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SUPERSYMMETRIC RENORMALIZATION GROUP FLOWS

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The functional renormalization group equation for the quantum effective action is a powerful tool to investigate nonperturbative phenomena in quantum field theories. We discuss the application of manifest supersymmetric flow equations to the $\mathcal{N} = 1$ Wess–Zumino model in two and three dimensions and the linear $O(N)$ sigma model in three dimensions in the large-N limit. The former is a toy model for dynamical supersymmetry breaking, the latter for an exactly solvable field theory.

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INTRODUCTION

Supersymmetry (SUSY) is an important ingredient for most theoretical developments beyond the Standard Model, including supergravity and string theory. To study SUSY field theories at intermediate and strong couplings, nonperturbative methods are required. Unfortunately, most methods are either perturbative or break SUSY explicitly. For example, any lattice regularization breaks supersymmetry and one needs to fine-tune the bare parameters to recover supersymmetry in the continuum limit [3–5, 8]. Only for particular SUSY models with extended supersymmetry one may find a nilpotent combination of the supercharges and this nilpotent charge may be used to recover a supersymmetric continuum limit. Clearly what we need is an alternative, universally applicable and complementary nonperturbative method. In the past functional renormalization group methods have been successfully applied to many problems in strongly coupled quantum field theories [1, 2, 6, 10], and thus we decided to adapt functional method to supersymmetric systems. Actually, it is possible to formulate renormalization group equations in superspace, such that the flow equations yield supersymmetric effective actions on all scales [11, 12]. Here we consider supersymmetric Yukawa models built from a real superfield $\Phi(x,\theta) = \phi(x) + \overline{\theta}\gamma_*\psi(x) + (1/2)(\overline{\theta}\gamma_*\theta)F(x)$ consisting of a scalar field ϕ, a Majorana spinor ψ and an auxiliary field F. The (super)covariant derivatives acting on Φ are given by $D = \partial/\partial\overline{\theta} + i(\gamma^\mu\theta)\partial_\mu$ and $\overline{D} = -\partial/\partial\theta - i(\overline{\theta}\gamma^\mu)\partial_\mu$. For more details, we refer to our previous works on Wess–Zumino models in [7, 9, 13, 14].

The functional renormalization group can be formulated as a flow equation for the effective average action Γ_k. This scale-dependent functional interpolates between the classical action $S = \Gamma_{k=\Lambda}$ at the UV-cutoff Λ and the full quantum effective action $\Gamma = \Gamma_{k=0}$ that includes all quantum fluctuations (see Fig. 1, a). For a given initial condition Γ_Λ at the cutoff the effective average action is determined by the Wetterich equation [15],

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \partial_k R_k \right\}, \tag{1}$$

where k denotes the momentum scale. Here $(\Gamma_k^{(2)})_{ab} = \dfrac{\overrightarrow{\delta}}{\delta \Psi_a} \Gamma_k \dfrac{\overleftarrow{\delta}}{\delta \Psi_b}$ is the second functional derivative of Γ_k, where the indices a, b summarize all field components (internal and Lorentz indices, space-time or momentum coordinates). Here Ψ denotes the collection of component fields and not the superfield. The flow equation contains an infrared regulator R_k derived from a cutoff action quadratic in the fields. A general supersymmetric cutoff action has the form $\Delta S_k = (1/2) \int d^d x \ \Phi R_k \Phi|_{\overline{\theta}\gamma_* \theta}$, where the supersymmetric regulator is a function of supercovariant derivatives, $R_k \equiv f(\overline{D}\gamma_* D)$. Using the properties of D and $\bar{D}$, one proves that $R_k = r_1(\Delta) + r_2(\Delta)\overline{D}\gamma_* D$. The first term is a momentum-dependent mass and the second a kinetic term with momentum-dependent coefficient. A consistent approximation scheme to solve Eq. (1) is given by an expansion in supercovariant derivatives of the superfield. The truncation of such an expansion preserves supersymmetry.

