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The nonperturbative effects in the processes involving strongly interacting particles are systematically studied within the instanton liquid model (ILM) of the QCD vacuum. The detailed analysis of the nonperturbative contributions to the *electromagnetic quark form factor* is presented. We consider the structure of the instanton induced effects in the evolution equation describing the high-energy behaviour of the form factor and demonstrate that the instanton contributions result in the finite renormalization of the subleading perturbative result and numerically are characterized by small factor reflecting the diluteness of the QCD vacuum. The consequences of the IR renormalon induced effects as well as various analytization procedures of the strong coupling constant in the IR domain are considered. The role of the instanton induced effects in *high-energy diffractive quark-quark scattering*, in particular, in formation of the soft Pomeron, is discussed. The further applications of the developed approach to the study of the nonperturbative effects in high-energy hadronic processes are discussed.

Дан систематический анализ непертурбативных эффектов в различных процессах сильновзаимодействующих частиц в рамках модели вакуума КХД как инстантонной жидкости. Представлены результаты для вкладов в электромагнитный формфактор кварка при высоких энергиях. Обсуждается структура инстантонных эффектов в уравнении эволюции. Показано, что инстантонные вклады сводятся к конечной перенормировке следующего за лидирующим пертурбативного вклада и численно характеризуются малым параметром, отражающим разреженность КХД-вакуума. Рассматриваются также эффекты инфракрасных ренормалонов и различные варианты аналитизации сильной константы связи в инфракрасной области. Рассмотрена роль инстантонных эффектов в дифракционном кварк-кварковом рассеянии при высоких энергиях и, в частности, в образовании мягкого померона. Обсуждаются дальнейшие применения развитого подхода в изучении непертурбативных эффектов в адронных процессах при высоких энергиях.

INTRODUCTION

The very powerful methods of the perturbative Quantum ChromoDynamics (pQCD) have been developed in order to describe the processes involving strong interacting particles at high energies (for a review and comprehensive description of the methods, see, e.g., [1]). The total cross section of the e^+e^- annihilation and the logarithmic violation of scaling in deep inelastic scattering became the classical tests of pQCD already in the lowest orders of expansion in the strong coupling constant [2], and nowadays there are no doubts that the QCD Lagrangian provides a proper basis for a quantum field theory of strong interactions.

At the same time, the present status of pQCD does not allow one to consider it as the only tool for investigation of the hadronic properties even at the highest energies accessible at modern machines. The perturbative methods should be supplied by certain information that cannot be obtained (at least, in the present stage of development) directly from pQCD calculations. For instance, a nontrivial situation arises when at high energy two or more hard scales appear. In that case in order to make predictions reliable it is necessary to resum the soft part of the quark-gluon interaction to all orders. Moreover, in several situations the applicability of pQCD can be definitely justified only at asymptotically high energies, while in experimentally accessible region the nonperturbative effects could be important and even dominant. Therefore, the study of the role of the nonperturbative input in investigation of the processes with strongly interacting particles is not only an interesting theoretical problem, but also an important task for phenomenology of hadronic physics.

While the explicit perturbative calculations even in high energy domain require certain nonperturbative supplement, the hadron processes at the low and moderate energies appear to be a natural ground for development of nonperturbative methods. Coming down in energy, more and more powers of the strong coupling constant have to be taken into account. Moreover, in the intermediate energy region the power corrections come into play that are very sensitive to the intrinsic hadron structure. Typically, the coefficients of the expansion in powers of the coupling constant and the inverse momentum transfer squared are the quark-gluon matrix elements taken at hadronic energy scale and have to be found by nonperturbative methods. The dependence of these matrix elements on the energy scale is governed by the evolution equations that are determined within pQCD for different hard processes. These equations start to be applicable at momentum transfer squared of order 1 GeV^2 or higher where the strong coupling constant becomes small. So, it is necessary to find the initial data for the evolution equation which is essentially a nonperturbative problem. The presence of the nonperturbative effects may be important in this energy region. There are different approaches to treat manifestations of nonperturbative phenomena at high energies: QCD sum rules, lattice QCD, quark models, etc.

On the other hand, a great progress has been made in the study of the QCD ground state, its vacuum, and a number of important results has been obtained that relate the properties of the vacuum to the characteristics of hadrons.

Although the QCD vacuum is known to play an important role in the high-energy collisions, the direct investigation of these effects remains a difficult task. The idea that the nontrivial vacuum structure could be relevant in high-energy hadronic processes was first explicitly formulated for the soft Pomeron case in [3–5], and further developed using the eikonal approximation and the Wilson integral formalism in [6].

Here, we will consider the nonperturbative effects originating from the nontrivial structure of the QCD vacuum treating the latter within the framework of the instanton liquid model (ILM) [7–11]. The approach based on similar principles was successfully developed within the stochastic vacuum model, where some important and interesting results were obtained (see, e.g., [12]). However, since the correspondence between both pictures is not completely clear at the moment, we restrict ourselves by the ILM only.

Considering QCD vacuum as an ensemble of instantons, one can describe a number of low-energy phenomena in strong interactions on qualitative and quantitative levels [8, 10]. The importance of the instanton induced effects in the strong interaction is also supported by lattice simulations [8, 13]. The instanton picture is generally considered as a fruitful and perspective framework for hadronic physics. The role of instantons in the hard hadronic processes has been studied intensively, both theoretically and experimentally. The contact with the perturbative QCD results is possible providing the unique information about the quark-gluon distribution functions in the QCD vacuum and hadrons at low-energy normalization point. The perspectives for an unambiguous experimental detection of instanton contributions are believed to be optimistic and promising.

In the present review, we report recent results on investigation of the instanton induced effects in a number of processes involving strongly interacting particles in high-energy regime. All considered cases manifest, in a sense, a similar structure and are studied within the unified framework — the Wilson integral approach, which allows one to study both perturbative and nonperturbative effects on the same footing. The method of path-ordered Wilson integrals is known as a powerful (and sometimes unique) tool in QCD which reformulates the theory in terms of the gauge invariant quantities — the Wilson loops — while the gauge fields are considered as chiral fields in the space of all possible loops [14]. The Green functions, amplitudes and cross sections can be expressed completely in terms of the Wilson integrals over the contours with geometry determined by specific kinematics, in an intrinsically nondiagrammatic (that is, nonperturbative) fashion [16–18]. We apply the Wilson loop formalism to the study of both the perturbative and nonperturbative contributions to the following quantities: first, we analyze the nonperturbative contributions in the simplest case — the amplitude of the quark elastic scattering in an external color singlet gauge field, that is, the electromagnetic quark form factor. Then, we investigate the role of instantons in the diffractive quark-quark scattering and formation of the soft Pomeron.

1. ELECTROMAGNETIC QUARK FORM FACTOR

1.1. Introduction. The behaviour of the form factors in various energy domains is one of the most important questions in the theory of hadronic exclusive processes. The electromagnetic quark form factors are determined via the elastic scattering amplitude of a quark in an external color singlet gauge field:

$$\mathcal{M}_\mu = F_q\left[(p_1-p_2)^2\right]\bar{u}(p_1)\gamma_\mu v(p_2) - \\ - G_q\left[(p_1-p_2)^2\right]\bar{u}(p_1)\frac{\sigma_{\mu\nu}(p_1-p_2)_\nu}{2m}v(p_2), \quad (1)$$

where $u(p_1)$, $v(p_2)$ are the spinors of outgoing and incoming quarks, and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$. In high-energy regime the Pauli form factor G_q is power suppressed and will be neglected in the present consideration. However, it should be emphasized that in low and moderate energy domains it becomes important and there arise interesting perturbative and nonperturbative effects (see, e.g., recent works [42, 43]). The kinematics is described by the two invariants (see Fig. 1, *a*):

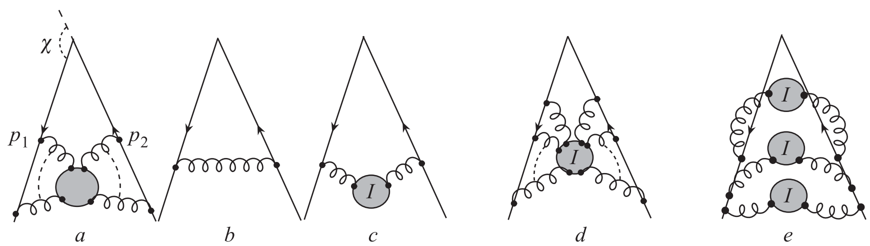


Fig. 1. The notations for the quark momenta and the total cusp-dependent part of the Wilson loop integral for the quark form factor (*a*); the leading order contributions of the perturbative (*b*) and nonperturbative (single-instanton) (*c*) fields; the all-order single instanton result (*d*); the exponentiation of the single instanton result (*e*)

$$m^2 = p_1^2 = p_2^2, \quad (p_1 p_2) = m^2 \cosh\chi, \quad (2)$$

or

$$s = (p_1+p_2)^2 = 2m^2(1+\cosh\chi), \quad t = -Q^2 = (p_1-p_2)^2 = 2m^2(1-\cosh\chi). \quad (3)$$

We assume that both the momentum transfer $-t$ and the total centre-of-mass energy s are large compared to the quark mass:

$$(p_1 p_2) \gg m^2, \quad \text{or} \quad (s+t) \gg (s-t), \quad \text{or} \quad \cosh\chi \gg 1. \quad (4)$$

We take the reference frame where the scattering point is in the origin and introduce the scattering vectors of quark as

$$v_1 = (1, 0, 0_\perp), \qquad v_2 = (\cosh\chi, i\sinh\chi, 0_\perp), \\ v_1^2 = v_2^2 = 1, \qquad (v_1 v_2) = \cosh\chi, \quad (5)$$

where the velocities $v_1 = p_1/m$ and $v_2 = p_2/m$ define the scattering plane.

The colour singlet quark form factor is one of the simplest and convenient objects for investigation of the double logarithmic behaviour of the amplitudes in QCD in the high-energy regime. From the methodological point of view, it requires a perturbative resummation procedure beyond the standard renormalization group techniques. Besides this, the resummation methods developed for this particular case can be applied to many other processes which possess the logarithmic enhancements near the kinematic boundaries. On the other hand, in spite of the evident theoretical significance, the computation of the quark form factor has important phenomenological applications. The quark form factor enters into the cross sections of a number of high-energy hadronic processes [2]. For example, the total cross section of the Drell–Yan process (normalized to deep inelastic (DIS) one) is determined by the ratio of the time-like and space-like form factors [19, 20]:

$$\frac{\sigma_n^{\rm DY}}{\sigma_n^{\rm PM}} \sim \left| \frac{F_q(Q^2)}{F_q(-Q^2)} \right|^2, \tag{6}$$

where $\sigma_n^{\rm PM}$ is the nth moment of the cross section calculated within the parton model. Similar resummation approach is also used in the study of the near-forward quark-quark scattering and the evaluation of the soft Pomeron properties [21]. In the latter case, the nonleading logarithmic terms become quite important.

Recently, it was shown how the experimental and phenomenological investigations of the electromagnetic quark form factors at low and moderate energies can shed a light on the problem of scaling violation in DIS and the structure of constituent quarks [22]. The energy regime, where Q^2 is larger than the typical hadronic scale determined by $\Lambda^2_{\rm QCD}$ and lower than the characteristic scale of the chiral symmetry breaking: $\Lambda^2_{\rm QCD} \sim \Lambda^2_{\rm conf} < Q^2 < \Lambda^2_{\chi{\rm SB}}$, is the most convenient one for detection of the nonperturbative phenomena, like the instanton induced effects. It was shown that the size of a constituent quark consistent with the data is about 0.2–0.3 fm while the mean instanton size in ILM is also about 0.3 fm. The complicated interplay of nonperturbative effects can lead in this regime to formation of the constituent quark which is, in a sense, an intermediate object between color-neutral hadron and pointlike structureless partons, associated with the fundamental QCD particles — quarks and gluons. The form factors of constituent quarks may be extracted [22] from the JLab experiment [24] data on the inelastic Nachtmann moments [23] of the unpolarized proton structure function, $F_2^p(x, Q^2)$.

The first example of large logarithm resummation was given by Sudakov for the case of *off*-shell fermion in the external Abelian gauge field in the leading logarithmic approximation (LLA), where the terms of order of $(\alpha_s^n \ln^{2n} Q^2)$ are taken into account while the contributions from $O(\alpha_s^n \ln^{2n-1} Q^2)$ are neglected. The exponentiation of the leading double logarithmic result was found [25].

This exponential decreasing of the form factor at large-Q^2 means that the elastic scattering of a quark by a virtual photon is suppressed at asymptotically large momentum transfer. The exponentiation for the *on*-shell form factor in the Abelian case was obtained in LLA in [26]. As expected, the non-Abelian gauge theories appeared to be more complicated: first, the LLA terms in the QCD perturbative series were found to be consistent with the exponentiation in [27] (the inelastic *on*-shell form factor with emission of one and two gluons was calculated in the same context in [28]; the role of the quark Sudakov form factor in the description of e^+e^- one-photon annihilation in quarks and gluons was considered in LLA in [29]), and the all-order LLA non-Abelian exponentiation has been proved in [30]. In LLA, the exponentiated form factor behaves as a rapidly decreasing at high momentum transfer function, but the question if the nonleading logarithmic terms could upset the LLA behaviour required a further work. The all-(logarithmic)-order resummation was performed in the Abelian case and the exponentiation was demonstrated in [31]. In the paper [32], the non-Abelian all-order exponentiation for the so-called hard part of the *on*-shell form factor has been shown first within the powerful factorization approach. Note, that in this work the case where a time-like photon with large invariant mass decays into a quark-antiquark pair was considered, however it can be easily shown that the results remain true as well for our case of a quark scattering in an external EM field.

