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KINETICS OF SYSTEM OF EMITTERS AND NONEQUILIBRIUM ELECTROMAGNETIC FIELD

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A new step in research into the Dicke model kinetics is proposed on the basis of the Bogoliubov reduced description method. A more precise description of the emitter subsystem and electromagnetic field is considered. New reduced description parameters — emitter energy density and binary correlation functions of electric and magnetic field amplitudes — open the way to investigating nonuniform systems of two-level emitters and nonequilibrium electromagnetic field with its correlation properties. Kinetic equations for such systems have been obtained in terms of electrodynamics of continuous media consisting of two-level emitters. Material equations for corresponding media are analyzed.

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INTRODUCTION

Theoretical investigations of superfluorescent systems are based traditionally on the concept of a dynamic system of emitters interacting with the electromagnetic field considered as an equilibrium one [1]. Different approaches lead to the same description of the dynamic system kinetics [2–4]. A superfluorescent pulse is detected through the behavior of the total quasi-spin of the subsystem consisting of two-level emitters which is described with the Rehler–Eberly equation [5]. Nowadays, correlation properties of near-coherent emission generated by the Dicke system of emitters are of interest due to quantum optics development. The consequent way of kinetics construction on the basis of the Bogoliubov reduced description method [6] used in our previous papers [4, 7, 8] allows one to obtain a more precise picture of the process in the system under consideration including nonequilibrium states of electromagnetic field and various excitations in the emitter subsystem. Thus we come to electrodynamics of nonequilibrium medium consisting of emitters.

1. REDUCED DESCRIPTION PARAMETERS FOR A NONUNIFORM AND NONEQUILIBRIUM SYSTEM

We proceed from the following generalization of the Dicke quasi-spin model [9]:

$$\hat{H} = \hat{H}_0 + \hat{H}_1,\ \hat{H}_0 \equiv \sum_{k\alpha} \hbar\omega_k c^+_{k\alpha} c_{k\alpha} + \hbar\omega \sum_{1\leqslant a\leqslant N} \hat{r}_{az},\quad \hat{H}_1 \equiv -\int d^3x \hat{E}^t_n(x)\hat{P}_n(x) \tag{1}$$

($\hat{r}_{an}$ is a quasi-spin operator; a is emitter's number; n is vector sub). Electromagnetic (EM) field modes are described with boson operators $c_{k\alpha}$, $c^+_{k\alpha}$, and operators of transversal EM field are expressed via them as

$$\hat{E}^t_n(x) = i\sum_{k\alpha}\left(\frac{2\pi\hbar\omega_k}{V}\right)^{1/2} e_{\alpha n}(k)(c_{\alpha k} - c^+_{\alpha,-k})\,\mathrm{e}^{ikx},$$
$$\hat{B}_n(x) = i\sum_{k\alpha}\left(\frac{2\pi\hbar\omega_k}{V}\right)^{1/2} [\tilde{k}, e_\alpha(k)]_n(c_{\alpha k} + c^+_{\alpha,-k})\,\mathrm{e}^{ikx} \quad (\tilde{k}_l \equiv k_l/k) \tag{2}$$

with standard commutation relations

$$[\hat{E}^t_n(x), \hat{E}^t_l(x')] = 0, \quad [\hat{B}_n(x), \hat{B}_l(x')] = 0,$$
$$[\hat{B}_n(x), \hat{E}^t_n(x')] = e_{nlm}4\pi i\hbar c\frac{\partial\delta(x-x')}{\partial x_m}. \tag{3}$$

Formula (1) contains density of dipole moment (polarization) of emitters

$$\hat{P}_n(x) = 2\sum_{1\leqslant a\leqslant N} d_{an}\hat{r}_{ax}\delta\,(x - x_a). \tag{4}$$

