ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2004. Т. 35. ВЫП. 7

УДК 539.12: 539.171

SMALL-x BEHAVIOUR OF THE NONSINGLET AND SINGLET STRUCTURE FUNCTIONS g_1

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Explicit expressions for the nonsinglet and singlet structure functions g_1 at the small-x region are obtained. They include the total resummation of double-logarithmic contributions and accounting for the running QCD coupling effects. We predict that asymptotically the singlet $g_1 \sim x^{-\Delta_S}$, with the intercept $\Delta_S = 0.86$, which is approximately twice larger than the nonsinglet intercept $\Delta_{\mathrm{NS}} = 0.4$. The impact of the initial quark and gluon densities on the sign of g_1 at $x \ll 1$ is discussed.

Получены явные выражения для несинглетной и синглетной g_1 в области малых x. Они включают полное суммирование дважды логарифмических вкладов и учет эффектов бегущей константы КХД. Предсказывается, что асимптотически синглетная $g_1 \sim x^{-\Delta_S}$ с интерсептом $\Delta_S = 0{,}86$, примерно вдвое большим несинглетного $\Delta_{\mathrm{NS}} = 0{,}4$. Обсуждается влияние начальных кварковых и глюонных плотностей на знак g_1 при $x \ll 1$.

INTRODUCTION

Deep inelastic scattering (DIS) is one of the basic processes for investigating the structure of hadrons. As is well known, all information about the hadrons participating into DIS comes from the hadronic tensor $W_{\mu\nu}$. The imaginary part of $W_{\mu\nu}$ is proportional to the forward Compton amplitude when the deeply off-shell photon with virtuality q^2 scatters off an on-shell hadron with momentum p. For the electron-hadron DIS, the spin-dependent part, $W^s_{\mu\nu}$, of $W_{\mu\nu}$ is

$$W^s_{\mu\nu} = \imath\epsilon_{\mu\mu\lambda\rho}\frac{q_\lambda m}{pq}\left[S_\rho g_1 + \left(S_\rho - \frac{(Sq)}{pq}p_\rho\right)g_2\right] \approx \approx \imath\epsilon_{\mu\mu\lambda\rho}\frac{q_\lambda m}{pq}\left[S^{||}_\rho g_1 + S^{\perp}_\rho\big(g_1+g_2\big)\right] \quad (1)$$

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where m is the hadron mass; $S_\rho^{||}$ and $S_\rho^{\perp}$ are the longltudinal and transverse (with respect to the plane formed by p and q) components of the hadron spin S_ρ; g_1 and g_2 are the spin structure functions. Both of them depend on $x = -q^2/2pq$, $0 < x \leqslant 1$, and $Q^2 = -q^2 > 0$. Obviously, small x corresponds to $s = (p+q)^2 \approx 2pq \gg Q^2$. In this case, $S_\rho^{||} \approx p_\rho/m$ and therefore the part of $W_{\mu\nu}^s$ related to g_1 does not depend on m. Then if $Q^2 \gg m^2$, one can assume the factorization and regard $W_{\mu\nu}^s$ as a convolution of two objects (see Fig. 1). The first of them is the probability Φ ($\Phi = \Phi_q$ in Fig. 1, *a* and $\Phi = \Phi_g$ in Fig. 1, *b*) to find a polarized parton (a quark or a gluon) in the hadron. The second one is the partonic tensor $\widetilde{W}_{\mu\nu}^s$ defined and parametrized similarly to $W_{\mu\nu}^s$.

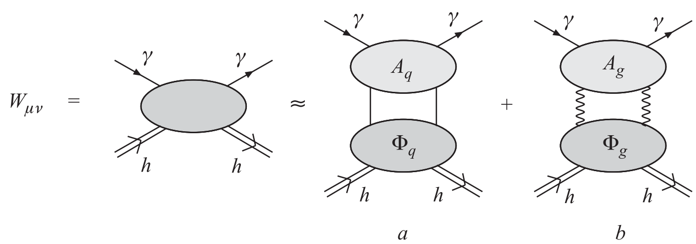


Fig. 1. Representation of the hadronic tensor as the covolution of the fragmentation functions $\Phi_{q,g}$ and the partonic tensor

