УДК 539.168.8

ANGULAR DISTRIBUTION OF RADIATIVE GAMMA QUANTA IN RADIATIVE BETA DECAY OF NEUTRON

R.U.Khafizov[1]

Russian Scientific Centre «Kurchatov Institute», Moscow

The present report is dedicated to consideration of spectrum and angular distribution of radiative gamma quanta for radiative neutron decay, which has not been discovered experimentally. The angular distribution of gamma quanta obtained will allow one to conduct new correlation experiments with the take-off of the fourth particle — the gamma quantum. Besides, as will be shown in the report, this distribution plays a significant role in suppressing the correlated background of bremsstrahlung gamma quanta, which simulates the effect completely. This background is impossible to remove even using the triple gamma quanta, electron and recoil proton coincidences. However, it is namely this unhomogeneous distribution by the angle of the outgoing radiative gamma quanta that allows one to use the space resolution and eliminate this dangerous background, thus making the conduction of an experiment on radiative neutron decay perfectly feasible.

Анализируется спектр и угловое распределение гамма-квантов при радиационном бета-распаде нейтрона, который пока экспериментально не обнаружен. Измерение угловых распределений гамма-квантов позволяет проводить корреляционные эксперименты нового типа с регистрацией четвертой частицы — гамма-кванта. Кроме того, как показано в работе, такие угловые распределения важны для подавления фона от сопутствующих фотонов тормозного излучения, которые могут полностью имитировать искомый эффект. Данный фон невозможно подавить за счет только тройных совпадений гамма-кванта, электрона и протона отдачи. Однако именно это неравномерное распределение по углам вылета радиационных гамма-квантов позволяет использовать пространственное разрешение и полностью исключить такой потенциально опасный фон, что, в свою очередь, открывает возможность успешного проведения поиска и измерения радиационного бета-распада нейтрона.

Among the many branches of any elementary decay with charged particles in the final state, the radiative branch, when the decay occurs with the creation of an additional particle — the gamma quantum, is usually the most intensive, as the smallness order for the branching ratio (BR) of this mode is determined by the fine structure constant that has the order of 10^{-2}. The aim of this work is the research for radiative decay of the free neutron. As we can see from the Particle Data Group information [1], no experimental measurements of BR for this decay exist to date. On the other hand, however, for other elementary particles this rare decay is experimentally well investigated. This fact makes the discovery of radiative neutron decay one of the most actual tasks in the present day neutron physics.

Moreover, this rare decay mode is directly connected with several fundamental problems. First of all there is the problem of radiative corrections for the neutron lifetime and correlation

[1]rash@pretty.mbslab.kiae.ru

coefficients. It is well known that if one neglects these corrections, the present experimental values for the neutron lifetime and correlation coefficients will not be in agreement in the framework of standard theory of the weak interaction and lead to incorrect values for the fundamental constants of this theory. Part of these corrections (the so-called outer radiative corrections) can be estimated directly from the experiment on the radiative decay of the free neutron that is proposed here.

Research of the radiative neutron decay mode is a task particularly actual today also because of the precision of the experiments on the ordinary neutron decay parameters such as lifetime and correlation coefficients [2], which has reached 10^{-3}. The point is that already at this level of accuracy it is necessary to consider the influence of the fourth outgoing particle — the gamma quantum on the final state. The influence of the radiative decay mode on the value of neutron lifetime is obvious, but this influence also shows in correlation experiments. As will be shown later, the angular distribution of the outgoing gamma quanta is not homogeneous and the radiative gamma quantum normally takes off at a sharp angle to decay electron. This, in turn, means that it is not only the value of the experimental precision that changes but also the measured mean value itself. The value of such a shift, which by estimation is no larger than 10^{-3}, should be considered in the concrete geometry of the correlation experiment, therefore, consideration of the radiative decay branch becomes actual when precision values of 10^{-3} and better are achieved.

In addition, the characteristic peculiarity of this experiment is the possibility of conducting BR measurements for gamma quanta of very low energies, which is impossible in the decay of other elementary particles, where the lower experimental limit for gamma-quanta energy is typically a few MeV. This fact makes experimental research on the radiative decay mode of the free neutron extremely perspective. In principle, it is possible to study the gamma-quanta energy spectrum from the endpoint down to the region of low energies (up to light region!), thereby creating a unique opportunity to check the infrared catastrophe which is given by the standard electrodynamics for all the radiative decays of the elementary particles.

