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# GROUP-THEORETICAL CLASSIFICATION OF BPS STATES IN $D = 4$ CONFORMAL SUPERSYMMETRY: THE CASE OF $1/N$-BPS*

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In an earlier paper we gave the complete group-theoretical classification of BPS states of the $N$-extended $D = 4$ conformal superalgebras $su(2, 2/N)$, but not all interesting cases were given in detail. In the present paper we spell out the interesting case of $1/N$-BPS and possibly protected states.

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## INTRODUCTION

Recently, superconformal field theories in various dimensions have been attracting more interest, especially in view of their applications in string theory. Thus, the classification of the UIRs of the conformal superalgebras is of great importance. For some time such a classification was known only for the $D = 4$ superconformal algebras $su(2, 2/1)$ [1] and $su(2, 2/N)$ for arbitrary $N$ [2] (see also [3,4]). Then, more progress was made with the classification for $D = 3$ (for even $N$), $D = 5$, and $D = 6$ (for $N = 1, 2$) in [5] (some results being conjectural), then for the $D = 6$ case (for arbitrary $N$) it was finalized in [6]. Finally, the cases $D = 9, 10, 11$ were treated by finding the UIRs of $osp(1/2n)$ [7].

After we have known the UIRs, the next problem to address is to find their characters since these give the spectrum which is important for the applications. This was done for the UIRs of $D = 4$ conformal superalgebras $su(2, 2/N)$ in [8]. From the mathematical point of view, this question is clear only for representations with conformal dimension above the unitarity threshold viewed as irreps of the corresponding complex superalgebra $sl(4/N)$. But for $su(2, 2/N)$ even the UIRs above the unitarity threshold are truncated for small values of spin and isospin. Moreover, in the applications the most important role is played by the representations with «quantized» conformal dimensions at the unitarity threshold and at

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discrete points below. In the quantum field or string theory framework, some of these correspond to operators with «protected» scaling dimension and therefore imply «non-renormalization theorems» at the quantum level, cf., e.g., [9, 10]. Especially important in this context are the so-called BPS states, cf., [10–16, 18].

These investigations require deeper knowledge of the structure of the UIRs. Fortunately, most of the needed information is contained in [2–4, 19]. We also use more explicit results on the decompositions of long superfields as they descend to the unitarity threshold [8].

In the paper [20], we gave the complete group-theoretical classification of the BPS states, but not all interesting cases were given in detail. In the present paper, motivated by the paper [21], we spell out the interesting case of $1/N$-BPS states.

## 1. PRELIMINARIES

**1.1. Representations of $D = 4$ Conformal Supersymmetry.** The conformal superalgebras in $D = 4$ are $\mathcal{G} = su(2, 2/N)$. The even subalgebra of $\mathcal{G}$ is the algebra $\mathcal{G}_0 = su(2, 2) \oplus u(1) \oplus su(N)$. We label their physically relevant representations of $\mathcal{G}$ by the signature:

$$\chi = [d;\, j_1,\, j_2;\, z;\, r_1, \ldots, r_{N-1}], \tag{1.1}$$

where $d$ is the conformal weight; $j_1, j_2$ are nonnegative (half-)integers which are Dynkin labels of the finite-dimensional irreps of the $D = 4$ Lorentz subalgebra $so(3, 1)$ of dimension $(2j_1 + 1)(2j_2 + 1)$; $z$ represents the $u(1)$ subalgebra which is central for $\mathcal{G}_0$ (and is central for $\mathcal{G}$ itself when $N = 4$), and $r_1, \ldots, r_{N-1}$ are nonnegative integers which are Dynkin labels of the finite-dimensional irreps of the internal (or $R$) symmetry algebra $su(N)$.

We recall the root system of the complexification $\mathcal{G}^{\mathbb{C}}$ of $\mathcal{G}$ (as used in [4]). The positive root system $\Delta^+$ is comprised of $\alpha_{ij}$, $1 \leqslant i < j \leqslant 4 + N$. The even positive root system $\Delta_{\bar{0}}^+$ is comprised of $\alpha_{ij}$, with $i, j \leqslant 4$ and $i, j \geqslant 5$; the odd positive root system $\Delta_{\bar{1}}^+$ is comprised of $\alpha_{ij}$, with $i \leqslant 4, j \geqslant 5$. The generators corresponding to the latter (odd) roots will be denoted as $X^+_{i,4+k}$, where $i = 1, 2, 3, 4$, $k = 1, \ldots, N$.