Operator evolution equations for operators of the field and emitter polarization

$$\dot{\hat{E}}^t_n(x) = c\,\mathrm{rot}_n\hat{B}(x), \;\; \dot{\hat{B}}_n(x) = -c\,\mathrm{rot}_n(\hat{E}^t(x) - 4\pi\,\hat{\mathrm{P}}(x)), \;\; \dot{\hat{P}}_n(x) = \hat{J}_n(x)$$
$$(\hat{J}_n(x) \equiv -2\omega\sum_a d_{an}\hat{r}_{ay}\delta(x - x_a), \quad \dot{\hat{A}} \equiv \frac{i}{\hbar}[\hat{H}, \hat{A}]) \tag{5}$$

give us operator Maxwell equations:

$$\dot{\hat{E}}_n(x) = c\,\mathrm{rot}_n\hat{B}(x) - 4\pi\hat{J}_n(x), \quad \dot{\hat{B}}_n(x) = -c\,\mathrm{rot}_n\hat{E}(x)$$
$$(\hat{E}_n(x) \equiv \hat{E}^t_n(x) - 4\pi\hat{P}_n(x)), \tag{6}$$

where the operators of complete electric field $\hat{E}_n(x)$ and current density $\hat{J}_n(x)$ are used. Emitter energy density $\hat{\varepsilon}(x)$ obeys the evolution equation

$$\dot{\hat{\varepsilon}}(x) = \hat{J}_n(x)\hat{E}^t_n(x) \quad (\hat{\varepsilon}(x) = \hbar\omega\sum_{1\leqslant a\leqslant N}\hat{r}_{az}\delta(x - x_a)) \tag{7}$$

which is the Joule heat exchange between the emitter and field subsystems.

Since the operators $\hat{E}^t_n(x)$, $B_n(x)$, $\hat{\varepsilon}(x)$ satisfy conditions

$$[\hat{H}_0, \hat{E}^t_n(x)] = -ic\hbar\,\mathrm{rot}_n\hat{B}(x), \;\; [\hat{H}_0, \hat{B}_n(x)] = ic\hbar\,\mathrm{rot}_n\hat{E}^t(x), \;\; [\hat{H}_0, \hat{\varepsilon}(x)] = 0, \tag{8}$$

their average values can be the basis for building the system kinetics in the framework of the Peletminsky–Yatsenko scheme [6].

Let us define correlation functions $(a^x b^{x'})_t$ of local values $\hat{a}(x)$, $\hat{b}(x)$ by the formula

$$(a^x b^{x'})_t \equiv \frac{1}{2} \operatorname{Sp} \rho(t)\{\hat{a}(x), \hat{b}(x')\} - \operatorname{Sp} \rho(t)\hat{a}(x) \operatorname{Sp} \rho(t)\hat{b}(x'). \qquad (9)$$

More precise description of the system kinetics should be based on the reduced description parameters (RDP) $\hat{\varepsilon}(x)$, $\hat{\gamma}_\mu$, where $\hat{\gamma}_\mu$ notes the following set of operator variables:

$$\hat{E}^t_n(x), \quad \hat{B}_n(x), \quad \frac{\{\hat{E}^t_n(x), \hat{E}^t_l(x')\}}{2}, \quad \frac{\{\hat{E}^t_n(x), \hat{B}_l(x')\}}{2}, \quad \frac{\{\hat{B}_n(x), \hat{B}_l(x')\}}{2}.$$

Analogously to (8), operators $\hat{\gamma}_\mu$ satisfy the Peletminsky–Yatsenko condition $[\hat{H}_0, \hat{\gamma}_\mu] = \hbar \sum\limits_{\mu'} c_{\mu\mu'} \hat{\gamma}_{\mu'}$.

2. KINETIC EQUATIONS FOR THE GENERALIZED DICKE SYSTEM AT THE REDUCED DESCRIPTION

The quantum Liouville equation

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)] \equiv \mathbf{L}\rho(t) \qquad (10)$$

supplemented with the Bogoliubov functional hypothesis [10]

$$\rho(t) \xrightarrow[t \gg \tau_0]{} \rho(\varepsilon(t), \gamma(t)) \quad (\text{with} \ \ \operatorname{Sp}, \rho(t)\hat{\varepsilon}(x) \xrightarrow[t \gg \tau_0]{} \varepsilon(x,t),$$
$$\operatorname{Sp} \rho(t)\hat{\gamma}_\mu \xrightarrow[t \gg \tau_0]{} \gamma_\mu(t)) \qquad (11)$$