Besides, the presence of the fourth outgoing particle — the gamma quantum, allows one to conduct new correlation experiments, which will measure the angular distribution of the radiative gamma-quanta take-off in relation to the beta-electron. Theoretical angular distribution of the gamma quanta, calculated within the framework of the standard model and quantum electronics is the aim of this report. Another question under research is the question of conducting new correlated experiments for radiative decay of polarized neutrons. Next interesting question is the research of the radiative gamma-quanta polarization, particularly in the light region.

Presented above is only a brief outline of the possible research plan of radiative neutron decay branch. Actually, a wide and perspective programme of experimental research for radiative branch of neutron decay could be developed here, but the first step in this programme is, of course, the experimental discovery of the radiative neutron decay and the measurement of its BR itself. However, it is already this first step of the experimental research, to which this project is dedicated, that presents us with the serious problem of correlated bremsstrahlung gamma-quanta background. This report is primarily dedicated to this very question.

The matrix element M of the radiative β decay consists of two terms [3]:

$$M = \frac{eg_V}{\sqrt{2}} \left[\overline{u}_e \frac{2(p_e e) + \widehat{e}\widehat{K}}{2(p_e K)} \gamma_\rho (1+\gamma_5) u_\nu \overline{u}_p \gamma_\rho (1+\lambda\gamma_5) u_n - \right.$$

$$-\overline{u}_e\gamma_\rho(1+\gamma_5)u_\nu\overline{u}_p\frac{2(p_pe)+\widehat{e}\widehat{K}}{2(p_pK)}\gamma_\rho(1+\lambda\gamma_5)u_n\Bigg],$$

where $p_e(E_e,\mathbf{p}_e)$, $K(K,\mathbf{K})$, $p_\nu(E_\nu,\mathbf{p}_\nu)$, $p_p(E_p,\mathbf{p}_p)$ are the four-momenta of the electron, γ quantum, neutrino and proton; e is the γ-quantum polarization. The first term describes the electron γ-quanta emission and the second term describes proton γ-quanta emission. Both terms possess the infrared divergence, and if diagrams with γ-quanta exchange are taken into account, this divergence will be canceled. This cancellation was demonstrated by Sirlin [4], Geshkenbein and Popov [5], who used Ward's identity for this purpose. In contrast to the ordinary muon decay, here the g_V is not equal to g_A, therefore due to the γ-quanta exchange, the total radiative corrections depend also on ultraviolet cut-off parameter Λ. However, in the first order of perturbation theory the radiative beta decay does not depend on this parameter. Thus in the radiative beta decay of the neutron one can measure only part of these corrections, which is connected with the so-called outer radiative corrections.

After standard calculations the differential cross section consists of two terms:

$$\frac{d\Gamma}{dE_edK}=\frac{1}{8\pi^4}\alpha g_V^2\left(\frac{1}{12}\right)\left(1-\frac{m_p^2}{Q_0^2}\right)^2\int QdQ\Bigg\{Q_0^23\left(1+\frac{m_p^2}{Q_0^2}\right)(1+3\lambda^2)\times$$

$$\times\left[\frac{1}{(p_eK)^2}\left[m_e^2(E_e+K)-(p_eK)K\right]+\frac{1}{K}+\frac{E_e}{K^2}-\frac{2E_e(E_e+K)}{K(p_eK)}\right]+$$

$$+(1-\lambda^2)2Q_0\left(1+2\frac{m_p^2}{Q_0^2}\right)\left[\left(\frac{m_e^2}{(p_eK)^2}-\frac{1}{(p_eK)}\right)Q^2-\right.$$

$$\left.-\frac{E_e}{(p_eK)K}[2(\mathbf{p}_e\mathbf{Q})+(\mathbf{KQ})]+\frac{1}{K^2}(\mathbf{p}_e\mathbf{Q})+\frac{1}{K}4(Q_0-m_p)\right]\Bigg\},$$

here the momentum $Q=p_e+K; Q=|\mathbf{Q}|$ is associated with total energy $Q_0=m_n-E_e-K$. The first term is proportional to $1+3\lambda^2$ and has a large value. The second term is proportional to $1-\lambda^2$ and has a value of Q_0/m_N, which is very small. If one wants to investigate the γ radiation directly from the weak vertex, one should keep this small term. For the radiative neutron β decay one can take only the first term. To calculate the electron spectrum from the radiative neutron decay it is necessary to integrate the leading term over the energy of outgoing quanta from the upper limit of $m-E_e$ to a lower cut-off energy ω. This lower limit ω is indeed the lower experimental cut-off for γ quanta, which one can vary directly in the experiment. Finally, one arrives at [6]:

$$(d\Gamma/dE_e)\approx\frac{g_V^2}{2\pi^3}\frac{\alpha}{2\pi}(1+3\lambda^2)\int\limits_{\omega}^{\Delta m-E_e}dK(\Delta m-E_e-K)^2\times$$

$$\times 2\left[KN+2E_e(N-\beta)+2E_e^2\frac{(N-\beta)}{K}\right]=$$

$$= \frac{g_V^2}{2\pi^3}\frac{\alpha}{2\pi}(1+3\lambda^2)w_0(E_e)\left(g_e(\beta)+g_p(\beta)\right),$$

$$g_e(\beta) = \frac{N}{\beta}\frac{(E_{em}-E_e)^2}{6E_e^2} - 2\left[\frac{E_{em}-E_e}{3E_e} - \frac{3}{2} + \ln\frac{E_{em}-E_e}{\omega}\right],$$

$$g_p(\beta) = 2\left(\frac{2N}{\beta}-1\right)\left[\frac{E_{em}-E_e}{3E_e} - \frac{3}{2} + \ln\frac{E_{em}-E_e}{\omega}\right].$$

After the integration one can compare these results with the previous ones and first of all with the results obtained by Christian and Kuhnelt [7]. If we take Sirlin's function for total radiative corrections $g(\beta)$ [4]:

$$r(E_e) = (\alpha/2\pi)g(\beta); \quad \beta = |p_e|/E_e = v_e/c,$$

$$g(\beta) = 3\ln\frac{\Lambda}{m_e} - \frac{3}{4} + 4\left(\frac{N}{\beta}-1\right)\left[\frac{E_{em}-E_e}{3E_e} - \frac{3}{2} + \ln\frac{E_{em}-E_e}{m_e/2}\right] +$$

$$+\frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{N}{\beta}\left[2(1+\beta^2) - 4N + \frac{(E_{em}-E_e)^2}{6E_e^2}\right],$$

$$N = \frac{1}{2}\ln\frac{1+\beta}{1-\beta}; \quad L(z) = \int_0^z dt\frac{\ln|1-t|}{t}; \quad \frac{e^2}{4\pi} = \alpha \approx \frac{1}{137},$$

$$E_{em} = \Delta - (\Delta^2 - m_e^2)/2m_n; \Delta = m_n - m_p,$$

and subtract our result, the rest of Sirlin's function will be identical to Christian and Kuhnelt's one [7]. It is the so-called soft photon approximation [7]:

$$g(\beta,\omega) = 3\log\left(\frac{\Lambda}{m_e}\right) + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) - \frac{3}{4} + 2\frac{N}{\beta}(1+\beta^2-2N) + 4\left(\frac{N}{\beta}-1\right)\log\frac{2\omega}{m_p},$$

where $\Lambda = m_p$ is an ultraviolet cut-off parameter.

Another referring point is the so-called KUB approximation [8]. There are two directions in the problem under consideration. The first direction is connected with the radiative corrections and the second one is connected with the radiative β decay of nuclei. Both of them have developed separately and the KUB approximation is widely used in the field of nuclei radiative β decay. Under the KUB approximation with the nuclear matrix element equal to $1+3\lambda^2$, one obtains the following main term of the cross section:

$$\frac{d\Gamma}{dE_e dK} \approx \frac{g_V^2}{2\pi^3}\frac{\alpha}{2\pi}(Q_0-m_p)^2\left[2KN + 4E_e(N-\beta)\left(\frac{E_e}{K}+1\right)\right]|M|^2,$$

$$|M|^2 = 1+3\lambda^2.$$

Finally, we can calculate the dependence of BR for the radiative β decay of the free neutron on the experimental cut-off parameter. One can see from Fig. 1 that BR has significant value for experimental cut-off parameter less than 100 keV. Figure 2 shows this region in more detail. Here it is necessary to mention one experimental paper [2], which we consider as very important. In this work, the electron asymmetry was measured, and electrons and protons from the neutron β decay were registered by the method of accidental coincidence. However, alongside with the main proton peak the authors observed a small peak of momentary, or, as they put it, false coincidences. The ratio of the number of false events to the number of the true decays was about 0.001. If one assumes that protons in this experiment were registered with the help of microchannel plates, which can also register γ quanta of very low energies (about 10 keV) and compares the ratio with calculated BR given in Fig. 2, then one can arrive at a conclusion, that with high probability this false peak is due to the radiative neutron decays. Of course, this is only a hypothesis and one needs to investigate the peak more carefully, for example, by means of a set-up in a vacuum chamber with extra γ detector, installed together with proton and electron detectors.