In the work [32], the detailed study of hard part of the form factor (which is responsible for the ultraviolet (UV) properties) was performed, while the status of the soft part, containing all the infrared (IR) and collinear singularities and, as a consequence, all possible nonperturbative effects, remained unclear. The important results on the IR properties of the QCD vertex functions were obtained in [34] within the Wilson integral approach. In these works, the soft part of the form factor had been presented as the vacuum averaged ordered exponent of the path integral of a gauge field over the contour of a special form — an angle with sides of infinite length. The use of the gauge and renormalization group invariance allowed one to derive the perturbative evolution equation describing the high energy behaviour of the form factor taking into account all (not power suppressed) parts of the factorized amplitude, both for the *on*- [33] and *off*-shell [35] cases. It was shown that the leading asymptotic is controlled by the cusp-anomalous dimension which arises due to the multiplicative renormalization of the soft part, and can be calculated within the Wilson integral formalism up to the two-loop order [34]. It is worth noting that within the Wilson integral approach, the non-Abelian exponentiation can be proved independently [36], what is another important advantage of this framework. The efficiency of the Wilson integrals approach had been successfully demonstrated in a series of works [18,37,38]. In these papers, the nondiagrammatic framework is developed what allows one to calculate the fermionic Green's functions, Sudakov form factors, amplitudes and cross sections in QED and QCD completely in terms of world-line integrals, and thus avoid complicated diagrammatic factorization analysis.

The results presented above allow one to conclude that the leading high-energy behaviour of the quark form factor in non-Abelian gauge theory is completely determined by the perturbative evolution equation, and is given by the fast decreasing exponent:

$$\sim \exp\left[-\frac{2C_F}{\beta_0}\ln Q^2 \;\ln\ln Q^2 + O(\ln Q^2)\right]. \tag{7}$$

This rapid fall-off is not changed by any other logarithmic contributions [32,33,35, 38]. However, the nonleading logarithmic corrections are nevertheless important for evaluation of the numerical value of the form factor. Some of them are of a purely perturbative origin (higher loop corrections and subleading logarithmic terms), while the others can be attributed to the nonperturbative phenomena. The usual approach to treatment of the latter is developed within the IR renormalon picture (there are plenty of papers on this subject, for the most recent reviews see [39]). However they could only give the power-suppressed terms, which become, of course, important in low energy domain, but can be neglected at asymptotically large momenta. Here we should note, that this conclusion is to be changed for processes with two scales (such as quark-quark scattering, Drell–Yan process, etc.): then the corrections proportional to the powers of a smaller scale must be also involved in the game [40]. In the present work, we try to advocate the point of view that the true (not connected directly to renormalons) nonperturbative effects can be taken into account consistently in the evolution equation, and therefore they yield the nonvanishing subleading (perhaps, parametrically suppressed, but still logarithmic) contributions $\sim \ln\ Q^2$ to the high-energy behaviour. Further, we analyze another possible source of contributions which can be considered as «nonperturbative» — the IR renormalon ambiguities. We demonstrate explicitly that they produce the corrections with different IR structure compared to that one generated by instantons. Moreover, as can be shown these direct renormalon effects disappear in the dimensional regularization [20] and in the analytical perturbation theory [41], what means that one could hardly expect a considerable signature of the IR renormalon effects in this process.

In this Section we describe the consequences of the RG invariance properties of the factorized form factor, and derive the linear evolution equation considering the nonperturbative input as the initial value for perturbative evolution. Then, these nonperturbative effects are estimated in the weak-field approximation within the instanton model of QCD vacuum. The all-orders instanton contribution is evaluated using the Gaussian simulation of the instanton profile function. The large-Q^2 behaviour of the form factor is analyzed taking into account the leading perturbative and instanton induced contributions. The consequences of the IR renormalon ambiguities of the perturbative series and their relevance within the context of some analytization procedures are also studied.

1.2. Evolution Equation for the Quark Form Factor and Nonperturbative Effects. The classification of the diagrams with respect to the momenta carried by their internal lines allows one to express the form factor F_q in the amplitude (1) in the factorized form [31–33] (compare with the world-line expression for the three-point vertex in [38])

$$F_q(q^2) = F_H(q^2/\mu^2, \alpha_s) F_S(q^2/m^2, \mu^2/\lambda^2, \alpha_s) F_J(\mu^2/\lambda^2, \alpha_s), \tag{8}$$

where the hard, soft, and collinear (jet) part are separated. Note, that in the present paper, all dimensional variables are assumed to be expressed in units of the QCD scale Λ, so that $q^2 = Q^2/\Lambda^2_{\rm QCD}$, etc. The arbitrary scale μ^2 stands for the boundary value of the internal momenta squared which divides the different parts, and is assumed to be equal to the UV normalization point.

The total form factor F_q is the renormalization invariant quantity:

$$\mu^2 \frac{d}{d\mu^2}\ F_q(\mu^2, \alpha_s(\mu^2)) = \left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) F_q(\mu^2, \alpha_s(\mu^2)) = 0, \tag{9}$$

that in the large-q^2 regime leads to the following relations

$$\mu^2 \frac{d}{d\mu^2} \left[\frac{\partial \ln F_H}{\partial \ln q^2} \right] = -\mu^2 \frac{d}{d\mu^2} \left[\frac{\partial \ln F_S}{\partial \ln q^2} \right] = \frac{1}{2}\ \Gamma_{\rm cusp}(\alpha_s). \tag{10}$$

For convenience, we work with logarithmic derivatives in q^2. This allows us to avoid the problems with additional light-cone singularities in the soft part [33,44]. The collinear part F_J, being independent of q^2, does not contribute to these equations.

Within the eikonal approximation, the resummation of all logarithmic terms coming from the soft gluon subprocesses allows us to express F_S in terms of the vacuum average of the gauge invariant path ordered Wilson integral [14, 15]

$$F_S(q^2/m^2, \mu^2/\lambda^2, \alpha_s) = \\ = W(C_\chi; \mu^2/\lambda^2, \alpha_s) = \frac{1}{N_c} {\rm Tr} \left\langle 0 \middle| \mathcal{P} \exp \left\{ ig \int_{C_\chi} dx_\mu \hat{A}_\mu(x) \right\} \middle| 0 \right\rangle. \tag{11}$$

In Eq. (11) the integration path corresponding to the considered process goes along the closed contour C_χ: the angle (cusp) with infinite sides. We parameterize the integration path $C_\chi = \{z_\mu(t);\ t = [-\infty, \infty]\}$ as follows

$$z_\mu(t) = \begin{cases} v_1 t, & -\infty < t < 0, \\ v_2 t, & 0 < t < \infty. \end{cases} \tag{12}$$

The gauge field $\hat{A}_\mu(x) = T^a A^a_\mu(x)$ $(\mathrm{Tr}\,[T^a T^b] = (1/2)\delta^{ab})$ belongs to the Lie algebra of the gauge group $SU(N_c)$, while the Wilson loop operator $\mathcal{P}\exp\left(ig\int dx A(x)\right)$ lies in its fundamental representation. The cusp anomalous dimension $\Gamma_{\rm cusp}$ is found from the multiplicative renormalization of the Wilson integral (11) [16, 34]:

$$W(C_\chi;\mu^2/\lambda^2,\alpha_s(\mu^2)) = Z_{\rm cusp}(C_\chi;\bar{\mu}^2/\mu^2,\alpha_s(\mu^2))W_{\rm bare}(C_\chi;\bar{\mu}^2/\lambda^2,\alpha_s), \tag{13}$$

where $\bar{\mu}^2$ is the UV cut-off; μ^2 is the normalization point, and λ^2 is the IR cut-off. The presence of the IR divergence in (13) is a common feature of *on*-shell amplitudes in massless QCD. Since $W_{\rm bare}$ knows nothing about the normalization point (the latter is fixed by choosing a concrete $Z_{\rm cusp}$), one can write:

$$\frac{1}{2}\,\Gamma_{\rm cusp}(C_\chi;\alpha_s(\mu^2)) = -\mu^2\frac{d}{d\mu^2}\ln W(C_\chi;\mu^2/\lambda^2,\alpha_s(\mu^2)) = \\ = -\mu^2\frac{d}{d\mu^2}\ln Z_{\rm cusp}(C_\chi;\bar{\mu}^2/\mu^2,\alpha_s(\mu^2)). \tag{14}$$

It can be shown that the cusp anomalous dimension (10) is linear in the scattering angle χ to all orders of perturbation theory in the large-q^2 regime [34]:

$$\Gamma_{\rm cusp}(C_\chi;\alpha_s) = \ln q^2\ \Gamma_{\rm cusp}(\alpha_s) + O(\ln^0 q^2). \tag{15}$$

Then, from Eqs. (10), (14), (15) one finds after simple calculations [33]:

$$\frac{\partial \ln F_H(q^2)}{\partial \ln q^2} = \int_{q^2}^{\mu^2}\frac{d\xi}{2\xi}\Gamma_{\rm cusp}(\alpha_s(\xi)) + \Gamma(\alpha_s(q^2)), \tag{16}$$

$$\frac{\partial \ln F_S(q^2)}{\partial \ln q^2} = -\int_{\lambda^2}^{\mu^2}\frac{d\xi}{2\xi}\Gamma_{\rm cusp}(\alpha_s(\xi)) + \frac{\partial \ln W_{np}(q^2)}{\partial \ln q^2}, \tag{17}$$

where the «integration constant» of the hard part reads

$$\Gamma(\alpha_s(q^2)) = \left.\frac{\partial \ln F_H(q^2)}{\partial \ln q^2}\right|_{\mu^2=q^2}, \tag{18}$$

and W_{np} arises as the initial value of the soft part:

$$\frac{\partial \ln W_{np}(q^2)}{\partial \ln q^2} = \left.\frac{\partial \ln F_S(q^2)}{\partial \ln q^2}\right|_{\mu^2=\lambda^2}, \tag{19}$$

and is the only quantity where, according to our suggestion, the nonperturbative effects take place [46, 47]. Then we get the q^2-evolution equation of the total form factor at large q^2:

$$\ln \frac{F_q(q^2)}{F_q(q_0^2)} = -\int_{q_0^2}^{q^2} \frac{d\xi}{2\xi} \left[\ln \frac{q^2}{\xi}\, \Gamma_{\rm cusp}(\alpha_s(\xi)) - 2\Gamma(\alpha_s(\xi))\right] - \\ - \ln \frac{q^2}{q_0^2} \int_{\lambda^2}^{q_0^2} \frac{d\xi}{2\xi}\, \Gamma_{\rm cusp}(\alpha_s(\xi)) + \ln \frac{W_{np}(q^2)}{W_{np}(q_0^2)}. \quad (20)$$

In the next subsection, we explicitly calculate the perturbative quantities entering Eq. (20) in the one-loop approximation.

1.3. Analysis of the Perturbative Contributions to the Wilson Integral. The analysis of the hard contributions [32, 33] at large q^2 yields:

$$\frac{\partial \ln F_H(q^2/\mu^2, \alpha_s)}{\partial \ln q^2} = -\frac{\alpha_s}{2\pi} C_F \left(\ln \frac{q^2}{\mu^2} - \frac{3}{2}\right) + O(\alpha_s^2), \quad (21)$$

where $C_F = (N_c^2 - 1)/2N_c$. For the hard «integration constant» (18) one has:

$$\Gamma(\alpha_s(q^2)) = \frac{3}{4} \frac{\alpha_s(q^2)}{\pi} C_F. \quad (22)$$

The expression (21) is IR-safe, while the low-energy information is accumulated in the soft part of the quark form factor F_S. The Wilson integral (11) can be presented as a series:

$$W(C_\chi) = 1 + \frac{1}{N_c} \langle 0| \sum_{n=2} (ig)^n \int_{C_\chi} \int_{C_\chi} \cdots \int_{C_\chi} dx_{\mu_n}^n\, dx_{\mu_{n-1}}^{n-1} \cdots dx_{\mu_1}^1 \times \\ \times \theta(x^n, x^{n-1}, \ldots, x^1)\ {\rm Tr} \left[\hat{A}_{\mu_n}(x^n) \hat{A}_{\mu_{n-1}}(x^{n-1}) \cdots \hat{A}_{\mu_1}(x^1)\right] |0\rangle, \quad (23)$$

where the function $\theta(x)$ orders the color matrices along the integration contour.