Whereas $\Phi_{q,g}$ are essentially nonperturbative objects, the partonic tensor $\widetilde{W}_{\mu\nu}^s$, i.e., the partonic structure functions g_1 and g_2, can be studied within perturbative QCD. The lack of knowledge of Φ is usually compensated by introducing initial parton distributions that are found from phenomenological considerations. On the contrary, there are regular perturbative methods for calculating the structure functions in the partonic tensor $\widetilde{W}_{\mu\nu}^s$. The best known instrument to calculate the structure functions to all orders in α_s is the DGLAP [1] approach. Once applied to the description of the experimental data, DGLAP provides good results [2]. The extrapolation into the small-x region of DGLAP predicts an asymptotical behaviour $\sim \exp\left(\sqrt{C \ln(1/x) \ln\ln Q^2}\right)$ for all DIS structure functions (with different factors C). However, from the theoretical point of view, such an extrapolation is rather doubtful. In particular, it neglects in a systematical way contributions $\sim (\alpha_s \ln^2(1/x))^k$ which are small when $x \sim 1$ but become large when $x \ll 1$. The total resummation of these double-logarithmic (DL) contributions made in Refs. 3, 4 for the nonsinglet ($g_1^{\rm NS}$) and singlet g_1, respectively, leads to the Regge (power-like) asymptotics $g_1(g_1^{\rm NS}) \sim (1/x)^{\Delta^{\rm DL}}((1/x)^{\Delta^{\rm DL}_{\rm NS}})$, with $\Delta^{\rm DL}, \Delta^{\rm DL}_{\rm NS}$ being the intercepts calculated in the double-logarithmic approximation (DLA). The weakest point of Refs. 3, 4 is the fact that they keep α_s fixed (at some unknown scale). It leads therefore to the intercepts $\Delta^{\rm DL}, \Delta^{\rm DL}_{\rm NS}$ explicitly depending on this unknown coupling, whereas α_s is well known to be running. The results of Refs. 3, 4 led many authors (see, e.g., [5]) to suggest that the DGLAP parametrization $\alpha_s = \alpha_s(Q^2)$ has to be used. However, according to the results of Ref. 6, such a parametrization is correct only for $x \sim 1$ and cannot be used for $x \ll 1$. The explicit dependence of α_s suggested in Ref. 6 has been used to calculate both $g_1^{\rm NS}$ and g_1 at small x in Ref. 7. The present talk is based on the results obtained in those papers.

Instead of a direct study of g_1, it is more convenient to consider the forward Compton amplitude A related to g_1 as follows:

$$g_1(x, Q^2) = \frac{1}{\pi} \Im_s A(s, Q^2). \tag{2}$$

We cannot use DGLAP for studying g_1 or A at small x because it does not account for double-log (and single-log) contributions which are independent of Q^2. In order to account for the double-logs of both x and Q^2, we need to construct two-dimensional evolution equations that would combine the x and Q^2 evolutions. On the other hand, these equations should sum up the contributions of the Feynman graphs involved to all orders in α_s. Some of those graphs have either ultraviolet or infrared (IR) divergencies. The ultraviolet divergencies are regulated by the usual renormalization procedure. In order to regulate the IR ones, we have to introduce an IR cut-off. We use the IR cut-off μ in the transverse momentum space for momenta k_i of all virtual quarks and gluons:

$$\mu < k_{i\perp}, \tag{3}$$

where $k_{i\perp}$ stands for the transverse (with respect to the plane formed by the external momenta p and q) component of k_i. This way of regulating the IR divergencies was suggested by L. N. Lipatov and used first in Ref. 8 for quark–quark scattering. Using this cut-off μ, A acquires a dependence on μ. Therefore, one can evolve A with respect to μ, constructing thereby some Infrared Evolution Equations (IREE). As $A = A(s/\mu^2, Q^2/\mu^2)$,

$$-\mu^2 \partial A/\partial \mu^2 = \partial A/\partial \rho + \partial A/\partial y, \tag{4}$$

where $\rho = \ln(s/\mu^2)$ and $y = \ln(Q^2/\mu^2)$. Equation (4) is the l.h.s. of the IREE for A. In order to write the r.h.s. of the IREE, it is convenient to use the Sommerfeld–Watson transform

$$A(s, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (s/\mu^2)^\omega \xi(\omega) F(\omega, Q^2), \tag{5}$$

where $\xi(\omega)$ is the negative signature factor; $\xi(\omega) = [1 - \mathrm{e}^{-\imath\pi\omega}]/2 \approx \imath\pi\omega/2$. It must be noted that the transform inverse to Eq. (5) involves the imaginary parts of A:

$$F(\omega, Q^2) = \frac{2}{\pi\omega} \int_0^\infty d\rho\, \mathrm{e}^{-\rho\omega}\, \Im\, A(s, Q^2). \tag{6}$$

Notice that contrary to the amplitude A, the structure function g_1 does not have any signature and therefore $\xi(\omega) = 1$, when transform (5) is applied directly to g_1.

