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VECTOR MESON PHOTOPRODUCTION AND PROBLEM OF GAUGE INVARIANCE

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We discuss the problem of gauge invariance of the vector meson photoproduction at small x within the two-gluon exchange model. It is found that the gauge invariance is fulfilled if one includes the graphs with higher Fock states in the meson wave function. The obtained results are used to estimate the amplitudes with longitudinal and transverse photon and vector meson polarization.

Обсуждается проблема калибровочной инвариантности в фоторождении векторных мезонов при малых x в рамках модели двухглюонного обмена. Показано, что калибровочная инвариантность может выполняться, если учитываются графы с высшими фоковскими состояниями в мезонной волновой функции. Полученные результаты использованы для модельных оценок амплитуд с продольной и поперечной поляризацией фотона и векторного мезона.

Investigation of vector meson photoproduction at small x is a problem of considerable interest. We are interested in the low-x region, where the predominant contribution is determined by the two-gluon exchange and the vector meson is produced via the photon-two-gluon fusion. The factorization of diffractive vector meson production with longitudinally polarized photons into the hard part and parton distribution was shown in [1]. Thus, such processes, can be an excellent tool to study the generalized parton distribution [2]. Moreover, they should give important information on the vector meson wave function. The spin-density matrix elements which were studied at DESY (see [3] and references therein) should be sensitive to the vector meson wave function. To analyze spin effects in the $\gamma^* \to V$ transition, it is necessary to calculate the amplitude with transverse polarization of a vector meson. For the light meson production, this transition amplitude is not well defined because of the present end-point singularities [4]. One of the possible ways to regularize these end-point divergences is to include the transverse quark motion, as it was done, e.g., in [5–8].

Unfortunately, such higher-twist effects can result in the loss of the gauge invariance (GI) of the amplitude. In this report, we study the $\gamma^* \to V$ transition amplitude for different polarization of photon and vector meson at small x and

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check the GI of our results. The vector meson production can be described in terms of the kinematic variables which are the following:

$$q^2 = (L - L')^2 = -Q^2,\ r_P^2 = (P - P')^2 = t,\ x_P = \frac{q(P - P')}{qP},\ s = (q + P)^2, \tag{1}$$

where L, L' and P, P' are the initial and final lepton and proton momenta, respectively; Q^2 is the photon virtuality; r_P is the momentum carried by the two-gluons; x_P is the part of proton momentum carried by the two-gluon system and s is the photon–proton energy squared. The vector meson is produced by the photon-two-gluon fusion, and the momentum $V = (q + r_P)$ is on the mass shell. The x_P variable which is equivalent to skewedness ζ is determined by

$$x_P \sim \zeta \sim \frac{M_V^2 + Q^2 + |t|}{s}. \tag{2}$$

Within the two-gluon exchange model we calculate the $L \to L$, $T \to T$ and $T \to L$ amplitudes which are of importance in analyses of spin density matrix elements. In calculations the k-dependent wave function [9] is used

$$\hat{\Psi}_V = g\left[(\not{V} + M_V)\not{E}_V + \frac{2}{M_V}\not{V}\not{E}_V\not{K} - \frac{2}{M_V}(\not{V} - M_V)(E_V K)\right]\phi_V(k,\tau). \tag{3}$$

Here V is a vector meson momentum and M_V is its mass; E_V is a meson polarization vector and K is a quark transverse momentum. The first term in (3) represents the standard wave function of the vector meson. The leading twist contribution to the longitudinal vector meson polarization is determined by the $M_V\not{E}_V$ term in (3). The k-dependent terms of the wave function are essential for the transverse amplitude of the light mesons. Wave function (3) has quite a general form and can reproduce results of the most models [6–8]. The other model for the wave function which has a structure similar to (3) was considered in [10]. The GI of the vector meson production amplitude was discussed in [5,11]. It was found that the $\gamma^* \to V$ transition amplitude at zero momentum transfer should vanish as $l_\perp^2$ for $l_\perp^2 \to 0$, where $l_\perp$ is the transverse part of the gluon momentum. The importance of the higher Fock states of the wave function in GI of the vector meson production was shown in [11]. These results were obtained in the two-gluon model exchange in the Feynman gauge.

