ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
2014. Т. 45. ВЫП. 3

THE ZFITTER PROJECT

A. A. Akhundov [1], *A. B. Arbuzov* [2,3], *S. Riemann* [4], *T. Riemann* [4]

[1] Institute of Physics, Azerbaijan National Academy of Sciences, Baku

[2] Joint Institute for Nuclear Research, Dubna

[3] Department of Higher Mathematics, Dubna University, Dubna, Russia

[4] 15711 Königs Wusterhausen, Germany

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The ZFITTER project is aimed at the computation of high-precision theoretical predictions for various observables in high-energy electron–positron annihilation and other processes. The stages of the project development are described. The emphasis is made on applications to the analysis of LEP data. The present status of the project and perspectives are given as well.

Проект ZFITTER нацелен на получение высокоточных теоретических предсказаний для различных наблюдаемых в электрон-позитронной аннигиляции и других процессах физики высоких энергий. Описаны этапы развития проекта. Особое внимание уделено приложениям к анализу экспериментальных данных LEP. Также представлены современный статус проекта и перспективы его развития.

PACS: 12.15.-y; 12.15.Lk; 13.66.-a

1. INTRODUCTION

It is difficult to unambiguously define the starting date of the ZFITTER project. The first papers on electroweak loop calculations by D. Bardin and O. Fedorenko date back to 1976. In September 1983, the Dubna–Zeuthen group started an activity, due to the beginning of the four-year long stay of S. Riemann and T. Riemann at JINR, Dubna. The name ZFITTER was invented in 1989. It replaced the former name ZBIZON of our software project. Finally, we can mention the year 1985 when the article «Hunting the Hidden Standard Higgs» was published [1]. With this study, we began to take into account the finite nonzero top quark mass m_t in radiative corrections in the context of e^+e^- annihilation. To our knowledge, the paper contains the first plot confronting two important LEP observables: the weak mixing angle $\sin^2\theta_W$ and the Z-boson mass M_Z with their dependence on the unknown top quark mass m_t and on the also-unknown Higgs boson mass M_H in the Standard Model [2–5]. We reproduce the plot here in Fig. 1, *a*. Both top quark and Higgs boson were not yet discovered at that time, and the actual experimental values for M_Z and $\sin^2\theta_W$ had too huge errors to be included in the plot [6]: $M_Z = (92.9 \pm 1.6)$ GeV and $\sin^2\theta_W = 0.23 \pm 0.015$.

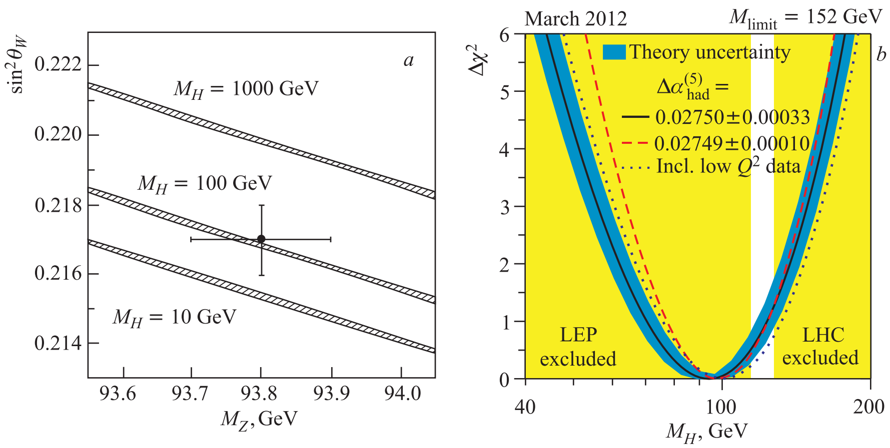

Fig. 1 (color online). *a*) The first ever plotted LEP observables' dependence on the Higgs mass in the Standard Model (reprinted from [1], with permission from Elsevier.) *b*) Blue-band plot of the LEPEWWG [7] with a Standard Model Higgs boson mass prediction based on combined world data from precision electroweak measurements

The numbers in the figure are based on the one-loop Standard Model prediction for Δr, the weak correction to G_μ, deserving a few lines of a Fortran code. For the sake of curiosity, we remark that from 1985 to 2011, the article was quoted only once (by authors outside our group).

The LEP/SLC Collaborations made exciting measurements of the Z-boson resonance profile and of its mass, width, the weak mixing angle, etc., with an unexpected final accuracy [8]:

$$M_Z = (91.1876 \pm 0.0021) \quad \text{GeV}, \tag{1.1}$$

$$\Gamma_Z = (2.4952 \pm 0.0023) \quad \text{GeV}, \tag{1.2}$$

$$\sin^2\theta_{\rm weak} = 0.22296 \pm 0.00028, \tag{1.3}$$

$$\sin^2\theta^{\rm eff}_{\rm lept} = 0.23146 \pm 0.00012, \tag{1.4}$$

$$\sin^2\theta^{\rm MS}_Z = 0.23116 \pm 0.00012, \tag{1.5}$$

$$N_\nu = 2.989 \pm 0.007. \tag{1.6}$$

For the Z-boson mass, this implies $\Delta M_Z/M_Z \approx 10^{-5}$. For the various definitions of the weak mixing angle, see Sec. 10 of [8]. And N_ν denotes the number of light neutrinos.

Figure 2 shows the rise of accuracy for M_Z due to LEP. Since the beginning of the nineteen–nineties, the true scientific standard is the so-called blue-band

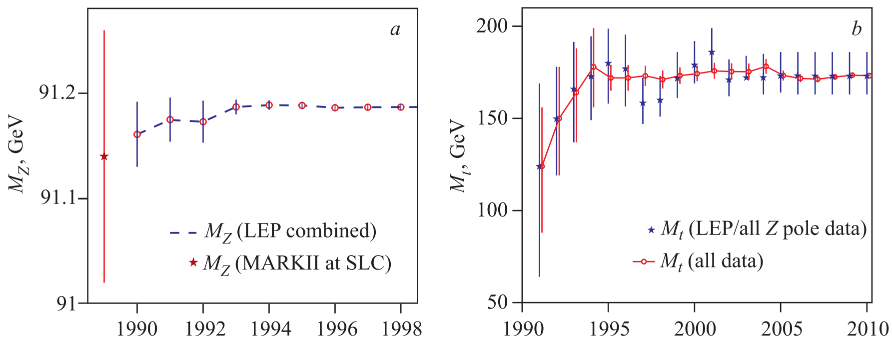

Fig. 2. *a*) Z-boson mass measurements at LEP. Earlier measurements are from UA1, UA2 at SPS (CERN) (see the text, not shown in plot) and from MARKII at SLC (SLAC). *b*) Top-quark mass measurements

plot of the LEPEWWG*, based on ZFITTER [9–13] and one other Standard Model package TOPAZ0 [14–16]**. The March 2012 version of the plot is reproduced in Fig. 1, *b*. Both ZFITTER and TOPAZ0 are huge software packages with tens of thousands lines of Fortran codes aiming at covering the complete known radiative corrections to the Z-resonance peak in the reaction $e^+e^- \to \bar{f}f$. The top quark was predicted by M. Kobayashi and T. Maskawa in 1973 [17] and discovered in 1995 with a mass of about 173 GeV [18, 19]. Top-quark mass data from precision electroweak measurements and from direct searches are collected in Fig. 2, *b*. After the discovery of the top quark, the LEP data were no more competitive. The agreement of the direct measurements (in «all data») and the indirect measurements (in «all Z-pole data») supports the validity of the Standard Model at the quantum loop level. Over the years, the predictive power of the indirect searches for the Higgs boson mass was improved considerably, and the discovery of the top quark gave a crucial input. This is described in Fig. 3. In 2012, the LHC collaborations reported the discovery of a scalar particle with a mass of about 125 GeV [20, 21], which fits into the expectations from the indirect searches. The general belief is that this particle is (or similar to) the one predicted by Peter Higgs in 1964 [22–24]. Within less than a year, in October 2013, Peter Higgs and Francois Englert were awarded the Nobel Prize in Physics «... for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed

*http://lepewwg.web.cern.ch/LEPEWWG/

**Reference [11] (1992) appeared as a CERN preprint because, at that time, we considered this to be more prestigious than, e.g., a paper in the journal «Computer Physics Communications» devoted to software publications. It was submitted to the Internet archive hep-ph in 1994.

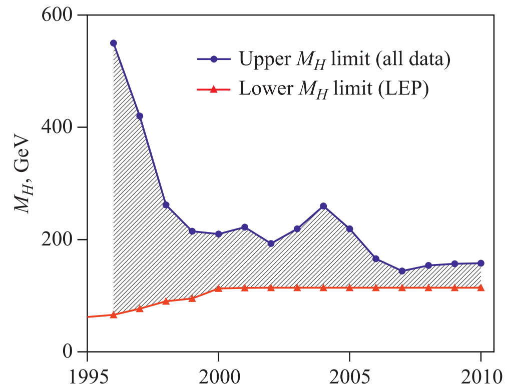

Fig. 3. Higgs boson mass measurements. The upper limits and the fit values for M_H derived from a combination of virtual corrections to LEP and similar data, top and W-mass measurements, performed by the LEPEWWG. The lower mass limit is due to LEP direct searches. The lower limits from data combinations are not shown

through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider» [25]. The accompanying advanced public information «Scientific Background on the Nobel Prize in Physics 2013: The BEH-Mechanism, Interactions with Short Range Forces and Scalar Particles», compiled by the Class for Physics of the Royal Swedish Academy of Sciences [26], reproduces the blue-band plot (March 2012) of the LEPEWWG on page 16. This plot relies on ZFITTER v.6.43.

