equal to $d\mathbf{f}_1 = -\mathbf{n}dI_n/c$. Integrating over all directions (over total solid angle), we get the expression for the force $\mathbf{f}_1$ (details for calculation see in [4]):

$$\mathbf{f}_1 = -\frac{I}{c}\boldsymbol{\beta}, \quad I = \frac{2e^2}{3c}\gamma^4\left(\dot{\beta}^2 + \gamma^2\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)^2\right). \tag{5}$$

Here I is the instantaneous power of radiation, being a relativistic invariant and having the form [3, 5]

$$I = -\frac{2}{3}ce^2\frac{du^k}{ds}\frac{du_k}{ds}. \tag{6}$$

In this formula $u^k = dx^k/ds$ is the four-velocity and $ds = cdt/\gamma$ is the Minkowskian interval (we follow the notations in the book [3]).

Now we turn to the force $\mathbf{f}_2$. Here it is necessary to take into account the contribution of both summands in formula (2). The calculations are too long and, as is easy to see, lead to integrals, divergent at both small and long distances. The latters are related to the divergences of the self-energy and momentum for the point charge field. To avoid these difficulties, we shall act as follows. Let's write a three-dimensional equation of motion $d\mathbf{p}/dt = \mathbf{f} = \mathbf{f}_1 + \mathbf{f}_2$ in the four-dimensional (covariant) form

$$\frac{dp^i}{dt} = g^i = g_1^i + g_2^i, \tag{7}$$

by entering the four-dimensional momentum $p^i = mcu^i = (\gamma mc, \mathbf{p})$ and force $g^i = \left(\frac{\gamma}{c}\mathbf{f}\boldsymbol{\beta}, \gamma/c\mathbf{f}\right)$. In formula (7) it is necessary to define g_2^i. Taking into account (5) and (6), it is easy to see that g_1^i has the form

$$g_1^i = \frac{2e^2}{3c}\frac{du^k}{ds}\frac{du_k}{ds}u^i. \tag{8}$$

As follows from the definition of the force $\mathbf{f}_2$ and formula (2), where the vectors $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$ enter only, four-dimensional vector g_2^i can be expressed through the vectors u^i, du^i/ds and d^2u^i/ds^2 only. The first possibility disappears as for $\mathbf{v} = \mathrm{const}$, should be $g_2^i = 0$. The summand containing du^i/ds is united with a left-hand side of equation (7) and leads to the renormalization of the charged particle mass, so that there remains the possibility $g_2^i = \alpha d^2u^i/ds^2$, where $\alpha = 2e^2/3c$ is a number (four-dimensional scalar) which is determined from the requirement that for an arbitrary four-dimensional force g^i should be $g^i u_i = 0$ (to see this it is necessary to use identity $u^i u_i = 1$ and its consequences as well). Hence

$$g_2^i = \frac{2e^2}{3c}\frac{d^2u^i}{ds^2}. \tag{9}$$

From (9) the expression for three-dimensional force $\mathbf{f}_2$ follows which we give for the reference purposes

$$\mathbf{f}_2 = \frac{2e^2}{3c^2}\gamma^2\left\{\ddot{\boldsymbol{\beta}} + \gamma^2\dot{\beta}^2\boldsymbol{\beta} + 3\gamma^2\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)\dot{\boldsymbol{\beta}} + \gamma^2\left(\boldsymbol{\beta}\ddot{\boldsymbol{\beta}}\right)\boldsymbol{\beta} + 4\gamma^4\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)^2\boldsymbol{\beta}\right\}.$$

The formulas (7)–(9) lead to the well-known expression (see, for example, [3]) for the four-dimensional self-action force g^i

$$g^i = \frac{2e^2}{3c^2}\gamma^2\left(\frac{d^2u^i}{ds^2} + \frac{du^k}{ds}\frac{du_k}{ds}u^i\right).$$

Hence, for the three-dimensional self-action force $\mathbf{f}$ we find (compare to the corresponding formulas in [6, 7])

$$\mathbf{f} = \frac{2e^2}{3c^2}\left\{\mathbf{A} + \left[\boldsymbol{\beta}\left[\boldsymbol{\beta}\mathbf{A}\right]\right]\right\}, \tag{10}$$

where $\mathbf{A} \equiv \gamma^4\left(\ddot{\boldsymbol{\beta}} + 3\gamma^2\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)\dot{\boldsymbol{\beta}}\right)$.