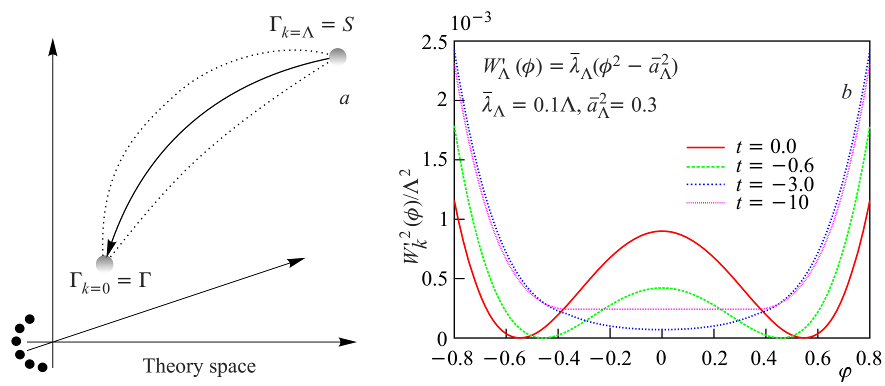

Fig. 1. a) A trajectory of the scale-dependent effective action in theory space. b) The flow of the scalar potential of the $\mathcal{N} = 1$ Wess–Zumino model in two dimensions with $\lambda = 0.1\,\Lambda$ and $a^2 = 0.3$

1. WESS–ZUMINO MODEL IN TWO AND THREE DIMENSIONS

Let us consider the two- and three-dimensional Wess–Zumino model with one supersymmetry as a toy model for dynamical SUSY breaking [16]. The action in superspace is given by

$$S = \int d^d x \left(-\frac{1}{2}\overline{D}\Phi\gamma_* D\Phi + W(\Phi) \right)\Big|_{\overline{\theta}\gamma_*\theta}. \tag{2}$$

After eliminating the auxiliary field F by its algebraic equation of motion the on-shell action contains a real scalar field ϕ with bosonic potential $V(\phi) = W'(\phi)^2/2$ and a Majorana fermion field ψ with Yukawa-type interaction to the scalar field. Unbroken SUSY is characterized by a vanishing ground state energy and it depends on the superpotential whether SUSY-breaking is possible or not. For $W(\phi) \sim \phi^{2n}$ SUSY is always unbroken, whereas for $W(\phi) \sim \phi^{2n+1}$ SUSY breaking is possible. We will focus on the more interesting latter case in these proceedings.

In the lowest order truncation $\Gamma_k[\phi, F, \overline{\psi}, \psi] = \int d^d x(-(1/2)\overline{D}\Phi\gamma_* D\Phi + W_k(\Phi))\big|_{\overline{\theta}\gamma_*\theta}$ and in the following we calculate the flow of the effective superpotential W_k. A nex-to-leading field independent wave function renormalization Z_k can be implemented via $\Phi \to Z_k\Phi$ in the kinetic term, implying a nonvanishing anomalous dimension $\eta = -\partial_t \ln Z_k^2$.

The flow equation for the superpotential is obtained by projecting Eq. (1) onto the part linear in the auxiliary field. To allow for spontaneous SUSY breaking, we consider superpotentials at the cutoff with odd highest power. Actually, the regulator function r_1 amounts to just a ϕ-independent shift so that we can set it to zero without loss of generality. Thus, in what follows we choose $r_1 = 0$ and $r_2 = (|k/p| - 1)\,\theta(1 - p^2/k^2)$ for which the momentum integration can be done analytically, and the flow equation simplifies to

$$\partial_k W_k(\phi) = -\frac{k^{d-1}}{A_d}\frac{W_k''(\phi)}{k^2 + W_k''(\phi)2}, \quad A_2 = 4\pi, \quad A_3 = 8\pi^2. \tag{3}$$

Expanding the superpotential into a power series, $W_k' = \lambda_k(\phi^2 - a_k^2) + \sum\limits_{n=2} b_{2n,k}\phi^{2n}$, the flow equation turns into an infinite system of coupled ordinary differential equations for the scale-dependent coefficients. As initial conditions at the cutoff we take $b_{2n,\Lambda} = 0$ and a double-well potential corresponding to unbroken SUSY. The flow of the bosonic potential $V_k(\phi) = W_k'(\phi)^2/2$ in two dimensions with $\lambda_\Lambda = \Lambda/10$ and $a_\Lambda^2 = 3/10$ is shown in Fig. 1, *b*. The flow of the potential in three dimensions is very similar. As the scale k is lowered to the infrared, a single-well potential emerges and we end up in the phase without SUSY. For larger values of a_Λ we end up in the supersymmetric phase. In the supersymmetric phase the scalar mass is given by $Z_k^4 m^2_{k,\mathrm{boson}} = W_k''^2(\chi_{\min}/Z_k) = Z_k^4 m^2_{k,\mathrm{fermion}}$ and in the broken phase by $Z_k^4 m^2_{k,\mathrm{boson}} = W_k'(0)W_k'''(0) \sim k^{1+\eta/2}$. We find that in the regime with broken SUSY the curvature of the bosonic potential at the minimum and therefore the bosonic mass goes to zero with the RG scale k as $m(k) \sim k^{1/\nu}$. This behavior is governed by a critical exponent ν which obeys the superscaling relation

