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PAIR CORRELATIONS AND THE TIME-DEPENDENT HARTREE–FOCK–BOGOLIUBOV METHOD IN THE THEORY OF NUCLEAR STRUCTURE

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It is shown that an idea of the superfluidity of nuclear matter and the u–v transformation of Bogoliubov, time-dependent Hartree–Fock–Bogoliubov method have made a strong impact on the development of the modern nuclear theory. Some applications of the Bogoliubov methods to describe properties of the low-lying collective nuclear states and the giant resonances are demonstrated.

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INTRODUCTION

Several ideas related to the name of N. N. Bogoliubov have made a strong impact on the development of nuclear theory. They are:

- the idea of superfluidity of nuclear matter;
- u–v transformation;
- time-dependent Hartree–Fock–Bogoliubov method;
- idea of broken symmetries of the self-consistent mean field;
- conception of quasi-averages.

In 1958 N. N. Bogoliubov [1] was the first to indicate a possibility of superfluidity of nuclear matter. Then A. Bohr, B. Mottelson and D. Pines [2] formulated a problem of existence of the superfluid state of atomic nuclei. The theory of pair correlations of superfluid type in atomic nuclei has been developed independently by S. T. Belyaev and V. G. Soloviev.

From that time the theory of pair correlations not only explains many nuclear properties which have not been understood before. That was a beginning of the modern stage in the development of the nuclear theory — microscopic approach to the nuclear structure. Due to simplicity of the u–v Bogoliubov transformation, this theoretical technique was used practically by all theoreticians and experimental groups for interpretation of the experimental data.

The interest in pair correlations of nucleons in atomic nuclei was recreated in connection with studies of the properties of nuclei far from the stability valley. With the new radioactive beam facilities, nuclides far from the beta stability are

now produced in an amount sufficiently large for detailed experimental studies. It became possible for the first time to investigate nuclear properties at large charge asymmetry and weak binding. There have been observed halo nuclei. Their study has shown that the properties of halo nuclei are quite different from the properties of stable nuclei. The existence of halo nuclei points to the necessity to revise a traditional approach to nuclear structure.

1. PAIRING IN NUCLEI NEAR DRIP-LINE

The decrease of separation energy enhances considerably the coupling between bound and continuum configurations. As a result, nuclei change from the well-bound to the open quantum systems. These changes are more dramatic for the valence particles and just between them act residual pairing forces.

In the well-bound nuclei, pairing only slightly influences the global properties. In the case of the weakly bound nuclei, binding properties are determined in a significant way by the interaction among a few valence nucleons. The main reason for this changing is the strong enhancement of the isospin effects.

In stable nuclei, pairing contributes only to description of the spectroscopic details: excitation energies of the low-lying states, spectroscopic factors for the one-nucleon transfer reactions, electromagnetic moments. The situation is different for nuclei near drip-line. In this case the existence of a nucleus as a bound system depends crucially on pairing. For instance, in the case of ^{6}He and ^{11}Li, removing one neutron leads to particle unstable nuclei. Thus, when one neutron is added to these nuclei the additional pairing gives enough energy to lower the total energy below the threshold for particle emission. Mainly it is the pairing interaction between the last two neutrons. However, the interaction of the last two neutrons with the core continues to be important.

2. PAIRING IN THE CONTINUUM

Pairing is the most important component of the residual forces acting among valence nucleons. It is indicated by the free nucleon–nucleon interaction that the strong correlations in the singlet even–even channel support the formation of $S = 0$, $T = 1$ pairs. In the case of the drip-line nuclei it is necessary to take into account pairing in the continuum. In this case it is more suitable to base a consideration on the equations including Bogoliubov's anomalous averages given in the coordinate space (Gorkov's equations):

$$(h - l_+)\,\Phi_+ - \Delta\Phi_- = 0,$$
$$(h - l_-)\,\Phi_- - \Delta^+\Phi_+ = 0,$$

where $l = \lambda \pm E$; λ is the chemical potential; E is the quasiparticle energy, $\Phi_{+(-)}$ is a particle (hole) wave function. It is an important property of this system of equations that hole-like component decreases exponentially at large distances providing that system is particle stable ($\lambda < 0$).

