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## ABOUT ONE POSSIBILITY OF RELATIVISTIC DESCRIPTION OF POLARIZED DEUTERON FRAGMENTATION

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In the framework of the light-front quantum theory developed by Karmanov et al. an analysis of the experimental data on the tensor analyzing power of the nuclear fragmentation of relativistic deuterons with the large transverse momentum proton emission has been made. With Karmanov's wave function taken in system, in which  $z$  axis is directed along the deuteron beam, we have managed to explain the existing data without invoking additional to nucleons degrees of freedom.

Анализ данных по тензорной анализирующей способности ядерной фрагментации релятивистских дейтронов с испусканием протона с большим поперечным импульсом выполнен в рамках квантовой теории на световом фронте, развитой Кармановым и др. С использованием волновой функции Карманова в системе, направленной по дейтронному пучку, удается объяснить данные без привлечения нуклонных степеней свободы.

The experiments with the polarized deuteron beams made in Saclay [1–4] and Dubna [5–10] have led one to recognize that at the relativistic momenta of deuteron there is something wrong either with the theory of  $A(d, p)X$  reactions or with the structure of the deuteron at short distances between nucleons. At first, it was proved out that the experimental dependence of the analyzing power  $T_{20}$  on  $k$  — internal momentum of nucleons in the deuteron — does not change sign at  $k \sim 0.5$  GeV/ $c$ , as it followed from theoretical calculations. Further [10], the pion-free deuteron-breakup process  $dp \rightarrow ppn$  in the kinematical region close to that of backward elastic  $dp$  scattering at a given value of  $k$  depends on the incident momentum. This forces one to suggest that in the description of this quantity an additional variable is required. This additional variable does not appear in the usual schemes of calculations. At last, the recent measurements of the tensor analyzing power  $A_{yy}$  of the breakup of relativistic deuterons on nuclei at large

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transverse momenta of emitted protons [11, 12] show also that something unusual takes place in the theory of this reaction since the measured  $A_{yy}$  values at fixed value of longitudinal proton momentum show a pronounced dependence on the transverse proton momentum, that does not appear in the calculations.

The theoretical considerations of  $A(d, p)X$  reaction were carried out in different lines and on the whole the situation with the description of this reaction is contradictory. The most popular of theoretical approaches is one of the light-front dynamics and in this paper we follow it. In general this approach in the approximation of a simple mechanism with the pole in  $t$  channel with using the standard deuteron wave functions satisfactorily describes the differential cross section data [13, 14] (see, for example, [15, 16]). On the other hand, the calculations of polarization observables in the same approach [17], as a rule, do not reproduce the experimental data; the exception is the paper [18], where the data on the  $T_{20}$  of nuclear fragmentation of relativistic deuterons with the proton emission at  $0^\circ$  are described.

The most simple statement would be to say that this discrepancy between the theory and experiment is due to the oversimplified mechanism of the reaction. But, in our opinion, at the experimental accuracy achieved the possibilities of this simple and thus valuable mechanism have not yet been exhausted.

In all previous papers concerned with the analysis of the polarization observables of the  $A(d, p)X$  reaction, the deuteron wave function has been presumed to be the superposition of the  $S$  and  $D$  waves, each represented in the momentum space as a product of angular and radial functions. In particular, this is true for one of relativistic versions of theory — relativistic quantum mechanics [19]. This superposition implies a definite relationship between the transverse and longitudinal components of the momentum of the internal motion of nucleons in a deuteron [17]. However, the dependence of the wave function on the transverse and longitudinal components in the light-front dynamics may be different from that dictated by the  $S$ - and  $D$ -wave combination. The attention to this possibility was called in [20, 21], where the relativistic hard collision model of composite hadrons [22] was generalized to the case of relativistic nucleus–nucleus collisions. It is precisely this possibility that is explored in the present paper.

The light-front dynamics [23] has many important benefits for the description of high-energy experiments. Probably the main physical achievement of this approach is the prediction and explanation [24] of behaviour of the ratio of the proton's elastic electromagnetic form factors [25]. The light-front dynamics is usually used also for description of the deep inelastic phenomena. A special feature of this dynamics is that the contribution of diagrams going back in time vanish.

