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QUASI-PARTICLE PHONON NUCLEAR MODEL FOR ODD-MASS NUCLEI — RECENT DEVELOPMENTS

*S. Mishev**, *V. V. Voronov***

Joint Institute for Nuclear Research, Dubna

An enhanced theory, based on the Extended Boson Approximation, for the lowest-lying states in odd-mass nuclei is presented. Our approach is built on the Quasi-particle Phonon Model extending it to take into account the ground state correlations due to the action of the Pauli principle more accurately than in the conventional theory.

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INTRODUCTION

The Quasi-Boson Approximation (QBA) underlying the Random Phase Approximation (RPA) [1] stimulates a discussion concerning its applicability towards approaching the problem of correctly taking into account the ground state correlations (GSC) in even–even nuclei. An enhanced version to this approximation, referred to as Extended RPA (ERPA), which was proposed long time ago [2] and later developed by [3, 4], proved successful in improving the theoretical results for most measurable quantities in the nuclear ground states as, for example, the transition charge densities in the interior region. Other microscopic approaches aiming to improve the RPA with respect to adding correlations to the ground states of even–even nuclei have also been attempted, as, for example, in [1, 5–8].

In the present work, we follow the ERPA approach, extending it to provide a refined version to the Quasi-particle Phonon Model (QPM) for odd–even nuclei [9–11, 13]. The interaction strengths between the quasi-particles and phonons in the presented model depend on the number of quasi-particles in the ground state. In this way the core–particle equations couple with the generalized equations describing the pairing correlations and the excited vibrational states of the even–even core thus forming a large nonlinear system. This model is applicable to open-shell spherical and transitional odd-A nuclei. Our research descends from the studies in [12, 13]. There it has been shown that the backward amplitudes in the wave functions of these nuclei play a very important role for the better

*E-mail: mishev@theor.jinr.ru
**E-mail: voronov@theor.jinr.ru

agreement with the experimentally measured spectroscopic factors. However, the theory in the latter research is based on the QBA which we intend to improve by taking into account the action of the Pauli principle more precisely due to the Extended Boson Approximation (EBA) we use.

1. EVEN–EVEN NUCLEI

This section aims to mark the basic building blocks of the QPM and its QBA extension for one-phonon states.

In ERPA one defines the quantities ρ_j (referred to as «blocking factors»), which are proportional to the quasi-particle occupation numbers on the level j

$$\rho_j = \frac{1}{\sqrt{2j+1}} \sum_m \langle|\alpha^\dagger_{jm}\alpha_{jm}|\rangle, \tag{1}$$

where α denotes a quasi-particle

$$\alpha_{jm} = u_j a_{jm} - (-)^{j-m}\, v_j\, a^\dagger_{j-m}. \tag{2}$$

The other key constituent of the theory is the phonon operators, defined as

$$Q^\dagger_{\lambda\mu i} = \frac{1}{2}\sum_{jj'} [\psi^{\lambda i}_{jj'}\, A^\dagger(jj';\lambda\mu) - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}\, A(jj';\lambda-\mu)]. \tag{3}$$

Here and below we follow the notations used in [4] and [13].

If the pairing vibrations are not taken into account, then one can obtain [4] the following modified QPM equations describing the states in even–even nuclei:

$$\frac{1}{2}\sum_j (2j+1)\left\{1 - \frac{\boxed{(1-2\rho_j)}(E_j-\lambda)}{\sqrt{(E_j-\lambda)^2+\Delta^2}}\right\} = n, \tag{4}$$

$$\frac{G}{4}\sum_j \frac{2j+1}{\sqrt{(E_j-\lambda)^2+\Delta^2}}\boxed{(1-2\rho_j)} = 1, \tag{5}$$

$$\frac{\kappa_\lambda}{2\lambda+1}\sum_{jj'} \boxed{(1-\rho_{jj'})}\frac{(f^\lambda_{jj'}u^+_{jj'})^2(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2_{\lambda i}} = 1, \tag{6}$$

$$\sum_{jj'} \boxed{(1-\rho_{jj'})}[(\psi^{\lambda i}_{jj'})^2 - (\varphi^{\lambda i}_{jj'})^2] = 2, \tag{7}$$

$$\boxed{\rho_j = \frac{1}{2}\sum_{\lambda i j'} \frac{2\lambda+1}{2j+1}(1-\rho_{jj'})(\varphi^{\lambda i}_{jj'})^2}. \tag{8}$$

The emergence of the blocking factors requires one to solve the equations above as a system of coupled equations. As it was found in [4], this extended version of the QRPA leads to a better reproduction of the experimentally measured charge transition densities.