3. DENSITY DEPENDENCE OF THE PAIRING INTERACTION

The physical criterium to define the interaction in the far exterior is that it produces a large scattering length. However, the interaction determined in this way is too strong in the nuclear interior. The problem is solved by an introduction of the phenomenological density dependence of the pairing interaction:

$$V_{\text{eff}}(\mathbf{r}_1, \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[V_0 + V_\rho \left(\frac{\rho((\mathbf{r}_1 + \mathbf{r}_2)/2)}{\rho_0} \right)^P \right],$$

where ρ is the nuclear density, $\rho_0 = 0.16$ fm^{-3};

$$V_0 \left(\frac{m\omega_0}{2\pi\hbar} \right)^{3/2} \approx -14 \text{ MeV}, \quad \hbar\omega_0 = 16 \text{ MeV},$$
$$V_\rho \approx -930 \text{ MeV} \cdot \text{fm}^3, \quad P \approx 1.2.$$

The density dependence of the pairing was confirmed by the experimental data. A significant increase of the pairing up to two times was found in nuclei when they approach a proton drip-line having the fixed number of neutrons.

Averaged over a large number of nuclei near the valley of β stability, the energy gap $\bar{\triangle}$ is approximated by the expression

$$\bar{\triangle} = \frac{12}{\sqrt{A}} \text{ MeV}.$$

However, this expression does not describe pairing in unstable nuclei.

The attempts to use a more detailed dependence of $\bar{\triangle}$ on neutron excess $I = (N - Z)/A$ in addition to the mass number dependence

$$\bar{\triangle} = \frac{C\left(1 - dI^2\right)}{A^{1/3}}$$

with $C = 7.2$ MeV, $d = 6.11$ were not successful. This parametrization underestimates essentially an increase of the proton energy gap for nuclei close to proton-drip line.

The empirical increase of the proton pairing near the proton-drip line can be described by introduction of the density dependence of the pairing forces,

resulting in an essential difference between the values of the pairing amplitudes inside the nucleus and in an external region [3]. The pairing interaction defined as the second derivative of the Lagrangian with respect to the anomalous density depends linearly on the density

$$F(\bar{R}) = C_0 \left(a + b \frac{\rho(\bar{R})}{\rho_0} \right), \quad C_0 = 300 \text{ MeV} \cdot \text{fm}^3, \quad a = -1.26, \quad b = 1.42,$$

$\bar{\triangle}_{p,i}$, which is a diagonal matrix element of the pairing function in the single particle state i, is obtained by averaging over the volume of the nucleus

$$\bar{\triangle}_{p,i} = \int d^3 R \Delta^p(\bar{R}) \left| \Psi_i(\bar{R}) \right|^2 .$$

In heavy nuclei single particle levers close to Fermi surface have predominantly large quantum numbers, and, therefore, they are located near the nuclear surface. Hence, the value of $\bar{\triangle}^p$ is most *sensitive* to $\bar{\triangle}_{p,i}(\bar{R})$ in the *periphery* region of a nucleus. When one approaches the proton-drip line, a distance at which a contribution of the single particle states close to Fermi surface has a maximum increases stronger than the radius of the total nuclear density, which is determined mostly by a contribution of deeper states. Because of the density dependence of the pairing interaction, the role of the attractive term is increased and that of the repulsive term is *weakened*. This leads to the increase of the proton pairing energy $\bar{\triangle}^p$.

Modern nuclear structure theory is rapidly expanding from the description of phenomena in stable nuclei towards regions of exotic short-lived nuclei far from stability. It goes not only for light but also for heavy nuclei.

The Hartree–Fock–Bogoliubov method is a reliable tool for a microscopic self-consistent description of nuclei which can be based now on the modern Energy Density Functionals.