Difficulties of the light-front dynamics are in breaking of rotational invariance as a result of selecting a particular direction in space for the orientation of the light front. One point related to this is that angular momentum operators  $J_x$ ,  $J_y$  in the light-front dynamics become dynamical operators, i.e. they are dependent

on an interaction. This leads to the difficulties associated with the determination of the spin of a composite system. References to papers devoted different aspects of light-front dynamics can be found in the review [26].

These difficulties with rotational invariance were circumvented by Karmanov and coworkers [27]. They found the relativistic deuteron wave function with correct internal spin. This function depends on two vector variables: on the momentum  $\mathbf{k}$  of nucleons in deuteron in their rest frame and on the extra variable  $\mathbf{n}$  — the unit normal to the light-front surface.

The general statement by Karmanov is: the final results when particles are on mass shell do not depend on the choice of the plane of quantization. But this is right, generally speaking, only in that case when one makes fully accurate calculations. Really, however, one makes only approximate calculation and therefore there arises some dependence on the choice of the light-front surface. We play on this dependence. Since the direction along deuteron beam is most evidently favoured, we direct  $z$  axis along the beam. As a result the wave function of relativistic deuteron with right spin becomes nontrivially dependent on longitudinal and transverse components of internal momentum and provides new possibilities in the description of experimental data. We should like to point out that without the accurate amplitudes of the process it is difficult to put the serious argument in the favour directing  $z$  axis along the deuteron beam. Therefore besides the intuitive arguments, only the argument of the agreement with experiment for several calculated spin characteristics can be put forward.

Karmanov's wave function is determined by six invariant functions instead of two ones in the nonrelativistic case, each of them depending on two scalar variables  $k$  and  $z = \cos(\widehat{\mathbf{k}\mathbf{n}})$  and has the following form:

$$\Psi_{\sigma_2\sigma_1}^M = w_{\sigma_2}^* \psi^M(\mathbf{k}, \mathbf{n}) \sigma_y w_{\sigma_1}, \quad (1)$$

where  $M = 0, \pm 1$  are the projections of spin  $\mathbf{J} = \mathbf{1}$  on the quantization axis, and

$$\begin{aligned} \psi(\mathbf{k}, \mathbf{n}) = & \frac{1}{\sqrt{2}} \sigma f_1 + \frac{1}{2} \left[ \frac{3}{k^2} \mathbf{k}(\mathbf{k}\sigma) - \sigma \right] f_2 + \frac{1}{2} [3\mathbf{n}(\mathbf{n}\sigma) - \sigma] f_3 + \frac{1}{2k} [3\mathbf{k}(\mathbf{n}\sigma) + \\ & + 3\mathbf{n}(\mathbf{k}\sigma) - 2\sigma(\widehat{\mathbf{k}\mathbf{n}})] f_4 + \sqrt{\frac{3}{2}} \frac{i}{k} [\widehat{\mathbf{k}\mathbf{n}}] f_5 + \frac{\sqrt{3}}{2k} [[\mathbf{k} \times \mathbf{n}]\sigma] f_6. \end{aligned} \quad (2)$$

Here  $\sigma$  are the Pauli matrices;  $w_{\sigma_1(\sigma_2)}$  are the spin functions of nonrelativistic nucleons, and  $f_1, \dots, f_6$  are the invariant-about-rotations functions of the kinematical variables, that define the deuteron state. Here

$$k = \sqrt{\frac{m_p^2 + \mathbf{p}_T^2}{4x(1-x)} - m_p^2}, \quad (\widehat{\mathbf{n}\mathbf{k}}) = \left( \frac{1}{2} - x \right) \sqrt{\frac{m_p^2 + \mathbf{p}_T^2}{x(1-x)}}, \quad (3)$$

where  $x$  is the fraction of the deuteron longitudinal momentum taken away by the proton in the infinite momentum frame.

