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CRITICAL PHENOMENA IN HIGH-ENERGY LEPTON- AND HADRON-INDUCED REACTIONS

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We argue that the nucleon seen in the Bjorken scaling region is a gas of quasi-free partons. As x decreases, for certain values of Q^2 the partons in a nucleon coalesce to a liquid (saturation). Experimentally this phenomenon is manifested as the turn-over of the structure function derivatives. The phenomenon can be quantified in the framework of statistical models, percolation and other approaches to collective effects in the strongly interacting matter. Similarities and differences between the case of lepton–hadron, hadron–hadron and heavy-ion collisions are discussed.

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INTRODUCTION

In [1] a thermodynamic interpretation of the saturation in nucleon structure functions was suggested. According to this, below we propose a novel approach to the saturation phenomenon in DIS and related processes based on the collective properties of the excited nucleon. Namely, we suggest that below the saturation region, the nucleon in DIS is seen as a gas of almost free partons. With their increasing density, the constituent gradually overlaps and, starting from a certain value of x and Q^2, the gas of free partons coalesces condensing in a liquid of quarks and gluons (Fig. 1). Saturation corresponds to the onset of the new phase.

The saturation regime is expected when certain values of low enough x and relevant Q^2 are reached. According to the dipole model of DIS, this regime has already been achieved and it is characterized by the «saturation radius» [2] $R_0^2 = (x/x_0)^\lambda/Q_0^2$, with $Q_0^2 = 1$ GeV^2, $x_0 = 3 \cdot 10^{-4}$ and $\lambda = 0.29$, found from a fit to the DIS data at $x < 0.01$. On more general grounds, saturation could be expected also from unitarity: the rapid (power-like) increase with $1/x$ of the structure functions/cross sections may suggest that unitarity corrections will tamper this rise, although formally the Froissart bound has never been proven for off-mass-shell particles, thus unitarity does not provide any rigorous limitation for such amplitudes [3, 4]. One more argument is physical: the rise of the structure function $F_2(x, Q^2)$ reflects the increase of the parton density (parton number in the nucleon). Since this number increases as a power, and the nucleon radius is

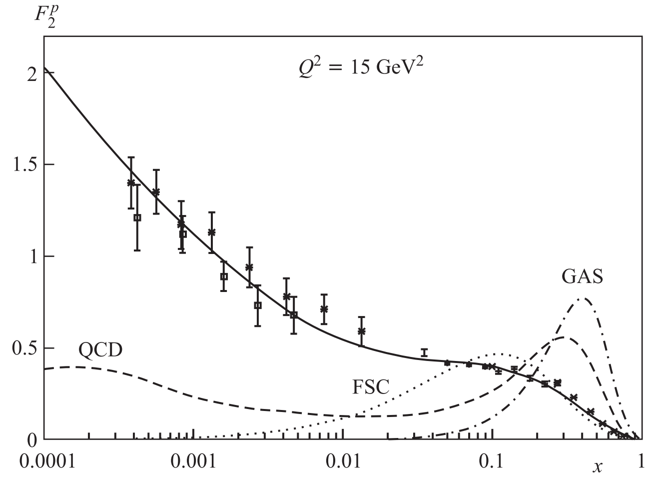

Fig. 1. Mapping different phases of the partonic gas/liquid onto the behavior of the structure function in the $x - Q^2$ plane, from [6], b

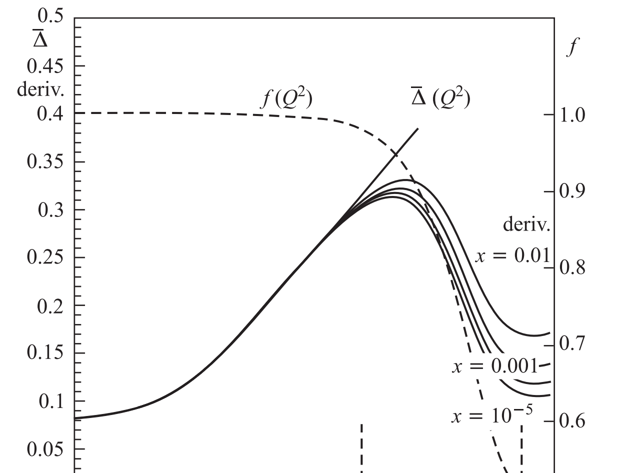

Fig. 2. The turn-over point in $B_Q(x, Q^2)$ calculated in [5] (for the relevant two-dimensional surface see [5] and Fig. 2 in [1])

known to increase as $\ln s$ (or, at most $\ln^2 s$) (shrinkage of the cone), the particle number density within a nucleon increases, inevitably, reaching a critical value where the partons start to coalesce (overlap, recombine, etc.). Quantitatively, the dynamics depends on many, poorly known, details, such as the properties of the constituents and their interaction within the nucleon. A model interpolating between different regimes in the whole kinematical range of x and Q^2, yet fitting the data, was suggested in [5]. The resulting critical point is shown in Fig. 2.