The properties of nuclei on the way from stable to drip line can change strongly with variation of the particle number. This requires a restoration, at least partial, of the symmetries broken by the mean field. In the case of pairing it means that the particle number conservation violated in the standard BCS wave function should be restored using the projection technique

$$\mid \Psi\rangle \equiv P^N \mid \Phi\rangle = \frac{1}{2\pi} \int_0^{2\pi} d\Phi \, \mathrm{e}^{i\Phi(\hat{N}-N)} \mid \Phi\rangle,$$

$$E^N(\rho, k) = \frac{\langle \Phi \mid H P^N \mid \Phi \rangle}{\langle \Phi \mid P^N \mid \Phi \rangle} = \frac{\int d\Phi \langle \Phi \mid H \, \mathrm{e}^{i\Phi(\hat{N}-N)} \mid \Phi \rangle}{\int d\Phi \langle \Phi \mid \mathrm{e}^{i\Phi(\hat{N}-N)} \mid \Phi \rangle},$$

where ρ is the particle-hole density and k is the pairing density.

4. TIME-DEPENDENT HFB METHOD IN NUCLEAR THEORY

At present it is difficult to imagine theoretical nuclear physics without such a notion as the self-consistent mean field. Numerous experimental data point out that the nucleons in the nucleus behave in a certain approximation as independent particles moving in a common potential well. However, this potential well fluctuates in time.

Owing to this fact it was reasonable to construct the nuclear theory, at least the theory of the low-lying excited states of nuclei basing on the concept of the self-consistent field plus pairing.

The main equations of the method were published by Bogoliubov in 1959.

The total Hamiltonian of the system taken in a general form is

$$H = \sum_{f,f'} T(f,f') a^+_f a_{f'} - \frac{1}{4} \sum_{f_1,f_2,f'_1,f'_2} G(f_1 f_2; f'_1 f'_2) a^+_{f_1} a^+_{f_2} a_{f'_2} a_{f'_1}.$$

The basic equations have been derived for the following quantities:

$$F(f_1,f_2) \equiv \langle a^+_{f_1} a_{f_2} \rangle$$

and

$$\Phi(f_1,f_2) \equiv \langle a_{f_1} a_{f_2} \rangle,$$

where the averaging is performed over the ground state of the system.

One can derive from the equations of motion the following exact equations:

$$i\frac{\partial}{\partial t} F(f_1,f_2) = \langle [a^+_{f_1} a_{f_2}, H] \rangle \equiv \mathcal{B}(f_1,f_2),$$
$$i\frac{\partial}{\partial t} \Phi(f_1,f_2) = \langle [a_{f_1} a_{f_2}, H] \rangle \equiv \mathcal{U}(f_1,f_2).$$

In the self-consistent field method, $\mathcal{B}(f_1,f_2)$ and $\mathcal{U}(f_1,f_2)$ can be expressed in terms of $F(f_1,f_2)$ and $\Phi(f_1,f_2)$.

To investigate the spectrum of the elementary excitations due to small deviations from the ground state, we should consider small additions to the stationary solutions F_0 and Φ_0:

$$F(f,f') = F_0(f,f') + \delta F(f,f'),$$
$$\Phi(f,f') = \Phi_0(f,f') + \delta\Phi(f,f'),$$
$$i\frac{\partial}{\partial t}\delta F(f,f') = \delta\mathcal{B}(f,f'), \quad i\frac{\partial}{\partial t}\delta\Phi(f,f') = \delta\mathcal{U}(f,f').$$