The invariant amplitude for the reaction  ${}^1\text{H}(d, p)X$  in the light-front dynamics is as follows:

$$\mathcal{M}_a = \frac{\mathcal{M}(d \rightarrow p_1 b)}{(1-x)(M_d^2 - M^2(k))} \mathcal{M}(bp \rightarrow p_2 p_3), \quad (4)$$

where  $\mathcal{M}(d \rightarrow p_1 b)$  and  $\mathcal{M}(bp \rightarrow p_2 p_3)$  are the amplitudes of the deuteron breakup into the particles  $p_1, b$  and of the reaction  $bp \rightarrow p_2 p_3$ , respectively. The ratio

$$\psi(x, p_{1T}) = \frac{\mathcal{M}(d \rightarrow p_1 b)}{M_d^2 - M^2(k)} \quad (5)$$

is nothing but the wave function in the channel  $(b, N)$ ; here  $p_{1T}$  is the component of momentum  $p_1$  transverse to the  $z$  axis, and  $M^2(k)$  is given by

$$M^2(k) = \frac{m^2 + p_{1T}^2}{x} + \frac{b^2 + p_{1T}^2}{1-x}, \quad (6)$$

where  $b^2$  is a four-momentum squared of the off-shell particle  $b$ .

The analyzing power of  $T_{\kappa q}$  is given by

$$T_{\kappa q} = \frac{\int d\tau \text{Sp} \{ \mathcal{M} \cdot t_{\kappa q} \cdot \mathcal{M}^\dagger \}}{\int d\tau \text{Sp} \{ \mathcal{M} \cdot \mathcal{M}^\dagger \}}, \quad (7)$$

where  $d\tau$  is the phase volume element, and the operator  $t_{2q}$  is defined by

$$\langle m | t_{\kappa q} | m' \rangle = (-1)^{l-m} \langle 1 m 1 - m' | \kappa q \rangle,$$

with the Clebsch–Gordan coefficients  $\langle 1 m 1 - m' | \kappa q \rangle$ .

The final expression for the analyzing power has the form

$$\begin{aligned} T_{2q} \left( \frac{p_{10} d\sigma}{d\mathbf{p}_1} \right)_{\text{un}} &= \frac{1}{2(2\pi)^3} \left\{ \frac{I(b, p)}{I(d, p) (1-x)^2} \rho_0(2, q) \sigma(bp \rightarrow X) + \right. \\ &\left. + \int \frac{dt d\mathbf{p}_{2T}}{2y(1-y) (1-y) I(d, p)} \rho_0(2, q) \frac{p_{10} d\sigma}{d\mathbf{p}_1} (bp \rightarrow p_2 p_3) [1 + \mathbf{P} \langle \sigma \rangle] \right\}, \quad (8) \end{aligned}$$

where  $I(b, p)$ ,  $I(d, p)$  are the invariant fluxes of the appropriate particles;  $\langle \sigma \rangle$  is the vector analyzing power of the  $NN$ -scattering;  $\sigma(bp \rightarrow X)$  is the total cross section of the  $NN$ -scattering;  $\mathbf{P}$  is the polarization vector of the nucleon in the deuteron that is characterized by indices  $(\kappa, q)$ :

$$\mathbf{P} = \text{Sp} \{ \rho(\kappa, q) \} / \rho_0(\kappa, q). \quad (9)$$

The first term in the curly brackets of (8) corresponds to the case when the spectator proton is detected, and the second term corresponds to the detecting of the proton scattered on the target. The differential cross section for an unpolarized beam entering in (8) is given by

$$\left(\frac{p_{10}d\sigma}{d\mathbf{p}_1}\right)_{\text{un}} = \frac{1}{2(2\pi)^3} \left\{ \frac{I(b,p)}{I(d,p)(1-x)^2} \rho_0 \sigma(bp \rightarrow p_2p_3) + \int \frac{dt d\mathbf{p}_{2T}}{2y(1-y)(1-y)I(d,p)} \rho_0 \frac{p_{10}d\sigma}{d\mathbf{p}_1}(bp \rightarrow p_2p_3) \right\}, \quad (10)$$

where

$$\rho_0 = 3[f_1^2 + f_2^2 + f_3^2 + f_2f_3(3z^2 - 1) + 4f_4(f_2 + f_3)z + f_4^2(z^2 + 3) + (f_5^2 + f_6^2)(1 - z^2)]. \quad (11)$$

