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ANOMALOUS SCALING IN HELICAL TURBULENT ENVIRONMENT

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The influence of helicity on the anomalous scaling of the single-time structure functions of a passive scalar advected by a non-Gaussian velocity field driven by the stochastic Navier–Stokes equation is investigated by the field theoretic renormalization group and the operator-product expansion within the second order of the perturbation theory (two-loop approximation). The set of composite operators with the minimal critical dimensions is identified, and their dependence on the helicity parameter is found. It is shown that the contribution to the critical dimensions of the structure functions of a passive scalar is given only by parts of the composite operators which are independent of the helicity parameter. Therefore, it is shown that the spatial parity violation has no impact on the anomalous scaling behavior of the passively advected scalar quantity in the turbulent environment.

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INTRODUCTION

During the last two decades considerable progress has been achieved in the understanding of intermittency and anomalous scaling of fluid turbulence [1–3]. Both natural and numerical experiments suggest that deviations from the phenomenological Kolmogorov–Obukhov theory [4, 5] are even more strongly pronounced for a passively advected scalar field (e.g., temperature or density of an impurity) than for the velocity field itself (see, e.g., [2, 3] and references cited therein). These deviations are manifested in a singular dependence of the corresponding correlation or structure functions on distances and on the external (integral) turbulence scale, and they are related to strong fluctuations of the energy flux (intermittency).

The central role in the studies of a passive advection is played by a simple model of a passive scalar quantity advected by a random Gaussian velocity field, white in time and self-similar in space, the so-called Kraichnan rapid-change model [6], and by various of its descendants. Namely, in the framework of the rapid-change model, for the first time, the anomalous scaling was established on the basis of a microscopic model and corresponding anomalous exponents were calculated within controlled approximations (see, e.g., [2, 3] and references cited therein).

An effective method for investigation of self-similar scaling behavior is the renormalization group (RG) technique [7, 8]. It was shown that within the field theoretic RG approach the anomalous scaling is related to the existence of the so-called «dangerous» composite operators with negative critical dimensions in the operator product expansion (OPE) of the scaling functions. The RG method allows us to construct a systematic perturbation expansion for the anomalous exponents and to calculate them to higher orders [8].

Although the models of the passive advection by the so-called synthetic velocity fields reproduce many anomalous features of genuine turbulent heat or mass transport, nevertheless, they are not able to describe some important properties, e.g., within the RG approach to the Kraichnan model it is impossible to study the influence of helicity (spatial parity violation) of a turbulent environment on the anomalous scaling. Thus, to be able to study such phenomena, it seems, that it is necessary to go beyond Gaussian statistics of the velocity field.

In the present paper, the field theoretic RG investigation of the anomalous behavior of the single-time structure functions of a scalar field passively advected by the velocity field which is governed by the stochastic Navier-Stokes equation with a given helical external random stirring force is done to the second order (two-loop) approximation in the corresponding perturbation theory. The main aim of the present work is to find possible dependence of the corresponding critical dimensions on the helicity parameter and to compare them to the results without presence of helicity [9] and to that obtained within the model with a Gaussian statistics [10].

1. THE MODEL AND ITS FIELD THEORETIC FORMULATION

The advection-diffusion equation for the scalar field $\theta \equiv \theta(t, \mathbf{x})$ has the form

$$\partial_t \theta = \kappa_0 \triangle \theta - (\mathbf{v}.\partial)\theta + f, \tag{1}$$

where $\partial_t \equiv \partial/\partial_t$ ∂ is d-dimensional gradient; $\triangle = \partial.\partial$ is the Laplace operator; κ_0 is the molecular diffusivity or thermal conductivity (subscript 0 denotes unrenormalized parameters of the theory), $\mathbf{v}(t, \mathbf{x})$ is a transverse (due to incompressibility) velocity field, and $f(t, \mathbf{x})$ is a random force with the correlator of the following form

$$D_\theta \equiv \langle f(t, \mathbf{x}) f(t', \mathbf{x}')\rangle = \delta(t - t') C(\mathbf{r}/L), \quad \mathbf{r} = \mathbf{x} - \mathbf{x}'. \tag{2}$$

The noise maintains the steady state of the system but, in what follows, the concrete form of the correlator is not essential. It is only important that C decreases rapidly for $r \equiv |\mathbf{r}| \ll L$, where L denotes an integral scale. In what follows, we shall suppose that the velocity field satisfies the stochastic Navier–Stokes equation

$$\partial_t \mathbf{v} = \nu_0 \triangle \mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{v} - \partial P + \mathbf{f}, \tag{3}$$

where ν_0 is the kinematic viscosity coefficient, and P is the pressure. The transverse random force per unit mass $\mathbf{f} = \mathbf{f}(t,\mathbf{x})$ simulates the energy pumping into the system on large scales to maintain the steady state. We assume that its statistics is Gaussian with zero mean and with pair correlation function of the following form

