ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2012. Т. 43. ВЫП. 5

SUPERSTRINGS IN AdS SUPERBACKGROUNDS AND THEIR INTEGRABILITY

D. Sorokin

INFN, Sezione di Padova, Padova, Italia,

Department of Theoretical Physics, the University of the Basque Country UPV/EHU, Bilbao, Spain, and

IKERBASQUE, Basque Foundation for Science, Bilbao, Spain

This contribution is mainly based on results of [1,2]. We review the structure of Green–Schwarz superstrings on AdS backgrounds with the emphasis on peculiarities of those cases which are not maximally supersymmetric. In particular, we discuss complications which one encounters with the proof of classical integrability of non-maximally supersymmetric string sigma-models and describe a method of the construction of a Lax representation of the equations of motion which is capable of providing evidence for the integrability of sigma-models whose target space is not a semi-symmetric supercoset manifold.

PACS: 11.25.-w; 11.30.Pb

INTRODUCTION

Superstring theories in backgrounds with AdS geometry underlie various instances of AdS/CFT correspondence [3]. Main examples include

• Type IIB string on $AdS_5 \times S^5$ which is maximally supersymmetric. It is invariant under 32 supersymmetries which generate the $PSU(2,2|4)$ isometry of the bulk superspace and gives rise to the AdS_5/CFT_4 correspondence.

• Type IIA string on $AdS_4 \times CP^3$. Preserves 24 (of 32) supersymmetries governed by the isometry group $OSp(6|4)$ and is holographically dual to a $D=3$, $\mathcal{N}=6$ superconformal Chern–Simons theory.

• Type IIB strings on $AdS_3 \times S^3 \times T^4$, which preserve 16 (of 32) supersymmetries generating $PSU(1,1|2) \times PSU(1,1|2)$ isometries, give rise to AdS_3/CFT_2 dualities. They are still poorly understood in the cases in which the background geometry is supported by Ramond–Ramond fluxes.

• Type IIA and IIB strings on $AdS_2 \times S^2 \times T^6$ preserving 8 (of 32) supersymmetries. Their underlying symmetry is $PSU(1,1|2) \times U(1)^6$, and they are related to the theory of extremal $4d$ Reissner–Nordstrom black holes and AdS_2/CFT_1 correspondence, which still remains to be less studied and less understood, especially from the CFT_1 side.

• Superstrings on other $AdS \times M$ backgrounds with less number of supersymmetries.

To study these theories and, in particular, to make computation of AdS/CFT-related quantities on the string theory side, one should know an explicit form of the string sigma-model actions in the AdS superbackgrounds. Since most of the cases mentioned above are backgrounds with Ramond–Ramond fluxes, a suitable formulation of corresponding superstring theories is the Green–Schwarz (GS) formulation, because coupling of strings to RR fields in the Ramond–Neveu–Schwarz (RNS) formulation is problematic*. The GS superstring action is known in a general (on-shell) supergravity background [5] and has the following form:

$$S = -\frac{1}{2}\int_{\Sigma} *E^A \eta_{AB} E^B(X,\Theta) + \int_{\Sigma} B_2(X,\Theta), \qquad (1)$$

where the pull-back to the $2d$ worldsheet Σ of the target-superspace vielbein

$$E^A(X,\Theta) = dX^M E_M{}^A(X,\Theta) + d\Theta^\mu E_\mu{}^A(X\,\Theta)$$

and the Neveu–Schwarz (NS) two-form gauge field $B_2(X,\Theta)$ is understood, the star $*$ denotes the Hodge dual operation on the worldsheet, and the wedge product of differential forms is implicit. $X^M(\xi)$ $(M = 0, 1, \ldots, 9)$ and $\Theta^\alpha(\xi)$ $(\alpha = 1, \ldots, 32)$ are, respectively, bosonic and fermionic coordinates of a type IIA (or IIB) $D = 10$ curved superspace into which the string worldsheet parametrized by ξ^i $(i = 0, 1)$ is embedded.

The action (1) is invariant under a local worldsheet fermionic symmetry with 16 independent parameters, called kappa-symmetry, which allows one to gauge away 16 of 32 string fermionic modes. Kappa-symmetry also requires the superbackground fields $E^A(X,\Theta)$ and $B_2(X,\Theta)$ to obey supergravity equations of motion.