We use lowest weight Verma modules $V^\Lambda$ over $\mathcal{G}^{\mathbb{C}}$, where the lowest weight $\Lambda$ is characterized by its values on the Cartan subalgebra $\mathcal{H}$ and is in 1-to-1 correspondence with the signature $\chi$. If a Verma module $V^\Lambda$ is irreducible, then it gives the lowest weight irrep $L_\Lambda$ with the same weight. If a Verma module $V^\Lambda$ is reducible, then it contains a maximal invariant submodule $I^\Lambda$ and the lowest weight irrep $L_\Lambda$ with the same weight is given by factorization: $L_\Lambda = V^\Lambda / I^\Lambda$ [22].

There are submodules which are generated by the singular vectors related to the even simple roots [4]. These generate an even invariant submodule $I_c^\Lambda$ present in all Verma modules that we consider and which must be factored out. Thus, instead of $V^\Lambda$, we shall consider the factor-modules:

$$\tilde{V}^\Lambda = V^\Lambda / I_c^\Lambda. \tag{1.2}$$

The Verma module reducibility conditions for the $4N$ odd positive roots of $\mathcal{G}^{\mathbb{C}}$ were derived in [3,4] adapting the results of Kac [22]:

$$\begin{gathered} d = d^1_{Nk} - z\delta_{N4}, \\ d^1_{Nk} \equiv 4 - 2k + 2j_2 + z + 2m_k - 2m/N, \end{gathered} \tag{1.3a}$$

$$\begin{gathered} d = d^2_{Nk} - z\delta_{N4}, \\ d^2_{Nk} \equiv 2 - 2k - 2j_2 + z + 2m_k - 2m/N, \end{gathered} \tag{1.3b}$$

$$\begin{gathered} d = d^3_{Nk} + z\delta_{N4}, \\ d^3_{Nk} \equiv 2 + 2k - 2N + 2j_1 - z - 2m_k + 2m/N, \end{gathered} \tag{1.3c}$$

$$\begin{gathered} d = d^4_{Nk} + z\delta_{N4}, \\ d^4_{Nk} \equiv 2k - 2N - 2j_1 - z - 2m_k + 2m/N, \end{gathered} \tag{1.3d}$$

where in all four cases of (1.3) $k = 1, \ldots, N$, $m_N \equiv 0$, and

$$m_k \equiv \sum_{i=k}^{N-1} r_i, \quad m \equiv \sum_{k=1}^{N-1} m_k = \sum_{k=1}^{N-1} k r_k. \tag{1.4}$$

Note that we shall also use the quantity $m^*$ which is conjugate to $m$:

$$m^* \equiv \sum_{k=1}^{N-1} k r_{N-k} = \sum_{k=1}^{N-1} (N-k) r_k, \tag{1.5}$$

$$m + m^* = N m_1. \tag{1.6}$$

We need the result of [2] (cf. part (i) of the Theorem there) that the following is the complete list of the lowest weight (positive energy) UIRs of $su(2,2/N)$:

$$d \geqslant d_{\max} = \max\,(d^1_{N1}, d^3_{NN}), \tag{1.7a}$$

$$d = d^4_{NN} \geqslant d^1_{N1}, \quad j_1 = 0, \tag{1.7b}$$

$$d = d^2_{N1} \geqslant d^3_{NN}, \quad j_2 = 0\ , \tag{1.7c}$$

$$d = d^2_{N1} = d^4_{NN}, \quad j_1 = j_2 = 0, \tag{1.7d}$$

where $d_{\max}$ is the threshold of the continuous unitary spectrum. Note that in case (d) we have $d = m_1$, $z = 2m/N - m_1$, and that it is trivial for $N = 1$.