$$\nu = \frac{2}{d-\eta}, \quad \eta = -k\partial_k \ln Z_k^2, \quad \nu_{d=2} \simeq 1.3, \quad \nu_{d=3} \simeq 0.7,$$

where η denotes the anomalous dimension and d the space-time dimension. We emphasize that any measurement (e.g., lattice simulations) involves an IR cutoff (e.g., lattice size). Hence we predict that any measurement will yield a bosonic mass proportional to the scale provided by this IR cutoff.

The coupling a_Λ^2 is a control parameter for SUSY breaking in both two and three dimensions. In Fig. 2 the phase diagram in the control-parameter plane $(\lambda_\Lambda,\ a_\Lambda^2\lambda_\Lambda)$ is shown for both cases. As a signal for SUSY breaking we use a nonvanishing ground-state energy. We find a maximal value for SUSY breaking at $\lambda_\Lambda a_\Lambda^2 \simeq 0.263$ in two dimensions. This agrees with a qualitative argument given by Witten [16] that spontaneous SUSY breaking is not possible for large values of a_Λ^2.

Let us turn to a discussion of the fixed points. For this we have to rescale the flow equation for the superpotential to dimensionless quantities $w_k(\phi) = W_k(\phi)/k$ and $t = \ln(k/\Lambda)$. In *two dimensions* the field ϕ is dimensionless. The fixed points are characterized by the condition $\partial_t w_* = 0$. In two dimensions this leads to a nonlinear ordinary differential equation with a singularity at $w_k''(\phi) = 1$. The superpotential has two relevant directions corresponding to the coefficients of the terms ϕ^0 and ϕ^1. As only the second derivative of the superpotential enters on the right-hand side of Eq. (3), it is sufficient to consider the second

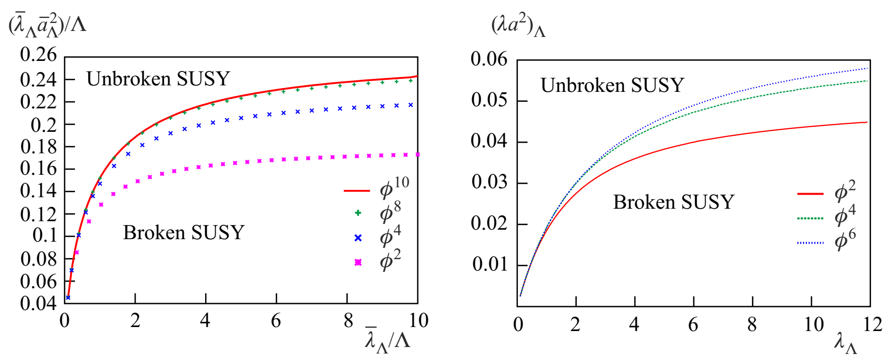

Fig. 2. Phase diagram in two (*a*) and three dimensions (*b*)

derivative of the fixed-point equation to get rid of the IR unstable directions. To leading order of the derivative expansion in two dimensions and with a polynomial expansion to order $2n$, we have $2n$ non-Gaussian fixed points and one Gaussian fixed point. These fixed points are labeled by the coupling λ_n^*. As the absolute value of λ_n^* decreases, the number of IR unstable directions increases. The largest value of λ_n^* has one IR unstable direction. The real parts of these critical exponents at truncation order 16 are given in Table 1. The fixed points found by a polynomial expansion correspond to Taylor expansions of periodic solutions with $|w''(0)| < 2\,|\lambda_{\text{crit}}| = 0.982$. The IR stable fixed point is given by $w''(0) = 2\lambda_{\text{crit}}$.