$$D^v_{ij} \equiv \langle f_i(t,\mathbf{x}) f_j(t',\mathbf{x}')\rangle = \\ = \delta(t-t') \int \frac{d\mathbf{k}}{(2\pi)^d} R_{ij}(\mathbf{k}) g_0 \nu_0^3 k^{4-d-2\varepsilon}\, \mathrm{e}^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}, \tag{4}$$

where the form of the tensor $R_{ij}(\mathbf{k})$, namely, $R_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2 + i\rho\,\varepsilon_{ijl} k_l/k$, represents the transition to a helical fluid. It consists of the nonhelical isotropic standard transverse projector $P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j/k^2$ and the helical transverse projector proportional to the helicity parameter ρ $(-1 < \rho < 1)$. Here, ε_{ijl} is the Levi-Civita completely antisymmetric tensor of rank 3; $g_0\nu_0^3 > 0$ is the positive amplitude and the physical value of formally small parameter $0 < \varepsilon \leqslant 2$ is $\varepsilon = 2$. It plays an analogous role as the parameter $\epsilon = 4 - d$ in the theory of critical behavior, and the parameter g_0 plays the role of the coupling constant of the model. In addition, g_0 is a formal small parameter of the ordinary perturbation theory (for more details see, e.g., [8]).

It can be shown [8] that the afore-mentioned stochastic problem is equivalent to the field theoretic model with doubled set of fields $\Phi \equiv \{\theta, \mathbf{v}, \theta', \mathbf{v}'\}$ with the following action functional

$$S(\Phi) = \frac{\theta' D_\theta \theta'}{2} + \frac{\mathbf{v}' D^v \mathbf{v}'}{2} + \theta'[-\partial_t + \kappa_0\triangle - (\mathbf{v}.\partial)]\theta + \mathbf{v}'[-\partial_t + \nu_0\triangle - (\mathbf{v}.\partial)]\mathbf{v}, \tag{5}$$

where θ' and $\mathbf{v}$ are auxiliary fields and all needed integrations are assumed.

The field theoretic formulation of the stochastic problem (1)–(4) by the action functional (5) corresponds to a standard Feynman diagrammatic technique with the following bare propagators (in the frequency-momentum representation):

$$\langle v_i v_j\rangle_0 = \frac{g_0\nu_0^3 k^{4-d-2\varepsilon} R_{ij}(\mathbf{k})}{(\omega^2 + \nu_0^2 k^4)}, \quad \langle v'_i v_j\rangle_0 = \frac{P_{ij}}{i\omega + \nu_0 k^2}, \quad \langle \theta'\theta\rangle_0 = \frac{1}{i\omega + \kappa_0 k^2}, \tag{6}$$

and two interaction vertices $-\theta' v_j \partial_j \theta = \theta' v_j V_j \theta$ and $-v'_i v_j \partial_j v_l = v'_i v_j W_{ijl} v_l/2$, where $V_j = ik_j$ and $W_{ijl} = i(k_l\delta_{ij} + k_j\delta_{il})$ (in the frequency-momentum representation).

Standard analysis of canonical dimensions of the model shows ultraviolet (UV) superficial divergent one-irreducible Green functions. The action functional formulation of the problem gives possibility to extract large-scale asymptotic behavior of the correlation functions after an appropriate renormalization procedure which is needed to remove the UV divergences.

Details of the two-loop RG analysis of the model which is rather huge will be given elsewhere. Here we shall give only necessary conclusions of the analysis which are important for further investigation of the anomalous scaling.

The model exhibits stable Kolmogorov-like scaling regime which is driven by the infrared (IR) stable fixed point of the corresponding RG equations. It can also be shown that to be able to consider the helicity effects in the present model it is necessary to go to the second order of the perturbation theory (two-loop approximation) (see also [11] for details). The main conclusion of the RG analysis is the fact that the presence of the helicity in the model does not disturb the stability of the scaling regime. This nontrivial fact is necessary condition for further investigation briefly described in the next section.

2. INERTIAL RANGE SCALING OF STRUCTURE FUNCTIONS AND ANOMALOUS SCALING

The existence of the stable fixed point in the model leads to the existence of the IR scaling behavior of various correlation functions within the inertial interval. Let us consider the following equal-time structure functions of the advected scalar field $\theta(t, \mathbf{x})$:

$$S_N(r) = \langle [\theta(t, \mathbf{x}) - \theta(t, \mathbf{x}')]^N \rangle, \quad r = |\mathbf{x} - \mathbf{x}'|. \tag{7}$$

The existence of IR stable fixed point implies that in the asymptotic region $r/l \gg 1$ (l is a characteristic inner (viscous) scale of the model) and at any fixed r/L, the function S_N has the following explicit form:

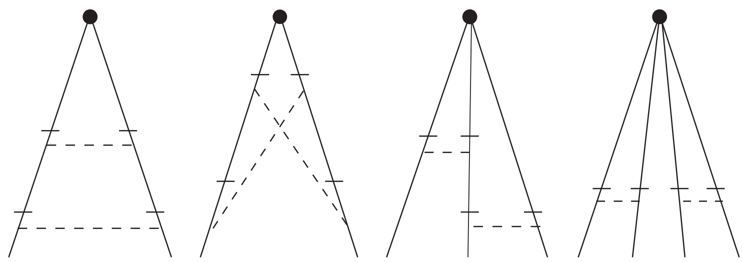
$$S_N(r) = (g_0 \nu_0^3)^{-N/2} r^{2N\varepsilon/3 + \gamma_N^*} R_N(r/L), \tag{8}$$

where γ_N^* denotes the fixed point values of the anomalous dimensions of the composite operator $F_N = (\partial_i \theta \, \partial_i \theta)^{N/2}$. The behavior of the scaling function $R_N(r/L)$ can be estimated by the OPE [8]. Here, the main contribution to the OPE is given by composite operators $F_{N,p} = ((\mathbf{n}.\partial)\theta)^p ((\partial\theta)^2)^n, N = 2n + p$, where the unit vector $\mathbf{n}$ represents the direction of the large scale anisotropy in the model (for details see, e.g., [9]). Then, the final asymptotic expression for the correlation functions has the form

$$S_N(r) \propto r^{N + \gamma_{N,p_N}^*}, \tag{9}$$

where γ_{N,p_N}^* means the smallest anomalous dimension of the set of operators $F_{N,p}$. Using the hierarchy relations between anomalous dimensions $\gamma_{N,p}^*$, it can be shown that the asymptotic behavior of the structure functions $S_N(r)$ is given by $\gamma^*_{N,0}$ for even values of N and by $\gamma^*_{N,1}$ for odd values of N. Therefore our aim is to find the dependence of the anomalous dimensions $\gamma^*_{2k,0}$ and $\gamma^*_{2k+1,1}$ for $k \geqslant 1$ on helicity parameter ρ. The general explicit expressions for them are rather huge and complicated functions which contain hypergeometric functions or integrals (see [9] where nonhelical version of the present model was studied). Therefore, the full discussion of the calculations will be not present here but it will be published elsewhere. Here, only some facts will be given which are important for the main result of the present paper.

In general, within the two-loop approximation, which is used in the present work, anomalous dimensions $\gamma^*_{N,p}$ are given by the explicit calculation of a set of the composite operators which contains 11 different two-loop operators. Nevertheless, as was shown in [9] by an analysis of the tensor structure of the composite operators together with the value of $\gamma^*_{2,0} = -2\varepsilon/3$ which is known exactly already at one-loop level approximation (i.e., it has no higher-loop corrections), only 4 of these 11 operators can give a contribution to the anomalous dimensions $\gamma^*_{N,p}$. Their graphical representation is shown in the Figure.



The graphical representation of the two-loop composite operators which contribute to the anomalous dimensions $\gamma^*_{N,p}$. The solid lines represent the propagator $\langle\theta\theta'\rangle$ (the end of the propagator with a slash corresponds to the field θ') and the dashed lines represent the propagator $\langle v_i v_j\rangle$. For more details see [9]

In two-loop approximation, anomalous dimensions $\gamma^*_{N,p}$ are given as series in ε in the following form:

$$\gamma^*_{N,p} = \gamma^{*1}_{N,p}\varepsilon + \gamma^{*2}_{N,p}\varepsilon^2, \tag{10}$$

where $\gamma^{*1}_{N,p}$ represents the one-loop contribution, and $\gamma^{*2}_{N,p}$ represents the two-loop contribution to the corresponding anomalous dimension. Their explicit form in the nonhelical case was found and discussed in [9]. As was already mentioned, the presence of helicity in the turbulent environment within the considered model can be manifested at the two-loop level of approximation. Therefore, our aim is to find explicit dependence of two-loop contribution $\gamma^{*2}_{N,p}$ on helicity parameter ρ.

To find this dependence it is necessary to calculate the composite operators shown in the Figure as functions of ρ. The corresponding analysis of the operators was done, and it was shown that the parts of the composite operators which give contributions to anomalous dimension $\gamma^{*2}_{N,p}$ are independent of ρ. Thus, our conclusion is that the anomalous dimensions $\gamma^{*}_{N,p}$ of the single-time structure functions of the passively advected scalar field do not feel the presence of spatial parity violation of the turbulent system. This conclusion is in accordance with the result obtained in [10] where the influence of helicity on the anomalous scaling of a passive scalar advected by the velocity field with a Gaussian distribution was investigated. Of course, the conclusion is right only at two-loop level approximation and the question, whether this conclusion is valid in general (at higher order approximations within the corresponding perturbation expansion), is still open.

CONCLUSION

We have investigated the scaling properties of the single-time structure functions of a scalar field passively advected by the turbulent velocity field driven by the stochastic Navier–Stokes equation with presence of helicity (spatial parity violation). The model was studied by the field theoretic renormalization group technique and by the operator-product expansion within two-loop approximation. First of all, it is shown that the presence of the helicity in the system does not destroy the stability of the scaling regime. Further, the analysis of needed composite operators for investigation of anomalous scaling of the single-time structure functions of advected field is done and their dependence on helicity parameter is found. It is shown that the final critical dimensions are independent of helicity parameter, i.e., the helicity does not influence the anomalous behavior of the model (at least at two-loop approximation).

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