To know an explicit complete form of the $D = 10$ type II string action (1) in a given superbackground means to know the explicit form of the expansion of $E^A(X,\Theta)$ and $B_2(X,\Theta)$ in power series of Θ whose terms are (combinations of) the supergravity fields (graviton, gravitino, dilaton, dilatino and NS–NS and RR tensor gauge fields) and their derivatives. The explicit form is known only for very few backgrounds including flat superspace, a specific 7-brane background with a magnetic RR flux [6], $AdS_5 \times S_5$ superspace [7] and the $AdS_4 \times CP^3$ superspace [8]. Less supersymmetries are preserved by a background, it is more

*There also exists a description of AdS superstrings in terms of a hybrid model [4], which makes use of GS and RNS features. The relation of the hybrid model to the GS formulation is well understood in the case of flat target superspace; however, in curved backgrounds this relation is still to be established.

difficult to reconstruct its full superspace structure. For instance, this is still an open problem of $AdS_3 \times S_3 \times T^4$ and $AdS_2 \times S_2 \times T^6$ superstrings.

The knowledge of the explicit structure of the string action and its equations of motion allows one to study such problems as its classical integrability. In what follows we shall discuss certain cases in which integrability has been proven and will show how the lack of certain amount of supersymmetry makes the search for integrability much more complicated since it involves string fermionic modes that are associated with broken supersymmetries in the bulk.

1. $AdS_5 \times S^5$ SUPERSTRING

Let us start with the most studied example of the string propagating in $AdS_5 \times S^5$ superspace. As we have already mentioned, this background possesses the maximum number 32 of supersymmetries and has the $SU(2,2|4)$ isometry. The unique superspace with this isometry, which has 32 fermionic directions and whose bosonic subspace is $AdS_5 \times S^5$, is the supercoset $K = \dfrac{SU(2,2|4)}{SO(1,4) \times SO(5)}$. The geometry of this supercoset space satisfies the type IIB supergravity constraints and hence is the proper superspace description of the $AdS_5 \times S^5$ background. This geometry is characterized by the $SU(2,2|4)$ Cartan form pulled-back on the supercoset $K(X,\Theta)$

$$K^{-1}dK(X,\Theta) = \Omega^{AB}(X,\Theta)M_{AB} + E^A(X,\Theta)P_A + E^\alpha(X,\Theta)Q_\alpha. \quad (2)$$

The $\dfrac{SU(2,2|4)}{SO(1,4) \times SO(5)}$ supervielbeins $E^A(X,\Theta)$, which enter the GS superstring action (1), are components of the Cartan form along the bosonic directions of the supercoset associated with the translation generators P_A of the $su(2,2|4)$ algebra. $E^\alpha(X,\Theta)$ are the spinorial supervielbeins or Cartan form components associated with the supersymmetry generators Q_α of $su(2,2|4)$, and $\Omega^{AB}(X,\Theta)$ is the spin connection of the supercoset taking values in the stability subalgebra $SO(1,4) \times SO(5)$ generated by M_{AB}.

Explicit expressions for the components of (2) were constructed by using a suitable realization of the coset element in [7]* and used for getting the following explicit sigma-model form of the $AdS_5 \times S^5$ superstring action:

$$S = -\frac{1}{2}\int (*E^A E^B \eta_{AB} - E^\alpha E^\beta\, C_{\alpha\beta}), \quad (3)$$

*The Cartan forms on supercosets of $SU(2,2|N)$ relevant to the construction of brane actions on AdS superbackgrounds were first calculated in the early 80s [9].

where $C_{\alpha\beta}$ is an $SO(1,4) \times SO(5)$-invariant symmetric constant matrix. The second term in (3) is a particular form of the B_2-term of (1), which was first constructed in [4].