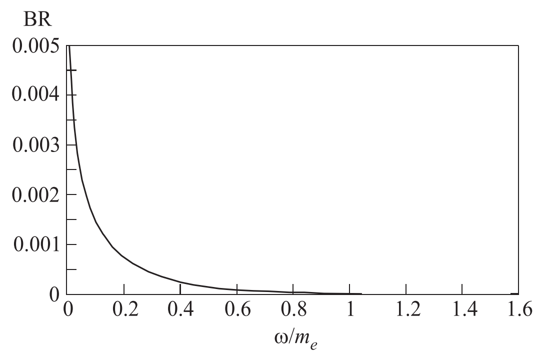

Fig. 1. The expected standard model branching ratio of the radiative neutron β decay as a function of ω in the case when the γ quanta with energy more than cut-off threshold ω is taken into account

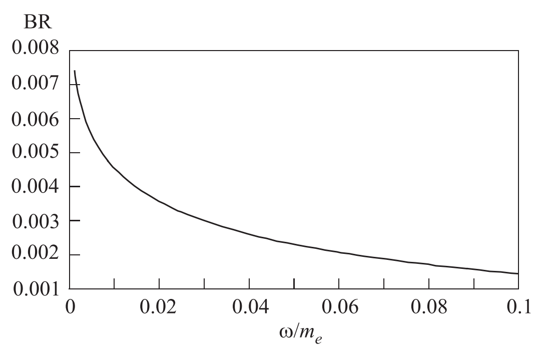

Fig. 2. The same as in Fig. 1 for other region of energy scale

The most interesting region is the so-called light domain. In Fig. 3 the x axis is expressed in angstroms and boundaries of this domain are indicated with arrows. One can see that BR for this light domain has a value of $1.09 \cdot 10^{-3}$. However, the main peculiarity of the light region is as follows. As was mentioned earlier, the radiative neutron decay allows us to principally test the value of the outer correction. At present, this correction is calculated with the experimental lifetime values for $0^+–0^+$ β transitions of a number of nuclei and the neutron. For all of them with great accuracy it is already established that the total outer radiative correction has the value of 1.5 % for the neutron, oxygen and 1.45–1.42 % for the other nuclei [9]. The value of the outer radiative correction is indicated in Fig. 3 with a dashed line. Comparing this line with the curve for BR, it becomes obvious that the light domain is beyond the limits of the total outer correction. This fact makes the research of the light domain particularly important, because experimental evaluation of BR in this region will force us to substantially reconsider our knowledge of the total outer radiative correction. It will also allow us to judge of the neutron structure. Besides, there are the latest evaluations of the full outer radiative correction, already on the level of 1.2 % [10], which makes the investigation of BR in the ultraviolet and light domains of γ-quantum energy especially important.

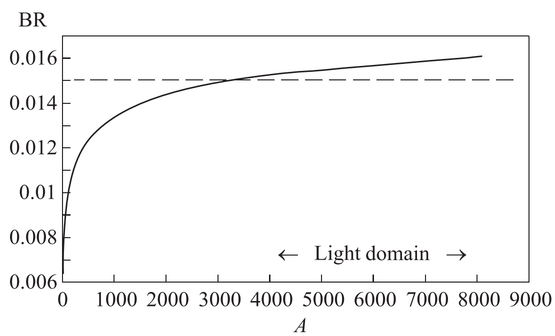

Fig. 3. The same as in Fig. 1 for the ultraviolet and light region; here cut-off threshold ω is expressed in angstroms

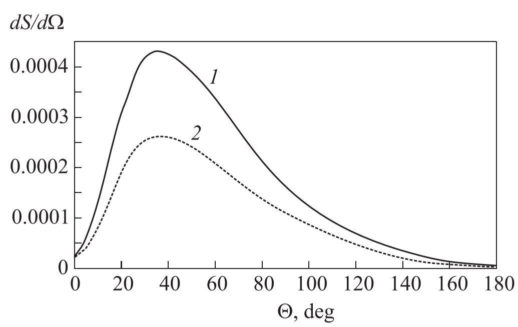

Fig. 4. Dependence of radiative decay spectrum on angle Θ between gamma quanta and electron moments (curve *1* for 25 keV threshold gamma energy, curve *2* for 50 keV threshold gamma energy)