If one introduces the density matrix in the spin space of the nucleon  $b$  at a given deuteron polarization characterized by indices  $(\kappa, q)$

$$\begin{aligned} \rho_{\mu\mu'}(\kappa, q) &= \sum_{\nu, M, M'} \psi_M(\nu, \mu) (-1)^{1-M'} \langle 1 M 1 - M' | \kappa q \rangle \psi_{M'}^*(\nu, \mu') = \\ &= \rho(\kappa, q) = \frac{1}{2} \rho_0(\kappa, q) (1 + \mathbf{P} \cdot \boldsymbol{\sigma}), \end{aligned} \quad (12)$$

then the density matrices  $\rho_0(\kappa, q)$  may be computed from the relations

$$\rho_0(\kappa, q) = \text{Sp} \{ \rho_{\mu, \mu'}(\kappa, q) \}. \quad (13)$$

The results of calculations of tensor analyzing power  $A_{yy}$  of the reaction  ${}^9\text{Be}(d, p)X$  at the initial deuteron momentum of 4.5 GeV/c and a proton emission angle of 80 mrad are compared with the experimental data in Fig. 1.

It is seen that the experimental data are qualitatively properly reproduced using Karmanov's relativistic deuteron wave function as opposed to the calculations with the standard deuteron wave functions [31, 32]; the last curves change sign at the proton momentum  $\sim 3.2$  GeV/c.

In Fig. 2 the experimental data on parameter  $A_{yy}$  of the reaction  ${}^{12}\text{C}(d, p)X$  at the initial deuteron momentum of 9 GeV/c and a proton emission angle of 85 mrad are compared with the calculations using different deuteron wave functions. It is seen that the momentum dependence calculated with the relativistic deuteron wave function is very close to the experimental points, whereas the curves calculated with the standard nonrelativistic deuteron wave functions are in sharp contradiction with the data.

Fig. 1. Parameter  $A_{yy}$  of the reaction  ${}^9\text{Be}(d,p)X$  at an initial deuteron momentum of 4.5 GeV/c and a proton emission angle of 80 mrad as a function of the detected proton momentum. Experimental data are from [12]. The calculations were made with the deuteron wave functions for the Bonn B [32] (dashed curve) and the Paris [31] (dash-dotted curve) potentials. The solid curve was calculated with Karmanov's relativistic deuteron wave function [27]

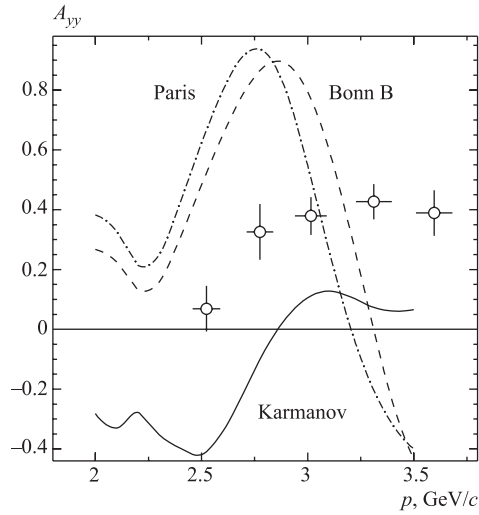
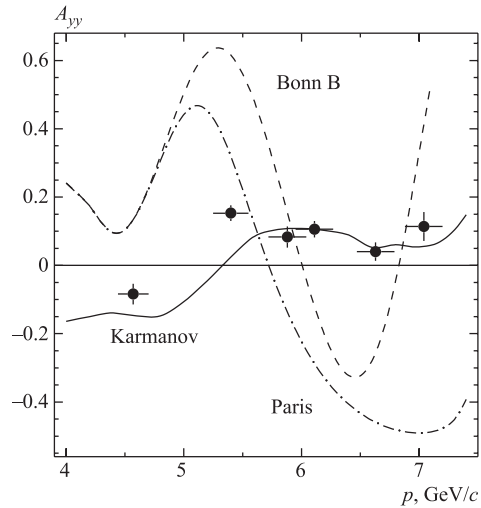