2. $AdS_4 \times CP^3$ SUPERSTRING

The type IIA string on $AdS_4 \times CP^3$ preserves 24 (of 32) supersymmetries and has $OSp(6|4)$ isometry. The supercoset $\frac{OSp(6|4)}{SO(1,3) \times U(3)}$, which might be associated with this background, has only 24 fermionic directions, while the complete superspace has 32 Grassmann-odd coordinates. Therefore, to construct a complete GS $AdS_4 \times CP^3$ superstring action, one should know the geometry of the full corresponding superspace. One may expect that the $\frac{OSp(6|4)}{SO(1,3) \times U(3)}$ supercoset should be a subspace of this superspace. If so, the complete GS action can reduce to an $\frac{OSp(6|4)}{SO(1,3) \times U(3)}$ sigma-model upon partial gauge fixing the fermionic kappa-symmetry by putting 8 non-coset fermionic modes to zero. The $\frac{OSp(6|4)}{SO(1,3) \times U(3)}$ sigma-model was constructed in [10, 11] by analogy with the $AdS_5 \times S^5$ one, and its action has the form similar to (3). However, in [10] it was noticed that in this model there are classical string configurations, for instance, when the string moves only in AdS_4 or forms a worldsheet instanton on CP^3 [12], which are singular in the sense that for such configurations the number of independent kappa-symmetries gets increased from 8 to 12, which means that in these regions there are only 12 fermionic physical degrees of freedom instead of 16 ones in the case of a general motion of the string in $AdS_4 \times CP^3$. This in turn indicates that for the singular regions the desired kappa-symmetry gauge fixing of the complete GS action, which would lead to the supercoset sigma-model, is not admissible.

To study these sectors of the theory, one should know the full GS action (1) in this superbackground. To this end, one should know the geometry of the complete $AdS_4 \times CP^3$ superspace with 32 fermionic directions. This superspace was constructed in [8], where it was shown that it is not a supercoset but rather a kind of fermionic fiber bundle over $\frac{OSp(6|4)}{SO(1,3) \times U(3)}$.

3. $AdS_2 \times S_2 \times T^4$ SUPERSTRINGS

As was mentioned in the introduction, $AdS_2 \times S_2 \times T^4$ backgrounds preserve 8 (of 32) supersymmetries and have $PSU(1,1|2) \times U(1)^6$ isometry. The associated supercoset $\dfrac{PSU(1,1|2)}{SO(1,1)\times U(1)}$ has only 8 fermionic directions and 4 bosonic coordinates. Moreover, this example also differs from the $AdS_4 \times CP^3$ case, since the 16-parameter kappa-symmetry is not enough to reduce the full GS superstring action on $AdS_2 \times S_2 \times T^4$ to the supercoset model. Upon kappa gauge fixing, there will always remain at least 8 string fermionic modes which do not belong to the supercoset.

Another complication of the $AdS_2 \times S_2 \times T^4$ case is that neither the non-coset fermionic modes nor the bosonic T^6-sector decouple from the supercoset model, the latter being only a consistent truncation of the full theory [2].

As we have demonstrated on the examples of $AdS_4 \times CP^3$ and $AdS_2 \times S_2 \times T^4$ backgrounds, the presence of the non-coset fermions associated with the supersymmetries broken by the AdS backgrounds makes the study of the superstring theory in the non-maximally supersymmetric AdS backgrounds much more complicated than, e.g., in the $AdS^5 \times S^5$ case. In particular, the proof of the integrability of these theories requires the development of more general (or alternative) methods for the construction of a zero-curvature Lax connection than those used in the case of the supercoset sigma-models. In what follows, we shall briefly discuss these issues.

4. CLASSICAL INTEGRABILITY OF $2d$ DYNAMICAL SYSTEMS

A $2d$ system is integrable if it has an infinite number of conserved charges (integrals of motion) which are in involution. These charges are generated by the Lax connection $L(z)$, a $2d$ one-form which depends on a spectral parameter z, takes values in a symmetry algebra and whose curvature vanishes:

$$dL + L \wedge L = 0 \tag{4}$$

when the fields describing the system satisfy the equations of motion. And vice versa, the zero-curvature condition implies the equations of motion.

The integrability is proven if one manages to construct $L(z)$. Though no generic prescription exists how to do this, for certain classes of models one can use one and the same techniques. The sigma-models on the supercosets with Z_4-grading called semi-symmetric superspaces (including the above examples) are of this kind. The receipt for constructing the Lax connection of the string sigma-models on such supercosets was proposed in [13].