Finally, the basic equations have been derived for the amplitudes $R^{(\mp)}(f_1, f_2)$ which are linearly related to δF and $\delta\Phi$. They are

$$\omega R^{(\mp)}(f_1, f_2) = (\mathcal{E}(f_1) + \mathcal{E}(f_2))\, R^{(\pm)}(f_1, f_2) - \\ - \sum_{f_1', f_2'} G^{\xi}(f_1 f_2; f_2' f_1') v^{(\pm)}_{f_1 f_2} v^{(\pm)}_{f_1' f_2'} R^{(\pm)}(f_1', f_2') - \\ - 2 \sum_{f_1', f_2'} G^{\omega}(f_1 f_2; f_2' f_1') u^{(\pm)}_{f_1 f_2} u^{(\pm)}_{f_1' f_2'} R^{(\pm)}(f_1', f_2'),$$

where $u^{(\pm)}_{ff'}$ and $v^{(\pm)}_{ff'}$ are related to the u–v Bogoliubov transformation. In this general form the equations are used up to now.

In studying the properties of the low-lying states of nuclei, we should have in mind that the residual interaction is used for different moments of the colliding particles.

Some collective effects associated with quadrupole, octupole and other correlations in the particle-hole channel are defined by the interaction with small-momentum transfer $G^{\omega}(f_1 f_2; f_2' 1 f_1' 2)$.

The other effects associated with the superconducting type pairing correlations are defined by the interaction with small total momentum of the colliding particles $G^{\xi}(f_1 f_2; f_1' f_2')$.

From the general equation the equations for the special type of the nuclear excitations have been derived by Soloviev and coworkers [4]:

- multipole isoscalar and isovector excitations;
- spin-multipole excitations;
- pairing-vibrational excitations;
- charge-exchange-type collective excitations.

It was also shown [5] that basic equations of the theory of the finite Fermi systems by Migdal [6] can be derived from Bogoliubov's equations.

The equations of the Migdal approach are written for the effective fields $V^{(\pm)}(f_1 f_2)$ and $d^{(\pm)}(f_1 f_2)$ generated in the atomic nucleus by some external field $V_0^{(\pm)}(f_1 f_2)$:

$$V^{(\pm)}(f_1 f_2) = V_0^{(\pm)}(f_1 f_2) + 2 \sum_{f_1', f_2'} G^{\omega}(f_1 f_2; f_2' f_1') \frac{u^{(\pm)}_{f_1' f_2'}}{(\mathcal{E}(f_1') + \mathcal{E}(f_2'))^2 - \omega^2} \times \\ \times \Big\{ (\mathcal{E}(f_1') + \mathcal{E}(f_2')) \left[u^{(\pm)}_{f_1' f_2'} V^{(\pm)}(f_1' f_2') + v^{(\pm)}_{f_1' f_2'} d^{(\pm)}(f_1' f_2') \right] + \\ + \omega \left[u^{(\mp)}_{f_1' f_2'} V^{(\mp)}(f_1' f_2') + v^{(\mp)}_{f_1' f_2'} d^{(\mp)}(f_1' f_2') \right] \Big\},$$

$$d^{(\pm)}(f_1 f_2) = \sum_{f_1', f_2'} G^{\xi}(f_1 f_2; f_2' f_1') \frac{v^{(\pm)}_{f_1' f_2'}}{\left(\mathcal{E}(f_1') + \mathcal{E}(f_2')\right)^2 - \omega^2} \times$$
$$\times \left\{ \left(\mathcal{E}(f_1') + \mathcal{E}(f_2')\right) \left[u^{(\pm)}_{f_1' f_2'} V^{(\pm)}(f_1' f_2') + v^{(\pm)}_{f_1' f_2'} d^{(\pm)}(f_1' f_2') \right] + \right.$$
$$\left. + \omega \left[u^{(\mp)}_{f_1' f_2'} V^{(\mp)}(f_1' f_2') + v^{(\mp)}_{f_1' f_2'} d^{(\mp)}(f_1' f_2') \right] \right\}.$$

The linearized time-dependent HFB method was at first applied to calculate the properties of the low-lying excitations using the phenomenological and rather simple separable residual nucleon–nucleon interactions. This is the basic ingredient of the quasiparticle-phonon model by Soloviev et al. The practical advantage of this approach is that it allows one to calculate nuclear excitations in a very large configurational space. On this basis, very detailed predictions have been done for nuclei away from the closed shells. However, in this approach, descriptions of the ground and excited states are completely decoupled.