(τ_0 is synchronization time depending on an initial state of the system) dictates the general structure of the quantum Liouville equation at the reduced description of the system

$$\sum_\mu \frac{\partial \rho(\varepsilon, \gamma)}{\partial \gamma_\mu} L_\mu(\varepsilon, \gamma) + \int dx \frac{\delta \rho(\varepsilon, \gamma)}{\delta \varepsilon(x)} L(x, \varepsilon, \gamma) = -\frac{i}{\hbar}[\hat{H}, \rho(\varepsilon, \gamma)]. \qquad (12)$$

The Bogoliubov boundary condition of the complete correlation weakening

$$\mathrm{e}^{\tau \mathbf{L}_0} \rho(\varepsilon, \gamma) \xrightarrow[\tau \to +\infty]{} \mathrm{e}^{\tau \mathbf{L}_0} \rho_f(\gamma) \rho_m(\varepsilon) w_d(d) w_\alpha(\omega) \quad (\mathbf{L}_0 \rho \equiv -\frac{i}{\hbar}[\hat{H}_0, \rho]), \quad (13)$$

where we use a quasi-equilibrium statistical operator (SO) of the field

$$\rho_f(\gamma) = \exp\left\{\Phi(\gamma) - \sum_{\mu} Z_\mu(\gamma)\,\hat{\gamma}_\mu\right\} \tag{14}$$

possessing the properties $\mathrm{Sp}_f\rho_f(\gamma) = 1$, $\mathrm{Sp}_f\rho_f(\gamma)\hat{\gamma}_\mu = \gamma_\mu$, a quasi-equilibrium SO of the internal degrees of freedom of emitters

$$\rho_m(\varepsilon) = \exp\left\{\Omega(\varepsilon) - \int dx\, Z(x,\varepsilon)\hat{\varepsilon}(x)\right\} \tag{15}$$

satisfying the conditions $\mathrm{Sp}_m\rho_m(\varepsilon) = 1$, $\mathrm{Sp}_m\rho_m(\varepsilon)\hat{\varepsilon}(x) = \varepsilon(x)$ ($T(x,\varepsilon) = 1/k_B Z(x,\varepsilon)$ is a local temperature of the emitter system), an isotropic distribution function of orientations of the dipole moments of the emitters w_d, and a phenomenological accounting of nonresonant interaction of emitters with electromagnetic field,

$$w_\alpha(\omega) \equiv c(\alpha)\frac{\alpha}{(\omega-\omega_0)^2+\alpha^2}, \qquad \int\limits_0^{+\infty} d\omega\, w_\alpha(\omega) = 1 \quad (\alpha \ll \omega_0),$$

results in the integral equation for SO of the system $\rho(\varepsilon,\gamma)$

$$\begin{aligned}\rho(\varepsilon,\gamma) = \rho_f(\gamma)\rho_m(\varepsilon)w_d w_\alpha + \\ + \int\limits_{-\infty}^{0} d\tau\, \mathrm{e}^{-\tau\mathbf{L}_0}\Bigg\{\mathbf{L}_1\rho(\varepsilon,\gamma) - \sum_\mu \frac{\partial\rho(\varepsilon,\gamma)}{\partial\gamma_\mu}M_\mu(\varepsilon,\gamma) - \\ - \int dx \frac{\delta\rho(\varepsilon,\gamma)}{\delta\varepsilon(x)}M(x,\varepsilon,\gamma)\Bigg\}_{\gamma\to\mathrm{e}^{\mathbf{ic}\tau}\gamma}\end{aligned} \tag{16}$$

to be solved in the iteration scheme.

This is our way to kinetic equations of the generalized Dicke system.

