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CHECKING LORENTZ-INVARIANCE RELATIONS BETWEEN PARTON DISTRIBUTIONS

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Lorentz-invariance relations connecting twist-3 parton distributions with transverse momentum dependent twist-2 distributions have been proposed previously. Naively, these relations can be extracted from a covariant decomposition of the quark–quark correlator. It is argued that the derivation of the Lorentz-invariance relations fails if the path-ordered exponential, which ensures gauge invariance, is taken into account in the correlator. Our model-independent analysis is supplemented by an explicit calculation of the corresponding parton distributions in perturbative QCD with a quark target, and in a simple model of Brodsky–Hwang–Schmidt.

Указывается, что соотношения между партонными распределениями различного твиста, следующие из лоренц-инвариантности, должны модифицироваться при учете глюонной экспоненты, обеспечивающей калибровочную инвариантность. Модельно-независимый анализ подтверждается явными вычислениями для модели Бродского–Хуанга–Шмидта.

INTRODUCTION

Transverse-momentum dependent parton distributions and twist-3 parton distributions play an important role in describing various hard processes like semi-inclusive deep inelastic scattering (DIS) or the Drell–Yan process. The twist-3 parton distributions are deeply connected to spin asymmetries in DIS or the Drell–Yan process [1, 2] which were measured recently [3, 4].

Lorentz-invariance relations (LI relations) were stated in [5, 6] and put constraints on the various parton distribution functions which can be used to eliminate unknown parton distributions in favor of known ones. Moreover, these relations were used to study the evolution of transverse-momentum dependent distribution functions [7].

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We argue that the LI relations are violated because in their derivation a special light-cone dependence was neglected [8]. Our model-independent analysis is supplemented by an explicit model calculation of the relevant parton distributions. The validity of the LI relations has already been questioned in [9], but the arguments given in this reference seem to be incomplete.

1. MODEL-INDEPENDENT ANALYSIS

The derivation of the LI relations as given in Ref. 5 can be understood easily by considering the correlator

$$\Phi_{ij}(P,S;k) \equiv \frac{1}{(2\pi)^4}\int d^4\xi\; \mathrm{e}^{ik\cdot\xi}\,\langle P,S|\,\overline{\Psi}_j(0)\mathcal{L}[0,\xi]\Psi_i(\xi)\,|P,S\rangle\,, \tag{1}$$

where the gauge-link is given by

$$\mathcal{L}[0,\xi|\mathrm{path}] = \mathcal{P}\,\exp\left\{-ig\int_0^{\xi} ds^{\mu}\; A_{\mu}(s)\right\}_{\mathrm{path}}. \tag{2}$$

This correlator has only a theoretical meaning because it doesn't enter a factorization theorem for a physical process. It can be decomposed in the most general Lorentz-invariant way which obeys hermicity and parity. This yields

$$\Phi_{ij}(P,S;k) = A_1 M + A_2 \not{P} + A_3 \not{k} + A_4\sigma^{\mu\nu}\frac{P_\mu k_\nu}{M} + \ldots, \tag{3}$$

where $A_i = A_i(k^2, kP)$ are unknown coefficient functions. We have only written the spin-independent structures of $\Phi_{ij}(P,S;k)$.

Now, various parton distributions can be extracted from $\Phi_{ij}(P,S;k)$ by integration over k^- and projection onto different Dirac matrices. The k^--integration of $\Phi_{ij}(P,S;k)$ leads to a physical correlator which is used in the calculation of the hadronic part of physical processes. The k^--integration of $\Phi_{ij}(P,S;k)$,

$$\Phi_{ij}(x,\mathbf{k}_T) = \int dk^-\; \Phi_{ij}(P,S;k)|_{k^+=xP^+} \tag{4}$$

leads to a correlator which is used in (4). The ($\mathbf{k}_T$-dependent) parton distributions are various Dirac projections of $\Phi_{ij}(x,\mathbf{k}_T)$. They can be written by means of Eq. (3) in terms of the coefficient functions A_i, e.g.,

$$f_1(x,\mathbf{k}_T) = 2P^+\int dk^-\;\; (A_2 + xA_3)|_{k^+=xP^+}, \tag{5}$$

$$h_1^{\perp}(x,\mathbf{k}_T) = 2P^+\int dk^-\;\; (-A_4)|_{k^+=xP^+}, \tag{6}$$

$$h(x,\mathbf{k}_T) = 2P^+\int dk^-\;\; \left(\frac{2kP - 2xM^2}{2M^2}\right) A_4\bigg|_{k^+=xP^+},\; \ldots \tag{7}$$

Comparing now these expressions for the parton distributions, one can find easily relations between those parton distributions which contain the same A_i's. These are the so-called LI relations. We list here the most important ones

$$
\begin{aligned}
g_T(x) &= g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x), && (8)\\
h_L(x) &= h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x), && (9)\\
f_T(x) &= -\frac{d}{dx} f_{1T}^{\perp(1)}(x), && (10)\\
h(x) &= -\frac{d}{dx} h_1^{\perp(1)}(x). && (11)
\end{aligned}
$$

The first two LI relations connect T-even parton distributions whereas the last ones are for T-odd parton distributions.