There exists another approach based on the Skyrme-type or Gogny interactions, which describes, throughout the periodic tables, the ground states in the framework of the HFB approximation and the excited states in RPA. However, a numerical application of this approach to description of nuclei removed from the closed shells meets serious difficulties.

This was a motivation to propose the finite rank approximation for the p–h interaction resulting from Skyrme-type forces. This program was realized and applied to calculate the properties of nuclei with open shells.

The various properties of vibrational nuclear states were successfully described in RPA which takes partly into account ground-state correlations.

The analysis of the properties of the vibrational states revealed that, by treating ground-state correlations more consistently than in the RPA, one can improve, for example, a description of transitional change densities of the vibrational states within the internal region of spherical nuclei.

For the 2_1^+ states in Zn isotopes the results of the calculations [7] of the transitional charge density have been improved owing to a more correct treatment of the Pauli exclusion principle. This leads to an approximately 20% suppression of the amplitude of the intrinsic peak of the transition charge density compared to that in the RPA and in a better agreement with the experimental data.

The linearized time-dependent Hartree–Fock–Bogoliubov method leads at the first step to the harmonic approximation. Close to the ground state fermion particle-hole excitations behave as bosons. The terms which, in the equation of motion, are related to the nonbosonic contributions of the commutation relations of pairs of fermions have random phase leading to cancelations which reduce the contributions of the corresponding terms justifying the harmonic approximation. However, although collective vibrations display small overlaps with each of the

particle-hole components of the wave function describing the mode, a certain amount of overcounting is unavoidable.

The discovery of the double giant dipole resonance (DGDR) in nuclei and the observation of small deviations from the harmonic picture concerning the excitation energy and the spreading width, combined with the large (up to a factor 2–3 enhancement) deviations of the associated Coulomb excitation cross sections measured in relativistic heavy ion collisions, call for a better understanding of the role played by the anharmonic effects in the spectrum of the DGDR.

To study the anharmonic properties of two-phonon excited states with a total spin J and its projection M, one can describe them by the wave function [8]

$$\mid \Psi^{\nu}_{JM}\rangle = \left\{ \sum_{i_2,i'_2} \frac{D^{\nu_2}_{i_2 i'_2} Q^{+}_{1^- i_2} Q^{+}_{1^- i'_2}}{\sqrt{1+\delta_{i_2,i'_2}}} + \right.$$

$$\left. + \sum_{\alpha_3\beta_3\gamma_3} \frac{T^{\nu_2}_{\alpha_3\beta_3\gamma_3} Q^{+}_{\alpha_3} Q^{+}_{\beta_3} Q^{+}_{\gamma_3}}{\sqrt{1+\delta_{\alpha_3,\beta_3}+\delta_{\alpha_3,\gamma_3}+\delta_{\beta_3,\gamma_3}+2\delta_{\alpha_3,\beta_3,\gamma_3}}} \right\} |\rangle_{\rm ph},$$

where Q^{+} is the phonon creation operator and $|\rangle_{\rm ph}$ is the phonon vacuum. With greek letters we denote the set of quantum numbers $\{\lambda, i\}$, where i is the RPA root number, while λ is the multipolarity of the mode. The ansatz is made that any combination $\alpha_n, \beta_n, \gamma_n$ appears in the sums only once.

We do not include a one-phonon term because the anharmonicity effects due to the coupling of 1p1h- to 2p2h-configurations are very small in the case of the double GDR [8].

Based on the linearized time-dependent Hartree–Fock–Bogoliubov description of the ground-state wave function, we have performed several investigations of the properties of the excited states, taking into account the effects outside the RPA.