INFRARED EVOLUTION EQUATIONS FOR g_1

When the factorization depicted in Fig. 1 is assumed, the calculation of g_1 (we will not use the superscript «s» for g_1 singlet, though we use the notaion g_1^{NS} for the nonsinglet g_1) is reduced to calculating the Feynman graphs contributing the partonic tensor $\widetilde{W}^s_{\mu\nu}$ depicted as the upper blobs in Fig. 1. Both cases (Fig. 1, *a*), when the virtual photon scatters off the nearly on-shell polarized quark, and (Fig. 1, *b*), when the quark is replaced by the polarized gluon, should be taken into account. Therefore, in contrast to Eq. (2), we need to introduce two Compton amplitudes: A_q and A_g corresponding to the upper blob in Fig. 1, *a* and *b*, respectively. The subscripts «q» and «g» refer to the initial partons. Therefore,

$$g_1(x, Q^2) = g_q(x, Q^2) + g_q(x, Q^2), \tag{7}$$

where

$$g_q = \frac{1}{\pi} \Im_s A_q(s, Q^2), \qquad g_g = \frac{1}{\pi} \Im_s A_g(s, Q^2). \tag{8}$$

Let us now construct the IREE for the amplitudes $A_{q,g}$ related to g_1. To this aim, let us consider a virtual parton with minimal $k_\perp$. We call such a parton the softest one. If it is a gluon, its DL contribution can be factorized, i.e., its DL contribution comes from the graphs where its propagator is attached to the external lines. As the gluon propagator cannot be attached to photons, this case is absent in IREE for $A_{q,g}$. The second option is when the softest partons are a t-channel quark–antiquark or gluon pair. It leads us to the IREE depicted in Fig. 2.

Applying the operator $-\mu^2 \partial/\partial\mu^2$ to it, combining the result with Eq. (4) and using (5), we arrive at the following system of equations:

$$\begin{aligned} \left(\omega + \frac{\partial}{\partial y}\right) F_q(\omega, y) &= \frac{1}{8\pi^2} \big[F_{qq}(\omega) F_q(\omega, y) + F_{qg}(\omega) F_g(\omega, y)\big], \\ \left(\omega + \frac{\partial}{\partial y}\right) F_g(\omega, y) &= \frac{1}{8\pi^2} \big[F_{gq}(\omega) F_q(\omega, y) + F_{gg}(\omega) F_g(\omega, y)\big]. \end{aligned} \tag{9}$$

The amplitudes F_q, F_g are related to A_q, A_g through transform (5). The Mellin amplitudes F_{ik}, with $i, k = q, g$, describe the parton–parton forward scattering.

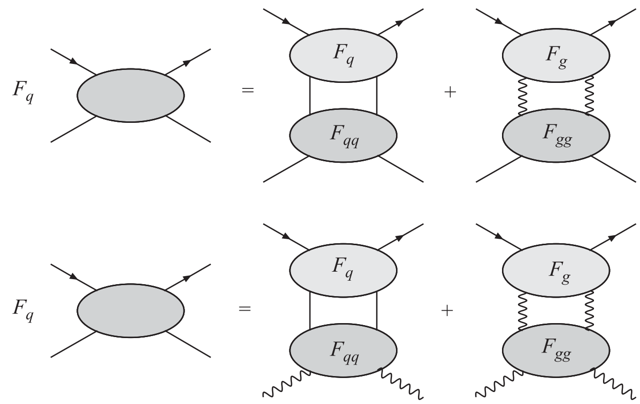


Fig. 2. Infrared evolution equations for the amplitudes A_q, A_g

They contain DL contributions to all orders in α_s. We can introduce the new anomalous dimensions $H_{ik} = (1/8\pi^2)F_{ik}$. The way of writing the subscripts «q, g» corresponds to the DGLAP notations. Solving this system of equations and using Eq. (8) leads to

$$g_q(x, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^\omega \Big[C_+(\omega)\, \mathrm{e}^{\Omega_+ y} + C_-(\omega)\, \mathrm{e}^{\Omega_- y} \Big],$$
$$g_g(x, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^\omega \left[C_+(\omega) \frac{X+\sqrt{R}}{2H_{qg}}\, \mathrm{e}^{\Omega_+ y} + C_-(\omega) \frac{X-\sqrt{R}}{2H_{qg}}\, \mathrm{e}^{\Omega_- y} \right]. \tag{10}$$