We work with the ZFITTER project for about 30 years now, and ZFITTER is yet in use for a diverse variety of applications, ranging from the global analyses of the LEPEWWG to many graduation papers, habilitations like, e.g., [27] and PhD theses like [28]. Thirty years are a long term. It takes similarly long to prepare the final results of big experiments at accelerators as LEP 1 (running 13 August 1989–1995), LEP 2 (running 1996–2 November 2000), HERA (running 1992–2007). The final analysis of the LEP 1 data for two-fermion production was published in 2005 [29] by the LEP collaborations and the LEPEWWG, using ZFITTER v.6.42. The corresponding enterprise for LEP 2 was finalized recently [30] with the help of ZFITTER v.6.43.

The big laboratories invented scientific programs for a dedicated long-term preservation of the experimental data, under the label «ICFA Study Group on Data Preservation and Long Term Analysis in High Energy Physics», http://www.dphep.org/. One might assume that this is a self-evident issue for any physics collaboration. Physics is the science of reproducible observations in Nature and of their explanations/descriptions, and the reproducibility deserves a storage. But a long-term storage is an unsolved problem, worth of any (reasonable) effort. DESY, as an example, founded in 2009 the «DESY Data Preservation Project» which is focusing mainly on the HERA experiments H1, Hermes, ZEUS. If such an effort is justified for data, then it is also needed for the analysis tools, which were used for an extraction of the model with its few parameters from the raw, or not-so-raw, data. To our knowledge, the Big Labs do not plan to support long-term maintenance of software like ZFITTER. We, as the authors, theoreticians and phenomenologists, have to mind by ourselves about maintenance of theory/phenomenology software. Everybody knows that the very details of a data analysis cannot be described in a few words. But for precision studies they are truly essential. Sometimes we say: «The description of the program is the program itself». This is a helpful statement if «the program itself» is preserved over a long term in its state of use. ZFITTER did and does a lot to fulfill such a demand, see its web-page http://zfitter.com.

Preservation of a code demands an effort. There are 17 people involved in the DESY Data Preservation Project. On the other hand, if a theoretician says: I care about the availability of my old software, people start to smile. This aim does not give credit points for a scientific carrier, in what phase of the carrier ever. In fact, not only the so-called main author of ZFITTER, D. Bardin, our «*primus inter pares*», tends to lose interest in active support of ZFITTER over the decades. This applies to all of us, mainly because of our interest in studying or inventing something new. Nevertheless, we collected in 2005 some volunteers into a ZFITTER support group, which submitted in that year ZFITTER v.6.42 and in 2008 ZFITTER v.6.43 [12, 13]. The ZFITTER v.6.44beta version dates in 2013.

Encouraged by the decreasing visibility of our ZFITTER support, in 2006 some experimentalists tried to reprogram in C++ in a year's time the Standard Model library of ZFITTER from the published literature. Not just for fun, but in order to do better than ZFITTER: use a more modern programming language than Fortran, with more modularity than ZFITTER, a bit updated, with a GUI. In order to retain ZFITTER for a longer term. The project was proprietary until August 2012, and it faced two major problems. It proved to be impossible to do so without using the ZFITTER software itself to a large extent. Further, without cooperation with ZFITTER authors and the community of theoreticians, including extensive numerical cross-checks, such a project cannot succeed*.

Finally, there is much influence by institutes' directors and by the editors and publishers of physics journals on the engagement of scientists in the development of software. Not all of them seem to mind about proper acknowledgement and quotation of software. Some even say that software has no genuine scientific value by itself and advocate an absolutely free use of any software as common

*See Subsec. 6.2.

habit. If this *would become* common habit, nobody with inspiration and ambition would invest time to write complicated software for the use by other people, like the ZFITTER group — and other groups as well — does. We live in an academic world and we are valued by our scientific results, their originality, importance, curiosity, usefulness, etc. Financing of our projects, of our working positions, our academic prestige depend on all that. We need proper quotation of our scientific results in case they are used. And we can only appeal and hope that the community understands this as a justified expectation, also for software. As a key feature of user-friendly support, we stored for many years all the relevant versions of ZFITTER at the project web-page for anonymous download. We collected about three dozen versions, covering more than 20 years. There are colleagues who take the freedom to use ZFITTER as if it were open-source software* in the strictest meaning of the word. Despite the facts that academic research deserves strict, proper quotation, and that there are license regulations (for ZFITTER this includes the CPC license**). In some countries there are even legal regulations***.

It is the aim of these notes to give an overview on the ZFITTER project. Maybe they can help to see theoretical software in particle physics as an intellectual enterprise like many other inventions of physics research like experimental set-ups, data, hypotheses, models, and theories.

We would like to finish the introduction with two quotes.

Several times we all thought that the ZFITTER project is in its final phase of dying out. See, for example, the remark of Dima Bardin at the symposium «50 Years of Electroweak Physics: a symposium in honor of Professor Alberto Sirlin's 70th Birthday», in the year 2000 [31]: «We would like to see the end of the ZFITTER project in the year 2000 and, therefore, a very natural question arises: What's next?».

In the same year, members of the ZFITTER group were granted the prestigious Award in Theoretical Physics of the Joint Institute for Nuclear Research, Dubna, Russia. The referee was Academician Prof. L. B. Okun from ITEP Moscow; he finished his estimate with the statement****:

«Overall, the project "ZFITTER Fortran Program" represents a unique theoretical tool of world class. The project formed the basis of a close cooperation of experimentalists and theoreticians (with a series of workshops at CERN). With the accumulation of experimental data, the accuracy of the programs has been increased. The project has always found great interest at conferences. Its im-

*http://en.wikipedia.org/wiki/Open_source_software

**http://cpc.cs.qub.ac.uk/licence/licence.html

***Due to controversial positions, we closed the links for anonymous download from ZFITTER web-pages in 2011; in 2012 the copies in the Andrew file system at CERN were removed.

****The original document is in Russian, see http://zfitter.com/jinr-prize-okun.pdf.

portance and the interest to it are shown with numerous references in articles, reviews and monographs. In the long term, with the advent of more precise experiments, ZFITTER will allow one to take into account all two-loop electroweak corrections.

The series of theoretical articles on precision tests of the Standard Model at electron–positron colliders certainly deserves the award of the JINR prize 2000. Academician L. B. Okun».

Our figures illustrate the development of mass predictions for Z boson (Fig. 2, *a*), top quark (Fig. 2, *b*), and the Higgs boson (Fig. 3). Here, ZFITTER has been useful until now. Okun's proposition that ZFITTER will be used also in the future is being fulfilled. We can only hope that our write-up might help to convince the present particle physics community that ZFITTER is worth some support by now and in the future.

At the end of the introduction, we would like to reproduce the long(est) list of ZFITTER authors, see also http://zfitter.com: A. Akhundov, A. Arbuzov, M. Awramik, D. Bardin, M. Bilenky, A. Chizhov, P. Christova, M. Czakon, O. Fedorenko (1951–1994), A. Freitas, M. Grünewald, M. Jack, L. Kalinovskaya, A. Olshevsky, S. Riemann, T. Riemann, M. Sachwitz, A. Sazonov, Yu. Sedykh, I. Sheer, L. Vertogradov, H. Vogt.

The list is not complete. According to the conventions of the software library of «Computer Physics Communications», we should also include here all the co-authors who helped to prepare the program descriptions in 1989, 1999, 2005 [10, 12, 13].

2. ZFITTER IN A NUTSHELL, OR: IS THERE A ZFITTER APPROACH?

We never used the label «ZFITTER approach». The reason is simple: There is no any ZFITTER approach. One can say that there is a kind of a Dubna approach, or of the Bardin's group approach.

Nevertheless, other people use this label. Let us outline some features which might make the origin of the popularity of ZFITTER, but also of one or the other of our scientific projects:

• Unitary gauge.

We are working in the unitary gauge when studying the renormalization of the Standard Model. Most of other groups use the 't Hooft–Feynman gauge. But when looking at observable quantities, there is no difference left, due to the gauge invariance of perturbation theory. So, if everything is correct, there is no difference for the users.

• On-mass-shell renormalization scheme.

We are applying the on-mass-shell renormalization scheme, with some modifications. Other groups do the same for electroweak corrections.

• Analytical treatment of QED corrections.

ZFITTER is not a Monte-Carlo program. The Dubna group has an enormous experience in the analytical treatment of QED corrections, allowing us, sometimes, to come relatively close to the experimental setups by dedicated analytical integrations. Several different approaches may be chosen by users. The necessary computational time for fits to data is small compared to that of other projects.

• Realistic observables and pseudo-observables.

There is a plethora of observables, of quite different polarized and nonpolarized cross-section combinations and asymmetries. Both the so-called realistic observables (including real corrections) and pseudo-observables (after unfolding the realistic observables) may be used. With the different interfaces one may optimize a study appropriately.