Table 1. **The critical exponents of the two-dimensional Wess–Zumino model at truncation $2n = 16$ for the different non-Gaussian fixed-point solutions labeled by the value of $|\lambda^*|$**

λ^*	$\text{Re}\,(\theta^{\text{I}})$ of non-Gaussian fixed points, truncation at $2n = 16$							
±0.9816	−1.54	−7.43	−18.3	−37.3	−68.9	−120	−204	−351
±0.8813	6.16	−1.64	−9.82	−25.6	−52.5	−96.9	−170	−300
±0.7131	21.4	4.37	−1.57	−11.1	−30.1	−63.3	−120	−223
±0.5152	28.7	13.3	3.33	−1.39	-11.6	−32.8	−71.7	−145
±0.3158	20.0	20.0	8.40	2.57	−1.14	−11.6	−34.3	−80.4
±0.1437	11.2	11.2	8.63	5.19	1.95	−0.842	−11.1	−35.7
±0.0322	4.20	4.20	2.86	2.72	2.72	1.47	−0.540	−10.5
±0.0003	1.57	1.57	1.43	1.43	1.14	0.542	0.542	−0.221

In *three dimensions* there is only one nontrivial fixed point (pair), the supersymmetric analogue of the Wilson–Fisher fixed point with a scalar potential that behaves like $V(\phi) \sim \phi^6$ for large values of ϕ. The critical exponents are listed in Table 2. Contrary to two dimensions, we observe a rapid convergence of the critical exponents with increasing order of truncation.

Table 2. **The critical exponents at different truncations for the three-dimensional Wess–Zumino model**

$2n$	Critical exponents for different truncations								
6	−0.799	−5.92	−20.9						
8	−0.767	−4.83	−14.4	−38.2					
10	−0.757	−4.35	−11.5	−26.9	−60.8				
12	−0.756	−4.16	−9.94	−21.4	−43.8	−89.0			
14	−0.756	−4.10	−9.13	−18.3	−35.1	−65.4	−123		
16	−0.756	−4.08	−8.72	−16.4	−29.9	−52.9	−91.9	−163	
18	−0.756	−4.08	−8.54	−15.2	−26.4	−45.0	−75.0	−124	−209

For the three-dimensional model at *finite temperatures* the integration in the timelike direction $\int dp_0$ is replaced by a summation over Matsubara frequencies. These sums can be performed analytically. As the Matsubara frequencies are

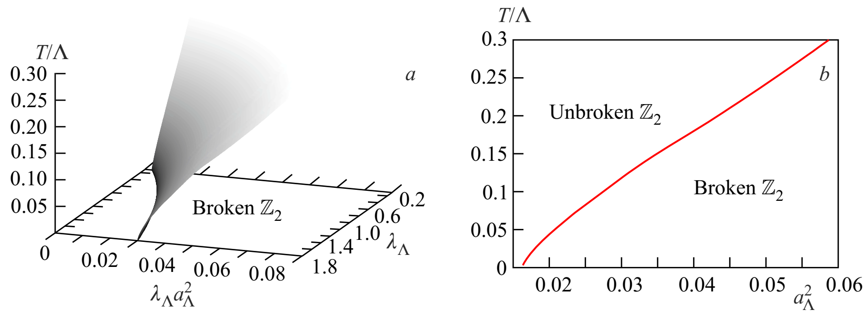

Fig. 3. The phase diagram at finite temperatures

different for bosons and fermions, the flow equation for the «superpotential» obtained from a projection on the bosonic or fermionic part of the flow equation differs by a temperature-dependent factor

$$\partial_k W_k'^{\,\mathrm{bos}} = -\frac{k^2}{8\pi^2} W_k''' \frac{k^2 - W_k''^2}{(k^2 + W_k''^2)2} \times F_{\mathrm{bos}}(T,k), \tag{4}$$

$$\partial_k W_k'^{\,\mathrm{ferm}} = -\frac{k^2}{8\pi^2} W_k''' \frac{k^2 - W_k''^2}{(k^2 + W_k''^2)2} \times F_{\mathrm{ferm}}(T,k) \tag{5}$$

with $F_{\mathrm{bos}} \sim T$ and $F_{\mathrm{ferm}} \to 0$ for large temperatures. For high temperatures the fermions decouple from the flow as they have no Matsubara zero mode. This is the manifestation of supersymmetry breaking at finite temperatures caused by thermal fluctuations. Although SUSY is always broken at finite temperatures, due to different interactions of fermions and bosons with the heat bath, there is still the $\mathbb{Z}_2$ symmetry. At zero temperature this symmetry is spontaneously broken in the supersymmetric phase and it is not broken in case supersymmetry is broken. The flow equation enables us to study the restoration of $\mathbb{Z}_2$ symmetry at finite temperatures. Figure 3 shows the corresponding phase diagram. The surface on the left shows the phase boundary in the space spanned by temperature T/Λ and the value of the couplings $(\lambda_\Lambda a_\Lambda^2)$ and λ_Λ at zero temperature. The curve on the right shows a slice of the phase boundary for fixed $\lambda_\Lambda = 0.8$. At sufficiently high temperature the $\mathbb{Z}_2$ symmetry is always restored.

