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GENERALIZED PARTON DISTRIBUTIONS FROM MESON LEPTOPRODUCTION

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Generalized parton distributions (GPDs) extracted from exclusive meson leptoproduction within the handbag approach are briefly reviewed. Only the GPD E is discussed in some detail. Applications of these GPDs to virtual Compton scattering (DVCS) and to Ji's sum rule are also presented.

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INTRODUCTION

The handbag approach to hard exclusive leptoproduction of photons and mesons (DVMP) off protons has extensively been studied during the last fifteen years. The handbag approach is based on factorization of the process amplitudes in a hard subprocess, e.g., $\gamma^* q \to \gamma(M) q$, and soft hadronic matrix elements parameterized in terms of GPDs. This factorization property has been shown to hold rigorously in the generalized Bjorken regime of large photon virtuality, Q, and large energy W but fixed x_B. Since most data, in particular the data from Jlab, are not measured in this kinematical regime, one has to be aware of power corrections from various sources. Which kinds of power corrections are the most important ones and have to be taken into account is still under debate. Nevertheless, progress has been made in the understanding of the DVCS and DVMP data. In this talk, I am going to report on an extraction of the GPDs from DVMP [1]. In this analysis the GPDs are constructed from double distributions (DDs) [2, 3] and the subprocess amplitudes are calculated taking into account quark transverse degrees of freedom as well as Sudakov suppression [4]. The emission and reabsorption of the partons by the protons are treated collinearly. This approach also allows one to calculate the amplitudes for transversely polarized photons which are infrared singular in collinear factorization. The transverse photon amplitudes are rather strong for $Q^2 \lesssim 10\ \mathrm{GeV}^2$ as is known from the ratio of longitudinal and transverse cross sections [5]. I am also going to discuss several applications of the extracted set of GPDs like the calculation of DVCS observables from them [6] or the evaluation of the parton angular momenta.

1. THE DOUBLE DISTRIBUTION REPRESENTATION

There is an integral representation of the GPD $F^i = H^i, E^i, \widetilde{H}^i, \ldots$ ($i = u, d, s, g$) in terms of DDs [2, 3]

$$F^i(x, \xi, t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho + \xi\eta - x)\, f_i(\rho, \eta, t) + D_i(x, t)\, \Theta(\xi^2 - x^2), \quad (1)$$

where D is the so-called D-term [7] which appears for the gluon and flavor-singlet quark combination of the GPDs H and E. The advantage of the DD representation is that polynomiality of the GPDs is automatically satisfied. A popular ansatz for the DD is

$$f_i(\rho, \eta, t) = F^i(\rho, \xi = 0, t)\, w_i(\rho, \eta), \quad (2)$$

where the weight function w_i that generates the skewness dependence of the GPD, is assumed to be

$$w_i(\rho, \eta) = \frac{\Gamma(2n_i + 2)}{2^{2n_i+1}\Gamma^2(n_i + 1)} \frac{[(1 - |\rho|)^2 - \eta^2]^{n_i}}{(1 - |\rho|)^{2n_i+1}} \quad (3)$$

(in [1] $n = 1$ for valence quarks; and 2, for sea quarks and gluons). The zero-skewness GPD is parameterized as its forward limit multiplied by an exponential in Mandelstam t

$$F^i(\rho, \xi = 0, t) = F^i(\rho, \xi = 0, t = 0)\, \exp\left[t p_{fi}(\rho)\right]. \quad (4)$$

The profile function, $p_{fi}(\rho)$, is parameterized in a Regge-like manner

$$p_{fi}(\rho) = -\alpha'_{fi} \ln \rho + B_{fi}, \quad (5)$$

where α' represents the slope of an appropriate Regge trajectory and B parameterizes the t dependence of its residue. This profile function is a simplified version of a more complicated one that has been proposed in [8]

$$p_{fi}(\rho) = \left(\alpha'_{fi} \ln 1/\rho + B_{fi}\right)(1 - \rho)^3 + A_{fi}\, \rho(1 - \rho)^2. \quad (6)$$

The profile function (5) approximates (6) for small ρ. Because of a strong $\rho - t$ correlation observed in [8], the profile function (5) can be applied at small $-t$. For the forward limits of the GPDs H, $\widetilde{H}$, and H_T, the corresponding parton distributions (PDFs) are used. The forward limits of the other GPDs are parameterized in a fashion analogously to the PDFs

$$F^i(\rho, \xi = t = 0) = c_i \rho^{-\alpha_i}(1 - \rho)^{\beta_i}. \quad (7)$$

For alternative parameterizations of the GPDs in terms of conformal $SL(2, R)$ partial waves, see [9, 10].

