ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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The paper presents short historical reviews of the processes of lepton-pairs production in peripheral interaction of leptons and ions at high energies. The orders of magnitude of the QED and QCD cross sections with the production of two and three jets are given. The technique of the analysis is described in detail based on the parameterization of Sudakov 4-momentum tasks and writing the amplitude in an explicit gauge-invariant form. Based on this formalism, the differential cross sections of the QCD processes $gp \to (ggg)p$; $qp \to (q\bar{Q}Q)p$; $gp \to (gQ\bar{Q})p$ were obtained, including the distribution on transverse momentum component of jets fragments. It was shown that the role of the contribution of «non-Abelian» nature may become dominant in a particular kinematics of the final particles. The kinematics, in which the initial particle changes the direction of motion to the opposite one, was considered in the case of heavy quark–antiquark pair production. In the appendices, the details of the calculations and the explicit form of the differential cross sections are given. Some extended comments on the frequently used cross sections of the pair production in the case of two-photon scattering are presented. In particular, the degree of the longitudinal polarization of the positron, at the interaction of polarized initial electron, was calculated. The method of calculating the cross sections of the $2 \to 2$ processes in QCD, based on the isolation of irreducible color structures, and the method CALCUL of spiral amplitudes were discussed in detail.

Дан краткий исторический обзор процессов рождения лептонных пар при периферическом взаимодействии лептонов и ионов при высоких энергиях. Приведены порядки величин сечений КЭД и КХД с рождением двух и трех струй. Детально изложена техника анализа процессов, основанная на параметризации Судакова 4-импульсов задачи и записи амплитуды в явно калибровочно-инвариантном виде. На основе этого формализма получены дифференциальные сечения процессов КХД $gp \to (ggg)p$; $qp \to (q\bar{Q}Q)p$; $gp \to (gQ\bar{Q})p$ и др., в том числе распределения по поперечным компонентам импульсов фрагментов струй. Показано, что роль вклада «неабелевой» природы может стать доминирующей в определенной кинематике конечных частиц. Рассмотрена кинематика, в которой начальная частица изменяет направление движения на обратное в случае рождения тяжелой кварк-антикварковой пары. Исследована кинематика протона отдачи. В приложениях даны детали вычислений и явный вид дифференциальных сечений. Даны комментарии к выводу часто используемых сечений образования пар для случая двухфотонного рассеяния. В частности, вычислена степень продольной поляризации позитрона при взаимодействии поляризованного начального электрона. Детально рассмотрены метод рас-

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чета сечений процессов $2 \to 2$ в КХД, основанный на выделении неприводимых цветовых структур, и метод CALCUL спиральных амплитуд.

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INTRODUCTION

It is known [1] that the differential cross sections of small-angle elastic (and inelastic) scattering processes do not fall with increasing the center-of-mass total energy $\sqrt{s}$, $s = 4E^2$. The reason for this is the contribution to the cross section from the photon exchange between charged particles. Similar phenomena take place as well in the strong interaction sector, where gluons take place instead of a photon.

The simplest processes of this kind are the scattering of a charged particle in the external field of nuclei and the elastic scattering of one sort of charged particles on the other one. The total cross sections of these processes do not exist due to contributions of large impact parameters, which correspond to small scattering-angles. The momentum of the virtual photon in the scattering channel (t channel) tends to the mass shell. So the virtual photon in the t channel becomes a real one. In the case of inelastic processes $a + b \to a + b + X$, with the set of particles x belonging to one of the directions in the center of mass a or b, the cross sections are finite [2–9]. Besides, the square of 4-momentum of a virtual photon is negative and restricted from below by the magnitude of some quantity of the created set of particles, invariant mass square of $(ax), (bx)$. The finiteness of the transfer momentum module caused the so-called Weizsäcker–Williams enhancement [7]. Namely, the region of small momentum transfer is realized in the appearance of a large logarithmic factor $L = \ln{(s^2/(m_1^2 m_2^2))}$. For modern colliders, this factor is of an order of 20. It often turns out that the consideration is restricted to the WW approximation. This means the accuracy of the order of $1 + O(1/L)$. The cross sections of inelastic peripheral processes are, as usuall, large.

The background caused by the events of the large-angle kinematics of produced particles determines the accuracy of peripheral cross sections

$$1 + O\left(\frac{\alpha}{\pi}, \frac{m^2}{s}\right). \tag{1}$$

So, the total accuracy of theoretical estimates is better than 5%.

The cross sections of interaction of photons with the target will also not fall with energy when taking into account the contributions of higher orders of perturbation theory (PT).

The main attention in our paper is paid to the double gluon emission and production of the pair of heavy quarks with subsequent jet production, in the fragmentation region of the incident particle.

Our paper is organized as follows.