In Figs. 1 and 2 we present sample distributions of the quasi-particle and particle occupation numbers on different levels in ^{130}Ba calculated within the

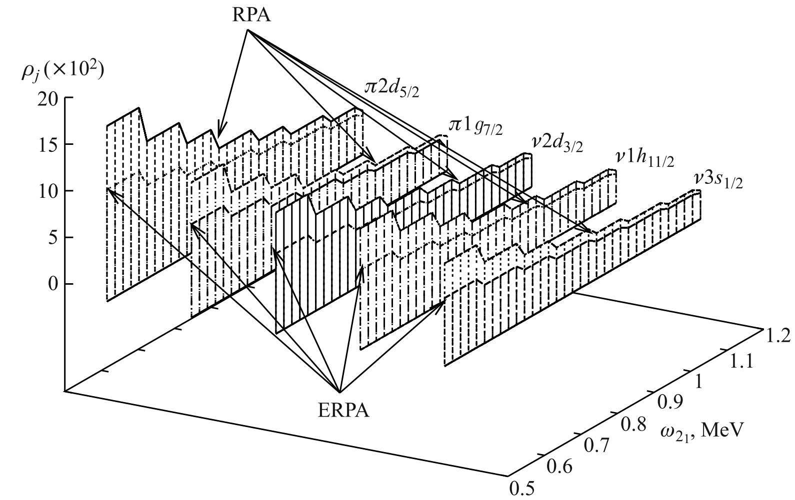

Fig. 1. Quasi-particle distribution in the ground state of ^{130}Ba

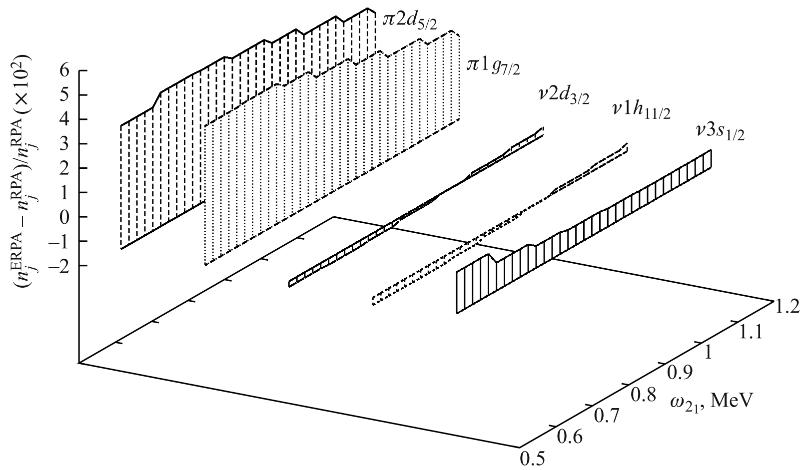

Fig. 2. Mean number of particle deviations $(n_j^{\rm ERPA} - n_j^{\rm RPA})/n_j^{\rm RPA}(\times 100)$ in the ground state of ^{130}Ba

ERPA and RPA. Both graphs illustrate the dependence of these quantities on the energy of the 2_1^+ state.

2. ODD–EVEN NUCLEI

The states in odd–even nuclei are described as mixed states composed of pure quasi-particle and quasi-particles⊗phonon states including backward-going amplitudes [12, 13]

$$\Psi_\nu(JM) = C_{J\nu}\alpha^+_{JM} + \sum_i D_j^{\lambda i}(J\nu)P^\dagger_{j\lambda i}(JM) - E_{J\nu}\tilde{\alpha}_{JM} - \\ - \sum_i F_j^{\lambda i}(J\nu)\tilde{P}_{j\lambda i}(JM)|\rangle, \quad (9)$$

where $P^\dagger_{j\lambda i}(JM) = [\alpha_j^\dagger Q^\dagger_{\lambda i}]_{JM}$ is the quasi-particles⊗phonon creation operator standing for time conjugation according to the convention $\tilde{a}_{jm} = (-1)^{j-m}a_{j-m}$.