Our crucial point of criticism of these LI relations is that the decomposition in Eq. (3) is incomplete because the presence of the gauge link leads to a dependence on an additional light-like vector [8]. To see this one has to keep in mind the appropriate gauge link structure of the correlator ($\tilde{\xi} = (\xi^-, 0, \boldsymbol{\xi}_T)$)

$$
\Phi_{ij}(x, \mathbf{k}_T) = \frac{1}{(2\pi)^3} \int d\xi^- d^2\xi_T \, \mathrm{e}^{ixP^+\xi^-} \, \mathrm{e}^{-i(\mathbf{k}_T \cdot \boldsymbol{\xi}_T)} \times \\
\times \langle P, S | \, \overline{\Psi}_j(0) \mathcal{L}[0, \tilde{\xi}] \Psi_i(\tilde{\xi}) \, | P, S \rangle \, . \quad (12)
$$

The link in (12) connecting the space-points 0 and $\tilde{\xi}$ runs straight from 0 along the light-cone in ξ^- direction up to infinity, then it goes in the transverse direction and finally comes back to the point $\tilde{\xi}$ [1, 10, 11]

$$
\mathcal{L}[0, \tilde{\xi}] = \mathcal{L}[0, (\infty, 0, \mathbf{0}_T)] \times \mathcal{L}[(\infty, 0, \mathbf{0}_T), (\infty, 0, \boldsymbol{\xi}_T)] \times \mathcal{L}[(\infty, 0, \boldsymbol{\xi}_T), \tilde{\xi}]. \quad (13)
$$

To parameterize this gauge link a light-like direction (beyond the direction given by the target momentum) is needed, which has to show up also in the unintegrated correlator in Eq. (1) due to the connection given in (4). Therefore, one has to add more covariant structures in (3) with new coefficient functions B_i indicating the light-cone dependence, namely

$$
\Phi_{ij}(P, S; k|n) = A_1 M + A_2 \not{P} + A_3 \not{k} + A_4 \sigma^{\mu\nu} \frac{P_\mu k_\nu}{M} + \ldots + \\
+ \frac{M^2}{Pn} \not{n} B_1 + B_2 \sigma^{\mu\nu} \frac{P_\mu n_\nu}{Pn} M + B_3 \sigma^{\mu\nu} \frac{k_\mu n_\nu}{Pn} M + \ldots \quad (14)
$$

The new terms don't change the extraction of parton distributions, however the explcit expressions of the parton distributions in terms of the coefficient functions look different. For instance, we now have

$$h_1^\perp(x,\mathbf{k}_T) = 2P^+ \int dk^- \, (-A_4), \tag{15}$$

$$h(x,\mathbf{k}_T) = 2P^+ \int dk^- \, \left(\frac{2kP - 2xM^2}{2M^2} A_4 + (B_2 + xB_3)\right), \ldots \tag{16}$$

Obviously the new terms B_i clearly spoil the LI relations (11). By considering spin-dependent structures also the violation of the relations in (8)–(10) can be shown.

2. MODEL CALCULATIONS

We now want to supplement our model-independent analysis by an explcit model calculation of relevant parton distributions.

The discussion for the LI relations involving T-odd functions is simple. It was shown in explicit model calculation [12,13] that $f_{1T}^\perp(x,\mathbf{k}_T)$ doesn't vanish in a simple diquark-spectator model. However, the T-odd ($\mathbf{k}_T$-independent) parton distribution $f_T(x)$ equals zero due to time-reversal symmetry. Therefore the LI relation (10) involving these parton distributions is clearly violated in this model. The same argument can be used for the discussion of the other T-odd LI relation (11). The $\mathbf{k}_T$-independent parton distribution $h_1^\perp(x,\mathbf{k}_T)$ has been calculated in [14, 15] and doesn't vanish whereas $h(x)$ equals zero due to time-reversal symmetry.

