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CRITICAL GRAVITIES IN $d \geqslant 3$

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Critical gravities originated in the context of three-dimensional massive gravities. They were conjectured to describe gravitational duals of two-dimensional logarithmic conformal field theories. In this talk, we show that critical gravities can also be studied in dimensions $d > 3$. As in three dimensions, higher-dimensional critical gravities exhibit logarithmic modes. We argue that the existence of these logarithmic modes leads one to conjecture that higher-dimensional critical gravities are dual to logarithmic conformal field theories in more than two dimensions.

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INTRODUCTION

Recently, three-dimensional massive gravities have received a lot of attention. These theories are extensions of ordinary Einstein–Hilbert gravity that include higher-derivative terms. The two best known examples are Topologically Massive Gravity (TMG) [1] and New Massive Gravity (NMG) [2]. In TMG, the higher-derivative term consists of a Lorentz–Chern–Simons term, whereas in NMG, one adds a specific curvature squared term to the usual Einstein–Hilbert action. The so-called General Massive Gravity (GMG) model is then obtained by combining TMG and NMG. Generically, one also adds a cosmological constant to be able to study massive gravities around AdS vacua. All of these models come with a parameter space, given essentially by a sign parameter $\sigma = \pm 1,\ 0$ in front of the Einstein–Hilbert term, a parameter λ determining the cosmological constant and finally mass parameters that accompany the higher-derivative terms. For certain regions in this parameter space, these models are perturbatively unitary, in the sense that they propagate one or two massive gravitons with the correct sign of the kinetic terms. Unfortunately, in these parameter regions massive gravities typically exhibit black hole solutions with negative energies, thus spoiling unitarity at the nonperturbative level.

When considered around AdS, for certain, the so-called «critical» values of the parameters, the massive gravitons become massless and the spectrum does not

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contain massive modes. Gravity theories at critical parameter values are referred to as critical gravities. Instead of the massive modes, a critical theory propagates a new kind of modes, that were dubbed logarithmic modes [3]. These are distinguished by a logarithmic fall-off behavior towards the AdS boundary, in contrast to massless and massive gravitons that obey the usual Brown–Henneaux boundary conditions. The existence of these logarithmic modes has interesting implications in the light of the AdS/CFT correspondence (see, e.g., [4–7]). It has been conjectured that three-dimensional critical gravities are no longer dual to ordinary two-dimensional conformal field theories. Instead the dual field theory, there is the so-called logarithmic conformal field theory (LCFT). LCFTs are characterized by a nondiagonalizable Hamiltonian. Although LCFTs are nonunitary, they have been studied by condensed matter physicists in a variety of contexts such as percolation, turbulence, and critical phenomena.

The purpose of this talk is to indicate that the notion of critical gravity is not a purely three-dimensional one, but also exists in higher dimensions. In particular, I will generalize critical NMG to arbitrary dimensions and show that the model again exhibits logarithmic modes. The existence and properties of these logarithmic modes lead to a higher-dimensional generalization of the conjecture that three-dimensional critical gravities are dual to two-dimensional LCFTs.

1. CRITICAL NMG IN ARBITRARY DIMENSIONS AND LOGARITHMIC MODES

The action of three-dimensional New Massive Gravity can be generalized to higher dimensions. In arbitrary dimension d, the action is explicitly given by [8–10]:

$$S = \frac{1}{\kappa^2} \int d^d x \sqrt{-g} \left\{ \sigma R - 2\lambda m^2 + \frac{1}{m^2} G^{\mu\nu} S_{\mu\nu} \right\}, \tag{1}$$

where

$$S_{\mu\nu} = \frac{1}{(d-2)} \left(R_{\mu\nu} - \frac{1}{2(d-1)} R g_{\mu\nu} \right), \tag{2}$$

and $G_{\mu\nu}$ denotes the Einstein tensor. The first term is the usual Einstein–Hilbert term, multiplied by a sign parameter σ that can take values $\sigma = 0, \pm 1$. The second term is necessary to include a cosmological constant Λ, and λ is referred to as the cosmological parameter. The last term is a specific curvature squared term, that is such that the action (1) only propagates spin-2 modes. The latter property can be made explicit by doing a linearized analysis. In order to perform the linearized analysis, it is convenient to turn the four-derivative action (1) in a two-derivative one, by introducing an auxiliary field, that corresponds to a symmetric two-tensor $f_{\mu\nu}$. One can then linearize the two-derivative action. The result is given by

$$\mathcal{L}_2 = -\frac{1}{2}\bar{\sigma} h^{\mu\nu}\mathcal{G}_{\mu\nu}(h) + \frac{2}{m^2(d-1)(d-2)} k^{\mu\nu}\mathcal{G}_{\mu\nu}(h) - \\ - \frac{1}{m^2(d-2)(d-1)^2}(k^{\mu\nu}k_{\mu\nu} - k^2), \quad (3)$$