The unknown factors $C_\pm(\omega)$ have to be specified. All other factors in Eq. (10) can be expressed in terms of H_{ik}:

$$X = H_{gg} - H_{qq}, \quad R = (H_{gg} - H_{qq})^2 + 4H_{qg}H_{gq},$$
$$\Omega_\pm = \frac{1}{2}\left[H_{qq} + H_{gg} \pm \sqrt{(H_{qq} - H_{gg})^2 + 4H_{qg}H_{gq}} \right]. \tag{11}$$

The anomalous dimension matrix H_{ik} was calculated in Ref. 7:

$$H_{gg} = \frac{1}{2}\Big(\omega + Y + \frac{b_{qq} - b_{gg}}{Y}\Big), \qquad H_{qq} = \frac{1}{2}\Big(\omega + Y - \frac{b_{qq} - b_{gg}}{Y}\Big),$$
$$H_{gq} = -\frac{b_{gq}}{Y}, \qquad H_{qg} = -\frac{b_{qg}}{Y}, \tag{12}$$

where

$$Y = \\ = -\sqrt{\Big(\omega^2 - 2(b_{qq}+b_{gg}) + \sqrt{[(\omega^2 - 2(b_{qq}+b_{gg}))^2 - 4(b_{qq}-b_{gg})^2 - 16 b_{qg} b_{gq}]}\Big)/2}, \tag{13}$$

$$b_{ik} = a_{ik} + V_{ik}, \tag{14}$$

$$a_{qq} = \frac{A(\omega)C_F}{2\pi},\ a_{gg} = \frac{2A(\omega)N}{\pi},\ a_{gq} = -\frac{n_f A'(\omega)}{2\pi},\ a_{qg} = \frac{A'(\omega)C_F}{\pi}, \tag{15}$$

and

$$V_{ik} = \frac{m_{ik}}{\pi^2} D(\omega), \tag{16}$$

with

$$m_{qq} = \frac{C_F}{2N},\ m_{gg} = -2N^2,\ m_{gq} = n_f \frac{N}{2},\ m_{qg} = -NC_F. \tag{17}$$

We have used here the notations $C_F = 4/3, N = 3$ and $n_f = 4$. The factors A and D account for running α_s. They are given by the following expressions:

$$A(\omega) = \frac{1}{b}\Big[\frac{\eta}{\eta^2 + \pi^2} - \int_0^\infty \frac{d\rho\, \mathrm{e}^{-\omega\rho}}{(\rho+\eta)^2 + \pi^2}\Big], \tag{18}$$

$$D(\omega) = \frac{1}{2b^2}\int_0^\infty d\rho\, \mathrm{e}^{-\omega\rho} \ln\big((\rho+\eta)/\eta\big)\Big[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} + \frac{1}{\rho+\eta}\Big] \tag{19}$$

with $\eta = \ln(\mu^2/\Lambda^2_{\rm QCD})$ and $b = (33 - 2n_f)/12\pi$. A' is defined as A with the π^2 term dropped out. Now we can specify the coefficients $C_\pm$ of Eq. (10). When $Q^2 = \mu^2$,

$$g_q = \tilde{\Delta} q(x_0), \qquad g_g = \tilde{\Delta} g(x_0), \tag{20}$$

where $\tilde{\Delta} q(x_0)$ and $\tilde{\Delta} g(x_0)$ are the input distributions of the polarized partons at $x_0 = \mu^2/s$. They do not depend on Q^2. Using Eq. (20) allows one to express $C_\pm(\omega)$ in terms of $\Delta q(\omega)$ and $\Delta g(\omega)$, which are related to $\tilde{\Delta} q(x_0)$ and $\tilde{\Delta} g(x_0)$ through the ordinary Mellin transform. Indeed,

$$C_+ + C_- = \Delta q, \quad C_+ \frac{X + \sqrt{R}}{2H_{qg}} + C_- \frac{X - \sqrt{R}}{2H_{qg}} = \Delta g. \tag{21}$$