First, we give an estimation of the magnitudes of the cross sections of several processes in high-energy $ep \to (eab)p$, $qp \to (qab)p$ collisions in the fragmentation region of a projectile e, q. In Sec. 1, we give a short historical introduction to the study of the processes of lepton-pair production in high-energy lepton–lepton, ion–ion collisions. In the attached Appendices we do comments to these results. In Sec. 2, the so-called «infinite momentum frame» method of description of high-energy processes based on the Sudakov parameterization of the 4-momenta of the problem is developed. The differential cross sections are expressed in terms of physically measurable energy fractions and the transverse component of the final particles. In Sec. 3, the simplest QCD processes with 2-jet production are presented. In Sec. 4, we consider the process of heavy quark–antiquark pair production in collisions of projectile with the colorless target. In Sec. 5, the QED process of double bremsstrahlung is studied. In Sec. 6, a similar QCD process of emission of two gluons is considered. In Sec. 7, the specific details of jet production on a fixed target are considered.

In Conclusion, we discuss the results and pay attention to the relation of the contributions of Abelian (QED) and non-Abelian nature. It seems that in the case of large magnitudes of transverse quark momenta, the role of non-Abelian contributions dominates. In Conclusion, we also discuss the «jet reflection» phenomena. It consists in the change of the direction of motion of the light projectile to the opposite one in the case of heavy-pair production.

In Appendices A–C, the explicit expressions beyond the WW approximation as well as in the WW ones are presented. In Appendices D, E, we give a short derivation of the famous formulae for the light- and heavy-pair production in lepton–(anti)lepton collisions. In Appendix F, we develop the method of description of heavy object production in the fragmentation region of one of projectiles. In Appendix G, this method is applied to the problem of transmission of the longitudinal polarization of the initial electron to the positron from the pair created. Appendix H contains the detailed derivation of the amplitudes and the differential cross sections of the simplest QCD $2 \to 2$ processes. The chiral amplitudes method is essentially used.

The cross section of the heavy-pair production in electron–proton and quark–proton collisions can be written as

$$\begin{aligned} \sigma^{ep\to(eQ\bar{Q})p} &\approx \frac{\alpha^4 L_q}{\pi M_Q^2} \approx 4.8 \text{ pb}; \\ \sigma^{qp\to(qQ\bar{Q})p} &\approx \frac{\alpha^2 \alpha_s^2 L_q}{\pi M_Q^2} \approx 20 \text{ nb}. \end{aligned} \tag{2}$$

We put here $\sqrt{s} = 3$ TeV, $M_Q = M = 1.5$ GeV. In this case $L_q \approx 30$. For a process of two-photon and two-gluon production we have

$$\begin{aligned}
\sigma^{ep \to (e\gamma\gamma)p} &\approx \frac{\alpha^4 L_q}{\pi k^2} \approx 10.8 \text{ pb};\\
\sigma^{qp \to (qgg)p} &\approx \frac{\alpha^2 \alpha_s^2 L_q}{\pi k^2} \approx 50 \text{ nb}, \quad k^2 = 1 \text{ GeV}^2
\end{aligned} \tag{3}$$

for typical transfer momentum squared $k^2 = 1$ GeV^2.

1. QED PERIPHERAL PROCESS, PAIR PRODUCTION

In 1934, the cross section of pair production in high-energy lepton collisions was calculated in the so-called double-WW approximation [8]

$$\sigma_{\bar{e}e \to \bar{e}e\bar{l}l} = \frac{28\alpha^4}{27\pi m_l^2} \left(\ln\left(\frac{s}{m_e^2}\right)\right)^2 \ln\left(\frac{s}{m_l^2}\right), \quad l = e, \mu, \quad s = 4E^2. \tag{4}$$

In 1937, G. Racah [9] published the total cross section of the process of pair creation in the collision of charged particles with the charges $Z_1 e$, $Z_2 e$; p_1, p_2 — the 4-momenta and m_1, m_2 — masses of the initial particles:

$$\begin{gathered}
\sigma_{Z_1 Z_2 \to Z_1 Z_2 e^+ e^-} = \frac{28(Z_1 Z_2 \alpha^2)^2}{27\pi m_e^2} (l^3 - Al^2 + Bl + C), \quad l = \ln \frac{2p_1 p_2}{m_1 m_2},\\
A = \frac{178}{28} \approx 6.36, \quad B = \frac{1}{28}(7\pi^2 + 370) \approx 15.7,\\
C = -\frac{1}{28}\left[348 + \frac{13}{2}\pi^2 - 21\xi(3)\right] \approx -13.8, \quad \xi(3) = 1.202.
\end{gathered} \tag{5}$$