A superalgebra has the Z_4 automorphism if its elements (or generators) can be endowed with Z_4-grading, i.e., are eigenvectors of the Z_4 group transformation Z:

$$\begin{gathered} Z^{-1}M_0Z = M_0, \quad Z^{-1}P_2Z = -P_2, \quad Z^{-1}Q_1Z = iQ_1, \\ Z^{-1}Q_2Z = -iQ_1, \quad Z^4 = 1. \end{gathered} \tag{5}$$

The above action of the Z_4-automorphism leaves the superalgebra invariant, if its generators satisfy the following schematic (anti)commutation relations:

$$
\begin{gathered}
[M_0, M_0] = M_0, \quad [M_0, P_2] = P_2, \quad [P_2, P_2] = M_0, \\
[M_0, Q_1] = Q_1, \quad [M_0, Q_3] = Q_3, \\
[P_2, Q_1] = Q_3, \quad [P_2, Q_3] = Q_1, \quad \{Q_1, Q_1\} = P_2, \\
\{Q_3, Q_3\} = P_2, \quad \{Q_1, Q_3\} = M_0.
\end{gathered}
\tag{6}
$$

The generators P_2 and M_0 are the same as in the definition of the Cartan form in (2), while Q_1 and Q_3 are the Z_4-graded decomposition of supersymmetry generators Q_α. So, with respect to Z_4 the Cartan form is decomposed as follows:

$$
K^{-1}dK(X,\Theta) = \Omega^0(X,\Theta)M_0 + E^2(X,\Theta)P_2 + E^1(X,\Theta)Q_1 + E^3(X,\Theta)Q_3. \tag{7}
$$

The Lax connection is constructed by taking the components of (7) with arbitrary numerical coefficients and adding to them the worldsheet dual $*E^2$ of E^2 [13]. The addition of the latter is prompted by the form of the first term of the superstring action (3), which contains the Hodge dual of E^A. Thus, the Lax connection has the following form:

$$
L(X,\Theta) = \Omega^0 M_0 + l_1 E^2 P_2 + l_2 * E^2 P_2 + l_3 E^1 Q_1 + l_4 E^3 Q_3. \tag{8}
$$

The curvature (4) of this connection, valued in the isometry superalgebra, vanishes if the worldsheet fields $X(\xi)$ and $\Theta(\xi)$ satisfy the equations of motion which follow from the action (3) and provided that the numerical coefficients $l_i = f_i(z)$ $(i = 1, 2, 3, 4)$ are certain functions of the single spectral parameter z.

The above construction of the Lax connection is only applicable to the cases in which the target superspace is semi-symmetrice, i.e., has the structure of a supercoset with Z_4-grading. It should be generalized, or an alternative procedure should be used for searching for Lax connections in the cases in which the superspace geometry is more involved, like, e.g., that of $AdS_4 \times CP^3$ and $AdS_2 \times S^2 \times T^6$. In the next section we shall describe such a more general procedure [1].

5. CONDITIONS FOR THE INTEGRABILITY OF THE GS SUPERSTRINGS ON GENERIC SUPERBACKGROUNDS WITH ISOMETRIES

Let us assume that the string propagates in a superspace which has isometries, but which is not the supercoset. In general, the superspace may have bosonic and fermionic isometries. The presence or the absence of the latter is just the manifestation whether the background under consideration is supersymmetric or not.

The first natural condition of the existence of a Lax connection for string σ-model in the target superspace is that the corresponding bosonic σ-model obtained from the former by putting to zero all fermionic fields is integrable. This restricts the choice of a possible bosonic subspace of the superbackground. A well-known class of integrable bosonic σ-models is those on symmetric spaces G/H. All the bosonic backgrounds considered above are of this kind. In the case of the G/H σ-models, the Lax connection can be constructed from their Noether currents as follows [14]. From the G/H σ-model action

$$S = \frac{1}{2}\int *e^A(X)\, e^B(X)\, \eta_{AB}, \tag{9}$$

where $e^A(X)$ is the worldsheet pullback of the G/H vielbein, we get the G-algebra valued Noether current

$$j(\xi) = e^A(X)\, K_A(X) = dX^M(\xi)\, e_M{}^A(X) K_A(X), \tag{10}$$

where $K_A(X)$ is the Killing vector generating the isometries of G/H.