• Form factors. Modularity.

We describe the effective Born cross section in the Standard Model approach by (essentially) four (complex-valued) gauge invariant form factors per production channel*. Plus a separated running QED coupling. This allows a modular programming, an efficient introduction of New Physics into the package, or a convenient export of Standard Model corrections into another approach to simulate particle interactions.

• Higher-order corrections.

Originally, we calculated the complete set of electroweak one-loop corrections to the Z-resonance physics. In due course, there became more and more electroweak, QCD, and mixed higher-order corrections available, and we had to implement them into ZFITTER. In the nineteen-nineties these implementations dominated in our efforts of ZFITTER development. It is not the genuine theoretical work we like, but it has to be done.

• Interfaces. Modularity.

ZFITTER is not a fitting program. From the very beginning, we were aware of the fact that a data analysis at, e.g., LEP may be based on different sets of assumptions, being incompatible with each other. The notion of interfaces was developed. The interfaces call the kernel of ZFITTER with different compositions of input variables, real corrections, and an effective Born cross section. The users of ZFITTER can choose among a few sample interfaces, or they can write their own ones.

• Flags.

The use of ZFITTER can be controlled by flags to be set by users. Although this implies problems for the authors in writing extensions and updates of the code, for users this is truly convenient.

*Massive top quark production deserves six form factors [32–34]. See also the in-depth discussion in [35].

• Descriptions.

ZFITTER is described for users at different levels of complexity. There are about 350 pages of instructions.

• Simplicity of file structure.

ZFITTER is easy to use. It has a simple file structure, it is self-contained, and it has a sample output. The installation on a computer is done and controlled within minutes. The installation of the user software, which is calling ZFITTER and performing data fits, writing tables and drawing figures, might be much more involved.

• Numerical cross-checks.

Because the availability of very precise data was typical for LEP physics, a careful numerical control of the theory software became mandatory. Here, a lot of colleagues, including competitors of ZFITTER, invested huge collective efforts. Without that, one could neither trust the impressive physical results of that era nor establish a long-term reliability of the codes.

• Source-open programming.

The scientific seriousness of ZFITTER is trusted because its source code is publicly available. And we expect that the usual academic conditions of use are respected, notably the CPC license, as we say, it is a source-open software. The meaning of the term «open-source software» is controversial and it should not be used for ZFITTER.

• Social aspects.

A software package of some complexity, written for use by other people, must be supported and, in case, updated. The authors need some contact with the users. And, last but not least, some license regulations have to be fixed if the authors want to get their academic credit, e.g., in the form of proper citations. Since the authors of ZFITTER are employed at some institutions distributed over several countries, it is of vital importance that these institutions interfere in a constructive way. We are happy that this did happen for a very long period, in view of several social restructurings of institutions and even countries.

ZFITTER is a Fortran library of Standard Model predictions for the scattering process

$$e^+e^- \to \bar{f}f \quad (+\gamma, \quad +n\,\gamma) \tag{2.1}$$

at energies in the range from $\sqrt{s} \approx 20$ to 150 GeV, i.e., above quark bound states (meson factories) and below the top quark production threshold. The package can be called by interfaces

— in the Standard Model,

— in several model-independent approaches,

— with Z' bosons and similar physics extensions,

— etc.

One may evaluate

— realistic observables: polarized and nonpolarized cross sections and cross-section asymmetries with a variety of cuts on the final state;

— (pseudo-)observables like M_Z, Γ_Z, $\sigma_{\rm had}^{\rm tot}$, $R_{\rm had}$, $A_{\rm FB}^{\rm lept}$, λ_τ, $\sin^2\theta_{\rm ew}^{\rm eff}$, ...

— and form factors which can be used in another program.

Different choices of input variables are possible, e.g.,

— M_Z, G_μ, m_t, M_H, $\alpha_{\rm em}$, α_s, ...

— M_Z, M_W, m_t, M_H, $\alpha_{\rm em}$, α_s, ...

3. ELECTROWEAK VIRTUAL CORRECTIONS

The first weak one-loop calculations were published as Dubna preprints by D. Bardin and his PhD student O. Fedorenko in 1978 [36–38]. Together with P. Christova, then also PhD student of D. Bardin, the by now famous articles on the complete on-mass-shell renormalization of fermion scattering amplitudes in the electroweak Standard Model were published in Nuclear Physics B [39, 40], see also [41]. The corresponding studies for weak boson production and fermion–boson scattering are unpublished [42, 43].

These calculations were complete, but assumed all fermions to be massless. When experiments showed that at least the top quark is not light, the top-mass dependence was included [1, 9, 44, 45]. Some studies of structural aspects in the renormalization of the Standard Model are [46, 47]. All this was done in the unitary gauge, while most of other groups usually worked with the 't Hooft–Feynman gauge. Later, this difference was of some value because an agreement of two calculations performed in truly quite different gauges established a powerful cross-check of the numerics. The first numerical program BFK (acronym for Bardin/Fedorenko/Khristova) was written in Fortran.

The Zeuthen partners, staying at Dubna from 1983 to 1987, worked out the renormalization of the electroweak Standard Model in the 't Hooft–Feynman gauge [48]. But because the corresponding numerical program was never created, the results of this work were more or less useless; they had a mere educational value. Nevertheless, the experiences from that activity were used in order to perform the first calculation of flavor-changing Z boson decays into different lepton flavors*. This work was unpublished [49,50]; see also [51]. An application to flavor-violating Z decays into different quark flavors was finally published [52]. Later, when we were working on precision predictions for LEP, the results could be easily transformed into the calculation of virtual top-mass corrections in (flavor-diagonal) $b\bar{b}$ production at LEP and in Z decay [44]. And yet later, they became

*We mention for curiosity that the numerics of this one-loop project was performed with a pocket calculator TI-57 with 50 program steps. The program had to be typed in after switching the calculator on. The price of the device was 120 DM in the CERN shop.

the starting point for studies of lepton number violation in e^+e^- annihilation with heavy neutrinos [53] and with supersymmetry [54]*.

3.1. Sirlin's Approach. The notion of form factors ρ and κ in the weak neutral current were, to our knowledge, introduced by A. Sirlin**:

— ρ contains electroweak corrections to the Fermi constant G_μ,

— κ contains electroweak corrections to the weak mixing angle $\sin^2\theta_W$.

This approach allows one to retain the Born definitions in the on-mass-shell renormalization scheme also in higher orders:

$$G_F^{\text{eff}} = \rho_Z G_\mu, \tag{3.1}$$

$$\sin^2\theta_W^{\text{eff}} = \kappa_Z \sin^2\theta_W, \tag{3.2}$$

where

$$\frac{G_\mu}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2}, \tag{3.3}$$

$$\sin^2\theta_W \equiv 1 - \frac{M_W^2}{M_Z^2}. \tag{3.4}$$

3.2. The HECTOR and ZFITTER Approach. For general 4-fermion scattering amplitudes, one needs a more general description. This was first introduced, to our knowledge, by the Dubna/Zeuthen group, in 1988, in the article «Electroweak Radiative Corrections to Deep Inelastic Scattering at HERA. Neutral Current Scattering» by D. Bardin, C. Burdik (Dubna), P. Khristova (Shoumen), T. Riemann (Zeuthen) [56]. The corresponding software is retained until today as the Fortran package HECTOR [57]. So, strictly speaking, one might call this the HECTOR approach.

We use four complex form factors ρ, κ_{ini}, κ_{fin}, and $\kappa_{\text{ini-fin}}$ for the parameterization of weak amplitudes including WW and ZZ box diagrams. In the article «A Realistic Approach to the Standard Z Peak» by D. Bardin, M. Bilenky, G. Mitselmakher (Dubna), T. Riemann, M. Sachwitz (Zeuthen) [9], we excluded the weak WW and ZZ box diagrams from the form factors, making them independent of the scattering angle. This is of advantage at LEP where these box diagrams have a minor numerical influence. When form factors are independent of the scattering angle, analytical phase space integrations become possible. In ZFITTER, there is an option to switch between the approaches.

The Born amplitude is factorized into two pieces with the vector coupling v_i and the axial vector coupling a_i of a fermion i to the Z boson; with $A_i = \gamma_\mu(v_i + a_i\gamma_5)$:

$$A_i \otimes A_f \equiv [\bar{u}_i\gamma_\mu(v_i + a_i\gamma_5)u_i][\bar{u}_f\gamma^\mu(v_f + a_f\gamma_5)u_f]. \tag{3.5}$$

*Several of the results in supersymmetry found in the literature turned out to be just wrong when we had a look at them.