Another example is a description of the lowest-lying 1^-_1 states in spherical and transitional nuclei. The experimental data demonstrate that the lowest-lying 1^-_1 states in the nondeformed nuclei have mainly the structure of «two-phonon» states arising as a result of coupling of the quadrupole and octupole modes. For this reason we suggest the following expression for the wave vector of the 1^-_1 state:

$$\mid 1^-_1, M\rangle \sim (\hat{Q}_2 \hat{Q}_3)_{1M} \mid 0^+_1\rangle,$$

where $\mid 0^+_1\rangle$ is the RPA ground-state which includes some ground-state correlations and $Q_{\lambda\mu}$ are the electric multipole moments.

With this expression for the 1^-_1 state wave vector, it is possible to explain several relations between the experimental data, using exact commutation relations of $Q_{\lambda\mu}$.

We have also calculated the neutron number dependence of the $E1$; $1^-_1 \to 0^+_1$ reduced transition probabilities for Xe isotopes [9] and have predicted

the appearance of the minimum in the neutron dependence of $B(E1; 1^-_1 \to 0^+_1)$ at $A = 128$. This minimum is a result of the cancellation of the proton and neutron contribution to the $E1$ transition matrix elements.

The other possibility to use the ground-state wave function determined by the static HFB method was realized in our investigations of the $E0$ transitional density for nuclei between spherical and deformed shape [10]:

$$\mid \Psi_i\rangle = \int d\beta g_i(\beta) \mid \beta; \mathrm{HFB}\rangle,$$

where $g_i(\beta)$, being an eigenfunction of the collective Hamiltonian, describes a distribution of $\mid \Psi_i\rangle$ over deformation β and $\mid \beta; \mathrm{HFB}\rangle$ is the HFB ground-state wave function corresponding to deformation β.

Using these wave functions, we have calculated the $E0$: $0^+_{\mathrm{g.s}} \to 0^+_\beta$ transitional densities for ^{150}Nd which will be measured in the nearest future in TU-Darmstadt.

At the present time the interests in nuclear structure investigations are shifted to the drip-line nuclei. Their properties differ from those nuclei belonging to stability valley. For instance, in the case of light nuclei even the magic numbers are changed with neutron excess. This explains why, first of all, the efforts in development of the theory have been concentrated on the nuclear mean field.

5. RELATIVISTIC MEAN FIELD APPROACH

The new approaches which are actively developed now are the Relativistic Mean Field theory and the Energy Density Functional [11, 12].

In the standard representation of the hadrodynamics the nucleus is described as a system of Dirac nucleons coupled to the mesons and the electromagnetic field. This coupling is described by the effective Lagrangian. The minimal set of the mesons which are necessary for a quantitative description of bulk and single-particle nuclear properties includes isoscalar scalar σ meson, isoscalar vector ω meson, and the isovector vector ρ meson.

The Lagrangian density is

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{\mathrm{int}}.$$

In this expression $\mathcal{L}_N$ is the free nucleon Lagrangian

$$\mathcal{L}_N = \bar{\Psi}\left(i\gamma^\mu\partial_\mu - m\right)\Psi,$$

where m is the bare nucleon mass,

$$\mathcal{L}_m = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m^2_\omega\omega_\mu\omega^\mu - \\ - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m^2_\rho\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where

$$\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad \mathbf{R}_{\mu\nu} = \partial_\mu \boldsymbol{\rho}_\nu - \partial_\nu \boldsymbol{\rho}_\mu,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad U(\sigma) = \frac{1}{2} m_\sigma \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4.$$

Here arrows mean isovectors. The minimal set of interactions is contained in $\mathcal{L}_{\rm int}$:

$$\mathcal{L}_{\rm int} = -\bar{\Psi}\Gamma_\sigma \sigma \Psi - \bar{\Psi}\Gamma^\mu_\omega \omega_\mu \Psi - \bar{\Psi}\boldsymbol{\Gamma}^\mu_\rho \boldsymbol{\rho}_\mu \Psi - \bar{\Psi}\Gamma^\mu_e A_\mu \Psi,$$
$$\Gamma_\sigma = g_\sigma, \quad \Gamma^\mu_\omega = g_\omega \gamma^\mu, \quad \boldsymbol{\Gamma}^\mu_\rho = g_\rho \boldsymbol{\tau} \gamma^\mu, \quad \Gamma^\mu_e = e\frac{1-\tau_3}{2}\gamma^\mu.$$