Next we note that if $d > d_{\max}$ the factorized Verma modules are irreducible and coincide with the UIRs $L_\Lambda$. These UIRs are called *long* in the modern literature, cf., e.g., [10, 18, 23–27]. Analogously, we shall use for the cases when $d = d_{\max}$, i.e., (1.7a), the terminology of *semishort* UIRs, introduced in [10, 23], while the cases (1.7b), (1.7c), (1.7d) are also called *short* UIRs, cf., e.g., [10, 18, 24–27].

Next consider in more detail the UIRs at the four distinguished reducibility points determining the UIRs list above: $d^1_{N1}$, $d^2_{N1}$, $d^3_{NN}$, $d^4_{NN}$. We note a partial ordering of these four points:

$$d^1_{N1} > d^2_{N1}, \quad d^3_{NN} > d^4_{NN}. \tag{1.8}$$

Due to this ordering, *at most two* of these four points may coincide.

First we consider the situations in which *no two* of the distinguished four points coincide. There are four such situations:

$$\begin{aligned}
&\mathbf{a}\colon\quad d = d_{\max} = d^1_{N1} = d^a \equiv 2 + 2j_2 + z + 2m_1 - 2m/N > d^3_{NN}, && \text{(1.9a)}\\
&\mathbf{b}\colon\quad d = d^2_{N1} = d^b \equiv z - 2j_2 + 2m_1 - 2m/N > d^3_{NN}, \quad j_2 = 0, && \text{(1.9b)}\\
&\mathbf{c}\colon\quad d = d_{\max} = d^3_{NN} = d^c \equiv 2 + 2j_1 - z + 2m/N > d^1_{N1}, && \text{(1.9c)}\\
&\mathbf{d}\colon\quad d = d^4_{NN} = d^d \equiv 2m/N - 2j_1 - z > d^1_{N1}, \quad j_1 = 0, && \text{(1.9d)}
\end{aligned}$$

where for future use we have introduced notations $d^a$, $d^b$, $d^c$, $d^d$, the definitions including also the corresponding inequality.

We shall call these cases *single-reducibility-condition (SRC)* Verma modules or UIRs, depending on the context. In addition, as already stated, we use for the cases when $d = d_{\max}$, i.e., (1.9a), (1.9c), the terminology of semishort UIRs, while the cases (1.9b), (1.9d) are also called short UIRs.

The factorized Verma modules $\tilde{V}^\Lambda$ with the unitary signatures from (1.9) have only one invariant odd submodule which has to be factorized in order to obtain the UIRs.

We consider now the four situations in which *two* distinguished points coincide:

$$\begin{aligned}
&\mathbf{ac}\colon\quad d = d_{\max} = d^{ac} \equiv 2 + j_1 + j_2 + m_1 = d^1_{N1} = d^3_{NN}, && \text{(1.10a)}\\
&\mathbf{ad}\colon\quad d = d^{ad} \equiv 1 + j_2 + m_1 = d^1_{N1} = d^4_{NN}, \quad j_1 = 0, && \text{(1.10b)}\\
&\mathbf{bc}\colon\quad d = d^{bc} \equiv 1 + j_1 + m_1 = d^2_{N1} = d^3_{NN}, \quad j_2 = 0, && \text{(1.10c)}\\
&\mathbf{bd}\colon\quad d = d^{bd} \equiv m_1 = d^2_{N1} = d^4_{NN}, \quad j_1 = j_2 = 0. && \text{(1.10d)}
\end{aligned}$$

We shall call these *double-reducibility-condition (DRC)* Verma modules or UIRs. The cases in (1.10a) are semishort UIR, while the other cases are short.

## 2. BPS AND POSSIBLY PROTECTED STATES

BPS states are characterized by the number $\kappa$ of odd generators which annihilate them — then the corresponding state is called $\kappa/4N$-BPS state. The most interesting case for BPS states is when $N = 4$ since it is related to super-Yang–Mills, cf., [10–18]. Also, group-theoretically, the case $N = 4$ is special since the $u(1)$ subalgebra carrying the quantum number $z$ becomes central and one can invariantly set $z = 0$. When $N \neq 4$ we can also set $z = 0$, though this does not have the same group-theoretical meaning as for $N = 4$.

In the paper [20], we gave the complete classification of the BPS states, but not all interesting cases were given in detail. In the present paper, motivated by the paper [21], we spell out the interesting case of $1/N$-BPS states, i.e., the cases when $\kappa = 4$.