In papers by Baier and Fadin [10] as well as Lipatov and Kuraev [11], the total cross section of the production process of an electron–positron pair in electron–positron collisions (only two exchanged photons) was obtained

$$\begin{aligned}
\sigma_{e^\pm e_- \to e^\pm e_- e_+ e_-} = \frac{\alpha^4}{\pi m_e^2} &\left[\frac{28}{27}\rho^3 - \frac{178}{27}\rho^2 - \left(\frac{164}{9}\frac{\pi^2}{6} - \frac{490}{27}\right)\rho + \right.\\
&\left. + \frac{401}{9}\xi(3) + \frac{52}{3}\frac{\pi^2}{6}\ln 2 + \frac{916}{27}\frac{\pi^2}{6} - \frac{676}{27}\right] \approx\\
&\approx \frac{\alpha^4}{\pi m_e^2}[1,03\rho^3 - 6.6\rho^2 - 11.7\rho + 104], \quad \rho = \ln\frac{s}{m_e^2}.
\end{aligned} \tag{6}$$

In the case of production of a muon pair we obtain

$$\sigma_{e^{\pm}e_{-}\to e^{\pm}e_{-}\mu_{+}\mu_{-}} = \frac{\alpha^4}{\pi m_\mu^2}\left[\frac{28}{27}\rho^3 - \frac{178}{27}\rho^2 - \left(\frac{535}{81} + \frac{14}{3}\frac{\pi^2}{6}\right)\rho + \frac{28}{9}\rho^2 l + \right.$$
$$\left. + \frac{14}{9}\rho l^2 - \frac{562}{27}\rho l - \frac{64}{9}l^2 - \left(\frac{56}{9}\frac{\pi^2}{6} - \frac{5855}{162}\right) l - 7\xi(3) + \frac{214}{27}\frac{\pi^2}{6} + \frac{51403}{486}\right] \approx$$
$$\approx \frac{\alpha^4}{\pi m_\mu^2}[1,03\rho^3 + 26.6\rho^2 - 56\rho - 342],$$
$$\rho = \ln\frac{s}{m_\mu^2}, \quad l = \ln\frac{m_\mu^2}{m_e^2} \approx 10.7, \quad \xi(3) = 1.202. \tag{7}$$

These formulae are in agreement with ones obtained by G. Racah [9]. The method used to obtain the cross section consists in imposing some cuts on the transverse momenta and energy fractions, which in principle can be used in experiment. Adding separate contributions, we obtain the results given above. In Appendix A, we give the sketch of derivation of the LL formula and discuss the experimental cuts.

Note that in the case of production of a heavy-muon pair, the corrections of the order $(m_\mu^2/s)L^n$ must be taken into account. Really, for $\sqrt{s} < 1$ GeV, the cross section calculated theoretically is negative.

In 1970, in paper by Brodsky, Kinoshita, and Terazawa, a special case of production of a heavy object by two virtual photons in electron–electron collisions was investigated [12]:

$$\sigma(s)^{ee\to eeF} = \left(\frac{\alpha}{\pi}\right)^2 \left(\ln\frac{E}{m_e}\right)^2 \int\limits_{4M^2}^{\infty} \frac{ds_1}{s_1}\sigma^{\gamma\gamma\to F}(s_1) f\left(\frac{s_1}{s}\right), \tag{8}$$

with $2M$ being the mass of a created system and

$$f(z) = (2+z)^2 \ln\frac{1}{z} - 2(1-z)(3+z). \tag{9}$$

The BKT formula in the modern language desribes the Drell–Yan process. Really, it consists in the probability P_e^γ to find the virtual photon in the electron:

$$dW_e^\gamma(\mathbf{k}_1, \beta_1) \sim 4\pi\alpha \frac{d\mathbf{k}_1^2 \cdot \mathbf{k}_1^2}{(\mathbf{k}_1^2 + m_e^2\beta_1^2)^2}\frac{d\beta_1}{1-\beta_1}P_e^\gamma, \quad P_e^\gamma = \frac{1-\beta_1}{\beta_1^2} + \frac{1}{2},$$

and the conversion of these probabilities with the cross section of a hard subprocess $\gamma\gamma \to F$ (see details in Appendix E).

In this step, it is useful to keep in mind the following integrals:

$$\int_z^1 \frac{d\beta_1}{\beta_1}\left[\frac{1-\beta_1}{\beta_1^2}+\frac{1}{2}\right]\left[\frac{1-\alpha_2}{\alpha_2^2}+\frac{1}{2}\right]=\frac{1}{4z^2}f(z), \quad \beta_1\alpha_2=z=\frac{s_1}{s}. \tag{10}$$

The parton language can also be applied to describe the processes in the fragmentation region. General formula for the cross section for the fragmentation region is derived in Appendix F. In Appendix G, this formalism is applied to the problem of transferring the longitudinal polarization of the initial electron to the positron.

Besides, the two-photon mechanism mentioned above, the so-called «bremsstrahlung» mechanism, must be taken into account. It consists in the emission of a light-like virtual photon by one of the initial particles with a subsequent conversion to the lepton pair. When calculating the differential and total cross sections, the effect of the Fermi–Dirac statistics must be taken into account.

Other QED peripheral processes, single and double bremsstrahlung, take into account the radiative corrections as well as the details of calculation and can be found in reviews [13–15].