The leading term of the amplitude of diffractive vector meson production is mainly imaginary. The imaginary part of the amplitude can be written as an integral over z and $k_\perp$. The leading over s term of the $\gamma^* \to V$ amplitude has the form

$$T_{\lambda_V,\lambda_\gamma} = N\int dz \int dk_\perp^2 \frac{\mathcal{F}_\zeta^g(\zeta,t)\,\phi_V(z,k_\perp^2)\,A^{l^2}_{\lambda_V,\lambda_\gamma}(z,k_\perp^2)}{\left(k_\perp^2 + \bar{Q}^2\right)\left(k_\perp^2 + |t| + \bar{Q}^2\right)^2}, \tag{4}$$

where N is normalization; $\bar{Q}^2 = m_q^2 + z\bar{z}Q^2$, $\bar{z} = 1 - z$ and m_q is a quark mass. Generally, the numerator of the hard scattering amplitude $A_{\lambda_V,\lambda_\gamma}$ can be written as follows:

$$A_{\lambda_V,\lambda_\gamma} = A^0_{\lambda_V,\lambda_\gamma} + A^{l^2}_{\lambda_V,\lambda_\gamma}\, l_\perp^2. \tag{5}$$

Only the second term in (5) obeys the GI and appears in (4). The imaginary part of the vector meson production amplitude (4) depends on the generalized gluon distribution $\mathcal{F}^g_\zeta(X = \zeta, \ldots)$. It can be connected with the unintegrated gluon distribution G through the integration over $l_\perp$

$$\mathcal{F}^g_\zeta(\zeta, t, k_\perp^2 + \bar{Q}^2 + |t|) = \int\limits_0^{l_\perp^2 < k_\perp^2 + \bar{Q}^2 + |t|} \frac{d^2 l_\perp (l_\perp^2)}{(l_\perp^2 + \lambda^2)((\mathbf{l}_\perp + \mathbf{r}_\perp)^2 + \lambda^2)} G(l_\perp^2, \zeta, \ldots). \tag{6}$$

Here $r_\perp$ is the transverse part of the r_P momentum; λ is some effective gluon mass. The distribution $\mathcal{F}^g_0(x, 0, q_0^2)$ is normalized to $(xg(x, q_0^2))$. The $l_\perp^2$ factor in the numerator of (6) appears from the second GI term of (5).

Unfortunately, in the model with the higher twist effects like the transverse quark motion, the sum of the graphs where gluons are coupled with the quarks in the loop does not obey GI. Let us discuss this problem in detail for the $L \to L$ amplitude. The GI term of the amplitude has the form

$$A^{l^2}_{L,L} = 4\frac{s}{\sqrt{Q^2}}\left[\bar{Q}^2 + k_\perp^2(1 - 4z\bar{z}) - 2m_q M_V z\bar{z}\right]\left(\bar{Q}^2 + k_\perp^2\right). \tag{7}$$

For the gauge-dependent term (GDT) we have

$$A^0_{L,L} = 2\frac{s}{\sqrt{Q^2}}\left[k_\perp^2(1 - 4z\bar{z}) + m_q\left(m_q - 2M_V z\bar{z}\right)\right]\left(\bar{Q}^2 + k_\perp^2\right)^2. \tag{8}$$

It can be seen that in the nonrelativistic limit $z = \bar{z} = 1/2$, $m_q = M_V/2$ the GDT $A^0_{L,L}$ is equal to zero. For light quarks, when $m_q = 0$, the $A^0_{L,L}$ term is equal to zero at $k_\perp^2 = 0$. At the same time, the $A^0_{L,L}$ term has additional power of $\left(\bar{Q}^2 + k_\perp^2\right)$ that compensates one propagator in (4) with respect to the GI term (7). As a result, the GDT $A^0_{L,L}$ of amplitude (4) is similar to the contribution of the higher Fock state. Really, here one gluon is coupled directly to the wave function and one quark propagator disappears. Let us suppose that we can write the sum of GDT and the contribution of the higher Fock state in the form

$$\tilde{A}_{L,L} \sim A^0_{L,L} + B(z, k_\perp^2)\,\Phi_{q\bar{q}g}\left(1 + C\,\frac{l_\perp^2}{Q^2}\right). \tag{9}$$

Here by the $C\,l_\perp^2/Q^2$ term in (9) we estimate the higher twist contributions in the $q\bar{q}g$ term of the wave function. Let us suppose that the higher Fock term

$B(z, k_\perp^2)\,\Phi_{q\bar{q}g}$ compensates the $A^0_{L,L}$ term in (9). In this case, the contribution proportional to $l_\perp^2$ in $\tilde{A}_{L,L}$ can be estimated as

$$\tilde{A}_{L,L} \sim -C\,\frac{l_\perp^2}{Q^2}\,A^0_{L,L}. \tag{10}$$

One can see that this GDT will be suppressed with respect to the GI contribution (7) as a power of Q^2. Really,

$$\frac{\tilde{A}_{L,L}}{l_\perp^2\,A^{l^2}_{L,L}} \propto \frac{m_q^2 + k_\perp^2}{Q^2}, \tag{11}$$

and we have GI of the model at sufficiently high Q^2.