**For a historical perspective, see reference [55].

This form is generalized by loop corrections to

$$A_{vv}\gamma\otimes\gamma + A_{av}\gamma\gamma_5\otimes\gamma + A_{va}\gamma\otimes\gamma\gamma_5 + A_{aa}\gamma\gamma_5\otimes\gamma\gamma_5, \tag{3.6}$$

or, equivalently,

$$B_{LL}\gamma(1+\gamma_5)\otimes\gamma(1+\gamma_5) + B_{\gamma L}\gamma\otimes\gamma(1+\gamma_5) + \\ + B_{L\gamma}\gamma(1+\gamma_5)\otimes\gamma + B_{\gamma\gamma}\gamma\otimes\gamma. \tag{3.7}$$

Amplitudes with Z boson and photon exchanges read

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z, \tag{3.8}$$

$$\mathcal{M}_\gamma \sim F_A[\gamma\otimes\gamma], \tag{3.9}$$

$$\mathcal{M}_Z \sim G_\mu\rho_Z[\gamma\gamma_5\otimes\gamma\gamma_5 + v_q\gamma\otimes\gamma\gamma_5 + v_l\gamma\gamma_5\otimes\gamma + v_{ql}\gamma\otimes\gamma]. \tag{3.10}$$

In the Born approximation, it is

$$v_{ql} \approx v_q \times v_l. \tag{3.11}$$

The form factors F_A, ρ, κ_q, κ_l, and κ_{ql} are complex-valued functions of s and t:

$$F_A(s) = \frac{\alpha_{\mathrm{QED}}(s)}{\alpha_{\mathrm{em}}}, \\ = 1 + \delta\alpha_{\mathrm{QED}}(s), \tag{3.12}$$

$$\alpha_{\mathrm{em}} = \frac{1}{137\ldots}, \tag{3.13}$$

$$a_f \equiv 1, \quad f = q, l, \tag{3.14}$$

$$v_f(s,t)^{\mathrm{eff}} = 1 - 4\sin^2\theta_w|Q_f|\kappa_f(s,t), \quad f = q, l, \tag{3.15}$$

$$v_{ql}(s,t)^{\mathrm{eff}} = v_q + v_l - 1 + 16\sin^4\theta_W|Q_qQ_l|\kappa_{ql}(s,t), \tag{3.16}$$

where we use $Q_e = -1$. From [58], Eq. (3.3.1), we quote:

$$\mathcal{A}_Z^{\mathrm{OLA}}(s,t) = i\,e^2\,4\,I_e^{(3)}I_f^{(3)}\frac{\chi_Z(s)}{s}\rho_{ef}(s,t)\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5) - \\ - 4|Q_e|s_W^2\kappa_e(s,t)\gamma_\mu\otimes\gamma_\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f(s,t)\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu + \\ + 16|Q_eQ_f|s_W^4\kappa_{ef}(s,t)\gamma_\mu\otimes\gamma_\mu\Big\}. \tag{3.17}$$

The form factors may be used, in analogy to the Z-decay matrix element of Sirlin, for the definitions of effective vector and axial vector couplings and for a generalization of the effective weak mixing angle:

$$G_\mu^{\text{eff}} = \rho_{ef} G_\mu, \tag{3.18}$$
$$\sin^2\theta_{W,e}^{\text{eff}} = \kappa_e \sin^2\theta_W, \tag{3.19}$$
$$\sin^2\theta_{W,f}^{\text{eff}} = \kappa_f \sin^2\theta_W, \tag{3.20}$$
$$\sin^2\theta_{W,ef}^{\text{eff}} = \sqrt{\kappa_{ef}} \sin^2\theta_W. \tag{3.21}$$

The unique definition of the effective weak mixing angle is absent.

The first applications of weak correction calculations by the Dubna group were applied, together with N. Shumeiko, to deep-inelastic scattering; see, e.g., [59, 60]. The calculations were requested by the NA-4 experiment at CERN with JINR participation. The form factors ρ and κ are simply related to the one-loop form factors introduced in the original renormalization articles by Bardin and Fedorenko (1978) [36–38] and Bardin, Christova, Fedorenko (1980) [39, 40]:

$$\rho_{ef} = 1 + F_{LL}(s,t) - s_W^2 \Delta r, \tag{3.22}$$
$$\kappa_e = 1 + F_{QL}(s,t) - F_{LL}(s,t), \tag{3.23}$$
$$\kappa_f = 1 + F_{LQ}(s,t) - F_{LL}(s,t), \tag{3.24}$$
$$\kappa_{ef} = 1 + F_{QQ}(s,t) - F_{LL}(s,t). \tag{3.25}$$

The corresponding relations of the form factors F_{ij} and the Z-boson matrix element are

$$\begin{aligned}\mathcal{A}_Z^{\text{OLA}} &= i\frac{g^2}{16\pi^2} e^2\, 4\, I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \times \\ &\times \Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5)F_{LL}(s,t) - 4|Q_e|s_W^2\gamma_\mu\otimes\gamma_\mu(1+\gamma_5)F_{QL}(s,t) - \\ &- 4|Q_f|s_W^2\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu F_{LQ}(s,t) + 16|Q_eQ_f|s_W^4\gamma_\mu\otimes\gamma_\mu F_{QQ}(s,t)\Big\}.\end{aligned} \tag{3.26}$$

So far we discussed matrix elements. The differential cross section for $e^+e^- \to f\bar{f}$ is

$$\begin{aligned}\frac{d\sigma}{d\cos\vartheta} = \frac{\pi\alpha_{\text{em}}^2}{2s}\Big\{&(1+\cos^2\vartheta)[K_T(\gamma) + \text{Re}\,(\chi(s)K_T(I)) + |\chi(s)|^2K_T(Z)] + \\ &+ 2\cos\vartheta[K_{\text{FB}}(\gamma) + \text{Re}\,(\chi(s)K_{\text{FB}}(I)) + |\chi(s)|^2K_{\text{FB}}(Z)]\Big\},\end{aligned} \tag{3.27}$$

with

$$\chi(s) = \frac{G_F}{\sqrt{2}}\frac{M_Z^2}{8\pi\alpha}\frac{s}{s - M_Z^2 + i\Gamma_Z M_Z}. \tag{3.28}$$

Here one has to care about the choice of the Z-boson width: either a constant one, γ_Z, or an s-dependent one, Γ_Z [61].

The effective couplings are

$$\begin{aligned}
K_T(\gamma) &= c_{\rm color} Q_i^2 Q_f^2 |F_\gamma(s)|^2, \\
&=_{\rm Born}\ c_{\rm color} Q_i^2 Q_f^2, \qquad (3.29)\\
K_T(I) &= 2c_{\rm color}|Q_i Q_f| F_\gamma(s)^* \rho_{if}(s,t) v_i v_f, \\
&=_{\rm Born}\ 2c_{\rm color}|Q_i Q_f| v_{B,i} v_{B,f}, \qquad (3.30)\\
K_T(Z) &= c_{\rm color}|\rho_{if}(s,t)|^2 (1+|v_i|^2+|v_f|^2+|v_{if}|^2), \\
&=_{\rm Born}\ c_{\rm color}(v_{B,i}^2+a_{B,i}^2)(v_{B,f}^2+a_{B,f}^2), \qquad (3.31)\\
K_{\rm FB}(\gamma) &= 0, \qquad (3.32)\\
K_{\rm FB}(I) &= 2c_{\rm color}|Q_i Q_f| F_\gamma(s)^* \rho_{if}(s,t), \\
&=_{\rm Born}\ 2c_{\rm color}|Q_i Q_f| a_{B,i} a_{B,f}, \qquad (3.33)\\
K_{\rm FB}(Z) &= 2c_{\rm color}|\rho_{if}(s,t)|^2 2\,{\rm Re}\,(v_i v_f + v_{if}), \\
&=_{\rm Born}\ 2c_{\rm color}(2v_{B,i} a_{B,i})(2v_{B,f} a_{B,f}). \qquad (3.34)
\end{aligned}$$

Here, i and f denote the initial state and the final state, respectively. For the Drell–Yan process $\bar{q}q \to l^+l^-$, we have $q = u, d$ and $f = l$. In the case of polarizations, (3.32) becomes nonvanishing [11]. The parameter $c_{\rm color}$ is the color factor, e.g., $c_{\rm color} = 3$ for the Drell–Yan process with the initial state quarks and the final state leptons.

A formula similar to (3.27) describes the special case of Bhabha scattering [12, 62–64]. The numerical comparison with W. Hollik in 1990 [63] seems to be the most precise prediction for the effective Born cross section of Bhabha scattering until now.

At the end of the subsection, we would like to emphasize that the notion of form factors is not unique. For purely phenomenological reasons, we split the matrix element into two pieces: a photon exchange amplitude and a Z boson one. The calculation of the running QED coupling is technically quite different from that of the weak loop diagrams. So this is reasonable. The gauge invariance justifies it, but only if it is handled with care. There are gauge-dependent diagrams which mix the photon and Z-boson amplitudes. So, in ZFITTER we decided to include all the corrections but the fermionic self-energy insertions, a bit arbitrary, into the Z-boson amplitude.

Such a separation of photonic and weak terms is wishful also for the charged current W-boson mediated amplitude. But a gauge-invariant separation of (virtual and real) photonic corrections from W-boson exchange is impossible. In HECTOR [57, 65], we found a way to do well-defined separations by considering logarithmic terms and just explicitly defining some rule. This really worked out.

Years later, when building a software for $e^+e^- \to \nu\bar{\nu}\gamma$, we could take over the weak charged current form factor into the Monte Carlo program of S. Jadach and Z. Was [66]. This reaction for $\nu = \nu_e$ is unique: it depends both on neutral current and charged current amplitudes*.

Similar problems have been discussed when the ZFITTER form factors were adapted to atomic violation measurements [67].