The radiative neutron β decay also allows us to perform correlation experiments with different angles between the γ- and the β-momentum directions. If one rewrites expression obtained, with the angle Θ between the electron p_e and γ-quanta K momenta, then the formula follows:

$$\frac{d\Gamma}{dE_e dK d\Omega} = \frac{\alpha}{4\pi^2}\frac{p_e}{f_0 K}\left[\frac{m_e^2(E_e+K)-(E_e-p_e\cos\Theta)K^2}{(E_e-p_e\ \cos\ \Theta)^2}+\right.$$

$$\left.+K+E_e-\frac{2E_e(E_e+K)}{(E_e-p_e\ \cos\ \Theta)}\right](Q_0-m_p)^2,$$

where

$$f_0=\int\limits_{m_e}^{Q_0} p_e E_e(Q_0-m_p)^2 dE_e.$$

It can be seen from this formula that as the angle between the β electron and the γ quantum increases, the probability of γ-quantum emission decreases. Such phenomenon was observed in all experiments of the nuclear radiative β decay. In these investigations a considerable exceed of the experimental probabilities of the γ-quantum radiation over the theoretical ones for large angles Θ was obtained [11]. Moreover, as mentioned by many authors, this difference remains considerable even if one takes into account the so-called detour transitions and the nuclear structure. Therefore these correlation investigations of the radiative decay of the neutron will allow us to figure out what the exceed comes from: the nuclear structure or the structure of the nucleon.

Now let us proceed to another possibility of investigation of the radiative β decay of the neutron, namely to the measurements of the polarization of the emitted γ quantum. Since a weak interaction vertex, which violates the parity, takes place in this process the γ quantum is polarized. In the field of the nuclear physics, from this point of view, the radiative electron K capture in nuclei is the best investigated topic (first theoretical and experimental study was done in 1958 [12, 13]).

Already in the pioneer works of Martin and Glauber it was marked that the polarization degree depends on the constant of weak interaction. If we now take the manifestly model of right-handed currents as the «minimal» deviation from the standard «minimal» $V-A$ model of weak interaction, then the γ-quantum polarization degree will depend on the mixing angle of the left W_L and right W_R bosons ζ:

$$W_1 = W_L\cos\zeta - W_R\sin\zeta,$$

$$W_2 = W_L\sin\zeta + W_R\cos\zeta$$

and the ratio $\eta = m^2(W_1)/m^2(W_2)$ of the mass squared of two bosons W_1 and W_2.

Provided that the absolute values of the polarization are determined experimentally with the accuracy $\varepsilon(P_\gamma)$, the limit on the parameters of right-handed currents is given by [14]:

$$2\left(\zeta^2+2\frac{\lambda^2-1}{\lambda^2+1}\zeta\eta+\eta^2\right)\leq\varepsilon(P_\gamma).$$

It can be seen from this limit that the boundary of the allowed region for the parameters of the manifestly right-handed current model is a closed curve — ellipse as in the muon decay case [15]. This result can easily be explained qualitatively if we consider the muon decay from the viewpoint of the nuclear physics. Actually, ordinary muon decay is the transition from one $1/2^+$ state to another $1/2^+$ state, and both allowed transitions (Fermi and Gammow–Teller) contribute to this process. Therefore, absolute measurements of the circular polarization for any allowed j–j transition ($j \neq 0$) can give results which are completely alternative to measurements of the asymmetry parameter of electron emission in ordinary muon decay [15].

Thus, from our point of view, the investigation of the radiative decay of the neutron is not only actual but also possible experimentally. The most interesting subjects to investigate, from the modern point of view, are the following:

— research of the γ spectum from radiative neutron beta decay, in particular in the light region;

— research of the correlation effects with measurements of the radiative γ-quantum momentum, first of all in the e–γ correlations and comparison of the data with corresponding data from nuclear beta decay;

— measurement of circular γ-quantum polarization in the radiative decay, which, in particular, would allow us to obtain new limits on right-handed currents parameters.

Acknowledgments. I am grateful to J.Deutsch and B.G.Yerozolimsky, our colleagues from «Kurchatov Institute» and Catholigue University of Leuven M.Beck, D.Beck, L.N.Bondarenko, Yu.A.Mostovoi and P.Schuurmans for fruitful discussions. This research was supported in part by the Russian Fundation for Basic Research under Grant No. 00-02-17383.

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