2. LINEAR SIGMA MODELS

As an example of a solvable flow equation we now discuss the three-dimensional linear sigma model in the large-N limit. The model is built upon a real superfield Φ with N components

$$\Phi^i(x,\theta) = \phi^i(x) + \bar\theta\psi^i(x) + \frac{1}{2}\bar\theta\theta F^i(x).$$

The superfield defines an $O(N)$-invariant composite superfield $R \equiv (1/2)\Phi^i\Phi_i$ withcomponents

$$R = \bar{\varrho} + (\bar{\theta}\psi_i)\phi^i + \frac{1}{2}\,\bar{\theta}\theta\left(\phi^i F_i - \frac{1}{2}\bar{\psi}^i\psi_i\right), \quad \bar{\varrho} = \frac{1}{2}\phi^i\phi_i.$$

The composite superfield is used to define the $O(N)$-invariant supersymmetric action

$$S = \int d^3x \left[-\frac{1}{2}\Phi^i\bar{\mathcal{D}}\mathcal{D}\Phi_i + 2N\,W\left(\frac{R}{N}\right)\right]\Big|_{\bar{\theta}\theta}.$$

Note that the on-shell action contains the bosonic potential $V(\bar{\varrho}) = \bar{\varrho}\,W'^2\,(\bar{\varrho}/N)$. Thus, for any polynomial W' we find $V(0) = 0$ and thus do not expect SUSY breaking in our RG studies.

In the following, we study the fixed-point structure of this model in the limit of many components $N \to \infty$ [9] and thus consider the rescaled dimensionless quantities $\rho = 8\pi^2\bar{\varrho}/(Nk)$ and $w(\rho) = 8\pi^2 W(\bar{\varrho}/N)/k^2$. We use an optimized cutoff function $r_2(p^2) = (k/|p|-1)\theta(k^2-p^2)$ for which the momentum integrals can be calculated analytically. This leads to the flow

$$\partial_t w - \rho w' + 2w = -\left(1-\frac{1}{N}\right)\frac{w'}{1+w'^2} - \frac{1}{N}\,\frac{(w'+2\rho w'')}{1+(w'+2\rho w'')^2}$$

of the dimensionless superpotential. Similar to the bosonic $O(N)$ model, the flow receives two specific contributions: one from the $N-1$ Goldstone modes (first term on the right) and one from the single radial mode (second term on the right). The terms on the left-hand side encode the canonical scaling of the superpotential and the fields. In the limit $N \to \infty$ the radial modes decouples and the flow equation simplifies to

$$\partial_t u + \partial_\rho u\left[1-\rho-u^2\,\frac{3+u^2}{(1+u^2)^2}\right] = -u,$$

where we have introduced $u = w'$ in order to simplify the notation. This nonlinear, first-order PDE can be solved analytically via the method of characteristics which yields

$$\frac{\rho-1}{u} - F(u) = G(ue^t), \qquad F(u) = \frac{u}{1+u^2} + 2\arctan u.$$

As initial condition we have to specify the superpotential $u(\rho)$ at the UV-scale $k = \Lambda$, thus fixing the RG time-dependent function $G(ue^t)$. The fixed-point solution