Similar calculations have been done for the amplitude with transversely polarized photons and vector mesons. The GI term of this amplitude has the form

$$A^{l^2}_{T,T} \sim \frac{2s}{M_V}\bar{Q}^2\left[k_\perp^2(1+4z\bar{z}) + M_V\left(2M_V z\bar{z} - m_q(1-4z\bar{z})\right)\right](e^\gamma_\perp e^V_\perp). \tag{12}$$

For light meson production the resulting amplitude is proportional to $k_\perp^2$. For heavy mesons, the term proportional to M_V^2 appears, too. In the transverse case, the GDT which does not vanish as $l_\perp^2$ in (5) takes place like for the longitudinal amplitude. If we suppose the same compensation of GDT as in (9), we find

$$\frac{\tilde{A}_{T,T}}{l_\perp^2\,A^{l^2}_{T,T}} \propto z\bar{z}. \tag{13}$$

This means that in the transverse case we do not find a Q^2 suppression of additional GDT, but we have only its numerical suppression. Really, it can be seen that the $T_{T,T}$ amplitude has additional divergence like $1/(z\bar{z})$ with respect to the $T_{L,L}$ amplitude (4). In the $\tilde{A}_{T,T}$ GDT the additional $z\bar{z}$ term in the numerator cancels this divergence and leads to the numerical suppression of the GDT contribution.

The GI term of the $T \to L$ transition amplitude is determined by

$$A^{l^2}_{L,T} \sim \frac{2s}{M_V}\bar{Q}^2\left[2M_V^2 z\bar{z} - k_\perp^2(1-2z)\right]\frac{(e^\gamma_\perp r_\perp)}{M_V}. \tag{14}$$

It can be found that in this case we have the numerical suppression of a possible GDT contribution like for the TT amplitude (13).

Thus, we have found that the GDT in the $\gamma^* \to V$ transition amplitudes are suppressed and one can use the GI terms (7), (12), and (14) to calculate spin-dependent amplitudes of the vector meson production. The average momentum transfer, which is used in (4), (14) is about $\langle|t|\rangle \sim 0.13$ GeV^2 [3]. The corresponding amplitudes were calculated for the k-dependent wave function (3) with the exponential form of $\phi_V(z, k_\perp^2)$ [12]

$$\phi_V(z, k_\perp^2) = H \exp\left(-\frac{k_\perp^2 b_V^2}{z\bar{z}}\right). \tag{15}$$

Here H is a normalization factor. Transverse momentum integration of (15) leads to the asymptotic form of a meson distribution amplitude $\Phi_V^{\rm AS} = 6z\bar{z}$. The model has one parameter b_V which determines the average value of $k_\perp^2$ and provides the regularization of the integrals in the end-point region. In our calculation, we use the value $b_V \sim 0.65$ GeV^{-1} which leads to a reasonable description of the σ_L cross section for ρ production [12]. Then the average $\langle k_\perp^2\rangle \sim 0.6$ GeV^2.

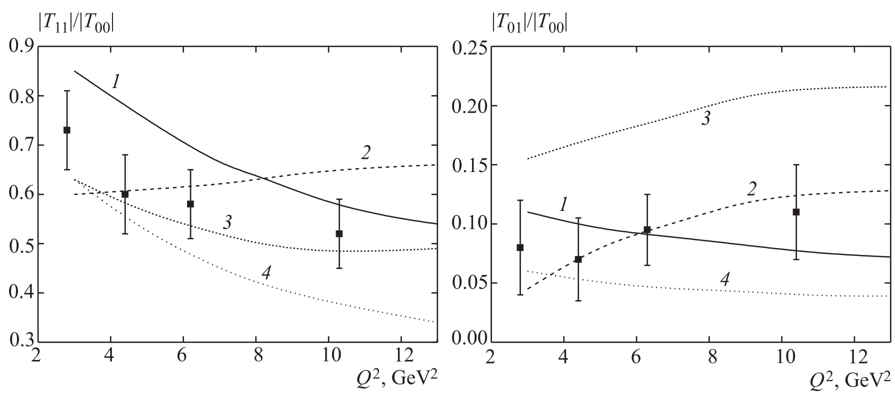

Fig. 1. Q^2 dependence of the ratio of helicity amplitudes $|T_{11}|/|T_{00}|$ [3]. *1* — our calculation; *2* — results of model [6]; *3, 4* — models [7] and [8], respectively