2. EXTRACTION OF THE GPDs

The GPDs are inserted into the convolution formula for vector mesons

$$\mathcal{F}_{\rm V}^{i}(\xi,t,Q^2)=\sum_{\lambda}\int_{x_i}^{1}dx\,\mathcal{A}^{i}_{0\lambda,0\lambda}(x,\xi,Q^2,t=0)\;F^{i}(x,\xi,t), \tag{8}$$

where $i=g,q$, $x_g=0$, $x_q=-1$ and F either H or E. A similar convolution formula holds for pseudoscalar mesons. The subprocess amplitude $\mathcal{A}$ for partonic helicity λ is to be calculated perturbatively using $k_\perp$ factorization. In collinear factorization, the cross section for the production of vector mesons drops as $1/Q^6(\log Q^2)^n$ with increasing Q^2 while experimentally [5] it approximately falls as $1/Q^4$. The required suppression of the amplitudes at low Q^2 is generated by the evolution of the GPDs and by $k_\perp$ effects. In [9], however, GPDs are proposed which have a much stronger evolution. At least for HERA kinematics, these GPDs lead to fair fits of the HERA data on vector meson electroproduction in collinear factorization.

In [1], parameters of the double distribution are fitted to the available data on ρ^0, ϕ, and π^+ production from HERMES, COMPASS, E665, H1, and ZEUS. The data cover a large kinematical range: $3\lesssim Q^2\lesssim 100$ GeV^2, $4\lesssim W\lesssim 180$ GeV. Data from the present Jlab are not taken into account in these fits because they are likely affected by strong power corrections at least in some cases (e.g., ρ^0 production). Constraints from nucleon form factors and from positivity bounds [8] are taken into account. Some parameters of the transversity GPDs needed for pion electroproduction, are fixed by lattice QCD results [11]. The fit leads to a fair description of all the mentioned data. What we learned about the GPDs is summarized in the Table.

Status of small-skewness GPDs as extracted from meson leptoproduction data. At present, no information is available on GPDs not appearing in the list. Except for H for gluons and sea quarks, all GPDs are only probed for scales of about 4 GeV^2. For comparison, five stars are assigned to PDFs

GPD	Probed by	Constraints	Status
H(val)	ρ^0,ϕ cross sections	PDFs, Dirac ff	***
H(g,sea)	ρ^0,ϕ cross sections	PDFs	***
E(val)	$A_{\rm UT}(\rho^0,\phi)$	Pauli ff	**
E(g,sea)	—	Sum rule for 2nd moments	—
$\widetilde{H}$ (val)	π^+ data	Polarized PDFs, axial ff	**
$\widetilde{H}$(g,sea)	$A_{LL}(\rho^0)$	Polarized PDFs	*
$\widetilde{E}$ (val)	π^+ data	Pseudoscalar ff	*
$H_T,\bar{E}_T$(val)	π^+ data	Transversity PDFs	*

3. DVCS

Because of universality, the GPDs extracted from DVMP may, for instance, be applied to DVCS. This process is calculated in [6] to leading-twist accuracy and leading-order of pQCD while the Bethe–Heitler (BH) contribution is worked out without any approximation. The DVCS part involves convolution integrals, the so-called Compton form factors, which read

$$\mathcal{F}(\xi,t) \,=\, \int\limits_{-1}^{1} dx \left[e_u^2 F^u + e_d^2 F^d + e_s^2 F^s\right] \left[\frac{1}{\xi - x - i\varepsilon} - \epsilon_f \frac{1}{\xi + x - i\varepsilon}\right], \quad (9)$$

where $\epsilon_f = +1$ for $F = H, E$; and -1, for $\widetilde{H}, \widetilde{E}$. With the GPDs at disposal, the convolution integrals can evaluated free of parameters. The corresponding amplitudes for DVCS can be combined with the BH amplitudes in those for leptoproduction of photons

$$|\mathcal{M}_{lp\to lp\gamma}|^2 \,=\, |\mathcal{M}_{\rm BH}|^2 + \mathcal{M}_I + |\mathcal{M}_{\rm DVCS}|^2. \quad (10)$$