3.3. Drell–Yan Processes. We went a bit into the details of a correct ansatz for the effective Born approximation in the Standard Model in e^+e^--annihilation. The situation in a Drell–Yan process is quite similar. One may study, e.g., the running of the weak mixing angle $\sin^2\theta_W^{\rm eff}(s')$ as a function of the scale s' from a hard cross section $\sigma_0(s')$:

$$\sigma_0(s') = \mathcal{L}_u\sigma_0(u\bar{u} \to l^+l^-) + \mathcal{L}_d\sigma_0(d\bar{d} \to l^+l^-), \tag{3.35}$$

where both hard scattering cross sections $\sigma_0(u\bar{u} \to l^+l^-)$ and $\sigma_0(d\bar{d} \to l^+l^-)$ depend on four complex valued, process-dependent form factors ρ_{ql}, κ_q, κ_l, and κ_{ql} with $q = u, d$. The cross section σ_0 depends on s' but also on the scattering angle θ. Further, we have not only the initial and final state photonic corrections, but also their interference.

An elegant way to cover at least a part of the complexity of all this in a modern QCD Monte Carlo program is the following:

— define a photon exchange amplitude;

— define a Z-exchange amplitude;

— split the v_{ql} into a Z part and a photon part:

$$v_{ql} \to (v_{ql} - v_q\ v_l) \quad + \quad v_q\ v_l; \tag{3.36}$$

— assume a Born-like structure with form factors ρ, v_q, v_l and put the deviation from that structure, which is contained in the difference $(v_{ql} - v_q v_l)$, into the photon amplitude.

In an unpublished paper of 1991 [68], A. Leike and T. Riemann worked out the influence of Z' physics on the evaluation of weak form factors. The idea of reshuffling matrix elements in form factors was invented there and it is now independently reused as a clever inclusion of ZFITTER's weak form factors into a Monte Carlo code, which was originally made for the description of QCD corrections to Drell–Yan processes**.

Evidently, once there are accurate data, one has to carefully understand how to model the correct physics ansatz with the minimal number of parameters. This is under study by experimentalists presently.

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*Bhabha scattering has also s- and t-channel exchanges, but only of the neutral current type.

**W. Sakumoto, private information and reference [69].

4. REAL CORRECTIONS, MOSTLY DUE TO QED

Around 1983 we began to envisage some contribution to the description of the Z-boson resonance as it was planned to be studied at LEP. There existed several articles on electroweak radiative corrections. Let us mention the electroweak study by Wetzel in 1982 [70] and that by Lynn and Stuart in 1984 [71], or the MC program MUSTRAAL by Berends, Kleiss, Jadach in 1982 [72]. It was not evident to us that we might contribute some novel results, and we decided therefore to study real photon emission first.

The Dubna group has an enormous experience in the analytical treatment of QED corrections, first mostly applied to t-channel exchange processes. This was pushed by the close contacts with Dubna experimentalists from the NA-4 Collaboration at CERN. The subtraction method for the treatment of infrared singularities was worked out in 1976 in a seminal paper [73]. The divergent part of the cross section is, in a simplified form, integrated over the whole phase space, and at the same time, subtracted from the exact squared matrix element. The difference can be integrated numerically, and the isolated term is sufficiently simple for an analytical treatment. In practice, this can become quite involved, see [74].

The first articles treated photonic corrections taking into account mass effects appeared at that time. The very first one was on pure QED corrections in e^+e^- annihilation, by A. Akhundov (Baku), D. Bardin (Dubna), O. Fedorenko (Petrozavodsk), T. Riemann (Dubna): «Some Integrals for Exact Calculation of QED Bremsstrahlung», an unpublished JINR Dubna preprint [75], followed by [76,77]. Then we extended the integration technology to experimental setups with Z-boson resonance phenomena, including mixing phenomena of Z boson and photon. This sounds easy, but there were several conceptual problems to be solved. As a result, ZFITTER relies now on several versions of semianalytical formulae with low-dimensional numerical phase space integrations left. At the time of LEP experiments, this was extremely useful. For an unfolding of measured cross sections into pseudo-observables, or for multidimensional fits, the computing time of an analysis code was absolutely decisive. The inclusion of certain kinematical cuts was a wish expressed by experimentalists. Computers were not so advanced. There were no personal computers, and workstations were also not yet on the market. In Dubna, there were one or two terminal stations for theoreticians, and we had to queue up every day. In Russian winter, the terminal room (with one terminal) was a bit cold at temperatures close to zero centigrades, because the windows did not close exactly. The upper left corner of the terminal screen was blind. Often, the terminal in the theory building was blocked by Riemann, Bardin, Akhundov from 9 to 12 in the morning. Not everybody was amused.

In the case of quark-pair production, or Z- or W-boson decays into quarks, the final state will get QCD modifications. These corrections are contained in the so-called radiator functions. Their implementation in ZFITTER relies on calculations by a variety of colleagues and it is described in various ZFITTER descriptions, notably in [9, 11–13]. Useful representations are also [78–81].

The treatment of the complete set of QED corrections related to real emission of photons in ZFITTER is quite specific. The higher-order corrections have been typically taken from the literature, as it is documented, notably in [12, 13]. An important example is reference [82]. The main work had to be performed at one-loop order, plus soft-photon exponentiation. It was clear that the numerical effects are important for the experimental analyses. There were several Monte-Carlo programs available, e.g., [72, 83–87] and the references therein. See also the report [88]. We aimed at an alternative, analytical integration of the three-dimensional photon phase space integrals. The necessary techniques were developed step by step over a long period. They originated to a large extent from studies for deep-inelastic scattering, e.g., $lN \to lX$ [74]. In the presence of the Z-boson resonance in the s channel, one is faced with the additional need to perform a correct treatment of the Breit–Wigner propagator, a truly complex function. Further, there is a mixing of photon and Z-boson exchange. This γ–Z mixing was studied, e.g., in [48, 52, 89, 90]; this issue was settled by a formal Dyson summation of the γ, Z propagator matrix. The Z-boson propagator with the finite width may become an issue for analytical integrations. In squared matrix elements, we are faced with $\gamma\gamma$, γZ, and ZZ interferences. The latter are dominating around the Z-boson pole, and they will contain squared Z-boson propagators. Performing analytical phase space integrations with such a term inside looks difficult. An important simple idea is to perform a partial fraction decomposition in order to linearize the integrand:

$$\left|\frac{1}{s - M_Z^2 + iM_Z\Gamma_Z}\right|^2 = \\ = \frac{-1}{2iM_Z\Gamma_Z}\left(\frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} - \frac{1}{s - M_Z^2 - iM_Z\Gamma_Z}\right) = \\ = \frac{-1}{M_Z\Gamma_Z}\,\mathrm{Im}\left(\frac{1}{s - M_Z^2 + iM_Z\Gamma_Z}\right). \quad (4.1)$$

At the first glance, this looks bizarre because the complete answer seems to carry an overall factor $s/(M_Z\Gamma_Z)$. Evidently, one may use complex integration theory, so this is good. The overall prefactor gets divergent for a vanishing Z width, but this is a technical expression of the well-known radiative tail, so this is also good.

We tried the approach and calculated the complete one-loop QED corrections for the total cross section and for the forward–backward asymmetry around the Z resonance without a cut. The results for the initial state radiation, the final state radiation, and the initial-final state interference were rather compact and looked explicitly reasonably behaving*. The results were published as a preprint in [92] and refined a bit in [93]. The paper could not be published in «Nuclear Physics B» because the referee found it not close enough to the experimental setup. Nevertheless, it is a nice piece of work and served for many years as an important numerical etalon for precision comparisons.

As a by-product, we understood that one may calculate photonic corrections to the initial-final state interference of the γZ interference as the arithmetical mean of the corrections to the ZZ and $\gamma\gamma$ initial-final state interferences:

$$R_{\text{ini-fin}}(Z, Z_2) = \frac{1}{2}\left[R_{\text{ini-fin}}(Z, Z) + R_{\text{ini-fin}}(Z_2, Z_2)\right], \quad Z_2 = \gamma. \tag{4.2}$$

Here, Z_2 is the second vector boson. For the proof, see [89]. This is not of utmost importance here. When we later studied QED corrections for Z, Z' production with a heavy Z' boson, then we had the newly appearing initial-final state part of the ZZ' interference at the disposal without a new calculation [94–98].

Later, we refined the techniques, and finally, ZFITTER enables the calculation of

— exact, completely integrated one-loop photonic corrections without cuts [93],

— convolution integrals for cross sections with soft-photon exponentiation [99],

— the corresponding angular distributions [100],

— convolution integrals with integrated angular cuts [101],

— convolution integrals with integrated acceptance cuts, combined further with an acollinearity cut [102].

The sophisticated final state phase space treatment with cut on the acollinearity final state fermions goes back to G. Passarino (1982) [103] and it is relatively close to realistic experimental cuts for lepton final states. The complete analytical QED corrections were worked out for this case by M. Bilenky and A. Sazonov [102] and became a part of ZFITTER. The truly nice paper remained unpublished, unfortunately. Later, we recalculated these corrections for ZFITTER from scratch (unpublished, see references [104–107]). We performed two minor corrections and got very nice, compact formulae for the special case of no cut for the fermion production angle [108].

Finally, all this was sufficiently close to what the experimentalists could derive from their Monte-Carlo simulations to confront the theory.