where $h_{\mu\nu}$, resp. $k_{\mu\nu}$, denote the perturbations of the metric and auxiliary field $f_{\mu\nu}$; $\mathcal{G}_{\mu\nu}$ denotes the linearized Einstein operator, a differential operator that, when acting on the metric perturbation h, gives the linearized Einstein tensor. The constant $\bar{\sigma}$ is given by

$$\bar{\sigma} = \sigma - \frac{\Lambda}{m^2}\frac{1}{d-1}. \quad (4)$$

When $\bar{\sigma} \neq 0$, the Lagrangian (3) can be diagonalized. The result is given by the sum of a linearized Einstein–Hilbert Lagrangian for a massless graviton and a linearized Fierz–Pauli Lagrangian for a massive spin-2 particle, with mass $M^2 = -m^2(d-2)\bar{\sigma}$. The signs in front of the Einstein–Hilbert part and the Fierz–Pauli part are opposite, showing that the model is generically not unitary, except in three dimensions, where the Einstein–Hilbert part does not describe any propagating degrees of freedom.

The case where $\bar{\sigma} = 0$ corresponds to «critical NMG». In this case, the Lagrangian (3) can no longer be diagonalized. Moreover, one also notices that the mass M^2 of the massive Fierz–Pauli gravitons goes to zero in this limit. Effectively, one finds that the massive gravitons become massless gravitons or pure gauge solutions when $\bar{\sigma} \to 0$. As in three dimensions, one can show that a new kind of solutions of the (linearized) equations of motion appears in the $\bar{\sigma} = 0$ case, that correspond to logarithmic modes. At the critical point, the linearized equations of motion correspond to stating that the metric perturbation h is annihilated by twice the action of the linearized Einstein operator:

$$\mathcal{G}_{\mu\nu}(\mathcal{G}(h)) = 0. \quad (5)$$

The solutions of (5) can be divided in three classes:

- Solutions for h that obey $\mathcal{G}_{\mu\nu}(h) = 0$. These correspond to massless gravitons.
- Solutions for h that obey $\mathcal{G}_{\mu\nu}(h) = 2\nabla_{(\mu}A_{\nu)}$, where A_μ is an arbitrary vector. These solutions correspond to logarithmic modes that were dubbed «Proca log modes».
- Solutions for h that obey $\mathcal{G}_{\mu\nu}(h) = k^{\perp}_{\mu\nu}$, where $k^{\perp}_{\mu\nu}$ is a nontrivial solution of the linearized Einstein equations. These solutions correspond to logarithmic modes that were called «spin-2 log modes».

The logarithmic modes can also be explicitly constructed using group theoretical techniques. In the following section, we will give the logarithmic modes in four dimensions and show how their properties lead to the conjecture that four-dimensional critical NMG is dual to a three-dimensional LCFT.

2. THE CRITICAL GRAVITY/LCFT CORRESPONDENCE

The logarithmic modes were obtained explicitly in four-dimensional critical NMG in [10]. We will work with the AdS_4 metric in global coordinates:

$$ds^2 = -d\tau^2 \cosh(\rho)^2 + d\rho^2 + \sinh(\rho)^2 (d\theta^2 + d\phi^2 \sin(\theta)^2). \tag{6}$$

There are then five logarithmic modes, labeled by a spin eigenvalue $s = -2, \ldots, 2$. The modes are explicitly given by

$$\psi_{\mu\nu}^{\log}(s) = f(\tau, \rho)\, \psi_{\mu\nu}^{(s)}(E_0 = 3), \tag{7}$$

where

$$f(\tau, \rho) = \frac{1}{2}(-2i\tau - \log(\sinh(2\rho)) + \log(\tanh(\rho))), \tag{8}$$

and $\psi_{\mu\nu}^{(s)}(E_0 = 3)$ for $s = -2, \ldots, 2$ denote five solutions of the linearized Einstein equations that are either pure gauge or massless gravitons. The behavior of these logarithmic modes towards the AdS boundary is similar to the boundary behavior exhibited by logarithmic modes in three dimensions. Moreover, it can be checked that the logarithmic modes are eigenstates under the AdS spin operator $-i\partial_\phi$:

$$-i\partial_\phi \psi_{\mu\nu}^{\log}(s) = s\psi_{\mu\nu}^{\log}(s). \tag{9}$$

Crucially, the logarithmic modes are no longer eigenstates of the AdS energy $i\partial_\tau$. Instead, one has

$$i\partial_\tau\ \psi_{\mu\nu}^{\log}(s) = 3\psi_{\mu\nu}^{\log}(s) + \psi_{\mu\nu}^{(s)}(E_0 = 3). \tag{10}$$

This implies that the AdS energy operator is no longer diagonalizable. In the context of the AdS/CFT correspondence, the AdS energy operator translates to the Hamiltonian of the dual field theory. The existence of the logarithmic modes thus implies that the Hamiltonian of the dual field theory is no longer diagonalizable. This is the property that in two dimensions defines a logarithmic CFT. In light of the above, it is thus natural to expect that critical gravities in more than three dimensions lead to dual field theories in more than two dimensions that can be seen as generalizations of two-dimensional LCFTs.

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