This leads to the following expressions for g_q and g_g:

$$g_q(x, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^\omega \Big[\big(A^{(-)}\Delta q + B\Delta g\big)\, \mathrm{e}^{\Omega_+ y} + \\ + \big(A^{(+)}\Delta q - B\Delta g\big)\, \mathrm{e}^{\Omega_- y}\Big], \tag{22}$$

$$g_g(x, Q^2) = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^{\omega} \Big[\big(E\Delta q + A^{(+)}\Delta g \big)\, \mathrm{e}^{\Omega_+ y} + \\ + \big(-E\Delta q + A^{(-)}\Delta g \big)\, \mathrm{e}^{\Omega_- y} \Big] \quad (23)$$

with

$$A^{(\pm)} = \Big(\frac{1}{2} \pm \frac{X}{2\sqrt{R}} \Big),\ B = \frac{H_{qg}}{\sqrt{R}},\ E = \frac{H_{gq}}{\sqrt{R}}. \quad (24)$$

Equations (22), (23) express g_1 in terms of the parton distributions $\Delta q(\omega)$ and $\Delta g(\omega)$. However, they are related to the distributions $\tilde{\Delta} q(x_0)$ and $\tilde{\Delta} g(x_0)$ at very low x: $x_0 \approx \mu^2/s \ll 1$. Therefore, they scarcely can be found from experimental data. It is much more useful to express g_q, g_g in terms of the initial parton densities $\tilde{\delta} q$ and $\tilde{\delta} g$ defined at $x \sim 1$. We can do it, using the evolution of $\tilde{\Delta} q(x_0)$, $\tilde{\Delta} g(x_0)$ with respect to s. Indeed, the s evolution of $\tilde{\delta} q$, $\tilde{\delta} q$ from $s \approx \mu^2$ to $s \gg \mu^2$ at fixed Q^2 ($Q^2 = \mu^2$) is equivalent to their x evolution from $x \sim 1$ to $x \ll 1$. In the ω space, the system of IREE for the parton distributions looks quite similar to Eqs. (9). However, the equations for Δq, Δg are algebraic because they do not depend on Q^2:

$$\begin{aligned} \Delta q(\omega) &= (\langle e_q^2 \rangle/2)\delta q(\omega) + (1/\omega)\left[H_{qq}(\omega)\Delta q(\omega) + H_{qg}(\omega)\Delta g(\omega) \right], \\ \Delta g(\omega) &= (\langle e_q^2 \rangle/2)\hat{\delta g}(\omega) + (1/\omega)\left[H_{gq}(\omega)\Delta q(\omega) + H_{gg}(\omega)\Delta g(\omega) \right], \end{aligned} \quad (25)$$

where $\langle e_q^2 \rangle$ is the sum of the quark electric charges ($\langle e_q^2 \rangle = 10/9$ for $n_f = 4$); δq is the sum of the initial quark and antiquark densities and $\hat{\delta g} \equiv -(A'(\omega)/2\pi\omega^2)\delta g$ is the starting point of the evolution of the gluon density δg. It corresponds to Fig. 1, *b* where the upper blob is substituted by the quark box. Solving Eqs. (25), we obtain

$$\Delta q = (\langle e_q^2 \rangle/2) \frac{\big[\omega(\omega - H_{gg})\delta q + \omega H_{qg}\hat{\delta g} \big]}{\big[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq}) \big]}, \quad (26)$$

$$\Delta g = (\langle e_q^2 \rangle/2) \frac{\big[\omega H_{gq}\delta q + \omega(\omega - H_{qq})\hat{\delta g} \big]}{\big[\omega^2 - \omega(H_{qq} + H_{gg}) + (H_{qq}H_{gg} - H_{qg}H_{gq}) \big]}. \quad (27)$$

Then Eqs. (22), (23), (26), (27) express g_1 in terms of the initial parton densities $\delta q, \delta g$.