$$\rho = 1 + H(u_*) + c\,u_*, \quad H(u_*) = u_*\,F(u_*)$$

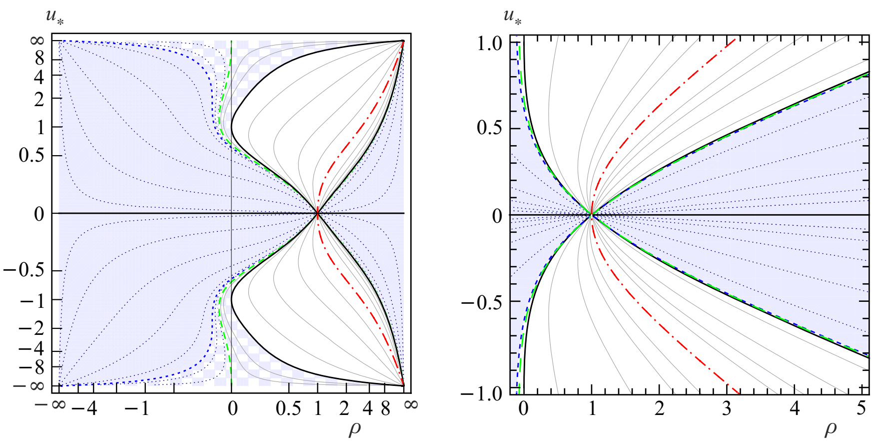

Fig. 4. Fixed-point solutions in the large-N limit. *a*) Global solution $u_*(\rho)$, where the axes are rescaled with $x \to x/(1+|x|)$; *b*) fixed-point solutions $u_*(\rho)$ in the vicinity of the node $\rho = 1$. Specific lines refer to $|c| = \infty$ (horizontal line), $|c| = \pi$ (green, long-dashed lines), $|c| = (\pi+3)/2$ (black, full lines) and $c = 0$ (red, dashed-dotted line)

only depends on the real constant parameter $c = G(ue^t)$. We conclude that the theory possesses a one-parameter family of nontrivial fixed-point solutions, solely parametrized by the constant c, representing the inverse of the linear superfield coupling. All solutions have a node at $\rho = 1$.

As Fig. 4 illustrates, there exist two classes of fixed-point solutions. Solution in the first class are globally well-defined for all values of $\rho \in (-\infty, \infty)$. Solutions in the second class are defined only on part of the $\rho - u$ plane. In the weakly coupled regime, where $|c| \geqslant \pi$, we find a unique fixed-point solution in the physical domain $\rho \geqslant 0$. The intermediate coupling regime with $(\pi+3)/2 \leqslant |c| < \pi$ features two fixed-point solutions, one with a node at $\rho = 1$ and one without a node. However, in the strong coupling regime with $|c| < (\pi+3)/2$ the slope of the potentials diverges at some $\rho_s > 0$ such that the solutions are not defined for all physical fields. Finally, we note that the solution with infinitely large coupling $c^{-1} = \infty$ is closely related to the Wilson–Fisher fixed point of the $3d$ bosonic $O(N)$ model.

In order to determine the universal critical scaling exponents, we consider the flow in the vicinity of a fixed point, i.e., $u(t,\rho) = u_*(\rho) + \delta u(t,\rho)$. Linearizing the flow equation in δu yields the fluctuation equation and explicit solution

$$\partial_t \, \delta u = \frac{u_*}{u'_*}\left(\partial_\rho - \frac{(u_* u'_*)'}{u_* u'_*}\right)\delta u \Rightarrow \delta u(t,u) = \sum_n C_n \, \mathrm{e}^{\lambda_n t} \, u_*^{\lambda_n + 1} \, u'_*, \qquad (6)$$

where λ_n denotes the set of possible eigenvalues. Regularity of the perturbations at $\rho = 1$ then requires nonnegative integer values for the exponent $\lambda_n + 1$. Since the critical exponents θ_n correspond to the negative eigenvalues, we find $\theta_n = 1 - n$, $n \in \mathbb{N}_0$. Interestingly, we obtained Gaussian critical exponents* for non-Gaussian fixed points.

3. SUMMARY AND OUTLOOK

The formulation in superspace is suitable to extend the functional renormalization group to supersymmetric theories. We were able to derive the phase diagram for SUSY breaking and to determine the fixed point structure in the local potential approximation with a constant wave function renormalization for the $\mathcal{N} = 1$ Wess–Zumino model in two and three dimensions. We predict a superscaling relation for the critical exponent corresponding to the ubiquitous IR unstable direction. Furthermore, we solved the three-dimensional supersymmetric $O(N)$ model exactly in the large-N limit and found a line of non-Gaussian fixed points, parametrized by the linear superfield coupling similar to the bosonic $(\phi^2)^3$ theory. This line is bounded by the Gaussian fixed point corresponding to vanishing coupling and a fixed point characterized by an infinitely large linear coupling and related to the fixed point of the $3d$ nonlinear sigma model as well as the Wilson–Fisher fixed point of the $3d$ spherical model.

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