Fig. 2. Q^2 dependence of the ratio of helicity amplitudes $|T_{01}|/|T_{00}|$ extracted from H1 and ZEUS measurements of the spin density matrix elements in [3]. Lines are the same as in Fig. 1

The results of calculations for the ratio of helicity amplitudes $|T_{TT}|/|T_{LL}|$ are compared in Fig. 1 with the data extracted in [3] from H1 and ZEUS measurements of the spin density matrix elements. It can be seen that experimental results are reproduced by the model quite well. The model gives a reasonable description of the ratio $R = \sigma_L/\sigma_T$. The results of the models [6–8] are shown in this graph, too. We can see that all the models describe experimental data satisfactorily.

The comparison of model results for the $|T_{LT}|/|T_{LL}|$ with experiment is presented in Fig. 2. It can be seen that the T_{LT} amplitude is more sensitive to the structure of the wave function. The best description of the $|T_{LT}|/|T_{LL}|$ ratio is found in our model and in the models [6, 8]. Note that the experimental errors in the spin-density matrix elements are quite large. This does not allow us to find out which model of the wave function describes experiment data adequately.

In this report, the results of the model for the $\gamma^* \to V$ transition amplitude which considers the transverse quark motion have been analyzed. These higher twist effects regularize the end-points singularities of the amplitudes but lead in the models to violation of GI. Note that a similar problem with GI should take place in the models [5, 6]. It is found that the contribution of GDT in the model should be small. This permits us to use the model results for the GI terms of the $\gamma^* \to V$ amplitudes for numerical calculations. Our results describe experimental data on the ratio of helicity amplitudes quite well. Unfortunately, the experimental errors in DESY experiments are large and all known models describe the experimental results qualitatively. To obtain more information on the form of the vector meson wave function it is important to reduce the experimental errors. We hope that the precise analyses of spin density matrix elements can be done in the COMPASS experiment at CERN.

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REFERENCES

1. *Collins J. C., Frankfurt L., Strikman M.* // Phys. Rev. D. 1997. V. 56. P. 2982.
2. *Müller D. et al.* // Fortschr. Physik. 1994. V. 42. P. 101;
   Radyushkin A. V. // Phys. Rev. D. 1997. V. 56. P. 5524;
   Ji X. // Ibid. V. 55. P. 7114.
3. *Clerbaux B. for the H1 and ZEUS Collab.* hep-ph/9908519.
4. *Mankiewicz L., Piller G.* // Phys. Rev. D. 2000. V. 61. P. 074013.
5. *Ginzburg I. F., Ivanov D. Yu.* // Phys. Rev. D. 1996. V. 54. P. 5523.
6. *Ivanov D. Yu., Kirschner R.* // Phys. Rev. D. 1998. V. 59. P. 114026.
7. *Royen I., Cudell J.-R.* // Nucl. Phys. B. 1999. V. 545. P. 505.
8. *Ivanov I. P., Nikolaev N. N.* // JETP Lett. 1999. V. 69. P. 294;
   Kuraev E. V., Nikolaev N. N., Zakharov B. G. // JETP Lett. 1998. V. 68. P. 696.
9. *Bolz J., Körner J., Kroll P* // Z. Phys. A. 1994. V. 350. P. 145.
10. *Goloskokov S. V.* // Proc. of XV Intern. Seminar on High Energy Physics Problems «Relativistic Nuclear Physics and Quantum Chromodynamics», Dubna, Sep. 25–29, 2000; hep-ph/0012307.
11. *Hebecker A., Landshoff P.V.* // Phys. Lett. B. 1998. V. 419. P. 393.
12. *Goloskokov S. V., Kroll P., Postler B.* // Proc. of DIS03 Workshop, St. Petersburg, April 23–27, 2003 (to appear); hep-ph/0308140.