However, the density dependence of the vertex functions g_σ, g_ω and g_p is also taken into account. The additional nonlinear terms in the potential U are needed because with only quadratic in the meson field term it is impossible to get a quantitative description of complex nuclear systems.

The model Lagrangian density leads to the following equation of motion for the nucleons:

$$\left[\gamma^\mu \left(i\partial_\mu + V_\mu\right) + m + S\right] \Psi = 0.$$

If we neglect retardation effects for meson fields, we come to the following time-dependent mean-field potentials:

$$S(\mathbf{r}, t) = g_\sigma \sigma(\mathbf{r}, t),$$
$$V_\mu(\mathbf{r}, t) = g_\omega \omega_\mu(\mathbf{r}, t) + g_\rho \boldsymbol{\tau} \boldsymbol{\rho}_\mu(\mathbf{r}, t) + eA_\mu(\mathbf{r}, t)\frac{(1-\tau_3)}{2}.$$

The meson fields are calculated at each step in time solving the stationary Klein–Gordon equation

$$-\Delta \Phi_m + U'(\Phi_m) = \pm \langle \bar{\Psi} \Gamma_m \Psi \rangle,$$

where «+» is for vector fields, «−» is for scalar fields $\Phi_m \equiv \{\sigma, \omega^\mu, \boldsymbol{\rho}^\mu, A^\mu\}$; U' is the functional derivative of the corresponding potential. The stationary approximation for the meson fields is justified by the large meson masses compared to the energies relevant for nuclear structure.

In the model formulated above the σ meson approximates large attractive scalar fields. The ω meson approximates the short-range repulsion between nucleons, and the ρ meson carries the isospin.

In the early applications to nuclear matter, the same effective Lagrangian was used both in the ph channel and in the pp channel. It was found, however, that the standard RMF effective interactions produce pairing correlations that are much too strong. However, in the effective theory there is no physical reason to

use the same interactions for both the pp and ph channels. It was found that the phenomenological potential provides a very reasonable description of pairing in nuclear matter [13].

The gap vanishes at very small and at very large densities. The maximum is located at $k_F = 0.8$ fm^{-1}. This value of k_F corresponds to relatively low density which in finite nuclei is found only in the surface region.

What is the reason to apply the Relativistic theory to description of the nuclear structure phenomena at relatively low excitation energies? There is no need in the relativistic kinematics because $\sqrt{p_F^2 + m_N^2} = m_N\sqrt{1 + 0.075}$, where 0.075 is a small term. Nonrelativistic theory of nuclear structure works well. There are several technical problems to apply a relativistic theory. However, in the relativistic approach appear large scalar and vector fields: $S \approx -400$ MeV, $V \approx 350$ MeV. This explains large spin-orbit splitting in nuclei. The saturation mechanism in nuclei was clarified within the relativistic approach too. In the framework of the relativistic approach it became clear why approximate pseudospin symmetry is observed in nuclei. Some possibilities for experimental investigations of the manifestations of the pseudospin symmetry in spectra of nuclei with $Z \geqslant 100$ have been indicated in [14].

SUMMARY

The ideas related to the name of N. N. Bogoliubov have made a strong impact on the development of the nuclear structure theory. They have formulated a theoretical basis for the microscopic approach to nuclear structure. For more than 40 years these methods have been used intensively in the nuclear structure theory. In the last years the interests in the nuclear structure investigations have been shifted to the drip-line nuclei. Their description requires a development of the former concepts and methods. However, the ideas of the mean field, its fluctuations, pair correlations continue to play the central role.

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