It results in a nonleading contribution. Really, the contribution from the diagram corresponding to the single-photon production mechanism [15] is

$$\sigma_{\rm br}=2\frac{\alpha^4}{\pi m^2}\left[\left(\frac{77}{54}\pi^2-\frac{1099}{81}\right)\rho-\frac{223}{18}\xi(3)-\frac{17}{9}\pi^2\ln 2+\frac{163}{108}\pi^2+\frac{5435}{486}\right]= \\ =2\frac{\alpha^4}{\pi m^2}(0.5\rho-1.7), \tag{11}$$

where factor 2 takes into account both the kinematic situations when a jet moves along both the initial directions.

The effect of identity of final particles taken into account, contributes to the total cross section (both directions are taken into account) [17]

$$\sigma_{\rm int}=2\frac{2\alpha^4\rho}{105\pi m^2}\left[-374\xi(3)-120\pi^2\ln 2+\frac{13591}{90}\pi^2-\frac{2729}{12}\right]= \\ =2\frac{\alpha^4}{\pi m^2}(-0.14)\rho, \tag{12}$$

here $\sigma_{\rm int}$ is the contribution from the interference term associated with the identity of particles in the final state.

For electron-pair production and muon-pair production, the specific effect of the charge-odd contribution to spectral distributions takes place. It is caused by the interference of 2-gamma mechanism and the bremsstrahlung one.

Similar effects take place in the process of bremsstrahlung and pair production by a gluon and a quark on the proton or the nuclei. We will restrict ourselves below only to the cases when a proton or nuclei remain to be a proton or nuclei. No excitation of the target is allowed.

In the case of large transverse momenta of the jet particle component, the subtle effect of the double logarithmic contributions in the fragmentation region disappears.

2. KINEMATICS OF PERIPHERAL PROCESSES, SUDAKOV PARAMETERIZATION

First, we remind the general Sudakov technique to study the peripheral kinematics of the QED process $e+p \to (e+l+\bar{l})+p$ of creation of a heavy charged lepton pair in high-energy electron–proton collisions in the fragmentation region of the electron,

$$\begin{gathered} e(p_1)+p(p_2) \to e(p_1')+l(q_-)+\bar{l}(q_+)+p(p_2'), \\ p_2^2 = p_2^{'2} = m_p^2, \quad p_1^2 = p_1^{'2} = m^2, \quad q_\pm^2 = M^2, \\ s = 2p_1p_2 \gg M^2 \sim m_p^2 \gg m^2. \end{gathered} \tag{13}$$

The peripheral kinematics or the electron fragmentation region is defined as

$$s \gg -q^2 = -(p_2-p_2')^2 \sim M^2. \tag{14}$$

It is convenient to use the Sudakov parameterization of momenta. For this aim, we introduce two light-like 4-vectors constructed from the momenta of the initial particles $\tilde{p}_2 = p_2 - p_1(m_p^2/s)$, $\tilde{p}_1 = p_1 - p_2(m^2/s)$ [17]

$$\begin{gathered} q = \alpha\tilde{p}_2 + \beta\tilde{p}_1 + q_\perp, \quad q_\pm = \alpha_\pm\tilde{p}_2 + x_\pm\tilde{p}_1 + q_{\perp\pm}, \quad p_1' = \alpha'\tilde{p}_2 + x\tilde{p}_1 + p_\perp, \\ a_\perp p_1 = a_\perp p_2 = 0, \quad a_\perp^2 = -\mathbf{a}^2 < 0, \quad \tilde{p}_1^2 = \tilde{p}_2^2 = 0, \quad 2p_1\tilde{p}_1 = m^2, \end{gathered} \tag{15}$$

where $\mathbf{a}$ is the two-dimensional vector transversal to the beam axis (direction of $\mathbf{p}_1$, center-of-mass reference frame implied), and $x, x_\pm$ are the energy fractions of the scattered electron and the heavy-lepton pair, and $x+x_-+x_+=1$. Below, we will omit the tilde sign. According to the energy-momentum conservation law, we also have

$$\mathbf{q} = \mathbf{p} + \mathbf{q}_- + \mathbf{q}_+, \quad \alpha = \alpha' + \alpha_+ + \alpha_- - \frac{m^2}{s}. \tag{16}$$

The on-mass shell condition for the scattered proton $p_2^{'2} - m_p^2 = 0$, being written in terms of the Sudakov variables, reads (one must take into account the relation $2p_2\tilde{p}_2 = m_p^2$)

$$(p_2-q)^2 - m_p^2 = s\alpha\beta - \mathbf{q}^2 - m_p^2\alpha - s\beta = 0, \quad s\beta = -\frac{\mathbf{q}^2 + m_p^2\alpha}{1-\alpha}. \tag{17}$$

One finds for $q^2 = s\alpha\beta - \mathbf{q}^2$:

$$q^2 = -\frac{\mathbf{q}^2 + \alpha^2 m_p^2}{1-\alpha} \approx -\left(\mathbf{q}^2 + \frac{s_1^2}{s^2} m_p^2\right). \tag{18}$$