The leading order of the expansion (23) (one-loop — for the perturbative gauge field and weak-field limit for the instanton) is given by expression (see Fig. 1, *b, c*):

$$W_{\rm bare}^{(1)}(C_\chi) = -\frac{g^2 C_F}{2} \int_{C_\chi} dx_\mu \int_{C_\chi} dy_\nu\ D_{\mu\nu}(x - y), \quad (24)$$

where the gauge field propagator $D_{\mu\nu}(z)$ in n-dimensional space-time ($n = 4 - 2\varepsilon$) can be presented in the form:

$$\left\langle 0 \middle| \mathcal{T} A_\mu^a(z) A_\nu^b(0) \middle| 0 \right\rangle = \delta^{ab} D_{\mu\nu}(z), \quad (25)$$

$$D_{\mu\nu}(z) = g_{\mu\nu} \partial_z^2 \Delta_1(\varepsilon, z^2, \bar{\mu}^2/\lambda^2) - \partial_\mu \partial_\nu \Delta_2(\varepsilon, z^2, \bar{\mu}^2/\lambda^2). \quad (26)$$

Here $\bar{\mu}^2$ is a parameter of dimensional regularization. The exponentiation theorem for non-Abelian path-ordered Wilson integrals [36] allows us to express (to one-loop accuracy) the Wilson integral (11) as the exponentiated one-loop term of the series (23):

$$W_{\rm bare}(C_\chi; \varepsilon, \bar{\mu}^2/\lambda^2) = \exp\left[W^{(1)}_{\rm bare}(C_\chi; \varepsilon, \bar{\mu}^2/\lambda^2) + O(\alpha_s^2)\right]. \qquad (27)$$

In general, the expression (24) contains UV and IR divergences, that can be multiplicatively renormalized in a consistent way [16]. In the present work, we use the dimensional regularization for the UV singularities, and define the «gluon mass» λ^2 as the IR regulator. The dimensionally regularized free propagator reads $(n = 4 - 2\varepsilon)$, $\varepsilon > 0$:

$$D_{\mu\nu}(z;\xi) =$$
$$= \mu^{4-n}\frac{1}{i}\int\frac{d^nk}{(2\pi)^n}{\rm e}^{-ikz}\left(\frac{g_{\mu\nu}}{k^2-\lambda^2+i0} - \xi\frac{k_\mu k_\nu/\left[k^2-(1-\xi)\lambda^2+i0\right]}{k^2-\lambda^2+i0}\right), \qquad (28)$$

where ξ is a gauge parameter. It is convenient to express the partial derivatives in terms of the derivative with respect to the interval z^2 in n-dimensional space-time

$$\partial_z^2 = 2n\partial_{z^2} + 4z^2\partial^2_{z^2}, \quad \partial_\mu\partial_\nu = 2g_{\mu\nu}\partial_{z^2} + 4z_\mu z_\nu\partial^2_{z^2} \qquad (29)$$

and the scalar products of the scattering vectors (5) in terms of the scattering angle χ

$$\begin{gathered} v^1_\mu v^2_\nu g_{\mu\nu} = \cosh\chi, \quad v^1_\mu v^2_\nu z_\mu z_\nu = z^2\ \cosh\chi + \sigma\tau\ \sinh^2\chi, \\ z^2 = (v_1\sigma + v_2\tau)^2 = \sigma^2 + \tau^2 + 2\sigma\tau\cosh\chi. \end{gathered} \qquad (30)$$

For the calculations of different terms we use the following integrals:

$$\int_0^\infty d\sigma\int_0^\infty d\tau\ \exp\left[-\alpha(\sigma^2+\tau^2+2\sigma\tau\cosh\chi)\right] = \frac{1}{2\alpha}\frac{\chi}{\sinh\chi} \qquad (31)$$

and

$$\int_0^\infty d\sigma\int_0^\infty d\tau\ \sigma\tau\ \exp\left[-\alpha(\sigma^2+\tau^2+2\sigma\tau\cosh\chi)\right] = -\frac{1-\chi\coth\chi}{4\alpha^2\sinh^2\chi}\ . \qquad (32)$$

Therefore, one has for the functions $\Delta_i(z^2)$ defined in (26) and (28):

$$\Delta_i^{\{k\}}(z^2) = (-)^k\int_0^\infty d\alpha\alpha^k\,{\rm e}^{-\alpha z^2}\,\bar{\Delta}_i(\alpha) \qquad (33)$$

and gets

$$\int_0^\infty d\sigma \int_0^\infty d\tau \ \Delta'(z^2) = -\frac{\chi}{2\sinh\chi}\Delta(0),$$
$$\int_0^\infty d\sigma \int_0^\infty d\tau \ z^2\Delta''(z^2) = \frac{\chi}{2\sinh\chi}\Delta(0), \tag{34}$$
$$\int_0^\infty d\sigma \int_0^\infty d\tau \ \sigma\tau\Delta''(z^2) = -\frac{1-\chi\coth\chi}{4\sinh^2\chi}\Delta(0).$$

Then, the dimensionally regularized formula for the leading order (LO) term (24) can be written as [46]:

$$W^{(1)}_{\rm bare}(C_\chi;\varepsilon,\bar\mu^2/\lambda^2,\alpha_s) = 8\pi\alpha_s C_F h(\chi)(1-\varepsilon)\Delta_1(\varepsilon,0,\bar\mu^2/\lambda^2), \tag{35}$$

where $h(\chi)$ is the universal cusp factor:

$$h(\chi) = \chi\coth\chi - 1, \tag{36}$$

which at large-q^2 is given by:

$$\lim_{\chi\to\infty} h(\chi) \to \chi \propto \ln\frac{q^2}{m^2}. \tag{37}$$

In Eq. (35), for the perturbative gauge field one has

$$\Delta_1(\varepsilon,0,\bar\mu^2/\lambda^2) = -\frac{1}{16\pi^2}\left(4\pi\frac{\bar\mu^2}{\lambda^2}\right)^\varepsilon \ \frac{\Gamma(\varepsilon)}{1-\varepsilon}. \tag{38}$$

The independence of the expression (35) of the function Δ_2 is a direct consequence of the gauge invariance. Then, in the one-loop approximation,

$$W_{\rm bare}(C_\chi;\varepsilon,\bar\mu^2/\lambda^2,\alpha_s) = 1 - \frac{\alpha_s}{2\pi}C_F h(\chi)\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi + \ln\frac{\bar\mu^2}{\lambda^2}\right), \tag{39}$$

and the cusp dependent renormalization constant, within the $\overline{MS}$ scheme which fixes the UV normalization point, reads:

$$Z_{\rm cusp}(C_\chi;\varepsilon,\bar\mu^2/\mu^2,\alpha_s(\mu^2)) = 1 + \frac{\alpha_s(\mu^2)}{2\pi}C_F h(\chi)\left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi\right) + O(\alpha_s^2). \tag{40}$$

Using Eq. (35), one finds the known one-loop result for the perturbative field, which contains the dependence on the UV normalization point μ^2 and IR cut-off λ^2 (e.g., [34,38]):

$$W^{(1)}_{\rm pt}(C_\chi;\mu^2/\lambda^2,\alpha_s(\mu^2)) = -\frac{\alpha_s(\mu^2)}{2\pi}C_F h(\chi)\ln\frac{\mu^2}{\lambda^2} + O(\alpha_s^2). \tag{41}$$

Therefore, in the leading order the kinematic dependence of the expression (24) is factorized into the cusp factor $h(\chi)$.

From the one-loop result (41), the cusp anomalous dimension which satisfies the RG equation (14) in one-loop order is given by:

$$\Gamma^{(1)}_{\rm cusp}(\alpha_s(\mu^2)) = \frac{\alpha_s(\mu^2)}{\pi} C_F. \tag{42}$$

Substituting into Eq. (20) the anomalous dimension (42) with the strong coupling constant given in the one-loop approximation, one finds

$$F_q^{(1)}(q^2) = \exp\left[-\frac{2C_F}{\beta_0}\left[\ln q^2 \left(\ln \frac{\ln q^2}{\ln \lambda^2} - 1 \right) - \right.\right.$$
$$\left.\left. - \frac{3}{2} \ln \frac{\ln q^2}{\ln q_0^2} + \ln q_0^2 \left(1 - \ln \frac{\ln q_0^2}{\ln \lambda^2} \right) \right] + W_{np}(q^2) \right] F^{(1)}(q_0^2). \tag{43}$$

Note, that the exponent in Eq. (43) has an unphysical singularity at $\lambda^2 = 1$ (in dimensional notations, $\bar\lambda^2 = \Lambda^2_{\rm QCD}$), i.e., where the one-loop coupling constant $\alpha_s(\bar\lambda^2)$ has the Landau pole. This feature can be treated in terms of IR renormalon ambiguities (see the next Section), and is often considered as a signal of nonperturbative physics. In the present paper, we will consistently separate the sources of nonperturbative effects which can be attributed to uncertainties of resummation of the perturbative series from the «true» nonperturbative phenomena. An important example of the latter is provided by instanton induced effects within the instanton model of QCD vacuum, which is considered in Subsec. 1.5.

1.4. IR Renormalon Induced Effects and Analytization of the Coupling Constant. As was pointed out at the end of the previous Subsection, the perturbative evolution equation (43) possesses an unphysical singularity at the point $\lambda^2 = 1$. Therefore, it is instructive to study the consequences of this feature. It is known that the presence of the Landau pole in the one-loop expression for the coupling constant leads to the IR renormalons [39] resulting in power suppressed corrections. In the present situation one can expect the corrections proportional to the powers of both scales: μ^2 and λ^2. We will treat here the power μ^2 terms to be strongly suppressed in large-q^2 regime, and focus on the power λ^2 corrections. To find them, let us consider the perturbative function $\Delta_1(\varepsilon, 0, \bar\mu^2/\lambda^2)$ in Eq. (35). The insertion of the fermion bubble 1-chain to the one-loop order expression (24) is equivalent to replacement of the frozen coupling constant g^2 by the running one $g^2 \to g^2(k^2) = 4\pi\alpha_s(k^2)$ [40] (for convenience, we work here in Euclidean space):

$$\widetilde\Delta_1(\varepsilon, 0, \bar\mu^2/\lambda^2) = -4\pi\bar\mu^{2\varepsilon} \int \frac{d^n k}{(2\pi)^n} \alpha_s(k^2) \frac{e^{ikz}\delta(z^2)}{k^2(k^2+\lambda^2)}. \tag{44}$$

By using the integral representation for the one-loop running coupling $\alpha_s(k^2) = \int_0^\infty d\sigma (1/k^2)^{\sigma b}$, $b = \beta_0/4\pi$, we find:

$$\widetilde{\Delta}_1(\varepsilon, 0, \bar{\mu}^2/\lambda^2) = \\ = -\frac{1}{\beta_0(1-\varepsilon)} \left(4\pi \frac{\bar{\mu}^2}{\lambda^2}\right)^\varepsilon \int_0^\infty dx \frac{\Gamma(1-x-\varepsilon)\Gamma(1+x+\varepsilon)}{(x+\varepsilon)\Gamma(1-\varepsilon)} \left(\frac{1}{\lambda^2}\right)^x . \quad (45)$$

To define properly the integral in r.h.s. of Eq. (45), one needs to specify a prescription to go around the poles, which are at the points $\bar{x}_n = n$, $n \in \mathbb{N}$. Thus, the result of integration depends on this prescription giving an ambiguity proportional to $\left(1/\lambda^2\right)^n$ for each pole. Then, the IR renormalons produce the power corrections to the one-loop perturbative result, which we assume to exponentiate with the latter [40]. Extracting from (45) the UV singular part in vicinity of the origin $x = 0$, we divide the integration interval $[0, \infty]$ in two parts: $[0, \delta]$ and $[\delta, \infty]$, where $\delta < 1$. This procedure allows us to evaluate separately the ultraviolet and the renormalon-induced pieces. For the ultraviolet piece, we apply the expansion of the integrand in Δ_1 in powers of small x and replace the ratio of Γ functions by $\exp(-\gamma_E \varepsilon)$:

$$\widetilde{\Delta}_1^{\rm UV}(\varepsilon, 0, \bar{\mu}^2/\lambda^2) = \\ = -\frac{1}{\beta_0(1-\varepsilon)} \sum_{k,n=0} (-)^n \frac{\left(\ln 4\pi - \gamma_E + \ln(\bar{\mu}^2/\lambda^2)\right)^k}{k! \varepsilon^{n-k+1}} \int_0^\delta dx\ x^n \left(\frac{1}{\lambda^2}\right)^x , \quad (46)$$

which after subtraction of the poles in the $\overline{MS}$ scheme becomes:

$$\widetilde{\Delta}_1^{\rm UV}(0, \mu^2/\lambda^2) = \frac{1}{\beta_0(1-\varepsilon)} \sum_{n=1} \left(\ln \frac{\mu^2}{\lambda^2}\right)^n \frac{(-)^n}{n!} \int_0^\delta dx x^{n-1} \left(\frac{1}{\lambda^2}\right)^x . \quad (47)$$

In analogy with results of [58], this expression may be rewritten in a closed form as

$$\widetilde{\Delta}_1^{\rm UV}(0, \mu^2/\lambda^2) = \frac{1}{\beta_0(1-\varepsilon)} \int_0^\delta \frac{dx}{x} \left[{\rm e}^{-x \ln \mu^2} - {\rm e}^{-x \ln \lambda^2} \right] . \quad (48)$$

Then, using the relation

$$\frac{\partial W^{(1)}(q^2)}{\partial \ln q^2} = 2C_F(1-\varepsilon) \widetilde{\Delta}_1^{\rm UV}(0, \mu^2/\lambda^2), \quad (49)$$

one finds

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(g) \frac{\partial}{\partial g}\right) \frac{\partial \ln W^{(1)}(q^2)}{\partial \ln q^2} =$$
$$= -\frac{1}{2} \Gamma^{(1)}_{\text{cusp}}(\alpha_s(\mu^2)) \left(1 - \exp\left[-\delta \frac{4\pi}{\beta_0 \alpha_s(\mu^2)}\right]\right). \quad (50)$$

The second exponent in the last equation yields the power suppressed terms $\left(1/q^2\right)^\delta$ in large-q^2 regime. In LLA, Eq. (49) is reduced to:

$$\frac{\partial W^{(1)}(q^2)}{\partial \ln q^2} = -\frac{2C_F}{\beta_0} \left(\ln \frac{\ln \mu^2}{\ln \lambda^2}\right). \quad (51)$$

The last expression obviously satisfies the perturbative evolution equation (43).

The remaining integral in Eq. (45) over the interval $[\delta, \infty]$ is evaluated at $\varepsilon = 0$ since there are no UV singularities. The resulting expression does not depend on the normalization point μ, and thus it is determined by the IR region including nonperturbative effects. It contains the renormalon ambiguities due to different prescriptions in going around the poles $\bar{x}_n$ in the Borel plane which yields the power corrections to the quark form factor.

After the substitution $\mu^2 = q^2$ and integration, we find in LLA (for comparison, see Eq. (43)):

$$F_q^{\text{ren}}(q^2) = \exp\left[-\frac{2C_F}{\beta_0} \ln q^2 \left(\ln \ln q^2 - 1\right) - \ln q^2 \Phi_{\text{ren}}(\lambda^2)\right] F^{\text{ren}}(q_0^2), \quad (52)$$

where the function $\Phi_{\text{ren}}(\lambda^2) = \sum_{k=0} \phi_k (1/\lambda^2)^k$ accumulates the effects of the IR renormalons. The coefficients ϕ_k cannot be calculated in perturbation theory and are treated often as «the minimal set» of nonperturbative parameters. It is worth noting that the logarithmic q^2 dependence of the renormalon induced corrections in the large-q^2 regime is factorized, and thus Eq. (52) corresponds to the structure of nonperturbative contributions found in the one-loop evolution equation (43), in a sense of its large-q^2 behaviour. On the other hand, the IR structures of the renormalon corrections and the instanton induced ones (78) are different. Moreover, it can be shown that careful account of partially resumed perturbative series yields, sometimes, cancellation of the leading power corrections associated with the leading renormalon contributions [40]. As the corresponding nonperturbative terms are calculated independently (e.g., by means of the ILM) their direct relation to the IR renormalon ambiguities should be questioned. In our point of view, it allows us to separate the true nonperturbative (e.g., instanton induced, but not only) effects from that ones related to ambiguities of the resumed perturbative series.