Making use of the equation of motion method and conforming to relation (1) when calculating the matrix elements, we obtain the following generalized eigenvalue problem:

$$\begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda'i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda'i') \end{pmatrix} \times \\ \times \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix} = \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-\mathcal{L}^*(Jj\lambda i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1-\mathcal{L}^*(Jj\lambda i) \end{pmatrix} \times \\ \times \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}. \quad (10)$$

For reasons of conciseness, we provide only the leading terms of the expressions for the matrix elements in a diagonal approximation for $\mathcal{L}$ [10]

$$V(Jj\lambda i) = \langle|\{[\alpha_{JM}, H], P^\dagger_{j\lambda i}\}|\rangle = -\frac{1}{\sqrt{2}}[1-\rho_j + \mathcal{L}^*(Jj\lambda i)]\Gamma(Jj\lambda i), \quad (11)$$

$$W(Jj\lambda i) = \langle|\{[\alpha^\dagger_{JM}, H], \tilde{P}^\dagger_{j\lambda i}\}|\rangle = \\ = \frac{\pi_\lambda}{\pi_J}\varepsilon_J\rho_j\varphi^{\lambda i}_{Jj} - \frac{1}{4}[1-\rho_j+\mathcal{L}^*(Jj\lambda i)]\frac{\pi_\lambda}{\pi_J}\sum_{i_1}\mathcal{A}(\lambda i_1 i)\varphi^{\lambda i_1}_{Jj}, \quad (12)$$

$$K_J(j\lambda i|j'\lambda' i') = \frac{1}{2}[I_J(j\lambda i|j'\lambda' i') + I_J(j'\lambda' i'|j\lambda i)] =$$
$$= \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}[1-\rho_j+\mathcal{L}^*(Jj\lambda i)](\varepsilon_j + w_{\lambda i}) -$$
$$- \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}(1+\mathcal{L}(Jj\lambda i))\,\frac{1}{4}\sum_{i_1}\mathcal{A}(\lambda i i_1)\mathcal{L}^*_{J|j}(j\lambda i|j\lambda i_1), \quad (13)$$

where

$$I_J(j\lambda i|j'\lambda' i') = \langle|\{P_{j\lambda i}(JM), [H, P^+_{j'\lambda' i'}(JM)]\}|\rangle, \quad (14)$$

$$\mathcal{L}^*_{J|j'}(j\lambda i|j'\lambda' i') = \pi_{\lambda\lambda'}\sum_{j_1}(1-\rho_{j_1 j'})\psi^{\lambda' i'}_{1j}\psi^{\lambda i}_{1j'}\left\{\begin{array}{ccc} j' & j_1 & \lambda \\ j & J & \lambda' \end{array}\right\}, \quad (15)$$

$$\mathcal{L}^*(Jj\lambda i) = \pi_{\lambda\lambda}\sum_{j_1}(1-\rho_{j_1 j'})\psi^{\lambda i}_{1j}\psi^{\lambda i}_{1j}\left\{\begin{array}{ccc} j & j_1 & \lambda \\ j & J & \lambda \end{array}\right\}. \quad (16)$$

The above matrix element has been calculated using a simple Hamiltonian in the form

$$H = \sum_{\tau}^{(n,p)}\left\{\sum_{jm}(E_j - \lambda_\tau)a^\dagger_{jm}a_{jm} - \right.$$
$$\left. - \frac{1}{4}G^{(0)}_\tau : (P^\dagger_0 P_0)^\tau : - \frac{1}{2}\sum_{\lambda\mu}\kappa^{(\lambda)} : (M^\dagger_{\lambda\mu}M_{\lambda\mu}) : \right\}$$

accounting for the mean field, the pairing and multipole–multipole interactions, respectively.

One appealing feature of the QPM and in particular of the version described in this paper is that the interaction strengths between the quasi-particles and phonons depend only on the parameters describing their internal structure without any additional free parameters. In the limit case, where the number of the quasi-particles in the ground state is set to zero, this system of equations decouples to reduce to the model obtained in [13].

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