3. MATERIAL COEFFICIENTS IN ELECTRODYNAMICS OF MEDIUM CONSISTING OF EMITTERS. ONSAGER PRINCIPLE

Important convenience is provided by the structure of $\rho_f(\gamma)$ allowing one to use the Bloch–Wick–de Dominicis theorem. Averages of linear and bilinear in the field expressions can be put down via local field values and binary correlation functions:

$$\mathrm{Sp}_f\rho_f(\gamma)\hat{E}^t_n(x) = E^t_n(x), \quad \mathrm{Sp}_f\rho_f(\gamma)\hat{B}^t_n(x) = B^t_n(x),$$

$$\mathrm{Sp}_f \rho_f(\gamma)\hat{E}^t_n(x)\hat{E}^t_l(x') = (E^{tx}_n E^{tx'}_l) + E^t_n(x)E^t_l(x'),$$
$$\mathrm{Sp}_f \rho_f(\gamma)\hat{B}^t_n(x)\hat{B}^t_l(x') = (B^{tx}_n B^{tx'}_l) + B^t_n(x)B^t_l(x'),$$
$$\mathrm{Sp}_f \rho_f(\gamma)\hat{E}^t_n(x)\hat{B}^t_l(x') = \frac{1}{2}[\hat{E}^t_n(x), \hat{B}^t_l(x')] + (E^{tx}_n B^{tx'}_l) + E^t_n(x)B^t_l(x').$$

Expressions for $Z(x,\varepsilon)$ and $\Omega(\varepsilon)$ are given by formulae

$$-\frac{\hbar\omega}{2}n(x)\,\mathrm{th}\left(\frac{\hbar\omega}{2}Z(x,\varepsilon)\right) = \varepsilon(x),$$
$$\Omega(\varepsilon) = -\sum_{1\leqslant a\leqslant N}\ln\left(2\,\mathrm{ch}\left(\frac{\hbar\omega}{2}Z(x_a,\varepsilon)\right)\right), \qquad (17)$$

where $n(x) \equiv \sum\limits_{1\leqslant a\leqslant N}\delta(x - x_a)$ is spatial density of emitters.

Thus we come to evolution equations for average electromagnetic field, i.e., the Maxwell equations

$$\partial_t E_n(x,t) = c\,\mathrm{rot}_n B(x,t) - 4\pi J_n(x,\varepsilon(t),\gamma(t)),$$
$$\partial_t B_n(x,t) = -c\,\mathrm{rot}_n E(x,t), \qquad (18)$$

and material equation for the current density in terms of the complete electric field

$$J_n(x,\varepsilon,\gamma) = \int dx'\sigma(x - x',\varepsilon(x))E_n(x') +$$
$$+ c\int dx'\chi(x - x',\varepsilon(x))Z_n(x') + O(\lambda^3) \qquad (19)$$

$(Z_n(x,t) \equiv \mathrm{rot}_n B(x,t))$. Here we consider emitter-field interaction as a small one $\hat{H}_1 \sim \lambda$ $(\lambda \ll 1)$ and

$$E_n(x,t) = E^t_n(x,t) + O(\lambda^2), \quad \sigma(k,\varepsilon) \sim \lambda^2, \quad \chi(k,\varepsilon) \sim \lambda^2. \qquad (20)$$

Material coefficients in (19) are conductivity $\sigma(k,\varepsilon)$ and magnetic susceptibility $\chi(k,\varepsilon)$

$$\sigma(k,\varepsilon) = -\frac{2\pi}{3}\frac{\varepsilon d^2}{\hbar^2}w_\alpha(\omega_k), \quad \chi(k,\varepsilon) = -\frac{4}{3}\frac{\varepsilon d^2}{\hbar^2}\int\limits_0^{+\infty} d\omega\, w_\alpha(\omega)P\frac{1}{\omega^2 - \omega_k^2}. \qquad (21)$$

Density of the dipole moment of the emitters depends on the same variables

$$P_n(x,\varepsilon,\gamma) = \int dx'\kappa(x - x',\varepsilon(x))E_n(x') +$$
$$+ c\int dx'\alpha(x - x',\varepsilon(x))Z_n(x') + O(\lambda^3) \qquad (22)$$

with coefficients

$$\kappa(k,\varepsilon)=\chi(k,\varepsilon), \quad \alpha(k,\varepsilon)=-\frac{\sigma(k,\varepsilon)}{\omega_k^2}. \tag{23}$$