The latter conclusion can be illustrated by considering the consequences of an analytization of the strong coupling constant [41] in the perturbative evolution equation. In this approach, the one-loop strong coupling constant $\alpha_s(\mu^2)$ is replaced by the expression which is analytical at $\mu^2 = 1$ (i.e., at $\Lambda^2_{\rm QCD}$ in dimensional variables):

$$\alpha_s^{\rm AN}(\mu^2) = \frac{4\pi}{\beta_0}\left(\frac{1}{\ln \mu^2} + \frac{1}{1-\mu^2}\right). \tag{53}$$

The direct substitution of (53) into the evolution equation (20) yields (for brevity, we assume $q_0^2 = \lambda^2$):

$$\begin{aligned} -\left(\frac{\beta_0}{2C_F}\right)\ln F_q^{\rm AN}(q^2) = \ln q^2 \ln \frac{\ln q^2}{\ln \lambda^2} - \ln\frac{q^2}{\lambda^2} - \frac{3}{2}\ln\frac{\ln q^2}{\ln \lambda^2} + \\ + \ln q^2\left(\ln\frac{q^2}{q^2-1} + \ln\frac{\lambda^2-1}{\lambda^2}\right) - \frac{1}{2}\left(\ln^2 q^2 - \ln^2\lambda^2\right) - \\ - {\rm Li}_2(1-q^2) + {\rm Li}_2(1-\lambda^2) + \frac{3}{2}\left(\ln\frac{q^2}{q^2-1} + \ln\frac{\lambda^2-1}{\lambda^2}\right). \end{aligned} \tag{54}$$

The functions ${\rm Li}_2$ in the resulting expression accumulate the power corrections of q^2 and IR scale λ^2, but do not exhibit a singularity at $\lambda^2 = 1$. Therefore, it gives no room for IR renormalons ambiguities, at least in the considered approximation. Nevertheless, the power corrections of nonperturbative origin do contribute to the large-q^2 behaviour, and the investigation of the correspondence between latter and the instanton corrections calculated in the next Section would be an interesting task. Note, that the consequences of the analytization of the strong coupling constant in the IR region have been studied earlier in the case of the Sudakov effects in the pion form factor and Drell–Yan cross section in the works [63].

Another possible way to avoid the Landau pole at the integration path was developed within the dimensional regularization scheme [20]. In this case, the running coupling reads

$$\alpha_s^{\rm DR}(\varepsilon;\mu^2) = \frac{4\pi\varepsilon}{\beta_0\left[(q^2)^\varepsilon - 1\right]}\ , \tag{55}$$

and for complex ε, Re $\varepsilon < 0$, it has the Landau pole at the complex value of μ^2, thus this singularity moves out of the integration contour. In the limit $\varepsilon \to 0$, the form factor reads [20] (for comparison, see Eq. (43)):

$$\begin{aligned} F_q^{\rm DR}(q^2) = \exp\Bigg[-\frac{2C_F}{\beta_0}\left(\frac{\zeta(2)}{\varepsilon} + \ln\varepsilon\left(\ln q^2 - \frac{3}{2}\right) + \ln q^2\left(\ln\ln q^2 - 1\right) - \right. \\ \left. - \frac{3}{2}\ln\ln q^2\right) + O(\varepsilon, \varepsilon\ln\varepsilon)\Bigg]. \end{aligned} \tag{56}$$

This expression also leaves no room for any renormalon induced effects. At the same time, the instanton induced contributions still take place since they enter into the «integration constant» W_{np}, (17), which is not directly related to the analytical properties of the coupling constant.

1.5. Large-q^2 Behaviour of the Instanton Induced Contribution. The instanton induced effects in the high-energy QCD processes have been actively studied since the seventies [50, 51]). Recently, the investigation of these effects was renewed with promising perspectives [43,45–49,52–55]. The Wilson integral formalism is considered as a useful and convenient tool in the instanton applications mainly due to significant simplification in the path integral calculations if an explicit form of the gauge field is known. Another important feature of this approach is the possibility of making a correct analytical continuation of the results obtained in the Euclidean space (where the instantons are only determined) to the physical Minkowski space-time where the scattering processes actually take place. Namely, one maps the scattering angle, χ, to the Euclidean space angle, γ, by analytical continuation [56]

$$\chi \to i\gamma, \tag{57}$$

and performs the inverse transformation to the Minkowski space-time in the final expressions in order to restore the q^2 dependence.

Let us consider the instanton induced contribution to the function $W_{np}(q^2)$ from Eq. (19). Each instanton is taken in the singular gauge

$$\hat{A}_\mu(x;\rho) = A^a_\mu(x;\rho)\frac{\sigma^a}{2} = \frac{1}{g}\mathbf{R}^{ab}\sigma^a\eta^{\pm b}_{\mu\nu}(x-z_0)_\nu\varphi(x-z_0;\rho) \tag{58}$$

with the profile function

$$\varphi_I(x) = \frac{\rho^2}{x^2\left(x^2+\rho^2\right)}, \tag{59}$$

where $\mathbf{R}^{ab}$ is the colour orientation matrix $(a = 1, \ldots, (N_c^2-1), b = 1, 2, 3)$ which provides an embedding of $SU(2)$ instanton field into $SU(3)$ colour group; σ^a's are the Pauli matrices, and $(\pm)$ corresponds to the instanton, or anti-instanton.

The averaging of the Wilson operator over the nonperturbative vacuum is performed by the integration over the coordinate of the instanton centre z_0, the color orientation and the instanton size ρ. The measure for the averaging over the instanton ensemble reads $dI = d\mathbf{R}\ d^4z_0\ dn(\rho)$, where $d\mathbf{R}$ refers to the averaging over color orientation, and $dn(\rho)$ depends on the choice of the instanton size distribution. Taking into account (58), we write the Wilson integral (11) in the single instanton approximation in the form:

$$w_I(C_\gamma) = \frac{1}{N_c}\langle 0|\mathrm{Tr}\exp\left(i\sigma^a\phi^a\right)|0\rangle, \tag{60}$$

where

$$\phi^a(z_0,\rho) = \mathbf{R}^{ab}\eta^{\pm b}_{\ \mu\nu} \int_{C_\gamma} dx_\mu \ (x-z_0)_\nu \varphi(x-z_0;\rho). \tag{61}$$

We omit the path ordering operator $\mathcal{P}$ in (60) because the instanton field (58) is a hedgehog in color space, and so it locks the color orientation by space coordinates.

As is explained in [34], $w(C_\gamma)$ may be treated as an amplitude of the elastic scattering of an on-mass-shell ($p^2 = 0$) one-dimensional fermion on a color singlet potential. The contributions to this amplitude are due to both the self-energy corrections $\Sigma(p) = \Sigma(0) + p\partial\Sigma(0)/\partial p + \ldots$ to the fermion lines and the vertex corrections $\Gamma(p,p';\chi)$. The latter satisfy the equality

$$\Gamma(0,0;0) = -\left.\frac{\partial\Sigma(p)}{\partial p}\right|_{p=0} \tag{62}$$

following from the gauge properties. Hence, the all-order single instanton contribution to the quark form factor is given by [46]

$$w_I(C_\gamma) = \int d^4z_0 \int dn(\rho) \ \left[\cos\,\phi(\gamma,z_0,\rho) - \cos\,\phi(0,z_0,\rho)\right], \tag{63}$$

and to calculate it one can consider only the vertex corrections. In Eq. (63), the squared phase $\phi^2 = \phi^a\phi^a$ may be written as

$$\phi^2(\gamma,z_0,\rho) = \sum_{i,j=1,2} \left[(v_iv_j)z_0^2 - (v_iz_0)(v_jz_0)\right] \times$$
$$\times \int_0^\infty d\sigma\varphi\left[((-1)^{i+1}v_i\sigma - z_0)^2;\rho\right] \int_0^\infty d\sigma'\varphi\left[((-1)^{j+1}v_j\sigma' - z_0)^2;\rho\right], \tag{64}$$

where $v_{1,2} = p_{1,2}/m$, and $v_{1,2}^2 = 1$, $(v_1v_2) = \cos\gamma$ in Euclidean geometry. Let us note that due to nonperturbative factor g^{-1} in the instanton field (58) the phase (64) is independent of the coupling constant.

1.6. Exponentiation of the Instanton Contributions in the Dilute Regime. On the basis of the exponentiation theorem [36] for the non-Abelian path-ordered exponentials it is well known that perturbative corrections to the Sudakov form factor are exponentiated to high orders in the QCD coupling constant. The theorem states that the contour average $W_P(C)$ can be expressed as

$$W_P(C) = \exp\left[\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{W\in W(n)} C_n(W)F_n(W)\right], \tag{65}$$

where summation in the exponential is over all diagrams W of the set $W(n)$ of the two-particle irreducible contour averages of nth order of the perturbative expansion. The coefficients $C_n(W) \propto C_F N_c^{n-1}$ are the «maximally non-Abelian» parts of the color factor corresponding to the contribution coming from a diagram W to the total expression (65) in the contour gauge, and the factor $F_n(W)$ is the contour integral presented in the expression for W. This means that the essential diagrams are only those, which do not contain the lower-order contributions as subgraphs and, as a result, the higher-order terms are non-Abelian.

Let us now demonstrate how the single instanton contribution is exponentiated in the small instanton density parameter, treating the instanton vacuum as a dilute medium [57]. The gauge field is taken to be the sum of individual instanton fields in the singular gauge, with their centres at the points z_j's. In this gauge, the instanton fields fall off rapidly at infinity, so the instantons may be considered individually in their effect on the loop. Moreover, the contribution of infinitely distant parts of the contour may be neglected and only those instantons will influence the loop integral, which occupy regions of space-time intersecting with the quark trajectories. Since the parameterization of the loop integral along rays of the angle plays the role of the proper time, a time-ordered series of instantons arises and has an effect on the Wilson loop. Thus, the contribution of n instantons to the loop integral $W_I(\gamma)$ can be written in the dilute approximation as

$$W_I^{(n)}(\gamma) = \mathrm{Tr}\left(U^1 U^2 \cdots U^n U^{n\dagger} \cdots U^{2\dagger} U^{1\dagger}\right), \tag{66}$$

where the ordered line integrals U_i's

$$U^j(\gamma) = T\left\{\exp\left(ig\int_0^\infty d\sigma\ v_1^\mu A_\mu(v_1\sigma - z_j)\right)\times \right. \\ \left. \times \exp\left(ig\int_{-\infty}^0 d\tau\ v_2^\mu A_\mu(v_2\tau - z_j)\right)\right\}$$

are associated with individual instantons with the positions z_j's. Because of the wide separation of the instantons in the dilute phase and rapid fall-off of fields in the singular gauge, the upper and lower limits of the line integrals are extended to infinity. The line integrals $U^{i\dagger}$'s take into account the part of the contour that goes at infinity from $+\infty$ back to $-\infty$ and in the singular gauge $U^{i\dagger} = 1$. For $U^j(U^{j\dagger})$, the integral is taken over the increasing (decreasing) time piece of the loop.

Then, the expression is simplified when averaging over the gauge orientations of instantons. The averaging is reduced to substitution of U^j by $g_j U^j g_j^{-1}$, where g_j is an element of colour group, and independent integration of each g_j over the properly normalized group measure is performed. Under this averaging one gets

$$U^n U^{n\dagger} \to \frac{1}{N_c}\mathrm{Tr}(U^n U^{n\dagger}), \tag{67}$$

which is just the single instanton contribution $w_I^{(n)}(\gamma)$ as it is given by Eqs. (60), (63). But then, if the averaging is done in the inverse order, from n down to 1, the entire loop integral collapses to a product of traces

$$W_I^{(n)}(\gamma) \to \lim_{n\to\infty} \prod_{j=1}^{n} w_I^{(j)}(\gamma). \tag{68}$$

Since the individual instantons are considered to be decoupled in the dilute medium, the total multiple instanton contribution to the vacuum average of the Wilson operator simply exponentiates the all-order single instanton term $w_I(\gamma)$ in (63), and one has

$$W_I(\gamma) = \lim_{n\to\infty} \left\{ 1 + \frac{1}{n} w_I(\gamma) \right\}^n = \exp\left[w_I(\gamma)\right]. \tag{69}$$

Thus, we prove that in the dilute regime the full instanton contribution to the quark form factor is given by the exponent of the all-order single instanton result (see Fig. 1, *e*). The exponentiation arises due to taking into account the multi-instanton configurations effect. As is well known, in QED there occurs the exponentiation of the one-loop result due to Abelian character of the theory. In the instanton case, the analogous result takes place since instantons belong to the $SU(2)$ subgroup of the $SU(3)$ color group and the path-ordered exponents coincide with the ordinary ones.

The following comments are in order. First, the nonperturbative exponentiated expressions are strictly correct only so long as the instanton density n_c is small. Second, it is supposed that $U_{\rm DY}(z_0)$ is evaluated using the singular gauge form of $A_\mu^{\rm inst}$. On the other hand, Tr $\left(U_{\rm DY} U_{\rm DY}^\dagger\right)$ is identically the ordered loop integral for a single instanton and is gauge invariant. It is therefore legal to use the nonsingular gauge form of $A_\mu^{\rm inst}$ in evaluating the trace (a more handing gauge for computation).