We conclude that in the case $s_1 \neq 0$, a virtual photon has a space-like 4-vector and, in addition $|q^2| > q^2_{\min} = m_p^2(s_1/s)^2$. The quantity $s_1 = 2qp_1 = (p_1' + q_+ + q_-)^2 - q^2 - m^2 = s\alpha$ in the WW approximation $\mathbf{q} = 0$ coincides with the square of the invariant mass of the jet moving in the initial quark momentum direction. Using the on-mass shell conditions for momenta of the scattered muon and the created pair of heavy quarks,

$$p_1'^2 = s\alpha' x - \mathbf{p}^2 = m_q^2 = m^2, \quad q_\pm^2 = s\alpha_\pm x_\pm - \mathbf{q}_\pm^2 = M^2, \quad x + x_+ + x_- = 1, \tag{19}$$

we find (in the WW approximation)

$$s_1 = s\alpha = \frac{1}{xx_+x_-}[x_-(1-x_-)\mathbf{q}_+^2 + x_+(1-x_+)\mathbf{q}_-^2 + \\ + 2x_-x_+\mathbf{q}_-\mathbf{q}_+ + m^2 x_+ x_- + x(1-x)M^2]. \tag{20}$$

The matrix element can be written as

$$M = \frac{(4\pi\alpha)^2}{q^2} g^{\mu\nu} J_\mu^{(e)}(p_1) J_\nu^{(p)}(p_2), \tag{21}$$

with $J^{(e,p)}$ being the currents associated with electron and proton blocks of the relevant Feynman diagram. The main contribution arises from the longitudinal components of the tensor $g^{\mu\nu} = g_{\mu\nu\perp} + (2/s)(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu)$:

$$g^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu. \tag{22}$$

So, we obtain for the squared module of the summed over spin states of the matrix element

$$\sum |M|^2 = (8\pi\alpha)^2 s^2 \frac{1}{(q^2)^2} \Phi^{(e)} \Phi^{(p)}, \\ \Phi^{(e)} = \sum \left| \frac{1}{s} J_\lambda^{(e)} p_2^\lambda \right|^2, \quad \Phi^{(p)} = \sum \left| \frac{1}{s} J_\sigma^{(p)} p_1^\sigma \right|^2. \tag{23}$$

The quantities $\Phi^{(e,p)}$ (the so-called impact factors) remain finite in the limit of high energies $s \to \infty$. In particular,

$$\Phi^{(p)} = \sum \left| \frac{1}{s} \bar{u}(p_2') \hat{p}_1 u(p_2) \right|^2 = 2. \tag{24}$$

The electron current obeys the gauge condition

$$q_\mu J^{(e)}(p_1)_\mu \approx (\alpha p_2 + q_\perp)_\mu J^{(e)}(p_1)_\mu = 0. \tag{25}$$

Using this relation we obtain for our process

$$\sum |M|^2 = 2\frac{s^2}{s_1^2}\frac{(4\pi\alpha)^2\mathbf{q}^2}{(q^2)^2}\sum\left|\frac{1}{s}J^{(e)}_\lambda e^\lambda\right|^2, \tag{26}$$

where $\mathbf{e} = \mathbf{q}/|\mathbf{q}|$ can be interpreted as a polarization vector of the virtual photon. To obtain the differential cross section,

$$d\sigma^{ep\to(e\,\mathrm{jet}_q)p} = \frac{1}{8s}\sum |M^{2\to 2+n}|^2 d\Gamma_{2+n}, \tag{27}$$

we must rearrange the phase volume of the final state (the electron remains to be a spectator, whereas the scattered muon is accompanied with n particles)

$$d\Gamma_{2+n} = (2\pi)^4\delta^4\left(p_1 + p_2 - p_1' - p_2' - \sum q_i\right)\times \\ \times\frac{d^3p_1'}{2E_1'(2\pi)^3}\frac{d^3p_2'}{2E_2'(2\pi)^3}\Pi_i\frac{d^3q_i}{2E_i(2\pi)^3}, \tag{28}$$

including the additional variable q as

$$d\Gamma_{2+n} \to d\Gamma_{2+n}d^4q\delta^4(p_2 - q - p_2'). \tag{29}$$

We use the Sudakov variables:

$$d^4q = \frac{s}{2}d\alpha d\beta d^2\mathbf{q}, \quad \frac{d^3q_\pm}{2E_\pm} = \frac{s}{2}d\alpha_\pm dx_\pm d^2\mathbf{q}_\pm\delta(s\alpha_\pm x_\pm - \mathbf{q}_\pm^2 - M^2). \tag{30}$$