Evolution equation for density of energy $\varepsilon(x,t)$ of the emitters is given by the expression

$$\partial_t\varepsilon(x,t)=L(x,\varepsilon(t),\gamma(t)),$$

$$L(x,\varepsilon,\gamma)=\int dx'\sigma(x-x',\varepsilon(x))\{(E_n^x E_n^{x'})+E_n(x)E_n(x')\}+$$
$$+c\int dx'\chi(x-x',\varepsilon(x))\{(E_n^x B_n^{x'})+E_n(x)B_n(x')\}+R(n(x))+O(\lambda^3) \tag{24}$$

in which the last term describes dipole radiation of the emitters

$$R(n)\equiv-\frac{2d^2}{3\pi c^3}n\int_0^{+\infty}d\omega\,\omega^4 w_\alpha(\omega). \tag{25}$$

For small α $w_\alpha(\omega)\to\delta(\omega-\omega_0)$ and $R(n)$ gives known expression

$$R(n)=-\frac{2d^2\omega_0^4}{3\pi c^3}n.$$

Evolution equations for correlation functions of the EM field in terms of the complete electric field can be written in the form

$$\begin{aligned}
\partial_t(E_n^x E_l^{x'})&=c\,\mathrm{rot}_n(B^x E_l^{x'})+c\,\mathrm{rot}'_l(E_n^x B^{x'})-4\pi(J_n^x E_l^{x'})-4\pi(E_n^x J_l^{x'}),\\
\partial_t(E_n^x B_l^{x'})&=c\,\mathrm{rot}_n(B^x B_l^{x'})-c\,\mathrm{rot}'_l(E_n^x E^{x'})-4\pi(J_n^x B_l^{x'}),\\
\partial_t(B_n^x E_l^{x'})&=-c\,\mathrm{rot}_n(E^x E_l^{x'})+c\,\mathrm{rot}'_l(B_n^x B^{x'})-4\pi(B_n^x J_l^{x'}),\\
\partial_t(B_n^x B_l^{x'})&=-c\,\mathrm{rot}_n(E^x B_l^{x'})-c\,\mathrm{rot}'_l(B_n^x E^{x'}).
\end{aligned} \tag{26}$$

Material equations for the current-field correlations in terms of correlations of the complete electric field are

$$\begin{aligned}
(E_n^x J_l^{x'})&=\int dx''\sigma(x'-x'',\,\varepsilon(x'))(E_n^x E_l^{x''})+\\
&\quad+c\int dx''\chi(x'-x'',\varepsilon(x'))(E_n^x Z_l^{x''})+S_{nl}(x-x',n(x'))+O(\lambda^3),\\
(B_n^x J_l^{x'})&=\int dx''\sigma(x'-x'',\varepsilon(x'))(B_n^x E_l^{x''})+\\
&\quad+c\int dx''\chi(x'-x'',\varepsilon(x'))(B_n^x Z_l^{x''})+T_{nl}(x-x',n(x'))+O(\lambda^3)
\end{aligned} \tag{27}$$

with

$$S_{nl}(k,n) \equiv -\frac{2\pi}{3} d^2 n\,(\delta_{nl} - \tilde{k}_n \tilde{k}_l)\omega_k^2 w_\alpha(\omega_k),$$

$$T_{nl}(k,n) \equiv \frac{4\pi i}{3} c d^2 n\, e_{nlm} k_m \int_0^{+\infty} d\omega\, w_\alpha(\omega) P \frac{\omega}{\omega^2 - \omega_k^2}.$$

Comparison of relations (19) and (27) shows that the Onsager principle is valid for the considered system.

Values $S_{nl}(k,n)$, $T_{nl}(k,n)$ define equilibrium correlations of the EM field.

CONCLUSION

The obtained equations (18), (24), and (26) completely describe nonuniform nonequilibrium states of the emitters interacting with EM field. Equation (24) is a generalization of the Rehler-Eberly equation. They will be applied to consideration of the following phenomena: 1) waves of correlations of EM field in equilibrium medium of emitters, which can be observed in the absence of average field; 2) connected waves of EM field and emitters; 3) pumping of emitters by EM field; 4) radiation of correlation waves by emitters; 5) super-radiance of emitter system; 6) stability problem of equilibrium states of emitters and EM field; 7) scattering of EM field in emitter medium; 8) influence of spatial distribution of emitters on phenomena in the system.

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