When we put $H_{qg} = H_{gq} = H_{gg} = 0$ and do not sum over e_q, we arrive at the expression for the nonsinglet structure function g_1^{NS}: Obviously, in this case $A^{(+)} = B = E = \Omega_- = 0$, $A^{(-)} = 1$, $\Omega_+ = H_{qq}$. However, the nonsinglet anomalous dimension H_{qq} should be calculated in the limit $b_{gg} = b_{qg} = b_{gq} = 0$. We denote such $H_{qq} \equiv H^{\mathrm{NS}}$. The explicit expression for it is

$$H^{\mathrm{NS}} = (1/2)\Big[\omega - \sqrt{\omega^2 - 4b_{qq}} \Big]. \quad (28)$$

Therefore, we arrive at

$$g_1^{\mathrm{NS}} = \frac{e_q^2}{2}\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}\Big(\frac{\omega\delta q}{\omega - H^{\mathrm{NS}}}\Big)\big(1/x\big)^{\omega}\big(Q^2/\mu^2\big)^{H^{\mathrm{NS}}}. \tag{29}$$

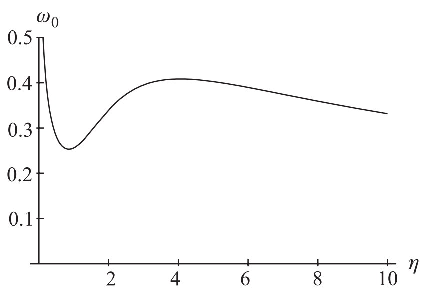
SMALL-x ASYMPTOTICS FOR g_1

When $x \to 0$ and $Q^2 \gg \mu^2$, one can neglect contributions with Ω_- in Eqs. (10). As is known, $g_1 \sim (1/x)^{\omega_0}$ at $x \to 0$, with ω_0 being the position of the leading singularity of the integrand of g_1. According to Eqs. (12), the leading singularity, ω^{NS} for g_1^{NS} is the rightmost root of the equation

$$\omega^2 - 4b_{qq} = 0 \tag{30}$$

while the leading singularity, ω_0 for g_1 is the rightmost root of

$$\omega^4 - 4(b_{qq} + b_{gg})\omega^2 + 16(b_{qq}b_{gg} - b_{qg}b_{gq}) = 0. \tag{31}$$



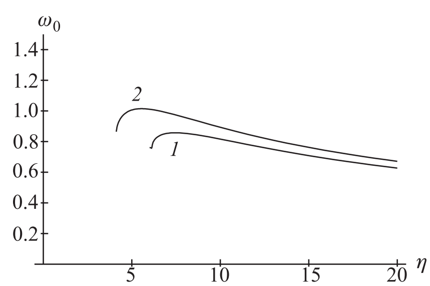


Fig. 3. Dependence of the intercept ω_0 on infrared cut-off $\eta = \ln(\mu^2/\Lambda_{\mathrm{QCD}})$ for g_1^{NS}

Fig. 4. Dependence on η of the rightmost root of Eq. (31), ω_0. Curve *2* corresponds to the case when gluon contributions only are taken into account; curve *1* is the result of accounting for both gluon and quark contributions

In our approach, all factors b_{ik} depend on $\eta = \ln(\mu^2/\Lambda_{\mathrm{QCD}})$, so the roots of Eqs. (30), (31) also depend on η. This dependence is plotted in Fig. 3 for ω^{NS} and in Fig. 4 for ω_0. Both the curve in Fig. 3 and curve *1* in Fig. 4 have a maximum. We denote this maximum as the intercept. Therefore,

$$\begin{aligned} g_1^{\mathrm{NS}} &\sim (e_q^2/2)\delta q\left(\frac{1}{x}\right)^{\Delta_{\mathrm{NS}}}\left(\frac{Q^2}{\mu^2}\right)^{\Delta_{\mathrm{NS}}/2}, \\ g_1 &\sim (1/2)[Z_1\delta q + Z_2\delta_g]\left(\frac{1}{x}\right)^{\Delta_S}\left(\frac{Q^2}{\mu^2}\right)^{\Delta_S/2}, \end{aligned} \tag{32}$$

and we find for the intercepts

$$\Delta_{\mathrm{NS}} \approx 0.4, \qquad \Delta_S \approx 0.86 \tag{33}$$

and $Z_1 = -1.2, \quad Z_2 = -0.08$. This implies that g_1^{NS} is positive when $x \to 0$, whereas g_1 can be either positive or negative, depending on the relation between δq and δg. In particular, g_1 is positive when

$$15\,\delta q + \delta g < 0. \tag{34}$$

otherwise it is negative. In other words, the sign of g_1 at small x can be positive if the initial gluon density is negative and large.