The current (10) is conserved (on the mass shell):

$$d * j = \partial_i j^i = 0, \tag{11}$$

and satisfies the Maurer–Cartan equations associated with the group G; i.e., it has zero curvature

$$dj + 2j \wedge j = 0. \tag{12}$$

It then follows from Eqs. (11) and (12) that the following Lax connection is flat:

$$L_{\mathcal{B}} = \left(\frac{2z^2}{1-z^2} e^A + \frac{2z}{1+z^2} e^A\right) K_A \Rightarrow dL_{\mathcal{B}} - L_{\mathcal{B}} \wedge L_{\mathcal{B}} = 0. \tag{13}$$

Let us now extend this construction to the case of the GS superstring, which in addition to the bosonic modes $X(\xi)$ has the fermionic modes $\Theta(\xi)$. The Noether current consists of a term corresponding to the bosonic isometries and a term associated with supersymmetry (if the background is supersymmetric):

$$J(X,\Theta) = J_{\mathcal{B}} + J_{\mathrm{SUSY}}. \tag{14}$$

The two terms are conserved separately:

$$d * J_{\mathcal{B}} = 0 = d * J_{\mathrm{SUSY}}. \tag{15}$$

One can show [1] that up to the second order in Θ the two terms of the Noether current have the following form:

$$J_{\mathcal{B}}(X,\Theta) = j(X) + J_1^A(X,\Theta) K_A(X) + J^{AB}(X,\Theta)[K_A(X), K_B(X)], \tag{16}$$

$$J_{\mathrm{SUSY}} = J^{\alpha}(X,\Theta)\Xi_{\alpha}(X), \tag{17}$$

where $j(X)$ and $K_A(X)$ are the same as in the bosonic case (10) and $\Xi_\alpha(X)$ is the Killing spinor on the bosonic space G/H which generates the supersymmetric part of the isometry.

The Lax connection constructed with the current components (14)–(17) has the following form:

$$\mathcal{L} = \alpha_1\, j + \alpha_2 {*} J_{\mathcal{B}} + \alpha_2^2\, J_2 + \alpha_1\alpha_2 {*} J_2 - \alpha_2(\beta_1 J_{\mathrm{SUSY}} - \beta_2 {*} J_{\mathrm{SUSY}}) + \mathcal{O}(\Theta^3), \tag{18}$$

where $\alpha_{1,2}(z)$ and $\beta_{1,2}(z)$ are certain functions of the spectral parameter z. Note that the parameter α_1 and α_2 are already fixed by the zero–curvature condition of the purely bosonic limit (13) of the Lax connection.

In addition to be on shell, the condition for the Lax connection (18) to have zero curvature fixes the spectral parameter dependence of $\beta_{1,2}(z)$ and requires the Noether currents to satisfy the following relations:

$$\begin{gathered} dJ_{\mathrm{SUSY}} = -2(J_{\mathcal{B}} \wedge J_{\mathrm{SUSY}} + J_{\mathrm{SUSY}} \wedge J_{\mathcal{B}}), \\ (\nabla J_2^{AB} - J_1^A \wedge j^B)[K_A, K_B] = -J_{\mathrm{SUSY}} \wedge J_{\mathrm{SUSY}}. \end{gathered} \tag{19}$$

These conditions on the Noether currents restrict possible superbackgrounds in which the superstring sigma-model is integrable. All the supersymmetric backgrounds listed in the Introduction are of this kind.

If a background is not supersymmetric, the components J_1 and J_2 of the bosonic isometry current should satisfy the relation

$$(\nabla J_2^{AB} - J_1^A \wedge j^B)[K_A, K_B] = 0. \tag{20}$$

Then, a natural question arises whether there exist integrable string sigma-models in superbackgrounds in which target-space supersymmetry is completely broken. The obvious examples to check are non-supersymmetric $AdS \times M$ backgrounds which are obtained from the supersymmetric ones by changing the sign of a supporting gauge field flux. It turns out, however, that for these backgrounds the condition (20) is not satisfied. The only example of the integrable superstring in non-supersymmetric background which is known so far is a $D = 4$, $\mathcal{N} = 2$ superstring in AdS_4 with completely broken supersymmetry [1]. This model is a consistent truncation to $D = 4$ of the $AdS_4 \times CP^3$ superstring in which only 8 non-supercoset fermionic modes are kept. The Lax connection for this model has been constructed in [1] to all orders in the string fermionic modes in a particular kappa-symmetry gauge of [15].

It can be shown [1,2] that the Lax connection (18) is related by an isometry group transformation to the Lax connection (8) generalized with terms containing the contribution of the non-coset fermionic modes. Such a generalization of (8) preserves the Z_4-invariance [16], which is of crucial importance for the application of Bethe-ansatz techniques.

CONCLUSION

We have reviewed some features of superstring theories in AdS superbackgrounds with a particular emphasis on the cases with less supersymmetries in which the string has physical fermionic modes associated with broken symmetries that do not admit the supercoset interpretation. The presence of the non-coset fermions makes the proof of the classical integrability of the theory much more difficult.