1. STATISTICAL MODELS OF PARTON DISTRIBUTIONS

The statistical model of parton distributions in DIS was considered and developed in quite a number of papers [6–8]. In its simplest version, one assumes [6] that inside the nucleon, the valence quarks, as well as the sea quarks and antiquarks and gluons form a noninteracting gas in equilibrium. This simple picture may be further developed in two directions: 1) introduction of the effect of the finite size (and its energy dependence!) of the nucleon on statistical expression for the number of states for the unit energy inerval; 2) account for the Q^2 evolution, that can be calculated either from the DGLAP equation or phenomenologically, as, e.g., in [5]. A likely scenario emerging [6, 7] for this high quark density $n_q = (n_q - n_{\bar{q}})/3$ system is that quarks form Cooper pairs and new condensates develop.

The thermodynamic properties of excited nucleon are characterized by its temperature, pressure, etc., and a relevant equation of state (EoS). The statistical treatment of partonic distributions is by far not new, see, e.g., [6–8]. What is new in our approach, is the interpretation of the saturation in DIS as a manifestation of the transition from a dilute partonic gas to a liquid. The details (nature) of this (phase?) transition are not known. It can be of the first, of second order, or, moreover, be a smooth cross-over phenomenon. Our main argument is that the volume of the nucleon confining the partons (quarks and gluons) in the interior increases slower, at most as $\sim \ln^6 s$ (more likely, as $\sim \ln^3 s$), while the volume occupied by the interior, quarks and gluons, increases as a power, thus resulting in a limiting behavior: a gas-to-liquid cross-over or a phase transition. The present contribution is a first step in understanding of this complex processus.

The nucleon of mass M consists of a gas of massless particles (quarks, antiquarks and gluons) in equilibrium at temperature T in a spherical volume V with radius $R(s)$ increasing with squared c.m.s. energy s as $\ln s$ (or $\ln^2 s$). The invariant parton number density in phase space is given by [11]

$$\frac{dn^i}{d^3p^i d^3r^i} = \frac{dn}{d^3p\, d^3r} = \frac{gf(E)}{(2\pi)^3}, \tag{1}$$

where g is the degeneracy ($g = 16$ for gluons and $g = 6$ for q and $\bar{q}$ of a given flavor), E, p is the parton four-momentum and $f(E) = \Big(\exp\left[\beta(E - \mu)\right] \pm 1\Big)^{-1}$ is the Fermi or Bose distribution function with $\beta \equiv T^{-1}$. Quantities in the infinite momentum frame (IMF) are labeled by the subscript i.

The invariant parton density dn^i/dx in the IMF is related to dn/dE and $f(E)$ in the proton rest frame as follows [6]:

$$\frac{dn^i}{dx} = \frac{gV(s)M^2x}{(2\pi)^2} \int\limits_{xM/2}^{M/2} dEf(E), \tag{2}$$

and the structure function

$$F_2(x) = x\sum_q e_q^2\Big[\Big(\frac{dn^i}{dx}\Big)_q + \Big(\frac{dn^i}{dx}\Big)_{\bar{q}}\Big]. \tag{3}$$

Without any account for the finite volume of the hadron, this SF disagrees with the data. Finite volume effects can be incorporated, following R. S. Bhalerao from [6], and the result is

$$\frac{dn}{dE} = gf(E)\bigg(\frac{VE^2}{2\pi^2} + aR^2E + bR\bigg), \tag{4}$$

where V and R are energy dependent and a, b, in front of the surface and curvature terms, are unknown numerical coefficients. Their values are important for the final result, but they cannot be calculated from perturbative QCD. A rather general method to calculate these important parameters can be found in [12]. We intend to come back to this point in a subsequent publication.

In a related paper, [9], the phase structure of the hadronic matter in terms of its temperature T and its baryochemical potential μ, was studied in the framework of the percolation theory. The percolation mechanism was used in [10] to obtain a limiting energy dependence of the hadronic matter at $s \to \infty$.

One considers two frames: the proton rest frame and the infinite momentum frame (IMF) moving with velocity $-v(\simeq -1)$ along a common z-axis. Of interest is the limit when the Lorentz factor $\gamma \equiv (1-v^2)^{-1/2} \to \infty$. The invariant parton number density in phase space is given by (quantities in the IMF are denoted by the index i)

$$\frac{dn^i}{d^3p\, d^3r^i} = \frac{dn}{d^3p\, d^3r} = \frac{g}{(2\pi)^3}\left[\frac{1}{\exp\left[\beta(E-\mu)\right] \pm 1}\right] \equiv f(E), \tag{5}$$

where $\beta \equiv T^{-1}$, g is the degeneracy ($g = 16$ for gluons and $g = 6$ for q or $\bar{q}$ of a given flavor), E, p is the proton four-momentum in the proton rest frame and $f(E)$ is the usual distribution for noninteracting fermions or bosons. In terms of the Bjorken scaling variable $x = p_z^i/(Mv\gamma)$, the phase space element can be written as