The three terms in (10) have the following harmonic structure in ϕ, the azimuthal angle of the outgoing photon with regard to the leptonic plane ($i =$BH, DVCS, interference):

$$|\mathcal{M}_i|^2 \propto L_i \sum_{n=0}^{3} \left[c_n^i \cos{(n\phi)} + s_n^i \sin{(n\phi)}\right], \quad (11)$$

where $L_i = 1/[-tP(\cos\phi)]$ for the Bethe–Heitler and the interference term and $L_{\rm DVCS} = 1$. Although there are only harmonics up to the maximal order 3 in the

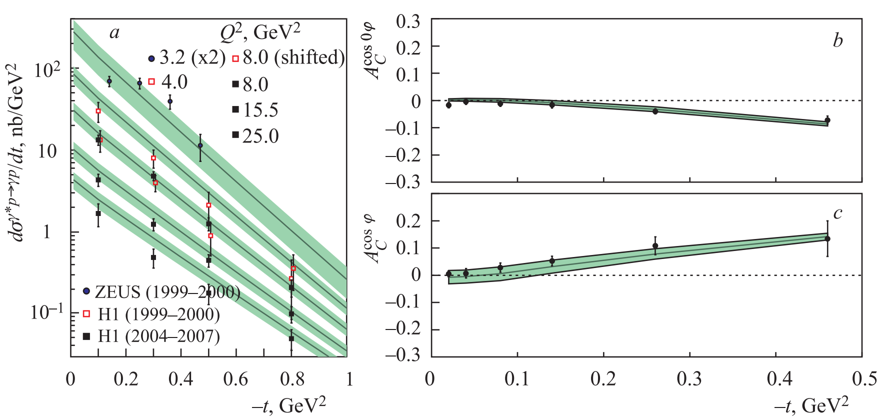

Fig. 1. The DVCS cross section (a) and the beam charge asymmetry (at $Q^2 \simeq 2.51$ GeV^2, $x_B \simeq 0.097$) (b, c). Data are taken from [14–16]. The theoretical results obtained in [6] are shown as solid lines with shadowed bands representing their uncertainties

sums, the additional $\cos\phi$ dependence from the lepton propagators, included in $P(\cos\phi)$, generates, in principle, an infinite series of harmonics for the BH and interference terms. A more detailed harmonic structure taking into account beam and target polarizations can be found, for instance, in [12].

A detailed comparison of this theoretical approach with experiments performed in [6], reveals reasonable agreement with HERMES, H1, and ZEUS data and a less satisfactory description of the large-skewness, small W Jlab data. The GPDs extracted in [1] are not optimized for this kinematical region. As examples, the DVCS cross section at HERA kinematics and the beam charge asymmetry are shown in Fig. 1. It should be mentioned that, in the same spirit, a DVCS analysis is performed in [9, 10].

4. WHAT DO WE KNOW ABOUT THE GPD E?

Let me now discuss the GPD E in some detail. The analysis of the nucleon form factors performed in [8] provided the zero-skewness GPDs for valence quarks with the profile function (6) which can be used as input to the DD representation (1). Since in 2004, data on the neutron form factors were only available for $Q^2 \lesssim 2$ GeV^2, the parameters of the zero-skewness GPD E were not well fixed; a wide range of values were allowed for the powers β_e^u and β_e^d. There is an ongoing reanalysis of the form factors [18] making use of the new data which for the neutron extend now to much larger values of Q^2. The new results for the valence-quark GPDs are similar to the 2004 ones but the powers β_e^q are better determined.

Not much is known about E^g and E^{sea}. There is only a sum rule for the second moments of E [17] at $t = \xi = 0$

$$\int\limits_0^1 dx\, E^g(x, \xi = 0, t = 0) = e_{20}^g = -\sum e_{20}^{q_v} - 2\sum e_{20}^{\bar{q}}. \qquad (12)$$

It turns out that the valence contribution to the sum rule is very small. Hence, the second moments of the gluon and sea-quark GPD E cancel each other almost completely. Since the parameterization (7) for the forward limit of E does not have nodes except at the end-points, this property approximately holds for other moments as well and even for the convolution (8).

For E^s, a positivity bound for its Fourier transform with respect to the momentum transfer [8] forbids a large strange quark contribution and, assuming a flavor-symmetric sea, a large gluon contribution, too:

$$\frac{b^2}{m^2}\left(\frac{\partial e_s(x,b)}{\partial b^2}\right) \leqslant s^2(x,b) - \Delta s^2(x,b), \qquad (13)$$

where s, Δs, and e_s are Fourier conjugated to H^s, $\widetilde{H}^s$, and E^s, respectively. The impact parameter $\mathbf{b}$ is canonically conjugated to the two-dimensional momentum transfer $\mathbf{\Delta}$ ($\Delta^2 = t$). The bound on E^s is saturated for $c_e^s = \pm 0{,}155$ ($\beta_e^s = 7$ in (7)) [13]. The normalization of E^g can subsequently be fixed from the sum rule (12) ($\beta_e^g = 6$). These results are inserted in (1) in order to obtain an estimate of $E^{\rm sea}$ and E^g.