CONCLUSION

The total resummation of the most singular ($\sim \alpha_s^n/\omega^{2n+1}$) terms in the expressions for the anomalous dimensions and the coefficient functions leads to the expressions of Eqs. (7), (22), (23), (29) for the singlet and the nonsinglet structure functions g_1. It guarantees the Regge (power-like) behaviour (32) of $g_1,\ g_1^{\mathrm{NS}}$ when $x \to 0$, with the intercepts given by Eq. (33). The intercepts $\Delta_{\mathrm{NS}}, \Delta_S$ are of course obtained with the running QCD coupling effects taken into account. The value of the nonsinglet intercept $\Delta_{\mathrm{NS}} = 0.4$ is now confirmed by several independent analyses [10] of experimental data. The value $\Delta_S = 0.86$ of the singlet intercept is in good agreement with the estimate $\Delta_S = 0.88$ obtained in Ref. 11 from analysis of the HERMES data.

Another interesting point to discuss is the sign of these structure functions. Equation (29) states that g_1^{NS} is positive both at $x \sim 1$ and at $x \ll 1$. The situation concerning the singlet g_1 is more involved: being positive at $x \sim 1$, the singlet g_1 can remain positive at $x \ll 1$ only if the initial parton densities obey Eq. (34), otherwise it becomes negative.

Acknowledgements. One of us (B. I. Ermolaev) is grateful to the Organizing Commettee of the Workshop for their hospitality. The work is supported by grants POCTI/FNU/49523/2002 and RSGSS-1124.2003.2

REFERENCES

1. *Altarelli G., Parisi G.* // Nucl. Phys. B. 1977. V. 126. P. 298;
Gribov V. N., Lipatov L. N. // Sov. J. Nucl. Phys. 1978. V. 15. P. 438; 675;
Dokshitzer Yu. L. // Sov. Phys. JETP. 1977. V. 46. P. 641.
2. *Altarelli G. et al.* // Acta Phys. Polon. B. 1998. V. 29. P. 1201; Nucl. Phys. B. 1999. V. 496. P. 337.
3. *Ermolaev B. I., Manaenkov S. I., Ryskin M. G.* // Z. Phys. C. 1996. V. 69. P. 259;
Bartels J., Ermolaev B. I., Ryskin M. G. // Ibid. V. 70. P. 273.

4. *Bartels J., Ermolaev B. I., Ryskin M. G.* // Z. Phys. C. 1996. V. 72. P. 627.
5. *Kwiecinski J.* // Acta Phys. Polon. B. 2001. V. 29. P. 1201;
Kotlorz D., Kotlorz A. // Ibid. V. 32. P. 2883;
Badalek B., Kiryluk J., Kwiecinski J. // Phys. Rev. D. 2000. V. 61. P. 014009;
Kwiecinski J., Ziaja B. // Phys. Rev. D. 1999. V. 60. P. 9802386;
Ziaja B. // Phys. Rev. D. 2002. V. 66. P. 114017; hep-ph/0304268.
6. *Ermolaev B. I., Greco M., Troyan S. I.* // Phys. Lett. B. 2001. V. 522. P. 57.
7. *Ermolaev B. I., Greco M., Troyan S. I.* // Nucl. Phys. B. 2001. V. 594. P. 71; 2000. V. 571. P. 137; hep-ph/0307128.
8. *Kirschner R., Lipatov L. N.* // Sov. Phys. JETP. 1982. V. 56. P. 266.
9. *Gribov V. N. et al.* // Sov. J. Nucl. Phys. 1968. V. 6. P. 95; 262.
10. *Soffer J., Teryaev O. V.* // Phys. Rev. D. 1997. V. 56. P. 1549;
Kataev A. L., Parente G., Sidorov A. V. CERN-TH-2001-058; Part. Nucl. 2003. V. 34. P. 20; Fiz. Elem. Chast. Atom. Yadra. 2003. V. 34. P. 43; Nucl. Phys. A. 2000. V. 666. P. 184;
Kotikov A. V. et al. // Eur. Phys. J. C. 2002. V. 26. P. 51;
Krivokhijine V. G., Kotikov A. V. hep-ph/0108224;
Kotikov A. V., Peshekhonov D. V. hep-ph/0110229.
11. *Kochelev N. I. et al.* // Phys. Rev. D. 2003. V. 67. P. 074014.