Fig. 2. Parameter  $A_{yy}$  of the reaction  ${}^{12}\text{C}(d,p)X$  at 9 GeV/c and a proton emission angle of 85 mrad versus the detected proton momentum. Experimental data from [11]. The calculations were made with the Bonn B [32] (dashed curve), Paris [31] (dash-dotted curve) and Karmanov's (solid curve) deuteron wave functions [27]



Since the relativistic deuteron wave function [27] has a rather complicated appearance, the question arises, what terms of this function help to describe qualitatively the experimental data on the tensor analyzing power of the nuclear fragmentation of the relativistic deuterons with emission of protons with large transverse momenta. To answer this question, the calculations of the momentum dependence of the parameter  $A_{yy}$  of the reaction  ${}^{12}\text{C}(d,p)X$  at 9 GeV/c and 85 mrad have been made, in which the terms  $f_2, \dots, f_6$  of function (2) have been taken into account successively. The results are shown in Fig. 3.

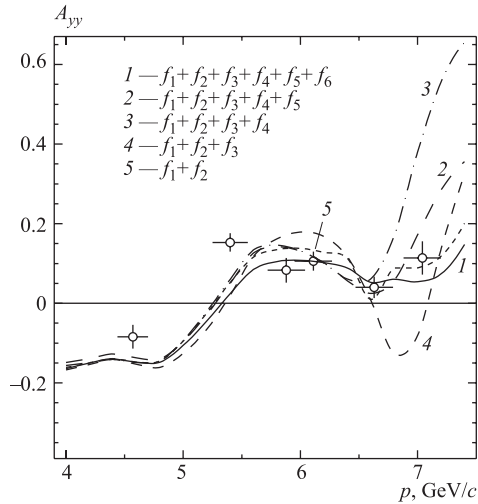


Fig. 3. Parameter  $A_{yy}$  of the reaction  $^{12}\text{C}(d,p)X$  at 9 GeV/c and a proton emission angle of 85 mrad as a function of the detected proton momentum. The different curves correspond to the successive taking into account the terms of  $f_i$  of the relativistic wave function [27]

It is seen that the two first terms of (2) give the dominating contribution to the  $A_{yy}$  dependence, and the remaining terms give only corrections; the role of these corrections increases with the momentum.

It is shown in [27] that in the nonrelativistic limit the functions  $f_1$  and  $f_2$  correspond to the  $S$ - and  $D$ -states of the deuteron. Hence it follows that the relation between the  $k_L$  and  $k_T$  in a moving deuteron differs essentially from that in the nonrelativistic case. The method of relativization proposed by Karmanov et al. [27] appears to reflect correctly this relation.

Two main conclusions can be made from this investigation. At first, it turns out rather unexpectedly that up to small relative distances corresponding to the internal momenta of nucleons  $k \sim 0.5\text{--}0.8$  GeV/c the deuteron can be considered as a two-nucleon system in the light-front of the quantum mechanics. A similar conclusion was made in [14] in connection with the measurements of the momentum spectra of protons emitted as a result of fragmentation of 9 GeV/c deuterons in the region of proton transverse momenta of 0.5–1 GeV/c. Secondly, in the fragmentation process the relativistic effects become significant very rapidly, and these effects can be taken into account in the most simple way through the use of the light-front dynamics.

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#### REFERENCES

1. Arvieux J. et al. // Phys. Rev. Lett. 1983. V. 50. P. 19; Nucl. Phys. A. 1984. V. 431. P. 613.
2. Perdrisat C. F. et al. // Phys. Rev. Lett. 1987. V. 59. P. 2840;  
Punjabi V. et al. // Phys. Rev. C. 1989. V. 39. P. 608.