1.7. Large-q^2 Behaviour of the Instanton Induced Contribution in the Weak Field Approximation. Although sometimes such integrals as in Eq. (64) can be evaluated explicitly, the full expression (63) requires numerical calculations. Thus, we restrict ourselves by the weak-field approximation which can be studied analytically. In this limit the leading instanton induced term (Fig. 1, *c*) reads

$$w_I^{(1)}(\gamma) = \\ = -\frac{g^2\lambda^{n-4}}{2} \int dn(\rho) \int_{C_\gamma} dx_\mu \int_{C_\gamma} dy_\nu \int \frac{d^n k}{(2\pi)^n}\ \tilde{A}_\mu^a(k;\rho) \tilde{A}_\nu^a(-k;\rho) {\rm e}^{-ik(x-y)}. \tag{70}$$

By using the Fourier transform of the instanton field

$$\tilde{A}_\mu^a(k;\rho) = -\frac{2i}{g} \eta^{\pm a}{}_{\mu\sigma} k_\sigma \tilde{\varphi}'(k^2;\rho), \tag{71}$$

Eq. (70) can be written in the form of Eq. (35) with the instantonic analogue of the function $\Delta_1(z^2)$:

$$\Delta_1(z^2) \to D_1^I(z^2) = -\frac{1}{g^2 C_F}\int dn(\rho) D_I(z^2;\rho,\lambda). \tag{72}$$

Above, $\tilde{\varphi}(k^2;\rho)$ is the Fourier transform of the instanton profile function $\varphi(z^2;\rho)$ and $\tilde{\varphi}'(k^2;\rho)$ is it's derivative with respect to k^2. Then, in case of the instanton field, the LO contribution in Minkowski space reads

$$w_I^{(1)}(C_\chi) = 2h(\chi)\int dn(\rho)\ \Delta_1^I(0,\rho^2\lambda^2), \tag{73}$$

where

$$\Delta_1^I(0,\rho^2\lambda^2) = -\int \frac{d^4k}{(2\pi)^4} \mathrm{e}^{ikz}\delta(z^2)\left[2\tilde{\varphi}'(k^2;\rho)\right]^2, \tag{74}$$

and we use the same IR cut-off λ^2, while the UV divergences do not appear at all due to the finite instanton size. Here, $\tilde{\varphi}(k^2;\rho)$ is the Fourier transform of the instanton profile function $\varphi(z^2;\rho)$, and $\tilde{\varphi}'(k^2;\rho)$ is it's derivative with respect to k^2. In the singular gauge one gets:

$$\Delta_1^I(0,\rho^2\lambda^2) = \frac{\pi^2\rho^4}{4}\left[\ln(\rho^2\lambda^2)\ \Phi_0(\rho^2\lambda^2) + \Phi_1(\rho^2\lambda^2)\right], \tag{75}$$

where

$$\Phi_0(\rho^2\lambda^2) = \frac{1}{\rho^4\lambda^4}\int_0^1 \frac{dz}{z(1-z)}\ \left[1+\mathrm{e}^{\rho^2\lambda^2} - 2\mathrm{e}^{z\rho^2\lambda^2}\right],\ \lim_{\lambda^2\to 0}\Phi_0(\rho^2\lambda^2) = 1, \tag{76}$$

and

$$\Phi_1(\rho^2\lambda^2) = \sum_{n=1}^{\infty}\int_0^1 dxdydz\ \frac{[-\rho^2\lambda^2(xz+y(1-z))]^n}{n!n}\,\mathrm{e}^{\rho^2\lambda^2[xz+y(1-z)]}, \quad \lim_{\lambda^2\to 0}\Phi_1(\rho^2\lambda^2) = 0 \tag{77}$$

are the IR-finite expressions.

At high energy the instanton induced contribution is reduced to the form:

$$\frac{\partial \ln W_I(q^2)}{\partial \ln q^2} = \frac{\pi^2}{2}\int dn(\rho)\ \rho^4\left[\ln(\rho^2\lambda^2)\ \Phi_0(\rho^2\lambda^2) + \Phi_1(\rho^2\lambda^2)\right] \equiv -B_I(\lambda^2). \tag{78}$$

Here we use the exponentiation of the single-instanton result in a dilute instanton ensemble (see [46] and the previous Subsection) and took only the LO term of the weak-field expansion (24): $W^{(1)} = w_I + (\text{higher order terms})$.

1.8. Numerical Estimate of the Instanton Effects. In order to estimate the magnitude of the instanton induced effect we consider the standard instanton size distribution [59] multiplied by the exponential suppressing factor which was suggested in [60] (and discussed in [61] in the framework of constrained instanton model) in order to describe the lattice data [13]:

$$dn(\rho) = \frac{d\rho}{\rho^5}\, C_{N_c} \left[\frac{2\pi}{\alpha_s(\mu_r)}\right]^{2N_c} \exp\left[-\frac{2\pi}{\alpha_s(\mu_r)}\right] (\rho\mu_r)^\beta \exp\left(-2\pi\sigma\rho^2\right), \quad (79)$$

where the constant $C_{N_c} = 4.6/\pi^2 \ \exp(-1.679 N_c)/\left[(N_c-1)!(N_c-2)!\right] \approx \approx 0.0015$; σ is the string tension; $\beta = \beta_0 + O(\alpha_s(\mu_r))$, and μ_r is the normalization point [62]. Given the distribution (79), the main parameters of the instanton liquid model — the instanton density $\bar{n}$ and the mean instanton size $\bar{\rho}$ — will read:

$$\bar{n} = \int_0^\infty dn(\rho) = \frac{C_{N_c}\Gamma(\beta/2-2)}{2}\left[\frac{2\pi}{\alpha_S(\bar{\rho}^{-1})}\right]^{2N_c}\left[\frac{\Lambda_{\rm QCD}}{\sqrt{2\pi\sigma}}\right]^\beta (2\pi\sigma)^2, \quad (80)$$

$$\bar{\rho} = \frac{\int_0^\infty \rho\ dn(\rho)}{\int_0^\infty dn(\rho)} = \frac{\Gamma(\beta/2-3/2)}{\Gamma(\beta/2-2)}\frac{1}{\sqrt{2\pi\sigma}}. \quad (81)$$

In Eq. (81) we choose, for convenience, the normalization scale μ_r of order of the instanton inverse mean size $\bar{\rho}^{-1}$, taking into account that the distribution function (79) is the RG-invariant quantity up to $O(\alpha_s^2)$ terms [62]. Note, that these quantities correspond to the mean size ρ_0 and density n_0 of instantons used in the model [7], where the size distribution (79) is approximated by the delta-function: $dn(\rho) = n_0\delta(\rho-\rho_0)d\rho$.

Thus, we find the leading instanton contribution (78) in the form:

$$B_I = K\pi^2\bar{n}\bar{\rho}^4 \ln\frac{2\pi\sigma}{\lambda^2}\left[1 + O\left(\frac{\lambda^2}{2\pi\sigma}\right)\right], \quad (82)$$

where

$$K = \frac{\Gamma(\beta_0/2)[\Gamma(\beta_0/2-2)]^3}{2\ [\Gamma(\beta_0/2-3/2)]^4} \approx 0.74, \quad (83)$$

and we used the one-loop expression for the running coupling constant

$$\alpha_s(\bar{\rho}^{-1}) = -\frac{2\pi}{\beta_0 \ln\ \bar{\rho}\Lambda}, \quad \beta_0 = \frac{11N_c - 2n_f}{3}. \quad (84)$$

The packing fraction $\pi^2\bar{n}\bar{\rho}^4$ characterizes diluteness of the instanton liquid and within the conventional picture its value is estimated to be 0.12 if one takes the model parameters as (see [8]):

$$\bar{n} \approx 1\ \text{fm}^{-4}, \quad \bar{\rho} \approx 1/3\ \text{fm}, \quad \sigma \approx (0.44\ \text{GeV})^2. \quad (85)$$

The leading logarithmic contribution to the quark form factor at asymptotically large q^2 is provided by the (perturbative) evolution governed by the cusp anomalous dimension (42). Thus, the instantons yield the subleading effects to the large-q^2 behaviour accompanied by a numerically small factor as compared to the perturbative term:

$$B_I \approx 0.02 \ll \frac{2C_F}{\beta_0} \approx 0.24. \tag{86}$$

Therefore, from Eqs. (78) and (52), we find the expression for the quark form factor at large-q^2 with the one-loop perturbative contribution and the nonperturbative contributions (the function W_{np} in Eq. (43)) which include both the instanton induced terms:

$$F_q(q^2) = \\ = \exp\left[-\frac{2C_F}{\beta_0}\ln q^2 \ln\ln q^2 - \ln q^2 \ \left(B_I - \frac{2C_F}{\beta_0}\right) + O(\ln\ln q^2)\right] F_0(q_0^2;\lambda^2). \tag{87}$$

It is clear, that while the asymptotic «double-logarithmic» behaviour is controlled by the perturbative cusp anomalous dimension, the leading nonperturbative corrections result in a finite renormalization of the subleading perturbative term (Fig. 2). Note, that the instanton correction has the opposite sign compared to the perturbative logarithmic term.

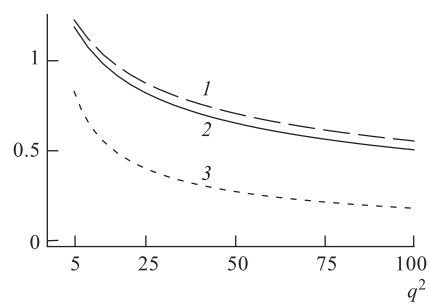


Fig. 2. The asymptotic behaviour of the quark form factor is shown as the function of the dimensionless variable $q^2 = Q^2/\Lambda^2$, up to terms $O(\ln\ln q^2)$. Curve *1* presents the contribution of one-loop perturbative terms; curve *2* — the total form factor including the instanton induced part, Eq. (87). For comparison, the leading ($\sim \ln q^2 \ln\ln q^2$) perturbative contribution is shown separately (curve *3*)

1.9. All-Order Calculations of the Wilson Loop for Gaussian Profile. Let us consider the properties of the single instanton contribution to the Wilson loop, $w_I(\chi)$, defined in Eq. (63). First, rewrite this quantity in the equivalent form (in the fixed size instanton model)

$$w^I(\chi) = 2n_c[w_c^I(\chi) + w_s^I(\chi) - w_c^I(0) - w_s^I(0)], \tag{88}$$

$$\begin{aligned} w_c^I(\chi) &= \int d^4 z_0 \, \cos\, \alpha(v_1, z_0) \cos\, \alpha(v_2, z_0), \\ w_s^I(\chi) &= -\int d^4 z_0 \, (\hat{n}_1^\alpha n_2^\alpha) \sin\, \alpha(v_1, z_0) \sin\, \alpha(v_2, z_0), \end{aligned} \tag{89}$$

where the phases corresponding to each scattering ray are defined as (from (64))

$$\begin{aligned} \alpha(v_1, z_0) &= s(v_1, z_0) \int_0^\infty d\lambda \varphi[(v_1\lambda - z_0)^2; \rho], \\ \alpha(v_2, z_0) &= s(v_2, z_0) \int_0^\infty d\lambda \varphi[(v_2\lambda + z_0)^2; \rho], \end{aligned} \tag{90}$$

where

$$s^2(v_i, z_0) = z_0^2 - (v_i z_0)^2. \tag{91}$$

Eq. (88) takes into account the subtraction of the self-energy parts of the quark form factor.

In order to have simpler analytical form for $w_I(\chi)$ we shall use Gaussian Anzatz for the profile function

$$\varphi_G(x^2) = \Lambda^2 \, \mathrm{e}^{-x^2\Lambda^2}, \tag{92}$$

with the parameter $\Lambda^2 \sim \rho^{-2}$ characterizing the vacuum field nonlocality. Below we keep the parameter Λ to unity. It is easy to reconstruct the dependence on it by simple dimension considerations.

From first glance it is easy to calculate the phases (90) corresponding to the quark form factor in the instanton background as

$$\alpha\,(v_{1,2}, z_0) = s(v_{1,2}, z_0) \frac{\sqrt{\pi}}{2} \, \mathrm{e}^{-s^2(v_{1,2}, z_0)} \mathrm{erfc}\,(\mp \mathrm{v}_{1,2} \mathrm{z}_0), \tag{93}$$

where with definitions from Eq. (12) one has

$$(v_1 z_0) = t, \qquad (v_2 z_0) = t \cosh\chi + i z_3 \sinh\chi, \tag{94}$$

$$s^2(v_1, z_0) = z_3^2 + z_\perp^2, \quad s^2(v_2, z_0) = (z_3 \cosh\chi - it \sinh\chi)^2 + z_\perp^2. \tag{95}$$

However, due to exponentially large oscillations at large χ that occur during integration over instanton position it is not easy to use the closed form for the phases (64) in general and for the Gaussian profile (92) in particular. We have to note that these complications do not arise in the case of elastic quark-quark scattering considered in [52]. In the latter case the angle dependence simply factorizes from the integrand (see also next Section).