Performing the integrations over the «small» Sudakov variables $\alpha, \alpha_\pm$, we obtain

$$d\Gamma_{2+n} = \frac{1}{sx}(2\pi)^4(2\pi)^{-3(2+n)}2^{-n-1}d^2\mathbf{q}\Pi_1^n\frac{dx_i}{x_i}d^2\mathbf{q}_i, x + \sum_1^n x_i = 1. \tag{31}$$

It can be noted that the cross section does not depend on s at large s and tends to zero in the limit of zero recoil momentum of the spectator electron $\mathbf{q}\to 0$. The last property is the consequence of gauge invariance of the theory. Once being integrated over the recoil momentum, the cross section reveals the so-called WW enhancement factor

$$L = \int_0^{Q^2}\frac{\mathbf{q}^2d\mathbf{q}^2}{(\mathbf{q}^2 + m_2^2\alpha^2)^2} = \ln\frac{Q^2s^2}{m_2^2s_1^2} - 1 = L_q - 1, \tag{32}$$

where $Q^2 \sim M^2$ is the scale of transfer momentum squared in the process.

3. 2-JET QCD PROCESSES IN QUARK (GLUON)–PROTON COLLISIONS

The differential cross sections of the processes

$$q(p_1) + p(p_2) \to q(p_1') + g(k) + p(p_2'), \tag{33}$$

$$e(p_1) + p(p) \to e(p_1') + \gamma(k) + p(p_2') \tag{34}$$

differ only by the color factors from similar expressions in QED.

$$d\sigma^{qp \to (g,q)p} = \frac{N^2-1}{2N} d\sigma^{ep \to (e\gamma)p}, \tag{35}$$

with

$$\begin{gathered} d\sigma^{ep \to e\gamma p} = \frac{2\alpha^3 d^2q\, d^2p'\, dx\bar{X}}{\pi^2 (q^2)^2 (DD')^2} R^\gamma [1 + \xi_3 B_3 + \xi_1 B_1], \\ R^\gamma = DD' q^2 (1+x^2) - 2xm^2 (D-D')^2, \end{gathered} \tag{36}$$

with

$$\begin{aligned} B_3 &= \frac{2x}{R_\gamma} [A^2 q^2 \cos(2\varphi_q) + B^2 p^2 \cos(2\varphi_p) + 2AB|\mathbf{q}||p| \cos(\varphi_q + \varphi_p), \\ B_1 &= \frac{2x}{R_\gamma} [A^2 q^2 \sin(2\varphi_q) + B^2 p^2 \sin(2\varphi_p) + 2AB|\mathbf{q}||p| \sin(\varphi_q + \varphi_p), \end{aligned} \tag{37}$$

and

$$A = \frac{1}{\bar{x}}(D' - xD), \quad B = \frac{1}{x}(D - D'), \tag{38}$$

and $B_{1,3}$ are the effective Stokes parameters of a gluon. Besides

$$D = m^2 \bar{x}^2 + (\mathbf{p} - \mathbf{q})^2, \quad D' = m^2 \bar{x}^2 + (\mathbf{p} - \mathbf{q}x)^2, \tag{39}$$

where $\mathbf{p}$ is the transverse component of the scattered electron momentum; $\mathbf{q}$ is the same value for the recoil proton; ϕ_p, ϕ_q are the azimuthal angles between the transverse component of a gluon and $\mathbf{p}, \mathbf{q}$.

For the process of quark–antiquark pair production by a gluon on a proton we have

$$\begin{gathered} d\sigma^{gp \to (Q\bar{Q})p} = \frac{1}{2} \frac{2\alpha^3}{\pi^2 (q^2)^2} \Phi^\gamma\, d^2q_+ d^2q\, dx_+, \\ \Phi^\gamma = \frac{1}{(D_+ D_-)^2} \left\{ 2m^2 x_+ x_- (D_+ - D_-)^2 + \mathbf{q}^2 (x_+^2 + x_-^2) D_+ D_- \right\}, \\ D_\pm = \mathbf{q}_\pm^2 + m^2, \quad \mathbf{q}_+ + \mathbf{q}_- = \mathbf{q}. \end{gathered} \tag{40}$$

The first factor is the color factor $\dfrac{1}{N^2-1} \dfrac{N^2-1}{2} = \dfrac{1}{2}$.

4. PRODUCTION OF HEAVY CHARGED LEPTON (QUARK) PAIRS IN ELECTRON (QUARK)–PROTON COLLISIONS

We will distinguish two mechanisms of the heavy-fermion pair creation, the so-called «bremsstrahlung mechanism» (see Fig. 1, a) and the «two-photon» one (Fig. 1, b). The matrix element of the process

$$\mu(p_1) + Y(p_2) \to \mu(p_1') + Q(q_-) + \bar{Q}(q_+) + Y(p_2'),$$
$$p_1^2 = p_1'^2 = m^2, \quad p_2^2 = p_2'^2 = m_Y^2, \quad q_\pm^2 = M^2 \tag{41}$$

in the kinematic region of μ particle fragmentation can be written as

$$M^{\mu Y \to (\mu Q\bar{Q})Y} = (4\pi\alpha)^2 \frac{2s}{q^2} N_2 \times$$
$$\times \left[\frac{1}{q_1^2 s} \bar{u}(p_1') Q_\mu u(p_1) \bar{u}(q_-)\gamma_\mu v(q_+) + \frac{1}{q_2^2 s} \bar{u}(q_-) R_\lambda v(q_+) \bar{u}(p_1')\gamma_\lambda u(p_1) \right], \tag{42}$$
$$q_1^2 = (q_- + q_+)^2, \quad q_2^2 = (p_1 - p_1')^2, \quad N_2 = \frac{1}{s} \bar{l}_2(p_2') \hat{p}_1 l_2(p_2).$$