In the nonrelativistic case ($\boldsymbol{\beta} \ll 1$), at first approximation over β from (10) we get the following expression for the self-action force (by the way, we shall indicate that there was an error in the formula (6) in article [5])

$$\mathbf{f} = \frac{2e^2}{3c^2}\ddot{\boldsymbol{\beta}} + \frac{2e^2}{c^2}\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)\dot{\boldsymbol{\beta}}. \tag{11}$$

This force differs from the conventional one $\mathbf{f}' = \dfrac{2e^2}{3c^2}\ddot{\boldsymbol{\beta}}$, in which the essential defect is inherent: for uniformly accelerated motion $\left(\ddot{\boldsymbol{\beta}} = 0\right)$, the force of radiation reaction $\mathbf{f}'$ is zero, while the radiation is not equal to zero $\left(\dot{\boldsymbol{\beta}} \neq 0\right)$. The force (11) is deprived of this defect and always is nonzero if the radiation is nonzero $\left(\dot{\boldsymbol{\beta}} \neq 0\right)$. If $\ddot{\boldsymbol{\beta}} \neq 0$ and the first summand in the right-hand side of (11) dominates, then $\mathbf{f} = \mathbf{f}'$; depending on the law $\boldsymbol{\beta}(t)$, the second summand can dominate. Generally, for $\beta \ll 1$, for self-action force it is necessary to use the formula (11).

The above mentioned allows us to state that the total self-action force acting on a radiating charge is determined by formula (10) and it is more appropriate to call a reaction force of radiation the force $\mathbf{f}_1$ determined by formula (5). This force is always nonzero when the particle moves with acceleration and hence radiates.

From this point of view let's consider again uniformly accelerated motion (for arbitrary velocities). It is known that the condition for uniformly accelerated motion has the form [7]

$$\frac{d^2u^i}{ds^2} + \frac{du^k}{ds}\frac{du_k}{ds}u^i = 0, \tag{12}$$

(thence $g^i = 0$) or in three-dimensional notations

$$\ddot{\boldsymbol{\beta}} + 3\gamma^2\left(\boldsymbol{\beta}\dot{\boldsymbol{\beta}}\right)\dot{\boldsymbol{\beta}} = 0. \tag{13}$$

As a result for this motion the vector $\mathbf{A}$ goes to zero and this is the case for the self-action force. However the radiation and radiation reaction force are nonzero, because the acceleration is nonzero. The latter can be easily obtained from the equation $d\mathbf{p}/dt = \mathbf{F}_0 + \mathbf{f}$ and is determined by the formula

$$mc\gamma\dot{\boldsymbol{\beta}} = \mathbf{F}_0 + \mathbf{f} - \boldsymbol{\beta}\left(\boldsymbol{\beta}\mathbf{f}_0\right) - \boldsymbol{\beta}\left(\boldsymbol{\beta}\mathbf{f}\right). \tag{14}$$

In our case for $\boldsymbol{\beta}||\mathbf{F}_0$, $\mathbf{F}_0 = \mathrm{const}$, the acceleration is equal to

$$c\dot{\boldsymbol{\beta}} = \frac{\mathbf{F}_0}{m\gamma^3}. \tag{15}$$

Hence, for the uniformly accelerated motion the only force acting on charge is the external force $\mathbf{F}_0$ (it can be easily checked that for the acceleration (15) the self-action force is zero). For $\boldsymbol{\beta} \to 1$ the acceleration tends to zero, and in the case $\boldsymbol{\beta} \to 0$ the acceleration, as it is expected, is equal to $\dfrac{\mathbf{F}_0}{m}$.

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