In order to kill exponentially large oscillations we need to integrate over the instanton position. Thus, in order to analyze the χ dependence of the instanton corrections to the quark form factor we expand the full expression (88) in powers of phases and change the order of integrations:

$$w_c^I(\chi) = \int d^4 z_0 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^{n+m} \frac{\alpha^{2n}(v_1, z_0)\, \alpha^{2m}(v_2, z_0)}{(2n)!\,(2m)!} = \\ = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-1)^{n+m} \frac{\left\langle \alpha_1^{2n} \alpha_2^{2m} \right\rangle(\chi)}{(2n)!\,(2m)!}, \quad (96)$$

$$\left\langle \alpha_1^{2n} \alpha_2^{2m} \right\rangle(\chi) = \int_0^{\infty} \prod_{i=1}^{2n} d\lambda_i \int_0^{\infty} \prod_{j=1}^{2m} d\lambda'_j \int d^4 z_0 s^{2n}(v_1, z_0)\, s^{2m}(v_2, z_0) \times \\ \times \mathrm{e}^{-\left[(v_1\lambda_i - z_0)^2 + \left(v_2\lambda'_j + z_0\right)^2\right]}, \quad (97)$$

$$w_s^I(\chi) = \int d^4 z_0 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^{n+m} \left(n_1^{\alpha} n_2^{\alpha}\right) \frac{\alpha^{2n}(v_1, z_0)\, \alpha^{2m}(v_2, z_0)}{(2n+1)!\,(2m+1)!} = \\ = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (-1)^{n+m} \frac{\left\langle \alpha_1^{2n+1} \alpha_2^{2m+1} \right\rangle(\chi)}{(2n+1)!\,(2m+1)!}, \quad (98)$$

$$\left\langle \alpha_1^{2n+1} \alpha_2^{2m+1} \right\rangle(\chi) = \\ = \int_0^{\infty} \prod_{i=0}^{2n+1} d\lambda_i \int_0^{\infty} \prod_{j=0}^{2m+1} d\lambda'_j \int d^4 z_0 s^{2n}(v_1, z_0)\, s^{2m}(v_2, z_0)\, s_{12}^2(z_0) \times \\ \times \mathrm{e}^{-\left[(v_1\lambda_i - z_0)^2 + \left(v_2\lambda'_j + z_0\right)^2\right]},$$

where we separate the sine and cosine terms from (89) and use the symmetry of the integrands with respect to change of variables

$$z \to z\cosh\chi + it\sinh\chi, \qquad t \to t\cosh\chi - iz\sinh\chi \quad (99)$$

to make one sum finite. Above we introduce notation for the color spin correlation factor

$$s_{12}^2(z_0) = \left(\eta_{\mu\nu}^{\alpha} v_1^{\mu} z_0^{\nu}\right)\left(\eta_{\rho\sigma}^{\alpha} v_2^{\rho} z_0^{\sigma}\right) = (v_1 v_2)\, z_0^2 - (v_1 z_0)\,(v_2 z_0) = \\ = z_3\,(z_3\cosh\chi - it\sinh\chi) + z_\perp^2\cosh\chi. \quad (100)$$

First, let us make change of variables λ by introducing the common lengths

$$\{\lambda_i\}_N \to \left\{ L = \sum_{i=1}^{N} \lambda_i, x_i = \frac{\lambda_i}{L} \right\},\ \{\lambda'_i\}_M \to \left\{ L' = \sum_{j=1}^{M} \lambda'_j, y_j = \frac{\lambda'_i}{L'} \right\} \quad (101)$$

with new measures given by

$$\int_0^\infty dL L^{N-1} \int d\{x\}_N, \qquad \int_0^\infty dL' L'^{M-1} \int d\{y\}_M, \qquad (102)$$

where the notation is used $\int d\{x\}_N = \int_0^1 dx_1 \cdots \int_0^{1-x_1-\ldots-x_{N-2}} dx_{N-1}$.

Then rearrange the power of exponents in Eqs. (96) and (98) to the form

$$\begin{aligned}
&\sum_i^N (v_1\lambda_i - z_0)^2 + \sum_j^M \left(v_2\lambda'_j + z_0\right)^2 = \\
&= \sum_i^N \lambda_i^2 + \sum_j^M \lambda_j'^2 + (N+M)z_0^2 - 2t(L - L'\cosh\chi) + 2iz_3L'\sinh\chi = \\
&\qquad = (N+M)(z_\perp^2 + t'^2 + z_3'^2) + \sum_i^N \lambda_i^2 + \sum_j^M \lambda_j'^2 + \\
&\qquad\quad + \frac{1}{N+M}\left(-L^2 - L'^2 + 2LL'\cosh\chi\right) = \\
&= (N+M)(z_\perp^2 + t'^2 + z_3'^2) + \frac{N+M-2}{N+M}\left(\sum_i^N \lambda_i^2 + \sum_j^M \lambda_j'^2\right) + \\
&+ \frac{1}{N+M}\left(L^2 + L'^2 + 2LL'\cosh\chi - 4\left(\sum_{i>j}^N \lambda_i\lambda_j + \sum_{i>j}^M \lambda'_i\lambda'_j\right)\right), \quad (103)
\end{aligned}$$

where $t' = t - \dfrac{L - L'\cosh\chi}{N+M}$ and $z_3' = z_3 + \dfrac{iL'\sinh\chi}{N+M}$. After these transformations we come to generic expression

$$\begin{aligned}
&\left\langle \alpha_1^N \alpha_2^M \right\rangle(\chi) = \\
&= \int_0^\infty dL \int_0^\infty dL'\, \mathrm{e}^{-(2/(N+M))LL'\cosh\chi} G_{N,M}(L, L', \chi) F_{N,M}(L) F_{M,N}(L'),
\end{aligned} \qquad (104)$$

with definitions

$$F_{N,M}(L) = L^{N-1} \int d\{x\}_N \exp\left\{-\frac{1}{N+M}\left[L^2 + \right.\right.$$
$$\left.\left. + (N+M-2)\sum_i^N \lambda_i^2 - 4\sum_{i>j}^N \lambda_i\lambda_j\right]\right\}, \quad (105)$$

$$G_{N,M}(L,L',\chi) = \int d^4 z_0 s^{2n}(v_1,z_0)\, s^{2m}(v_2,z_0)\, s_{12}^{2\eta}(z_0)\, \mathrm{e}^{-(N+M)\left(z_\perp^2 + t'^2 + z_3'^2\right)}. \quad (106)$$

The definition for $F_{M,N}(L')$ is similar to one of $F_{N,M}(L)$, the power η in $G_{N,M}(L,L',\chi)$ is equal to one for $w_s^I(\chi)$ and to zero for $w_c^I(\chi)$. In principle for the Gaussian profile the integral in $G_{N,M}(L,L',\chi)$ may be done analytically in any order of expansion, but in practice only few first terms may be analyzed. In asymptotic of large χ, as we see below, we are able to do the partial summation of the double sum. It is important that after d^4z_0 integration there is no more complex numbered variables. Thus the Euclidean (transverse) and Minkowskian (longitudinal on χ) dependencies get factorized.

However, with color spin factors included, the hierarchy disappears and all diagrams contribute to the leading asymptotic behavior. Furthermore, there is nontrivial cancellations between higher than leading asymptotics terms. Let us demonstrate these statements with more details.

Considering the first few order contributions (one-loop and two-loop) one has

$$w^I(\chi) = 2n_c\left(-\langle\alpha_1\alpha_2\rangle(\chi)|_S + \frac{1}{6}\langle\alpha_1^3\alpha_2\rangle(\chi)|_S + \right.$$
$$\left. + \frac{1}{6}\langle\alpha_1\alpha_2^3\rangle(\chi)|_S + \frac{1}{4}\langle\alpha_1^2\alpha_2^2\rangle(\chi)|_S + \ldots\right), \quad (107)$$

where $\langle\alpha_1^M\alpha_2^N\rangle(\chi)|_S = \langle\alpha_1^M\alpha_2^N\rangle(\chi) - \langle\alpha_1^M\alpha_2^N\rangle(0)$. The simplest one-loop diagram (Fig. 3, *a*) corresponds to $n=0$, $m=0$ term in the expression (98) for $w_s^I(\chi)$. In Eq. (104) it corresponds to $(N=M=1)$ term. All integrals may be done analytically and the final result reduces to the weak field expression (35) with the function Δ_1 given by

$$\Delta_1^G(x^2) = \frac{\pi^2}{4}\,\mathrm{e}^{-x^2} \quad (108)$$

and thus in the lowest order we have

$$w_G^{(1,1)}(\chi) = 2n_c\frac{\pi^2}{4}(\chi\coth\chi - 1),$$
$$w_G^{(1,1)}(\chi\to 0) = 2n_c\frac{\pi^2}{12}\chi^2, \quad w_G^{(1,1)}(\chi\to\infty) = 2n_c\frac{\pi^2}{4}\chi. \quad (109)$$

If one would neglected the spin factors in (98), one got another (exponentially suppressed) dependence on the scattering angle

$$w_{Gs}^{(1,1)}(\chi) = 2n_c \frac{\pi^2}{4} \frac{\sinh\chi - \chi}{\sinh\chi},$$
$$w_{Gs}^{(1,1)}(\chi \to 0) = 2n_c \frac{\pi^2}{24}\chi^2, \quad w_{Gs}^{(1,1)}(\chi \to \infty) = 2n_c \frac{\pi^2}{4}. \tag{110}$$

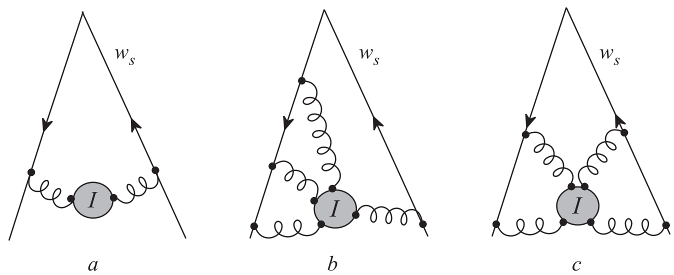


Fig. 3. Schematic representation of the lowest order instanton contributions

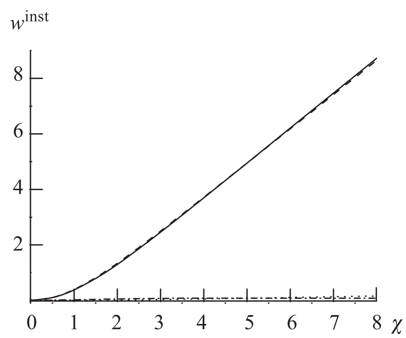
At two-loop level we have two diagrams corresponding to $n = 0$, $m = 1$ (Fig. 3, *b*) term in $w_s^I(\chi)$ $(N = 1,\ M = 3)$ and to $n = m = 1$ (Fig. 3, *c*) term in $w_c^I(\chi)$ ($N = M = 2$ in (104)). The functions $F_{N,M}(L)$ and $G_{N,M}(L, L', \chi)$ become

$$F_{1,3}(L) = \exp\left(-\frac{3}{4}L^2\right), \qquad F_{2,2}(L) = \sqrt{\frac{\pi}{2}} \exp\left(-\frac{1}{4}L^2\right) \mathrm{erf}\left(\frac{L}{\sqrt{2}}\right),$$
$$F_{3,1}(L) = L^2 \int_0^1 dx \int_0^{1-x} dx' \exp\left(-\frac{1}{4}L^2 \left[3 + 8(x^2 + x'^2 + xx' - x - x')\right]\right),$$

$$G_{2,2}(L, L', \chi) = \int d^4 z_0 s^2(v_1, z_0)\, s^2(v_2, z_0)\, \mathrm{e}^{-4\left(z_\perp^2 + t'^2 + z_3'^2\right)} =$$
$$= \frac{\pi^2}{2^{12}} \big[L'^2 \sinh^2\chi (L^2 \sinh^2\chi - 6) - 8LL' \sinh^2\chi \cosh\chi -$$
$$- 6L^2 \sinh^2\chi + 52 + 8\cosh^2\chi\big],$$

$$G_{1,3}(L, L', \chi) = \int d^4 z_0 s^2(v_1, z_0)\, s_{12}^2(z_0)\, \mathrm{e}^{-4\left(z_\perp^2 + t'^2 + z_3'^2\right)} =$$
$$= \frac{\pi^2}{2^{12}} \left[-LL'^3 \sinh^4\chi + 10L' \sinh^2\chi (L' \cosh\chi + L) - 60\cosh\chi\right].$$

The comparison of the one-loop, two-loop calculations and the full result at small χ is presented in Fig. 4.



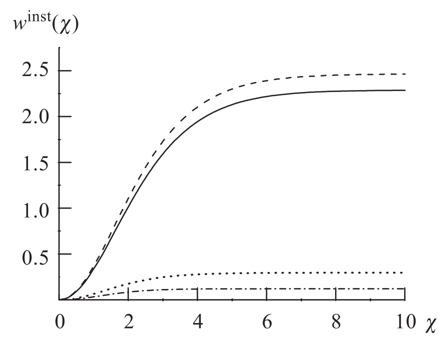


Fig. 4. Lowest orders instanton contributions to the Wilson integral with spin factors. The leading term $\langle\alpha_1^1\alpha_2^1\rangle(\chi)$ is the dashed line, next-to-leading terms $\langle\alpha_1^1\alpha_2^3\rangle(\chi)$ and $\langle\alpha_1^2\alpha_2^2\rangle(\chi)$ are the dotted and dash-dotted line, correspondingly. The sum of these contributions is the solid line

Fig. 5. The same, as in Fig. 4, but without spin factors

Figure 5 corresponds to the calculations without color spin factors, $s(v, z_0)$. In the latter case, the coordinate integral $\int d^4z$ may be performed easily

$$G^s_{N,M}(L, L', \chi) = \frac{\pi^2}{(M+N)^2}. \tag{111}$$

The next order calculations may be done in similar way. Finally, in the limit of large scattering angle χ the asymptotics may be found $w^I_{Gs}(\chi \to \infty) \sim$ const. Thus in this case one has weaker asymptotics than the asymptotics with color spin factors included.