Having at hand Lax connections which include the contribution of noncoset worldsheet modes, one can address the problem of how these modify the algebraic curve and Bethe ansatz equations for the full superstring theory in these backgrounds. This should lead to a more general approach to integrability of Green–Schwarz superstrings, which does not rely on having their supercoset sigma-model description.

For practical purposes, these results may be useful, in particular, for understanding the quantum spectrum of the $AdS_2 \times S_2 \times T^6$ superstring, which in turn should shed light on the structure of the dual CFT_1 theory.

Acknowledgements. The author would like to thank Alessandra Cagnazzo, Jaume Gomis, Pietro Grassi, Arkady Tseytlin, Konstantin Zarembo and especially Linus Wulff for the fruitful collaboration and stimulating discussions of topics reviewed in this contribution. This work was partially supported by the INFN Special Initiative TV12, the MIUR-PRIN contract 2009-KHZKRX and the grant FIS2008-1980 of the Spanish MICINN. The author is also grateful to the Department of Theoretical Physics of the Basque Country University for hospitality and the IKERBASQUE Foundation for a visiting fellowship.

REFERENCES

1. *Sorokin D., Wulff L.* Evidence for the Classical Integrability of the Complete $AdS(4) \times CP(3)$ Superstring // JHEP. 2010. V. 11. P. 143.

2. *Sorokin D. et al.* Superstrings in $AdS(2) \times S(2) \times T(6)$ // J. Phys. A. 2011. V. 44. P. 275401.

3. *Maldacena J. M.* The Large N Limit of Superconformal Field Theories and Supergravity // Adv. Theor. Math. Phys. 1998. V. 2. P. 231–252.

4. *Berkovits N. et al.* Superstring Theory on $AdS(2) \times S(2)$ as a Coset Supermanifold // Nucl. Phys. B. 2000. V. 567. P. 61–86.

5. *Grisaru M. T. et al.* $N = 2$ Superstrings in a Supergravity Background // Phys. Lett. B. 1985. V. 162. P. 116.

6. *Russo J.,Tseytlin A. A.* Green–Schwarz Superstring Action in a Curved Magnetic Ramond–Ramond Background // JHEP. 1998. V. 9804. P. 014.

7. *Metsaev R. R., Tseytlin A. A.* Type IIB Superstring Action in $AdS(5) \times S(5)$ Background // Nucl. Phys. B. 1998. V. 533. P. 109–126.

8. *Gomis J., Sorokin D., Wulff L.* The Complete $AdS(4) \times CP(3)$ Superspace for the Type IIA Superstring and D-branes // JHEP. 2009. V. 03. P. 015.

9. *Akulov V. P., Bandos I. A., Zima V. G.* Nonlinear Realization of Extended Superconformal Symmetry // Theor. Math. Phys. 1983. V. 56. P. 635–642.

10. *Arutyunov G., Frolov S.* Superstrings on $AdS_4 \times CP^3$ as a Coset Sigma-Model // JHEP. 2008. V. 09. P. 129.

11. *Stefanski B., Jr.* Green–Schwarz Action for Type IIA Strings on $AdS_4 \times CP^3$ // Nucl. Phys. B. 2008. V. 808. P. 80–87.

12. *Cagnazzo A., Sorokin D., Wulff L.* String Instanton in $AdS(4) \times CP(3)$ // JHEP. 2010. V. 05. P. 009.

13. *Bena I., Polchinski J., Roiban R.* Hidden Symmetries of the $AdS_5 \times S^5$ Superstring // Phys. Rev. D. 2004. V. 69. P. 046002.

14. *Eichenherr H. Forger M.* On the Dual Symmetry of the Nonlinear Sigma Models // Nucl. Phys. B. 1979. V. 155. P. 381.

15. *Grassi P. A., Sorokin D., Wulff L.* Simplifying Superstring and D-Brane Actions in $AdS(4) \times CP(3)$ Superbackground // JHEP. 2009. V. 08. P. 060.

16. *Cagnazzo A., Sorokin D., Wulff L.* More on Integrable Structures of Superstrings in $AdS(4) \times CP(3)$ and $AdS(2) \times S(2) \times T(6)$ Superbackgrounds. arXiv:1111.4197 [hep-th].