It is convenient to consider the case of general $N$ while treating separately $R$-symmetry scalars and $R$-symmetry nonscalars.

**2.1.** ***R*-Symmetry Scalars.** We start with the simpler cases of $R$-symmetry scalars when $r_i = 0$ for all $i$, which also means that $m_1 = m = m^* = 0$.

These cases are also valid for $N = 1$, however for $N = 1$ in all cases we have $\kappa < 4$ [20].

In fact, only three cases are relevant for $\kappa = 4$.

• **a** $d = (d^1_{N1})_{|m=0=z} = 2 + 2j_2 > 2 + 2j_1 = (d^3_{NN})_{|m=0=z}$. The last inequality leads to the restriction: $j_2 > j_1$, i.e., $j_2 > 0$, and then we have

$$\kappa = N, \quad m_1 = m = 0, \quad j_2 > 0. \tag{2.1}$$

These semishort UIRs may be called semichiral since they lack half of the antichiral generators: $X^+_{3,4+k}$, $k = 1, \ldots, N$.

• **c** $d = (d^3_{NN})_{|m=0=z} = 2 + 2j_1 >= (d^1_{N1})_{|m=0=z} \implies$

$$\kappa = N, \quad m_1 = m = 0, \quad j_1 > 0. \tag{2.2}$$

These semishort UIRs may be called semi-antichiral since they lack half of the chiral generators: $X^+_{1,4+k}$, $k = 1, \ldots, N$.

Thus, in both cases above, the interesting case $\kappa = 4$ occurs only for $N = 4$, as $1/4$-BPS.

• **ac**
$$\begin{aligned}
&d = d^{ac}_{|m=0} = 2 + j_1 + j_2, \quad z = j_1 - j_2,\\
&\kappa = 2N, \quad \text{if} \quad j_1, j_2 > 0,\\
&\kappa = N+1, \quad \text{if} \quad j_1 > 0, \quad j_2 = 0,\\
&\kappa = N+1, \quad \text{if} \quad j_1 = 0, \quad j_2 > 0,\\
&\kappa = 2, \quad \text{if} \quad j_1 = j_2 = 0.
\end{aligned}$$

Here, $\kappa$ is the number of mixed elimination: chiral generators $X^+_{1,4+k}$ ($k = 1, \ldots, N + (1-N)\delta_{j_1,0}$), and antichiral generators $X^+_{3,5+N-k}$ ($k = 1, \ldots, N + (1-N)\delta_{j_2,0}$). Thus, in the cases when $\kappa = 2N$ the semishort UIRs may be called semichiral-antichiral since they lack half of the chiral and half of the antichiral generators. The interesting case $\kappa = 4$ occurs only for $N = 2$, as 1/2-BPS.

**2.2. $R$-Symmetry Nonscalars.** Below we need some additional notation. Let $N > 1$ and let $i_0$ be an integer such that $0 \leqslant i_0 \leqslant N-1$, $r_i = 0$ for $i \leqslant i_0$, and if $i_0 < N-1$, then $r_{i_0+1} > 0$. Let now $i'_0$ be an integer such that $0 \leqslant i'_0 \leqslant N-1$, $r_{N-i} = 0$ for $i \leqslant i'_0$, and if $i'_0 < N-1$, then $r_{N-1-i'_0} > 0$.

The interesting cases of $1/N$-BPS states, i.e., when $\kappa = 4$, are given in the following list:

$$\bullet\ \mathbf{a} \qquad \begin{aligned} & d = d^a = 2 + 2j_2 + 2m^*/N, \quad N \geqslant 5, \\ & j_1 \text{ arbitrary}, \quad j_2 > 0, \quad i_0 = 3, \quad 0 \leqslant i'_0 \leqslant N-5, \\ & j_2 > j_1 + \sum_{k=4}^{N-1} (2k/N - 1) r_k. \end{aligned} \tag{2.3}$$

Here are eliminated four antichiral generators $X^+_{3,4+k}$, $k \leqslant 4$.