In the following we are interested in the limit $\chi \to \infty$, where the coefficient of χ is free of the light-cone singularities and therefore it has a well-defined limit as quark momenta go on-shell, $p_1^2 = p_2^2 = 0$. To find this limit we use the properties that $F_{N,M}(L \to 0) \neq 0$ and that $G_{N,M}(L, L')$ is polynomial in L and L' after integration in z_0. Then we have asymptotics for the L and L' integrations

$$\lim_{\chi\to\infty} \int_0^\infty dL \int_0^\infty dL' \, (LL')^n \, \mathrm{e}^{-\alpha LL' \cosh\chi} F_{N,M}(L) F_{M,N}(L') =$$
$$= \frac{n!\chi}{(\alpha\cosh\chi)^{n+1}} F_{N,M}(0) F_{M,N}(0),$$

$$\lim_{\chi\to\infty}\int_0^\infty dL\int_0^\infty dL'\,(LL')^n\,L^m\,\mathrm{e}^{-\alpha LL'\cosh\chi}F_{N,M}(L)F_{M,N}(L') = \\ = \frac{n!}{(\alpha\cosh\chi)^{n+1}}F_{M,N}(0)\int dLL^{m-1}F_{N,M}(L). \quad (112)$$

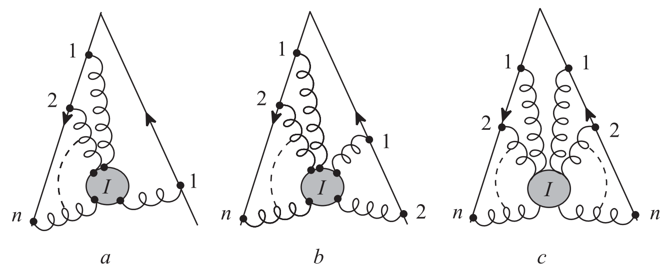


Fig. 6. Schematic representation of partial summation of the instanton contributions

Let us consider the contribution to the asymptotics from the $m=0$ and arbitrary n terms of $w_s^I(\chi)$ (Fig. 6, *a*). This contribution is reduced to the calculation of the element $\left\langle\alpha_1^{2n+1}\alpha_2^1\right\rangle$

$$\left\langle\alpha_1^{2n+1}\alpha_2^1\right\rangle = -\cosh\chi\int_0^\infty dLL^{2n}\int_0^\infty dL'\int d\{x\}_{2n+1}\,\mathrm{e}^{-L^2\sum_i^{2n}x_i^2-L'^2}\times \\ \times\int d^4z_0\left(z_3^2-iz_3t\tanh\chi+z_\perp^2\right)\left(z_3^2+z_\perp^2\right)^n\times \\ \times\exp\left(-\left[2(n+1)z_0^2-2t\left(L-L'\cosh\chi\right)+2iz_3L'\sinh\chi\right]\right). \quad (113)$$

The Gaussian integrals over z_0 are taken by

$$\int d^2z_\perp\left(z_\perp^2\right)^n\,\mathrm{e}^{-\alpha z_\perp^2} = n!\frac{\pi}{\alpha^{n+1}}, \\ \int_{-\infty}^\infty dyy^n\,\mathrm{e}^{-py^2-qy} = \sqrt{\frac{\pi}{p}}\,\mathrm{e}^{\frac{q^2}{4p}^2}\sum_{s=0}^{[n/2]}\overline{C}_s^n\left(-\frac{q}{2p}\right)^{n-2s}\frac{1}{p^s}, \quad (114)$$

where

$$\overline{C}_s^n = \frac{n!}{s!(n-2s)!2^{2s}} \quad (115)$$

and we use the binomial formula $\left(z_3^2+z_\perp^2\right)^n = \sum_{k=0}^n C_k^n z_\perp^{2k}z_3^{2(n-k)}$. The z_0

integration gives

$$\left\langle \alpha_1^{2n+1}\alpha_2^1 \right\rangle = -\pi^2 \cosh\chi \int_0^\infty dL L^{2n} \int_0^\infty dL' \int d\{x\}_{2n+1} \times$$
$$\times \exp\left(-L^2 \sum_i x_i^2 - L'^2\right) \exp\left(\frac{L^2 + L'^2 - 2LL'\cosh\chi}{2(n+1)}\right) \times$$
$$\times \sum_{k=0}^{n} C_k^n \frac{k!}{\left[2(n+1)\right]^{k+2}} \left[\frac{k+1}{2(n+1)} \left(\frac{iL'\sinh\chi}{2(n+1)}\right)^{2(n-k)} \sum_{s=0}^{n-k} \overline{C}_s^{2(n-k)} y^s + \right.$$
$$+ \left(\frac{iL'\sinh\chi}{2(n+1)}\right)^{2(n-k+1)} \sum_{s=0}^{n-k+1} \overline{C}_s^{2(n-k+1)} y^s - \left(\frac{iL'\sinh\chi}{2(n+1)}\right)^{2(n-k+1)} \times$$
$$\left. \times \sum_{s=0}^{n-k} \overline{C}_s^{2(n-k)+1} y^s + + \frac{iL\tanh\chi}{2(n+1)} \left(\frac{iL'\sinh\chi}{2(n+1)}\right)^{2(n-k)+1} \sum_{s=0}^{n-k} \overline{C}_s^{2(n-k)+1} y^s \right], \tag{116}$$

where we introduce notation

$$y = \frac{2(n+1)}{\left(iL'\sinh\chi\right)^2}. \tag{117}$$

Eq. (116) is a general expression and we are going to find its large χ limit. To do this, we need to analyze the coefficients of maximum powers of the leading diagonal terms $\left(LL'\right)^{2n}$. In the first and fourth terms in the brackets only terms in the sums with $k = s = 0$ provide the leading χ asymptotics and other one gives the subleading contributions. The second and third terms in the brackets give dominant asymptotics if $k + s \leqslant 1$ and their sum provides the leading χ asymptotics, while the terms of higher powers in $\chi\cosh\chi$ are canceled. Keeping only leading terms in (116) leads to

$$\left\langle \alpha_1^{2n+1}\alpha_2^1 \right\rangle\Big|_{\chi\to\infty} = -\frac{\pi^2\cosh\chi}{16(n+1)^3} \int_0^\infty dL L^{2n} \int_0^\infty dL' \int d\{x\}_{2n+1} \times$$
$$\times\, \mathrm{e}^{-L^2\sum_i x_i^2 - L'^2} \exp\left(\frac{\left[L^2 + L'^2 - 2LL'\cosh\chi\right]}{2(n+1)}\right) \times$$
$$\times \left(\frac{iL'\sinh\chi}{2(n+1)}\right)^{2n} \left(2n + 3 - \frac{LL'\sin^2(\chi)}{(n+1)\cos(\chi)}\right). \tag{118}$$

Under asymptotic condition all integrals can be done with the help of Eq. (112), and we get

$$\left\langle \alpha_1^{2n+1}\alpha_2^1 \right\rangle\Big|_{\chi\to\infty} = \frac{(-1)^{n+1}\pi^2(2n)!}{2^{4n+3}(n+1)^2}\chi. \tag{119}$$

Now, let us consider the contribution to the asymptotics from the $m = 1$ and arbitrary n terms of $w_s^I(\chi)$ (Fig. 6, *b*). This contribution is reduced to the element $\langle \alpha_1^{2n}\alpha_2^2 \rangle$

$$\langle \alpha_1^{2n}\alpha_2^2 \rangle = \cosh^2\chi \int_0^\infty dL L^{2n-1} \int_0^\infty dL' L' \int d\{x\}_{2n} \int_0^1 dy \times \\ \times \mathrm{e}^{-L^2\sum_i x_i^2 - L'^2\left(y^2+(1-y)^2\right)} \int d^4 z_0 \times \\ \times \left(\frac{z_\perp^2}{\cosh^2\chi} + z_3^2 + 2iz_3 t \tanh\chi - t^2 \tanh^2\chi \right) \times \\ \times \sum_{k=0}^n C_k^n z_\perp^{2k} z_3^{2(n-k)} \mathrm{e}^{-\left[2(n+1)z_0^2 + 2t\left(L - L'\cosh\chi\right) + 2iz_3\sinh\chi\right]}. \quad (120)$$

The Gaussian integration over z_0, Eq. (114), provides us with

$$\langle \alpha_1^{2n}\alpha_2^2 \rangle = \pi^2 \int_0^\infty dL \int_0^\infty dL' \,(LL')\, L^{2(n-1)} \int d\{x\}_{2n} \int_0^1 dy \times \\ \times \mathrm{e}^{-L^2\sum_i x_i^2 - L'^2\left(y^2+(1-y)^2\right)} \times \\ \times \exp\left(\frac{L^2 + L'^2 - 2LL'\cosh\chi}{2(n+1)} \right) \sum_{k=0}^n C_k^n \frac{k!}{[2(n+1)]^{n+3}} \left(\frac{L'^2\sinh^2\chi}{2(n+1)} \right)^{(n-k)} \times \\ \times \left[\sum_{s=0}^{n-k} \overline{C}_s^{2(n-k)} y^s \left(-\frac{L^2\sinh^2\chi}{2(n+1)} + \frac{3}{2} + k + s\cosh^2\chi \right) - \times \right. \\ \left. \times - \sum_{s=0}^{n-k-1} \overline{C}_s^{2(n-k)} (n-k-s) y^{s+1} LL' \sinh^2\chi \cosh\chi \right], \quad (121)$$

with y given by Eq. (117). Now, let us take the large χ limit of this expression. By analyzing the coefficients of the maximum powers of the leading diagonal terms $(LL')^{2n}$ and keeping only the leading terms, Eq. (121) is reduced to

$$\langle \alpha_1^{2n}\alpha_2^2 \rangle\big|_{\chi\to\infty} = \\ = \frac{(-1)^{n-1}\pi^2\cosh\chi}{[2(n+1)]^{2(n+2)}} \int_0^\infty dL \int_0^\infty dL' \left(LL'\sinh\chi\right)^{2n-1} \int d\{x\}_{2n} \int_0^1 dy \times \\ \times \mathrm{e}^{-L^2\sum_i x_i^2 - L'^2\left(y^2+(1-y)^2\right)} \exp\left(\frac{L^2 + L'^2 - 2LL'\cosh\chi}{2(n+1)} \right) \times \\ \times \left[n(n+1)(2n-1) - 4n(n+1)LL'\sinh\chi + (LL'\sinh\chi)^2 \right]. \quad (122)$$

Taking all integrals in the asymptotic regime one gets that the coefficient of leading asymptotic is equal to zero and thus one has

$$\left\langle \alpha_1^{2n} \alpha_2^2 \right\rangle\Big|_{\chi\to\infty} = \text{const.} \tag{123}$$

Moreover, it is possible to show that the leading asymptotic terms appear only if $n \geqslant 3$ and $m \geqslant 3$, but they are highly suppressed numerically.

From our analysis we find the leading correction to the quark form factor

$$w_G(Q^2) = n_c \log \frac{Q^2}{Q_0^2} \frac{\pi^2}{4\Lambda^4} \sum_{n=0}^{\infty} \frac{1}{16^n (n+1)^2 (2n+1)} = \\ = 1.0053 \frac{\pi^2 n_c}{4\Lambda^4} \log \frac{Q^2}{Q_0^2}. \tag{124}$$

Thus we prove that the weak field limit is a good approximation for the Gaussian profile function (92).

1.10. Discussion of the Results. We have to comment that the weak field limit used in the instanton calculations may deviate from the exact result. Nevertheless, we expect that using the instanton solution in the singular gauge which concentrates the field at small distances, leads to the reasonable numerical estimate of the full effect. Thus, the resulting diminishing of the instanton contributions with respect to the perturbative result appears to be a reasonable output. The analysis of the all-orders instanton contribution performed in the last part of this Section for a Gaussian profile function shows that the weak field approximation can be justified, but an additional investigation of this problem is required. It should be emphasized that in the present paper, all calculations in the weak field limit have been performed analytically while the evaluation of the all-orders instanton contribution required the numerical analysis. Besides this, the use of the singular gauge for the instanton solution allows us to prove the exponentiation theorem for the Wilson loop in the instanton field [46] which permits one to express the full instanton contribution as the exponent of the all-order single instanton result (63).

It is also important to note that the results obtained are quite sensitive to the way one makes the integration over instanton sizes finite. For example, if one used the sharp cut-off, then the instanton would produce strongly suppressed power corrections like $\propto (1/q)^{\beta_0}$. However, we think that the distribution function (79) should be considered as more realistic, since it reflects more properly the structure of the instanton ensemble modeling the QCD vacuum. Indeed, this shape of distribution was recently advocated in [60, 61] and supported by the lattice calculations [13] (for comparison, see, however, [64, 65]).

2. INSTANTON MODEL OF POMERON (LANDSHOFF–NACHTMANN MODEL)

Soft hadronic collisions are described successfully using Regge phenomenology, with the Pomeron exchange being dominating at high energy. The Pomeron is considered as an effective exchange in the t channel by the object with vacuum quantum numbers and with positive charge parity $C = +1$. That is why the idea of the nontrivial structure of the QCD vacuum is relevant in describing its mechanism.

To illustrate this let us consider high-energy diffractive quark-quark scattering, where there is hope that for small momentum transfer the nonperturbative effects give dominant contribution.

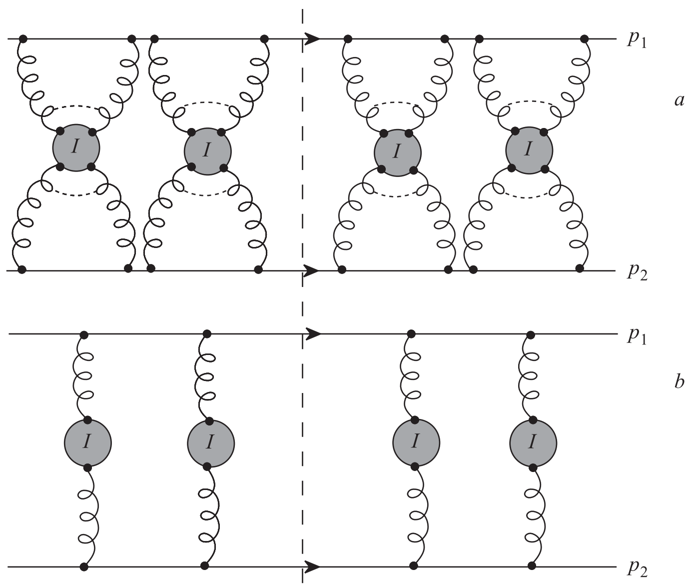


Fig. 7. Schematic representation of partial summation of the instanton contributions to the quark-quark scattering

At large energy, s, the invariant $\mathcal{T}$-matrix element of the quark-quark scattering exchanging by gluons is

$$\langle q(p_3)q(p_4)\,|\mathcal{T}|\,q(p_1)q(p_2)\rangle \underset{s\to\infty}{\longrightarrow} iI(q^2)\overline{u}(p_3)\gamma^\mu u(p_1)\overline{u}(p_4)\gamma^\mu u(p_2), \quad (125)$$

where the scattering amplitude is expressed in terms of the vacuum average of the gauge-invariant path ordered Wilson integrals (see Fig. 7)

$$I(q^2) = -\int d^2\mathbf{b}_\perp\, \mathrm{e}^{ib_\perp q} \frac{1}{N_c} \mathbf{Tr} \left\langle 0 \middle| \mathcal{P} \exp\left\{ ig \int_{C_{qq}} dx_\mu \hat{A}_\mu(x) \right\} \middle| 0 \right\rangle. \qquad (126)$$

In Eq. (126) the integration path corresponded to the quark scattering process goes along the closed contour C_{qq}: two infinite lines separated by transverse distance $b_\perp$ and having relative scattering angle γ. We parameterize the integration path $C_\chi = \{z_\mu(t); t = [-\infty, \infty]\}$ as follows

$$z_\mu(t) = \begin{cases} v_1 t, & -\infty < t < \infty, \\ v_2 t, & \infty < t < -\infty, \end{cases} \qquad (127)$$

with scattering vectors

$$\begin{aligned} v_1 &= (1, 0, 0_\perp)\,, \\ v_2 &= (\cosh\chi, i\sinh\chi, b_\perp)\,, \\ v_1^2 = v_2^2 &= 1,\ (v_1 v_2) = \cosh\chi, \end{aligned} \qquad (128)$$

where $v_1 = p_1/m$ and $v_2 = p_2/m$ are the velocities of quarks.