For T-even parton distribution we took another model, where the incoming nucleon is replaced by an incoming quark. This allows a calculation of the parton distributions in pQCD. In such a model the $\mathbf{k}_T$-dependent correlator can be written as

$$\Phi_{ij}(x,\mathbf{k}_T) = \int \frac{d\xi^-}{2\pi}\frac{d^2\xi_T}{(2\pi)^2}\, \mathrm{e}^{ixP^+\xi^-}\, \mathrm{e}^{-i(\mathbf{k}_T\boldsymbol{\xi}_T)} \times \\ \times \langle q;P,S,d|\, \overline{\Psi}_j(0)\, \mathcal{L}[0,\tilde{\xi}]\, \Psi_i(\tilde{\xi})\, |q;P,S,d\rangle\,. \tag{17}$$

In Feynman-gauge, only two parts of the link in (13) contribute because the part at light-cone infinity can be neglected. By introducing intermediate states one can expand the correlator up to first order in α_s

$$\Phi_{ij}(x,\mathbf{k}_T) = \int \frac{d\xi^-}{2\pi}\frac{d^2\xi_T}{(2\pi)^2}\, \mathrm{e}^{ixP^+\xi^-}\, \mathrm{e}^{-i(\mathbf{k}_T\boldsymbol{\xi}_T)} \times \\ \times \Big\{ \langle q;P,S,d|\, \overline{\Psi}_j(0)\mathcal{L}[0,(\infty,0,0)]\, |0\rangle\, \langle 0|\, \mathcal{L}[(\infty,0,\boldsymbol{\xi}_T),\tilde{\xi}]\Psi_i(\tilde{\xi})\, |q;P,S,d\rangle \, +$$

$$+\sum_{\beta,r'}\int\frac{d^3q}{2(2\pi)^3E_{\mathbf{q}}}\,\langle q;P,S,d|\,\overline{\Psi}_j(0)\mathcal{L}[0,(\infty,0,0)]\,|g;\mathbf{q},r',\beta\rangle\times$$

$$\times\times\langle g;\mathbf{q},r',\beta|\,\mathcal{L}[(\infty,0,\boldsymbol{\xi}_T),\tilde{\xi}]\Psi_i(\tilde{\xi})\,|q;P,S,d\rangle+\ldots\Big\}. \quad (18)$$

Up to $\mathcal{O}(\alpha_s)$ only gluons as intermediate particles are relevant. Because the 1-loop calculation diverges, one needs to regularize the corresponding expressions. Here we choose dimensional regularization. The resulting parton distributions are extracted by projecting $\Phi_{ij}(x,\mathbf{k}_T)$ onto Dirac matrices and integrating over $\mathbf{k}_T$. Up to $\mathcal{O}(\alpha_s)$ the parton distributions in Eq. (8) read

$$g_1(x)=\delta(1-x)+\frac{\alpha_s}{2\pi}C_F\frac{1+x^2}{1-x}\left\{\frac{1}{\varepsilon}-\gamma_E-\ln\left(\frac{m^2(1-x)^2}{4\pi\mu^2}\right)\right\}+\ldots, \quad (19)$$

$$g_T(x)=\delta(1-x)+\frac{\alpha_s}{2\pi}C_F\frac{1+2x-x^2}{1-x}\left\{\frac{1}{\varepsilon}-\gamma_E-\ln\left(\frac{m^2(1-x)^2}{4\pi\mu^2}\right)\right\}+\ldots, \quad (20)$$

$$g_{1T}^{(1)}(x)=-\frac{\alpha_s}{2\pi}C_Fx(1-x)\left\{\frac{1}{\varepsilon}-\gamma_E-\ln\left(\frac{m^2(1-x)^2}{4\pi\mu^2}\right)\right\}+\ldots \quad (21)$$

In order to check the LI relations it is sufficient to calculate only the terms of $\Phi_{ij}(x,\mathbf{k}_T)$ containing gluons as intermediate particles. The virtual contributions to order $\mathcal{O}(\alpha_s)$ are distinguished from the real gluon emission by an explicit factor of $\delta(1-x)$. The dots in Eqs. (19), (20), and (21) represent the omission of virtual contributions and higher orders.

The check whether the relation (8) holds can be made by comparing the pieces showing a $1/\varepsilon$-divergence. By inserting these divergencies into the LI relation one notices that this LI relation is also violated. We note that in leading order the relation is trivially satisfied, but is broken in order $\mathcal{O}(\alpha_s)$. With an analogous calculation it can be shown explicitly that also the relation (9) is violated.

In summary, we have shown by a model-independent analysis and by an expilcit model-calculation that the so-called Lorentz-invariance relations between parton distributions are violated.

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