$$d^3p^i d^3r^i = 2\pi\, p_T^i\ dp_T^i(Mv\gamma dx)\frac{d^3r}{\gamma} = 2\pi\left[Mxv^3 + \frac{Ev}{\gamma^2}\right]dEM\,dx\,d^3r,$$

where for $\gamma \to \infty$ the expression in square brackets becomes Mx. For fixed x the parton energy E varies between the kinematical limits $Mx/2 < E < M/2$, where the lower limit is attained when $p_T^i = 0$. Consequently, the parton number distribution dn^i/dx in the IMF is simply proportional to an integral of the rest-frame distribution $f(E)$:

$$\frac{dn^i}{dx} = 2\pi V M^2 x \int\limits_{xM/2}^{M/2} dE f(E),$$

where the factor V results from d^3r integration.

2. SATURATION

We define the saturation line (in the $x - Q^2$ plane) as the turning point (line) of the derivatives

$$B_Q(x,Q^2) = \frac{\partial F_2(x,Q^2)}{\partial(\ln Q^2)}, \quad B_x(x,Q^2) = \frac{\partial F_2(x,Q^2)}{(\partial \ln(1/x))}, \tag{6}$$

called B_Q or B_x slopes, where $F_2(x,Q^2)$ is a «reasonable» model for the structure function, i.e., one satisfying the basic theoretical requirements, yet fitting the data. For example, the model for $F_2(x,Q^2)$ of [5] interpolates between Regge behavior at small Q^2 and the solution of the DGLAP evolution equation at asymptotically large values of Q^2, practically for all values of x. The resulting two-dimensional projection of the Q slope can be found in [5], while the turn-down point (saturation) is shown in Fig. 2 of the present paper.

The main goal of this paper is the identification of this line(point) with the critical line(point) on the T, μ phase diagram of an excited nucleon viewed as a thermodynamic system. The thermodynamic approach to DIS may provide a new insight to this complex phenomenon. We are aware of the limited time scales in a deep inelastic scattering from the point of view of thermalization, a familiar problem relevant to any thermodynamic description of hadronic systems. Let us only remind that the thermodynamic approach to high-energy scattering and multiple production, originated by Fermi and Landau's papers, was applied to hadrons, rather than heavy ions.

3. PERCOLATION

Percolation as a model of phase transition from colourless hadrons to a quark gluon plasma (QGP) was studied in a number of papers, recently in [9]. Below we apply the arguments of that paper to the saturation inside a nucleon, where extended (dressed) quark and gluons, rather than mesons and baryons considered in [9], percolate into a uniform new phase of matter. Both objects are coloured particle inside a colourless nucleon.

The number of constituents in a nucleon can be found by integrating in b (impact parameter) the general parton distribution, e.g., that of [13]

$$q(x,b) = \frac{1}{2\pi}\int\limits_0^\infty \sqrt{-t}d\sqrt{-t}\mathcal{H}(x,t)J_0(b\sqrt{-t}).$$

Here, contrary to $q(x)$, $q(x,b)$ is dimensional, with a dimension of squared mass m^2, interpreted as the transverse size of the extended parton in the hadron, $m = R^{-1}$, $q(x,b) = m^2\tilde{q}(x,b)$, $\tilde{q}(x,b)$ being the partons number density [13]. In paper [13], m was a constant; here we choose it to depend on the photon virtuality: $m \to m_0 \ln(Q+m_\rho) = R^{-1}(Q)$, where m is the mass of the lightest vector meson, $m = m_\rho$. Note the inequality $q(x,b) \leqslant 1/S_q$, where $S_q(Q) = \ln^{-2}(Q+m_\rho)$.

Thus, the nucleon is composed of $N = 2\pi\int\limits_0^1 dx \int\limits_0^\infty b\, db\, \tilde{q}(x,b,Q)$ extended partons with the transverse area $S_q(Q)$.

CONCLUSIONS

In this talk a new approach to the saturation phenomena in deeply virtual processes — DIS, DVCS, VMP — diffractive and nondiffractive — is suggested. The basic idea is a physical one: Bjorken scaling implies that the nucleon in DIS and related processes is a system of weekly interacting partonic gas, that can be described by means of Bose–Einstein or Fermi statistics. As the density increases (with decreasing x and relevant Q^2), seen as the violation of Bjorken scaling, the system reaches a coalescence point where the gas condenses, eventually to a liquid. The thermodynamic properties of this transition can be only conjectured, and further studies are needed to quantify this phenomenon.

An alternative measure of the onset of saturation and expected change of phase can be related to nonlinear evolution equations. Saturation and a phase transition are expected when the nonlinear contribution overshoots the linear term.

The phenomenon discussed in the present note may have much in common with the colour glass condensate proposed in the context of heavy-ion collisions (see, e.g., [15] and earlier references therein). Apart from similarities (condensation of quarks and gluons as $x \to 0$) there are apparent differences: for example, hydrodynamical flow is not expected in DIS. In any case, the dynamics of the strong interaction is the same in lepton–hadron, hadron–hadron and heavy-ion collisions.

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