3. *Cheung E. et al.* // Phys. Lett. B. 1992. V. 284. P. 210.
4. *Punjabi V. et al.* // Phys. Lett. B. 1995. V. 350. P. 178.
5. *Ableev V. G. et al.* // Pis'ma Zh. Eksp. Teor. Fiz. 1988. V. 47. P. 558; JINR Rapid Commun. 1990. No. 4[43]. P. 5.
6. *Aono T. et al.* // Phys. Rev. Lett. 1995. V. 74. P. 4997.
7. *Azhgirey L. S. et al.* // Phys. Lett. B. 1996. V. 387. P. 37.
8. *Nomofilov A. A. et al.* // Phys. Lett. B. 1994. V. 325. P. 327.
9. *Kuehn B. et al.* // Phys. Lett. B. 1994. V. 334. P. 298;  
*Azhgirey L. S. et al.* JINR Rapid Commun. 1996. No. 3[77]. P. 23.
10. *Azhgirey L. S. et al.* // Phys. Lett. B. 1997. V. 391. P. 22; Yad. Fiz. 1998. V. 61. P. 494.
11. *Afanasiev S. V. et al.* // Phys. Lett. B. 1998. V. 434. P. 21.
12. *Ladygin V. P. et al.* // Few-Body Syst. 2002. V. 32. P. 127;  
*Azhgirey L. S. et al.* // Yad. Fiz. 2003. V. 66. P. 719.
13. *Ableev V. G. et al.* // Nucl. Phys. A. 1983. V. 393. P. 491; V. 411. P. 541(E); JETP Lett. 1983. V. 37. P. 233.
14. *Azhgirey L. S. et al.* // Nucl. Phys. A. 1991. V. 528. P. 621; Yad. Fiz. 1987. V. 46. P. 1134; 1991. V. 53. P. 1591.
15. *Azhgirey L. S., Razin S. V., Yudin N. P.* // Yad. Fiz. 1987. V. 46. P. 1657.
16. *Azhgirey L. S., Ignatenko M. A., Yudin N. P.* // Z. Phys. A. 1987. V. 343. P. 35.
17. *Azhgirey L. S., Yudin N. P.* // Yad. Fiz. 1994. V. 57. P. 160.
18. *Kobushkin A. P.* // Phys. Lett. B. 1998. V. 421. P. 53.
19. *Uzikov Yu. N.* // Part. Nucl. 1998. V. 29. P. 1405.
20. *Schmidt I. A., Blankenbecler R.* // Phys. Rev. D. 1977. V. 15. P. 3321.
21. *Wong Ch.-Y., Blankenbecler R.* // Phys. Rev. C. 1980. V. 22. P. 2433.
22. *Sivers D., Brodsky S. J., Blankenbecler R.* // Phys. Rep. C. 1976. V. 23. P. 1.
23. *Dirac P. A. M.* // Rev. Mod. Phys. 1949. V. 21. P. 392.
24. *Miller G. A.* nucl-th/0301041. 2003.
25. *Jones M. K. et al.* // Phys. Rev. Lett. 2000. V. 84. P. 1398.
26. *Miller G.* // Prog. Part. Nucl. Phys. 2000. V. 45. P. 83.
27. *Carbonell J., Karmanov V. A.* // Nucl. Phys. A. 1994. V. 581. P. 625.
28. *Karmanov V. A., Smirnov A. V.* // Ibid. V. 575. P. 520.
29. *Carbonell J. et al.* // Phys. Rep. 1998. V. 300. P. 215.
30. *Azhgirey L. S., Yudin N. P.* // Yad. Fiz. 2000. V. 63. P. 2280.
31. *Lacombe M. et al.* // Phys. Lett. B. 1981. V. 101. P. 139.
32. *Machleidt R. et al.* // Phys. Rep. 1987. V. 149. P. 1.