*In fact, it took us nearly half a year of heavy fighting with SCHOONSCHIP, because we did not agree, at the Z-boson peak, with the numerics of the Monte-Carlo program MUSTRAAL [72,91]. The MUSTRAAL was available via CPC, and we could run it at Dubna. The mistake was, as often, trivial, but influential. The final 5 digits agreement convinced us that our Breit–Wigner treatment makes sense and it is operational.

We wrote relatively monstrous programs in Veltman's SCHOONSCHIP [109, 110] to be run at a CDC-6500 main frame at JINR, Dubna. Bardin and Fedorenko were, in parallel with Vladimirov and Tarasov, among the first using SCHOONSCHIP at JINR in 1976*. Colleagues from Moscow came to JINR regularly in order to use the CDC-6500 main frame because comparable computers were subject of the US embargo policy and thus not available for civil use in the Soviet Union at that time. JINR, Dubna, as an international research center, was privileged in that respect**. A comprehensive review on the use of computer algebra at JINR is [111]. Later, FORM [112,113] was invented by Jos Vermaseren and we could run it at personal computers. The first article typeset in latex by our group was presumably [100], and the first article submitted to the hep-ph archive dates in 1994, remind that the hep-ph archive was opened in 1992.

At a certain moment, we realized that analytical integrations are fine; but if the sensitivity to the Z-boson width becomes sufficiently large, then it will matter whether the width is a pure constant γ_Z as in a normal Breit–Wigner function, or it arises from a quantum field theoretical calculation and will thus depend on the kinematics, $\Gamma_Z(s)$. In the latter case, it is (roughly speaking) the imaginary part of the Z-boson self-energy function, which is by itself s-dependent; and for the initial state corrections it is s'-dependent. The s' is one of the integration variables. We remembered that the s dependence is, to a very high accuracy, just $\Gamma_Z(s) = (s/M_Z)\ \Gamma_Z$, and this observation enables us to change the propagators into functions with a constant width, allowing not only a good estimate of the different approaches, but also further on the analytical integrations: The differences of mass and width in the two approaches are derived from the following identity [61]***:

$$\frac{1}{s - M_Z^2 + iM_Z\Gamma_Z(s)} \equiv c\frac{1}{s - m_Z^2 + im_Z\gamma_Z}, \tag{4.3}$$

with

$$m_Z = M_Z - \frac{\Gamma_Z^2}{2M_Z} = M_Z - 34\ \text{MeV}, \tag{4.4}$$

$$\gamma_Z = \Gamma_Z - \frac{\Gamma_Z^3}{2M_Z^2} = \Gamma_Z - 0.934\ \text{MeV} \approx \Gamma_Z - 1\ \text{MeV}. \tag{4.5}$$

*A. Akhundov, D. Bardin, L. Bobyleva, V. Gerdt, I. Zhidkova, W. Lassner, V. Rostovtzev, O. Tarasov, R. Fedorova, and D. Schirkov received in 1986 the JINR Award in Theoretical Physics for «Introduction, Development and Use of Computer Systems for Analytical Calculations at Central Computers of the Central Computing Installations of JINR». We are grateful to V. Gerdt for a clarifying e-mail exchange.

**We are grateful to Andrei Kataev reminding about this fact.

***The Z-boson mass shift was also discovered by a numerical study of the Z-boson peak in parallel to [61] in [114].

Here, $M_Z = 91.1876$ GeV and $\Gamma_Z = 2.4952$ GeV have to be chosen as the usual PDG values. Later, we worked out an approach to a model-independent Z-boson peak analysis inspired by the S-matrix theory, relying naturally on m_Z and γ_Z, not only for the Z-boson peak cross section but also for asymmetries. The point here again is the proper treatment of QED corrections [115–120]*.

In fact, the idea to use m_Z, γ_Z instead of M_Z, Γ_Z was born while listening to a talk on string theory at a conference, while reading a paper on QED corrections with complicated phase space cuts by Passarino [103].

The Z-boson parameter relations (4.4) and (4.5) become essential when two-loop electroweak corrections are determined in ZFITTER. This is carefully described in [121], where the complete electroweak two-loop corrections to the leptonic weak mixing angle have been calculated, see also Sec. 6. It is remarkable that the shift of the Z-boson width due to the change of the scheme (s-dependent or constant Z-boson width) amounts to 1 MeV and it is larger than the corresponding shift from the genuine weak NNLO corrections. Compared to the experimental error of 2.3 MeV, the shift is small. The authors of [121] did not take the correction into account because formally it is beyond the NNLO order and thus among the systematically neglected terms**. One should consider the term as an indication of the size of unknown higher-order terms.

What we describe here is about the state of real emission calculations in ZFITTER at the end of the nineteen-eighties. The treatment of the final state mass effect was refined in [122–124]. Some additional QED corrections, due to light fermion pair emission and higher-order photonic effects, needed for the proper treatment at LEP 2 energies were later added [125, 126], see also reference [127]. An extended discussion of higher-order QED effects in the leading and next-to-leading logarithmic approximations can be found in [128, 129].

Careful studies of ZFITTER physics updates originated in these years [80, 130, 131].

5. COMPETITION AND COOPERATION

5.1. 1989 — the First LEP Publications. In 1989, the world changed quite a bit. Participation at the Ringberg Workshop on LEP Physics in Germany became possible [132]. The NATO supported RADCOR conference on radiative corrections and their applications to experiments in Brighton, the first one of a series, was open for Eastern Country physicists [133, 134]. We remember the stimulating atmosphere of the 1989 LEP physics workshop at CERN [133, 135]. And LEP became operative in August 1989. The first months were exciting. A good

*The corresponding software package SMATASY is supported by Martin Grünewald.

**Ayres Freitas, private information.

knowledge of radiative corrections was needed from the very beginning, just in order to discriminate between trivial radiative effects and New Physics. Several relevant ZFITTER studies appeared in this period, e.g., [136–139]. In [139], approximate parameterizations of $O(\alpha\alpha_s)$ corrections [140] were derived in order to speed-up the numerics. The Fortran routines of B. Kniehl [141] improved this later further on. The LEP Collaborations performed the first Z-line shape analyses. We were closely related to the L3 Collaboration [142–149] and to the DELPHI one [150–154]. A review of the latter is [155].

Among the first DELPHI papers was [150]. From the ZFITTER group, D. Bardin and G. Mitselmakher were DELPHI authors. The paper quotes for the theory on the Z-line shape G. Burgers [156] and A. Borrelli et al. [157]. In [153], the Z-line shape analysis used the software packages ZAPPH and ZHADRO by G. Burgers [156]. In [152], March 1990, our papers [9, 102] are quoted. And in [154] the package ZFITTER/ZBIZON with a reference to the internal note DELPHI 89-71 PHYS 52 and to [9, 10] was used*.

A similar approach was implemented in the L3 Collaboration, where ZFITTER authors T. Riemann, M. Sachwitz, and H. Vogt were collaborating in 1989. The internal note L3-001 [142] quotes G. Burgers [156] and CERN 89-08**, but also our paper [61]. The Z-line shape analysis seems to be based on papers by Cahn [158] and Borrelli et al. [157]. In [143], internal note L3-003, our package ZBIZON is quoted with reference to the L3 Internal Note 679 as well as [61] and the Zeuthen preprint PHE 89-19 [100]. Back-up radiative corrections had been studied with ZBIZON. For the very Z-line shape fits, they used again Borrelli et al. [157], Cahn [158], and a paper by Jadach et al. [85], for Bhabha scattering. In [159], internal note L3-004, the paper on the Z-boson parameters [61], was quoted.

A bit later it became more and more common to use ZFITTER in DELPHI and L3, but also in OPAL. While ALEPH used the package BHM/WOH by F. Berends, M. Martinez, W. Hollik et al. [78, 160]. We mention these first papers on LEP physics results because they demonstrate that there was a true competition of the analysis packages and our ZBIZON/ZFITTER package was accepted step by step, but not from the very beginning.

5.2. 1992–2012 — LEPEWWG and Global Fits. The LEP Electroweak Working Group was founded in 1993***. Soon after the first measurements at LEP, the quest for combined data analyses with the fourfold statistics compared to a single experiment was expressed. Originally a group with members of the four LEP experiments, led by Jack Steinberger, investigated the combination of

*ZBIZON is the former version of ZFITTER.

**http://cds.cern.ch/record/116932/files/CERN-89-08-V-1.pdf

***We are grateful to Dorothee Schaile for private information.

the Z-line shape data [161]. In 1993, Dorothee Schaile was asked to take over the coordination of the group and she had then already ideas on the inclusion of other electroweak observables into a combined analysis. They called themselves the LEP EWWG*. The first publicly accessible document with this name is also the initial summary of the LEP results for the electroweak Summer Conferences in 1993, which then appeared annually [162–164]. The LEP EWWG was lead by D. Schaile in 1993–1996. When she became professor in Munich, Robert Clare took over the coordination of the LEP EEWG**. The present chair is Martin Grünewald. The final paper on LEP 1 data appeared in 2005 [29], nearly a decade after closing LEP 1 in 1996, while the analysis of LEP 2 data (finalized data taking in 2000) was finished in these days [30].

The ZFITTER group members, as well as the authors of other physics software packages used by the LEPEWWG are not members of the LEPEWWG. They are consulted in case.