By making steps similar to Subsec. 1.1 one arrives to the weak field expansion of the full amplitude (c.f. Eq. (107))

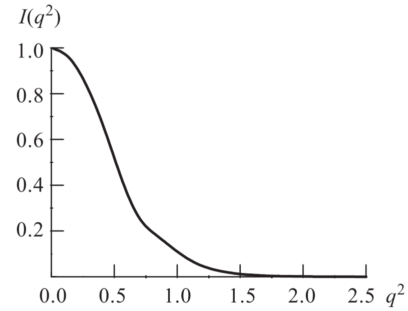


Fig. 8. Normalized quark-quark scattering amplitude Eq. (114)

$$w_{qq}^I(\chi, b_\perp) = -2n_c \left(\frac{1}{3} \langle \alpha_1^3 \alpha_2 \rangle (\chi, b_\perp)|_S + \frac{1}{4} \langle \alpha_1^2 \alpha_2^2 \rangle (\chi, b_\perp)|_S + \dots \right), \qquad (129)$$

where the phases used here are defined as

$$\alpha(v_1, z_0) = s(v_1, z_0) \int_{-\infty}^{\infty} d\lambda \varphi[(v_1\lambda - z_0)^2; \rho], \qquad (130)$$

$$\alpha(v_2, z_0) = s(v_2, z_0) \int_{-\infty}^{\infty} d\lambda \varphi[(v_2\lambda - b_\perp + z_0)^2; \rho], \qquad (131)$$

where $s^2(v_i, z_0)$ are defined as in (91). In Eq. (129) only the terms corresponding to the $C = +1$ exchange survive and the $C = -1$ terms vanish in the single instanton approximation.

In the weak field limit the model of the Pomeron reduces to the use of scattering of two instantons (Fig. 8):

$$w^I_{qq}(\chi, b_\perp) = -2n_c \int d^2\mathbf{b}_\perp\, \mathrm{e}^{ib_\perp q} \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} d\lambda' \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' s^4_{12}(z_0)\times$$
$$\times \left\langle 0 \middle| \varphi(v_1\lambda + v_2\eta - z_0 - b_\perp)\varphi(-z_0) \middle| 0 \right\rangle \left\langle 0 \middle| \varphi(v_1\lambda' + v_2\eta' - z_0 - b_\perp)\varphi(-z_0) \middle| 0 \right\rangle. \tag{132}$$

The vacuum brackets are related to the nonperturbative part of the gluon propagator given by (in the Feynman gauge)

$$\langle 0 \,|\!: A_\mu(x) A_\nu(0) :\!|\, 0 \rangle = g_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \mathrm{e}^{-ikx} D_{np}(k^2). \tag{133}$$

In the Abelian gauge model considered originally by Landshoff and Nachtmann [5] the nonperturbative gluon propagator $D_{np}(k^2)$ is related to the correlation function describing the gauge invariant gluon field strength correlator (nonlocal gluon condensate). In general non-Abelian case, this correlator has the form

$$\left\langle 0 \middle| : G_{\mu\nu}(x) \mathcal{P} \exp\left[ig \int_0^x dz^\alpha A_\alpha(z) \right] G_{\rho\sigma}(0) : \middle| 0 \right\rangle =$$
$$= \int \frac{d^4k}{(2\pi)^4} \mathrm{e}^{-ikx} \left\{ \left(D_0(k^2) + D_1(k^2) \right) k^2 \left(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \right) + \right.$$
$$\left. + D_1(k^2) \left(k_\mu k_\rho g_{\nu\sigma} - k_\mu k_\sigma g_{\nu\rho} + k_\nu k_\sigma g_{\mu\rho} - k_\nu k_\rho g_{\mu\sigma} \right) \right\}, \tag{134}$$

where the first tensor structure is called non-Abelian part and the second one is Abelian part. Indeed, in the Abelian gauge model without monopoles $D_0(k^2) \equiv 0$, and $D_1(k^2) = D_{np}(k^2)$. It is this property that has been used in [5] to relate the Pomeron properties to the value of the gluon condensate.

However, in the non-Abelian model one has opposite situation. Really, for the QCD instantons we find [61] $D_1(k^2) \equiv 0$ and $D_0(k^2)$ is nonzero. In the realistic model of the QCD vacuum, where the interaction with vacuum fields of large scale, R, is important, the instanton ceases to be an exact solution of the equations of motion, but the so-called constraint instanton approximate solution (CI) can be constructed [61]. This name is due to necessity to put constraints on the system to stabilize the instanton in the external vacuum medium. It was shown that the constraint instanton is exponentially decreasing at large distances ($\sim R$) asymptotics. The constraint instanton has topological number ± 1 as an instanton, however it is not self-dual field. Thus, in realistic QCD a small part of $D_1(k^2)$ appears. Very similar results have been found in the lattice simulations of the gluon field strength correlator [67].

Thus, within the non-Abelian models there is no direct connection of the gluon propagator to the gluon field strength correlator. So, let us directly consider the instanton part of the gluon propagator. The Fourier transform of the instanton field is defined as

$$\widetilde{A}^a_\mu(p) = \eta^a_{\mu\nu} p_\nu \widetilde{\varphi}(p^2), \tag{135}$$

where

$$\widetilde{\varphi}(p^2) = \frac{4\pi i}{p^2} \int_0^\infty ds s^3 J_2(|p|\, s) \varphi(s^2), \tag{136}$$

$\varphi(s^2)$ is the (constrained) instanton profile and $J_2(z)$ is the Bessel function. The explicit form of the Fourier transform of the pure instanton solution is well known (in the singular gauge)

$$\widetilde{\varphi}^I(p^2) = i\frac{(4\pi)^2}{p^4}\left[1 - \frac{(\rho p)^2}{2} K_2(\rho p)\right], \quad \widetilde{\varphi}^I(p^2) = \begin{cases} \dfrac{i\,(2\pi)^2\,\rho^2}{p^2}, & p^2 \to 0, \\ \dfrac{i\,(4\pi)^2}{p^4}, & p^2 \to \infty. \end{cases} \tag{137}$$

The constraint solution saves its form at short distances, but changes it at large ones:

$$\widetilde{\varphi}^{\mathrm{CI}}(p^2) = \begin{cases} \dfrac{i\pi^2}{4} R^4 I_{\mathrm{CI}}, & p^2 \to 0, \\ \dfrac{i(4\pi)^2}{p^4}, & p^2 \to \infty, \end{cases} \tag{138}$$

where the constant I_{CI} is given by

$$I_{\mathrm{CI}} = \int_0^\infty du u^2 \varphi(u). \tag{139}$$

Now, the Fourier transform of the single instanton contribution to the gluon propagator (in the Landau gauge) becomes

$$G^{ab}_{\mu\nu}(p) \equiv \int d^4x\, \mathrm{e}^{ipx} \langle 0\left|A^{a,I}_\mu(x) A^{b,I}_\nu(0)\right|0\rangle_I = \delta^{ab}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) G(p), \tag{140}$$

$$G(p) = -\frac{4}{N_c^2 - 1} p^2 \int dn(\rho) \rho^4 \widetilde{\varphi}^2(p^2), \tag{141}$$

where the effective instanton density takes into account averaging over the instanton size distribution. Thus, we see again that the nonperturbative gluon propagator and gluon field strength correlator are quite different functions, and the relation between them valid in the Abelian gauge model is destroyed in the non-Abelian case.

From (137) and (138) it is easy to deduce the asymptotics of the instanton part of the gluon propagator

$$
\begin{aligned}
G^I(p^2) &= \begin{cases} \dfrac{(2\pi)^4 \, n_c\rho^4}{N_c^2-1}\dfrac{1}{p^2}, & p^2 \to 0, \\ \dfrac{(4\pi)^4 \, n_c\rho^4}{N_c^2-1}\dfrac{1}{p^6}, & p^2 \to \infty, \end{cases} \\
G^{\mathrm{CI}}(p^2) &= \begin{cases} \dfrac{\pi^4 n_c R^4}{16(N_c^2-1)} I_{\mathrm{CI}}^2 p^2, & p^2 \to 0, \\ \dfrac{(4\pi)^4 \, n_c\rho^4}{N_c^2-1}\dfrac{1}{p^6}, & p^2 \to \infty. \end{cases}
\end{aligned} \tag{142}
$$

Calculating (in very similar way as in the Landshoff–Nachtmann model) at large energy, s, the invariant $\mathcal{T}$-matrix element of the quark-quark scattering exchanging by two gluons we get (Fig. 8)

$$
I(t) = \frac{1}{2}\int \frac{d\mathbf{k}_\perp}{(2\pi)^2} G\left[\left(\mathbf{k}_\perp + \frac{1}{2}\mathbf{q}_\perp\right)^2\right] G\left[\left(\mathbf{k}_\perp - \frac{1}{2}\mathbf{q}_\perp\right)^2\right], \tag{143}
$$

where $G(p^2)$ is defined in (141) with $p^2 \to \mathbf{p}_\perp^2$. Except numerical coefficient, this expression is in agreement with the Nachtmann–Landshoff formula. This agreement is due to specific features of the instanton induced interaction.

It is important to note that the original Wilson loop has essentially Minkowski light-cone geometry whereas the instanton calculations of a Wilson loop are performed in the Euclidean QCD. The analytical continuation from Minkowski space to Euclidean and back becomes possible since the dependence of the Wilson loop on the total energy s and transverse momentum $\mathbf{k}_\perp^2$ is factorized in (125) into two different pieces. The Q^2 dependence is given by a constant asymptotics which is the same in the perturbative and nonperturbative expressions. At the same time, the $\mathbf{k}_\perp^2$ dependence of I naturally comes through the nonperturbative instanton field propagator. Notice that in the original expression for the Wilson loop (126), the nonlocal instanton correlator was integrated over both space-like and time-like separations x^2 corresponding to the distance between different points on the contour C_{qq}, whereas the final expression (143) depends on the space-like vector squared $\mathbf{k}_\perp^2$. Thus we can do equivalently proceeding the formal calculations in Minkowski space and then make the Wick rotation $\mathbf{k}_\perp^2 \to -\mathbf{k}_\perp^2$, or, that is more natural from the point of view of the instanton model, to perform formal manipulations in the Euclidean space and then do analytical continuation by $\chi \to i\gamma$.

It is clear from the infrared behaviour of the instanton induced propagator (142) that $I(0)$ (143) is infinite for the pure instanton solution (137), but it is finite for the constraint instanton solution. This fact also noted recently in [60] was one of the arguments to the construct constraint instanton solution that modifies the profile of the instanton at large distances.

From (125) and the optical theorem it follows that the spin averaged total quark-quark cross section is constant at large energy:

$$\sigma_{qq} \sim (n_c \rho_c^4) R^2. \tag{144}$$

These results have been recently generalized in [60], where the growing part of the total cross section was also found

$$\sigma_{qq} \sim (n_c \rho_c^4) \Delta(t) \ln s, \tag{145}$$

due to inelastic quark-quark scattering in the instanton background.

As was discussed in detail in [5], already this simple model of the Pomeron explains many properties of the diffractive scattering: the effective vector-like exchange (125), the additive quark rule and the main features of the total cross section (144), (145).

CONCLUSION

Besides the considerable progress in investigation of the role of nonperturbative QCD vacuum structure (in particular, of the instanton induced phenomena) in low and moderate energy domains of hadronic physics, nowadays there is a lack of understanding of their role in high-energy processes which are intensively studied in modern experiments in particle physics. In this work we presented the results of the analysis of the structure of nonperturbative corrections in such important quantities as the quark form factor and the cross section of diffractive quark-quark scattering at high energy. The quark scattering process was considered in the background of QCD vacuum which is described within the instanton liquid model. The instanton induced contribution to the electromagnetic quark form factor is calculated in the large momentum transfer regime. We estimated analytically the weak field approximation for the instanton induced contribution, while the all-orders calculations require numerical analysis. Using the Gaussian simulation of the profile function, we calculated the all-orders instanton contribution and found that the leading contribution to high-energy asymptotic behavior is provided by the lowest order terms. Although the latter result could be treated as an argument in favor of validity of the weak field approximation, the further work has to be done in this direction since the results for Gaussian profile and the instanton in the singular gauge may be different in general.

The instanton induced effects are more interesting for theoretical study and more important for phenomenology of hadronic processes possessing two different energy scales. (For more detailed discussions see the works [70]). One of such situations — quark-quark diffractive scattering — was considered in the last Section. Here, the total centre-of-mass energy s (hard characteristic scale) is large while the squared momentum transfer $-t$ which is small compared to the latter: $-t \ll s$, but nevertheless larger than any IR scale. Besides this, the other cases of interest where the nonperturbative (including instanton induced) effects may be significant are the saturation in deep inelastic scattering (DIS) at small-x [66], and the transverse momentum distribution of vector bosons in the Drell–Yan process [40]. The latter is one of the most important objects of the experimental investigations (in particular, in the context of searches for New Physics and Higgs bosons — at future LHC and Tevatron experiments [68]), as well as theoretical studies of both the predictive power of pQCD at various energy scales and the role of nonperturbative physics (see, e.g., [69] and references therein). This problem will be one of the subjects of our forthcoming study.

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