The GPD E is probed by transverse target asymmetries

$$A_{\rm UT} \sim {\rm Im}\,[\mathcal{E}^*\mathcal{H}], \qquad (14)$$

for given H as extracted from the cross sections of vector–meson leptoproduction [1]. The data on ρ^0 production from HERMES [19] and COMPASS [20] are well fitted by the described parameterization of E. However, only E for valence quarks matters for $A_{\rm UT}$ since the sea and gluon contribution to E cancel to a large extent. Fortunately, the analysis of DVCS data [6] provides additional although not very precise information on $E^{\rm sea}$. To leading-oder of pQCD, there is no gluon contribution in DVCS and therefore $E^{\rm sea}$ becomes visible. The HERMES collaboration has measured the transverse target asymmetries for DVCS and for the BH-DVCS interference term [21]. The data are shown in Fig. 2 and compared to the results obtained in [6]. Despite the large experimental errors, it seems that a negative $E^{\rm sea}$ is favored. Independent information on E^g would be of interest. This may be obtained from a measurement of the transverse target polarization in J/Ψ photoproduction [22] .

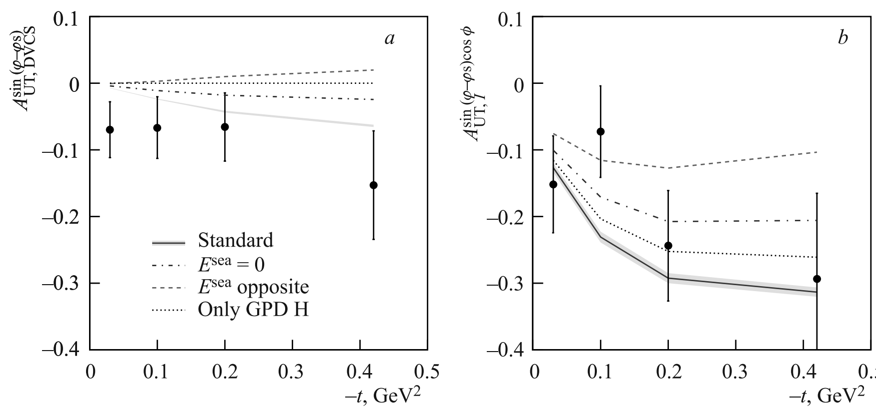

Fig. 2. The transverse target asymmetries for DVCS and the BH-DVCS interference. Data are taken from [21], theoretical results from [6]

5. SUMMARY

I have briefly summarized the recent progress in the analysis for hard exclusive leptoproduction of mesons and photons within the handbag approach. We learned that the data on both reactions are consistent with each other in so far as they can be described with a common set of GPDs. In fact, the GPDs constructed from double distributions and with parameters adjusted to the meson data allow for a parameter-free calculation of DVCS.

The knowledge of the GPDs allows for an evaluation of the angular momenta the partons inside the proton carry. At $\xi = t = 0$ they are given by the second moments of H and E

$$2J^a = [q_{20}^q + e_{20}^q], \quad 2J^g = [g_{20} + e_{20}^g]. \tag{15}$$

Taking the values of the H moments taken from the CTEQ6 PDFs [23], those for E from the form-factor analysis [8] and from the analysis of $A_{\rm UT}$ for DVMP [13] and DVCS [6], one obtains at the scale $4\ {\rm GeV}^2$

J^u	J^d	J^s	J^g	
0.250	0.020	0.015	0.214	(for $E^s = E^g = 0$)
0.225	−0.005	−0.011	0.286	(for $E^s < 0, E^g > 0$).

The main uncertainty comes from the badly known E^s contribution although it is strongly reduced due to the DVCS analysis which favors a negative E^s while from DVMP alone it could also be positive. The large value of J^g is no surprise. The value of g_{20} represents the familiar result that about 40% of the proton's momentum is carried by the gluons. Since E^g seems to be positive according to [6], it even increases J^g. New PDF analyses and the reanalysis of the nucleon form factors [18] will improve the results on J.

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