5.3. 1995 — The Electroweak Working Group Report. The work of the LEPEWWG and of the four LEP Collaborations relied on ZFITTER and TOPAZ0, and also on the BHM/WOH package, and on many other resources. Because of this, the role of establishing a kind of world standard, the community felt the need of careful numerical checks of their predictions. One is confronted with multiparameter problems: different calculation schemes, some freedom of input choices, the presence of approximations and dedicated omissions, misunderstandings and, sometimes, mistakes.

At a certain moment, the community has to set benchmarks. The result of a year-long workshop is the collection «Reports of the Working Group on Precision Calculations for the Z Resonance», edited by D. Bardin, W. Hollik, and G. Passarino. It was published as a CERN Yellow Report, CERN 95-03 (31 March 1995), http://cdsweb.cern.ch/record/280836/files/CERN-95-03.pdf.

A part of this document is the «Electroweak Working Group Report». Two years later it was submitted to the arxive/hep-ph [78]***. This work is one of the basics for the successful work of the LEP Electroweak Working Group. Until now it is one of the most important collections of higher-order corrections for e^+e^--annihilation in the Standard Model.

5.4. Higher-Order Corrections in ZFITTER. During the 1995 CERN workshop and shortly after, a lot of additional higher-order corrections were calculated and included into ZFITTER. We give here just a (presumably not complete) list of references and refer for details to the ZFITTER descriptions: [79, 90, 121, 141, 165–171]. Later, further improvements were added [172–181].

*http://lepewwg.web.cern.ch/LEPEWWG/

**We are grateful to J. Mnich for a clarification.

***Now it is also available as a pdf file at CERN, in CERN 95-03.

Until now, we did not yet include into ZFITTER the existing parameterization of the rather small *bosonic* two-loop weak corrections to the weak mixing angle [179]. The fermionic corrections are covered, as well as the complete weak two-loop corrections to the W-boson mass. For a complete treatment of the weak two-loop corrections to the Z-boson width, the corrections to the form factor ρ_Z are lacking yet. For this reason, the quite good agreement of the higher-order *approximations* to Γ_Z with the so far known pieces of the *complete* two-loop result are an indication that the final answer will be close to what we have already.

Generally speaking, we try to control about four to five digits of the predictions aiming at such a *physical* theory precision. One quote from the report [78] is interesting because it sheds some light on the progress of the so-called *technical* precision (precision under fixed, maybe not realistic conditions): «...compare results of independent calculations. Such a comparison has been done once for Δr, and an agreement of up to 12 digits (computer precision) was found [14]». Reference [14] is private communications of D. Bardin, B. Kniehl, and R. Stuart in 1992. This has to be compared to a three digits agreement between two Bhabha cross-section calculations in a comparison, performed few years earlier in 1990 [63]. Later, in 2002, a precision of up to 12 digits was reached in practice for complete virtual one-loop calculations, and of 5 digits with inclusion of real corrections [33, 182, 183].

6. ZFITTER 2013

6.1. From ZFITTER v.6.42 to ZFITTER v.6.44beta. The most recent publicly available ZFITTER version is ZFITTER v.6.43 (17 June 2008) [12, 13]. It agrees with ZFITTER v.6.42 up to a correction of a non-influential typo and was released by the ZFITTER support group (A. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. Grünewald, K. Mönig, S. Riemann, and T. Riemann, see http://zfitter.com). The ZFITTER group was reorganized in February 2012 and consists now of A. Akhundov, A. Arbuzov, D. Bardin, P. Christova, L. Kalinovskaya, A. Olshevksy, S. Riemann, and T. Riemann.

Recently, we have included into ZFITTER v.6.44beta (20 January 2013) the final results for the $\mathcal{O}(\alpha_s^4)$ QCD corrections to the Z-boson and W-boson quarkonic partial widths and to the so-called R ratio by P. Baikov et al. [181]. As may be seen from Fig. 4 and from Table 1, the numerical shifts in the widths amount to less than 0.3 MeV and are thus well below the experimental errors, e.g., at LEP or at an anticipated GigaZ option of the ILC [185]*. A fit formula for the complete electroweak two-loop corrections to the W-boson mass [175] was already included in ZFITTER v.6.42. The final exact results for the complete electroweak two-loop corrections to $\sin^2\theta_{\rm eff}^{f\bar{f}}$ for light fermions f [121] and the

*A detailed numerical study is in preparation.

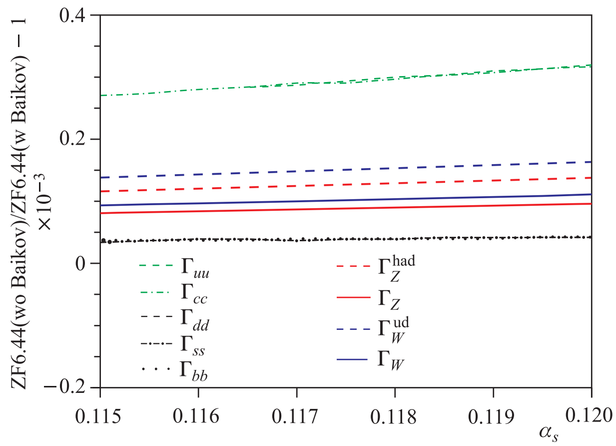

Fig. 4. The influence of the $\mathcal{O}(\alpha_s^4)$ QCD corrections [181] on the W- and Z-boson widths

Table 1. **ZFITTER v.6.44beta, with the input values $\alpha_s = 0.1184$, $M_Z = 91.1876$ GeV, $M_H = 125$ GeV, $m_t = 173$ GeV. The dependence on electroweak NNLO corrections is studied for IMOMS $= 1$ (input values are $\alpha_{\rm em}$, M_Z, G_μ). AMT4 $= 4$: with two-loop subleading corrections and resummation recipe of [23–28] of [13]; AMT4 $= 5$: with fermionic two-loop corrections to M_W according to [29, 30, 32] of [13]; AMT4 $= 6$: with complete two-loop corrections to M_W [37] and fermionic two-loop corrections to $\sin^2\theta_W^{\rm lept,eff}$ [52] of [13]. IBAIKOV $= 0$ (no α_s^4 QCD corrections) or IBAIKOV $=$ 2012 [181]**

AMT4					
	4	5	6	Diff.	Exp. Err.
IBAIKOV = 0					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9782	83.9748	83.9807	0.0059	0.086
Γ_Z, MeV	2494.7863	2494.6019	2494.8688	0.2669	2.3
$\Gamma_W(l\nu)$, MeV	226.3185	226.2877	226.2922	0.0308	1.9
Γ_W, MeV	2090.3308	2090.0465	2090.0882	0.2843	42
M_W, GeV	80.3578	80.3541	80.3546	0.0037	0.015
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231722	0.231791	0.231670	0.000121	0.00012
IBAIKOV = 2012					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9782	83.9748	83.9807	0.0059	0.086
Γ_Z, MeV	2494.5591	2494.3747	2494.6416	0.2669	2.3
$\Gamma_W(l\nu)$, MeV	226.3185	226.2877	226.2922	0.030	1.9
Γ_W, MeV	2090.1117	2089.8274	2089.8691	0.2843	42
M_W, GeV	80.3578	80.3541	80.3546	0.0037	0.015
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231722	0.231791	0.231670	0.000121	0.00012

two-loop electroweak fermionic corrections to $\sin^2\theta_{\rm eff}^{b\bar{b}}$ [180] have to be included yet into ZFITTER. They are known to be small compared to the fit formula [178] covered in ZFITTER since v.6.42. These corrections are also small compared to the present experimental errors for the gauge boson widths, see Table 1. For the leptonic weak mixing angle, they are of the order of the experimental error: compare the Particle Data Group value of (1.4) with the last row in Table 1. The comparison shows even a systematic deviation of the two values. This deviation traces back to the handling of the hadronic contributions to the photonic vacuum polarization. Changing the ZFITTER default by flag setting ALEM $= 2$ into a variable input and setting this to $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02750$ [184], produces a shift of the ZFITTER prediction towards the PDG value*. See the changes shown in Table 2. Just to mention, the influence of $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ on the Higgs mass prediction is visualized in Fig. 1, b. Here it is of a minor importance, but visible.