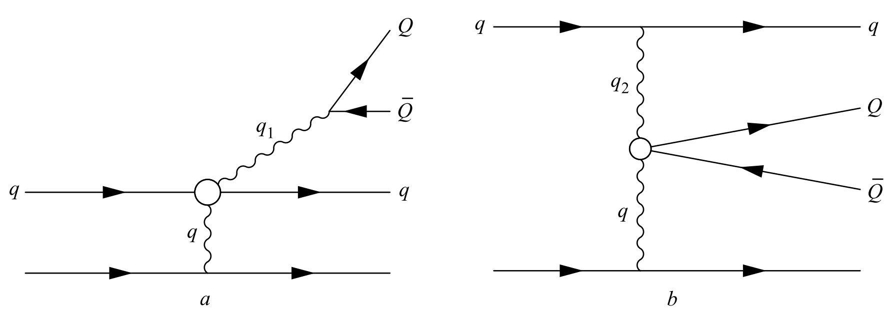

Fig. 1. Production of heavy-quark pair

Here we adopt Sudakov's parameterization of the 4-vectors

$$p_1' = \alpha' p_2 + x p_1 + p_\perp, \quad q_\pm = \alpha_\pm p_2 + x_\pm p_1 + q_{\pm\perp}, \quad q = \alpha p_2 + q_\perp. \tag{43}$$

The first term in the square brackets contains the Compton subprocess $e(p_1) + \gamma^*(q) \to e(p_1') + \gamma(q_1)$ amplitude $\bar{u}(p_1')Q_\mu u(p_1)$ with (we use here the on-mass shell conditions for the initial and final electrons (quarks))

$$Q_\mu = \frac{1}{D'}\gamma_\mu(\hat{p}_1 + \hat{q} + m)\hat{p}_2 - \frac{1}{D}\hat{p}_2(\hat{p}_1' - \hat{q} + m)\gamma_\mu =$$
$$= s\left(\frac{1}{D'} - \frac{x}{D}\right)\gamma_\mu + \frac{\gamma_\mu \hat{q}\hat{p}_2}{D'} + \frac{\hat{p}_2\hat{q}\gamma_\mu}{D},$$

where we use the notation

$$D' = (p_1+q)^2 - m^2 = \frac{1}{xx_+x_-}d', D = -[(p_1'-q)^2 - m^2] = \frac{1}{x_+x_-}d,$$
$$d' = d + \bar{x}x_+x_-\mathbf{q}^2 - 2x_+x_-\mathbf{q}(\mathbf{q}_+ + \mathbf{q}_-), \tag{44}$$
$$d = m^2x_+x_-\bar{x} + M^2x\bar{x} + \mathbf{q}_+^2x_-\bar{x}_- + \mathbf{q}_-^2x_+\bar{x}_+ + 2(\mathbf{q}_+\mathbf{q}_-)x_-x_+.$$

So we obtain

$$Q_\mu = \frac{x_+x_-}{dd'}[sx\rho\gamma_\mu + xd\gamma_\mu\hat{q}\hat{p}_2 + d'\hat{p}_2\hat{q}\gamma_\mu], \quad \rho = d - d'. \tag{45}$$

The two-photon amplitude contains a Dirac subprocess $\gamma^*(q_1)+\gamma^*(q) \to Q(q_-)+\bar{Q}(q_+)$ with the amplitude $\bar{u}(q_-)R_\lambda v(q_+)$

$$R_\lambda = -\gamma_\lambda\frac{\hat{q}-\hat{q}_+ + M}{D_+}\hat{p}_2 - \hat{p}_2\frac{\hat{q}_- - \hat{q} + M}{D_-}\gamma_\lambda. \tag{46}$$