$$\bullet\ \mathbf{b} \qquad \begin{aligned} & d = d^b = 2m^*/N, \quad N \geqslant 5, \\ & j_2 = 0, \quad j_1 \text{ arbitrary}, \quad i_0 = 1, \quad 0 \leqslant i'_0 \leqslant N-3, \\ & \sum_{k=2}^{[(N-1)/2]} (1 - 2k/N) r_k > j_1 + \sum_{[(N+1)/2]}^{N-1} (2k/N - 1) r_k. \end{aligned} \tag{2.4}$$

Here are eliminated four antichiral generators $X^+_{3,5+N-k}$, $X^+_{4,5+N-k}$, $k \leqslant 2$.

$$\bullet\ \mathbf{c} \qquad \begin{aligned} & d = d^c = 2 + 2j_1 + 2m/N, \quad N \geqslant 5, \\ & j_1 > 0, \ j_2 \text{ arbitrary}, \quad i'_0 = 3, \quad 0 \leqslant i_0 \leqslant N-5, \\ & j_1 > j_2 + \sum_{k=1}^{N-4} (1 - 2k/N) r_k. \end{aligned} \tag{2.5}$$

Here are eliminated four chiral generators $X^+_{1,4+k}$, $k \leqslant 4$.

$$\bullet\ \mathbf{d} \qquad \begin{aligned} & d = d^d = 2m/N, \quad N \geqslant 5, \\ & j_1 = 0, \quad j_2 \text{ arbitrary}, \quad i'_0 = 1, \quad 0 \leqslant i_0 \leqslant N-3, \\ & \sum_{k=1}^{[(N-1)/2]} (1 - 2k/N) r_k > j_2 + \sum_{[(N+1)/2]}^{N-4} (2k/N - 1) r_k. \end{aligned} \tag{2.6}$$

Here are eliminated four chiral generators $X^+_{1,4+k}$, $X^+_{2,4+k}$, $k \leqslant 2$.

$$\bullet\ \mathbf{ac} \qquad d = d^{ac} = 2 + j_1 + j_2 + m_1, \quad N \geqslant 4,$$
$$j_1 + m/N = j_2 + m^*/N,$$
$$j_1 j_2 > 0, \quad i_0 + i'_0 = 2, \tag{2.7a}$$
$$j_1 > 0, \quad j_2 = 0, \quad i_0 = 0, \quad i'_0 = 2, \tag{2.7b}$$
$$j_1 = 0, \quad j_2 > 0, \quad i_0 = 2, \quad i'_0 = 0. \tag{2.7c}$$

Here are eliminated four generators: chiral generators $X^+_{1,4+k}$, $k \leqslant 1 + i'_0(1 - \delta_{j_1,0})$, and antichiral generators $X^+_{3,5+N-k}$, $k \leqslant 1 + i_0(1 - \delta_{j_2,0})$.

$$\begin{aligned} \bullet\ \mathbf{ad} \qquad & d = d^{ad} = 1 + j_2 + m_1 = 2m/N, \quad N \geqslant 3, \\ & j_1 = 0, \quad j_2 > 0, \quad i_0 = 1, \quad i'_0 = 0. \end{aligned} \tag{2.8}$$

Here are eliminated two chiral generators $X^+_{1,5}$, $X^+_{2,5}$, and two antichiral generators $X^+_{3,5+N-k}$, $k = 1, 2$.

$$\begin{aligned} \bullet\ \mathbf{bc} \qquad & d = d^{bc} = 1 + j_1 + m_1 = 2m^*/N, \quad N \geqslant 3, \\ & j_2 = 0, \quad j_1 > 0, \quad i_0 = 0, \quad i'_0 = 1. \end{aligned} \tag{2.9}$$

Here are eliminated two chiral generators $X^+_{1,4+k}$, $k = 1, 2$, and two antichiral generators $X^+_{3,8}$, $X^+_{4,8}$.

$$\begin{aligned} \bullet\ \mathbf{bd} \qquad & d = d^{bd} = m_1, \quad N \geqslant 2, \\ & j_1 = j_2 = 0, \quad i_0 = i'_0 = 0. \end{aligned} \tag{2.10}$$

Here are eliminated two chiral generators $X^+_{1,5}$, $X^+_{2,5}$, and two antichiral generators $X^+_{3,8}$, $X^+_{4,8}$.