Presently, there are controversial positions concerning ZFITTER's «conditions of use» and the ZFITTER software license http://cpc.cs.qub.ac.uk/licence/licence.html granted to the authors by Elsevier's Computer Physics Communica-

Table 2. **IBAIKOV $= 0$ (no α_s^4 QCD corrections) or IBAIKOV $= 2012$ [181], AMT4 as described in Table 1. The difference to Table 1: Flag ALEM $= 2$ is chosen with input value** $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02750$

AMT4					
	4	5	6	Diff.	Exp. Err.
IBAIKOV $= 0$					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9875	83.9839	83.9900	0.0061	0.086
Γ_Z, MeV	2495.2859	2495.0958	2495.3662	0.2704	2.3
$\Gamma_W(l\nu)$, MeV	226.4020	226.3703	226.3745	0.0317	1.9
Γ_W, MeV	2091.1020	2090.8092	2090.8474	0.2928	42
M_W, GeV	80.3677	80.3639	80.3644	0.0038	0.015
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231532	0.231603	0.231481	0.000122	0.00012
IBAIKOV $= 2012$					
$\Gamma_Z(\mu^+\mu^-)$, MeV	83.9875	83.9839	83.9900	0.0061	0.086
Γ_Z, MeV	2495.0586	2494.8685	2495.1389	0.2704	2.3
$\Gamma_W(l\nu)$, MeV	226.4020	226.3703	226.3745	0.0317	1.9
Γ_W, MeV	2090.8828	2090.5901	2090.6283	0.2927	42
M_W, GeV	80.3677	80.3639	80.3644	0.0038	0.015
$\sin^2\theta_{\rm eff}^{\rm lept}$	0.231532	0.231603	0.231481	0.000122	0.00012

*Taking into account the uncertainty $\Delta\alpha^{(5)}_{\rm had}(M_Z) = 0.02750 \pm 0.00035$ [http://lepewwg.web.cern.ch/LEPEWWG/plots/winter2012/], the corresponding predictions in Table 2 vary: $\Gamma_Z(\mu^+\mu^-)$ by $\pm 6.7\cdot 10^{-5}$ GeV, Γ_Z by $\pm 1.2\cdot 10^{-4}$ GeV, $\Gamma_W(l\nu)$ by $\pm 2.2\cdot 10^{-4}$ GeV, Γ_W by $\pm 2.2\cdot 10^{-4}$ GeV, M_W by $\pm 7.5\cdot 10^{-5}$ GeV, $\sin^2\theta_{\rm eff}^{\rm lept}$ by $\pm 5.0\cdot 10^{-4}$. The latter is about the value of the experimental error.

tions Program Library — Programs in Physics & Physical Chemistry. For some details, see http://zfitter.com. Until the issue is settled, actualized versions of ZFITTER will stay at the beta level and cannot be released.

Sooner or later, the LHC is becoming a precision tool and the community feels some steady need of high-precision Standard Model predictions. They are required both for use in global fits and for specific cross-section predictions, notably of Drell–Yan processes via the Z resonance. This need would become even more pronounced if the ILC project would be substantiated [185].

Regrettably, we see today no alternative project to ZFITTER in the field of precision Standard Model predictions. In the mid-nineteen nineties there were three competing (and cooperating) projects at the disposal [78]: BHM/WOH by W. Hollik et al., TOPAZ0 by G. Passarino et al., and ZFITTER by D. Bardin et al. BHM/WOH was available on request, and the latter two are publicly available. To our knowledge, updating and user support have been minimized for TOPAZ0* and BHM/WOH [160].

6.2. A Comment on the Gfitter Project. Sometimes the Gfitter project is considered as an independent implementation of Standard Model predictions for some pseudo-observables, and as a true scientific alternative to ZFITTER (for these pseudo-observables). We do not share this opinion and would like to give a short, clarifying comment on the situation.

The Gfitter project was started in Summer 2006 and presented to the public in December 2007, at the kick-off meeting of the German «Helmholtz Alliance for Physics at the Terascale», see the slides at http://indico.desy.de/materialDisplay.py?contribId=36&sessionId=15&materialId=1&confId=477. Until August 2012, the Gfitter software was proprietary, but by private information** it became known that the Standard Model library of Gfitter, Gfitter/GSM, was relying on the FORTRAN package ZFITTER v.6.42 and was created to a large extent by copy-paste-adapt without any proper citation in the academic meaning of the word.

There are several versions of the program Gfitter.

• Gfitter/GSM (Summer 2006 – July 2011) is unpublished. It relies essentially and directly on the Standard Model implementation of the ZFITTER software.

*http://personalpages.to.infn.it/ giampier/topaz0.html

**Private information from and documentation by A. Akhundov, S. Riemann, T. Riemann, March to May 2011, http://zfitter.com. Further, a German ombuds person's report announces in July 2012: «A diploma thesis derives from ZFITTER in the sense that 8200 lines have been taken over by copying from ZFITTER». In the thesis work the kernel of the Gfitter/GSM software was written (in collaboration with others), and its text delivered basic building blocks for the so-called main article on Gfitter [186]. The third evidence for the confidential take-overs may be found in the unpublished version of Gfitter of July 2011, where about 100 to 200 identities are denoted, by the Gfitter/GSM authors, to originate from ZFITTER v.6.42. On occasion of the Erratum [187] to [186], ZFITTER authors wrote a letter to the Editorial Board of «European Physical Journal C» (14 September 2012), http://zfitter.com/letter-to-the-epjc-editors.pdf.

On top of that, Gfitter/GSM contains few add-ons. The *electroweak add-ons* of Gfitter/GSM, compared to ZFITTER v.6.42, are the bosonic two-loop corrections to the weak mixing angle in Awramik et al. [121]. They are small; see the discussion above. The complete two-loop parameterizations in [121], in turn, have been made with use of ZFITTER v.6.42. As a consequence, it is formally correct to quote the parameterization only [121], but one should have in mind that the ZFITTER numerics is also inside. There is also a *QCD add-on* of Gfitter/GSM (2011), compared to ZFITTER v.6.42 (2006), based on [188]. It is numerically small as well (see the discussion above) and it is implemented in ZFITTER v.6.44beta.

The use of this Gfitter version deserves a citation not only of [186], but also of [12, 13], for using ZFITTER v.6.42, according to ZFITTER's CPC license.

• Gfitter/GSM (August 2011 till August 2012) is unpublished. According to the authors, the program relies on a proprietary implementation of Standard Model corrections which are based on a parameterization tracing back to Cho et al. (1999) [189], which in turn is based on an electroweak one-loop calculation published in 1994 [190]. There have been made improvements later, and in a recent article by Cho et al. (2011) [191] the authors confirm the reliability of their parameterization by comparing them with ZFITTER v.6.42 predictions. These parameterizations are used in Gfitter further on, and overlaid with the most recent higher-order corrections mentioned.

• Gfitter_1.0 has been released publicly in September 2012. The Standard Model library Gfitter_1.0/gew relies presumably on the same parameterizations as Gfitter/GSM (2011).

The different versions of Gfitter rely, in one way or the other, on ZFITTER v.6.42. We further remark that without studying the numerical reliability of Gfitter, to four or five significant digits, the scientific value of the inclusion of NNLO weak and α_s^4 QCD corrections in Gfitter remains questionable. According to our standards, Gfitter simulates Standard Model predictions with unknown precision. It is a nice tool for the production of figures for the illustration of the Standard Model physics. Possibly, it is useful also for studies beyond the Standard Model.

7. CONCLUSIONS

A talk on history and features of the ZFITTER project was presented at LL2012, the eleventh «Loops and Legs» meeting. Its title was «ZFITTER — 20 years after»*. http://pos.sissa.it/archive/conferences/151/036/LL2012_036.pdf.

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*This text is an extended version of the talk. For the slides, see https://indico.desy.de/getFile.py/access?contribId=29&sessionId=10&resId=0&materialId=slides&confId=4362. The contribution to the proceedings of LL2012 in «Proceedings of Science» (PoS), by A. Akhundov et al., did not appear. See, for conference http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=151 and for contribution.

The «Loops and Legs» conference was founded by the Zeuthen Theory Group in 1992 when the Zeuthen Institute for High Energy Physics of the (then already former) East German Academy of Sciences became a part of DESY. We are glad that since then this conference regularly attracts colleagues who contribute to the progress in the field which is comprising both the branch of applied calculations and that of development of new theoretical methods.

ZFITTER is certainly one of the oldest source-open software projects in elementary particle physics with a permanent support. It comprises practically all the theoretical knowledge of relevance for a precise description of the Z-boson resonance in e^+e^- annihilation and for Z-boson's part in global fits within the Standard Model [192]. Obviously, today one would create such a project quite differently. We can only encourage our colleagues to try. Complex projects need (independent) duplication. As concerning the ZFITTER code, it is certainly of interest as a benchmark for SM calculations in the LEP energy range. In particular, it is used for cross-checks in development of new codes, see, e.g., [193].

Higher-order quantum field theoretical predictions face another problem: the solutions become so lengthy and complex that the idea of source-open software is, in practice, no longer a realistic option. This happens already with the $\mathcal{O}(\alpha_s^4)$ QCD corrections and the complete NNLO weak corrections in ZFITTER. They are mere parameterizations of huge partly unpublished expressions.

The LEP/SLC era gave to the scientific community an unprecedented precision in several fundamental quantities like M_Z, Γ_Z, the effective weak mixing angle $\sin^2\theta_W^{\rm eff}$, and the number of light neutrino flavors N_ν. The experimental confirmation of the Standard Model, a gauge theory with spontaneous symmetry breaking, a consistent quantum field theory with inclusion of higher orders of the perturbation theory, is of comparable importance.

At the present moment, the Standard Model remains being the most successful theory in description of the fundamental interactions. In fact, it possesses a huge predictive power and provides very accurate predictions for many observables which appear to be in agreement with experimental data. We see that also in post-LEP experiments at high-energy colliders like Tevatron and LHC as well as in high-precision low-energy experiments like searches for rare decays. Even so that we hardly believe that the Standard Model is the true theory of everything, it will certainly remain to be our working tool in the most relevant energy domain of the high-energy physics.

We are proud that we are being contributing to the establishment of the Standard Model.

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