Note that according to the results of [20], the following cases would not be protected: **ad** for $r_{N-1} > 2$, **bc** for $r_1 > 2$, **bd** for $r_1, r_{N-1} > 2$ when $N > 2$, and for $r_1 > 4$ when $N = 2$.

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REFERENCES

1. *Flato M., Fronsdal C.* // Lett. Math. Phys. 1984. V. 8. P. 159–162.
2. *Dobrev V. K., Petkova V. B.* // Phys. Lett. B. 1985. V. 162. P. 127–132.

3. *Dobrev V. K., Petkova V. B.* // Lett. Math. Phys. 1985. V. 9. P. 287–298.
4. *Dobrev V. K., Petkova V. B.* // Fortschr. Phys. 1987. V. 35. P. 537–572.
5. *Minwalla S.* // Adv. Theor. Math. Phys. 1998. V. 2. P. 781–846.
6. *Dobrev V. K.* // J. Phys. A. 2002. V. 35. P. 7079–7100.
7. *Dobrev V. K., Zhang R. B.* // Phys. At. Nucl. 2005. V. 68. P. 1660–1669.
8. *Dobrev V. K.* // Part. Nucl. (Fiz. Elem. Chast. At. Yadra). 2007. V. 38. P. 1079–1162 (564–609).
9. *Heslop P. J., Howe P. S.* // Class. Quant. Grav. 2000. V. 17. P. 3743–3768.
10. *Ferrara S., Sokatchev E.* // Intern. J. Theor. Phys. 2001. V. 40. P. 935–984.
11. *Andrianopoli L. et al.* // Adv. Theor. Math. Phys. 1999. V. 3. P. 1149–1197.
12. *Ferrara S., Sokatchev E.* // JHEP. 2000. V. 0005. P. 038; Intern. J. Mod. Phys. B. 2000. V. 14. P. 2315; New J. Phys. 2002. V. 4. P. 2–22.
13. *Ryzhov A. V.* // JHEP. 2001. V. 0111. P. 046; Operators in the $D = 4$, $N = 4$ SYM and the AdS/CFT Correspondence, 2003. UCLA Thesis. 169 p.; hep-th/0307169.
14. *D'Hoker E., Ryzhov A. V.* // JHEP. 2002. V. 0202. P. 047.
15. *Arutyunov G., Sokatchev E.* // Nucl. Phys. B. 2002. V. 635. P. 3–32; Class. Quant. Grav. 2003. V. 20. P. L123–L131.
16. *D'Hoker E. et al.* // JHEP. 2003. V. 0304. P. 038.
17. *Dolan F. A.* // Nucl. Phys. B. 2008. V. 790. P. 432–464.
18. *Eden B., Sokatchev E.* // Nucl. Phys. B. 2001. V. 618. P. 259–276.
19. *Dobrev V. K., Petkova V. B.* // Lect. Notes in Phys. V. 261. Berlin: Springer-Verlag, 1986. P. 291–299; 300–308.
20. *Dobrev V. K.* Group-Theoretical Classification of BPS and Possibly Protected States in $D = 4$ Conformal Supersymmetry. arXiv:1012.3685 [hep-th]; Preprint SISSA/86/2010/EP & ESI 2289. 2010; Nucl. Phys. B (in press).
21. *Bossard G. et al.* The Vanishing Volume of $D = 4$ Superspace. arXiv:1105.6087 [hep-th].
22. *Kac V. G.* Representations of Classical Lie Superalgebras // Lect. Notes in Math. V. 676. Berlin: Springer-Verlag, 1978. P. 597–626.
23. *Freedman D. Z. et al.* // Adv. Theor. Math. Phys. 1999. V. 3. P. 363–417.
24. *Ferrara S., Sokatchev E.* // Lett. Math. Phys. 2000. V. 52. P. 247–262.
25. *Arutyunov G., Eden B., Sokatchev E.* // Nucl. Phys. B. 2001. V. 619. P. 359–372.
26. *Bianchi M. et al.* // JHEP. 2001. V. 0105. P. 042.
27. *Heslop P. J., Howe P. S.* // Phys. Lett. B. 2001. V. 516. P. 367–375.