Again, with on-mass shell conditions for the heavy-fermion pair it can be written as

$$R_\lambda = s\gamma_\lambda r_1 - \frac{\gamma_\lambda\hat{q}\hat{p}_2}{D_+} + \frac{\hat{p}_2\hat{q}\gamma_\lambda}{D_-}, \quad r_1 = \frac{x_+}{D_+} - \frac{x_-}{D_-}, \tag{47}$$

with the definitions

$$D_+ = -[(q-q_+)^2 - M^2] = \frac{1}{xx_-}d_+, \quad D_- = -[(q-q_-)^2 - M^2] = \frac{1}{xx_+}d_-,$$
$$d_+ = d + xx_+(\mathbf{q}^2 - 2\mathbf{q}\mathbf{q}_-) + x_+x_-(\mathbf{q}^2 - 2\mathbf{q}(\mathbf{q}_+ + \mathbf{q}_-)),$$
$$d_- = d + xx_-(\mathbf{q}^2 - 2\mathbf{q}\mathbf{q}_+) + x_+x_-(\mathbf{q}^2 - 2\mathbf{q}(\mathbf{q}_+ + \mathbf{q}_-)), \tag{48}$$
$$q_2^2 = (p_1 - p_1')^2 = -\frac{1}{x}[\mathbf{p}^2 + \bar{x}^2m^2], \quad \mathbf{p} = \mathbf{q} - \mathbf{q}_- - \mathbf{q}_+.$$

With this notation we have

$$R_\lambda = \frac{x}{d_+d_-}[sx_+x_-\rho_1\gamma_\lambda - x_-d_-\gamma_\lambda\hat{q}\hat{p}_2 + x_+d_+\hat{p}_2\hat{q}\gamma_\lambda], \quad \rho_1 = d_- - d_+. \tag{49}$$

The square of the matrix element summed over spin states has the form

$$\sum|M^{\mu Y\to(\mu Q\bar{Q})Y}|^2 = \frac{8s^2}{(q^2)^2}16(4\pi\alpha)^4R^{Q\bar{Q}}, \quad R^{Q\bar{Q}} = R_{\rm br} + R_{2\gamma} + R_{\rm odd},$$
$$R_{\rm br} = \frac{1}{s^2(q_1^2)^2}\frac{1}{4}\,\mathrm{Sp}\,(\hat{q}_- + M)\gamma_\mu(\hat{q}_+ - M)\gamma_\nu\frac{1}{4}\,\mathrm{Sp}\,\hat{p}_1'Q_\mu\hat{p}_1Q_\nu^+,$$
$$R_{2\gamma} = \frac{1}{s^2(q_2^2)^2}\frac{1}{4}\,\mathrm{Sp}\,(\hat{q}_- + M)R_\mu(\hat{q}_+ - M)R_\nu^+\frac{1}{4}\,\mathrm{Sp}\,(\hat{p}_1' + m)\gamma_\mu(\hat{p}_1 + m)\gamma_\nu, \tag{50}$$
$$R_{\rm odd} = \frac{2}{s^2q_1^2q_2^2}\frac{1}{4}\,\mathrm{Sp}\,(\hat{q}_- + M)\gamma_\mu(\hat{q}_+ - M)R_\lambda^+\frac{1}{4}\,\mathrm{Sp}\,\hat{p}_1'Q_\mu\hat{p}_1\gamma_\lambda.$$

It is important to know that all the quantities entering into $R^{q\bar{Q}}$ do not depend on s and are proportional to $\mathbf{q}^2$ in the WW limit $\mathbf{q} \to 0$.

Keeping in mind that in the combinations $\hat{p}_2\hat{q}$ and $\hat{q}\hat{p}_2$ one can replace $\hat{q} \to \hat{q}_\perp$, one may use the relations needed in calculating the traces (we neglect the contributions of an order of m^2/M^2 compared to the ones of an order of unity)

$$\begin{gathered}
p_1^2 = p_1'^2 = p_2^2 = 0, \quad q_+^2 = q_-^2 = M^2, \quad q^2 = -\mathbf{q}^2, \\
2p_2p_1 = s,\ 2p_2p_1' = sx,\ 2p_2q_+ = sx_+,\ 2p_2q_- = sx_-,\ 2p_2q = 0,\ 2p_1q = 0, \\
qq_- = -\mathbf{q}\mathbf{q}_-, \quad qq_+ = -\mathbf{q}\mathbf{q}_+, \quad qp_1' = -\mathbf{q}\mathbf{p}, \\
2q_+p_1' = \frac{1}{xx_+}[x^2M^2 + (x_+\mathbf{p} - x\mathbf{q}_+)^2],\ 2q_-p_1' = \frac{1}{xx_-}[x^2M^2 + (x_-\mathbf{p} - x\mathbf{q}_-)^2], \\
2p_1p_1' = -q_2^2 = \frac{1}{x}\mathbf{p}^2, \quad 2p_1q_+ = \frac{1}{x_+}[M^2 + \mathbf{q}_+^2], \quad 2p_1q_- = \frac{1}{x_-}[M^2 + \mathbf{q}_-^2], \\
q_1^2 = \frac{1}{x_+x_-}[\bar{x}^2M^2 + \mathbf{r}^2], \quad \mathbf{r} = x_-\mathbf{q}_+ - x_+\mathbf{q}_-, \quad \mathbf{p} = \mathbf{q} - \mathbf{q}_+ - \mathbf{q}_-.
\end{gathered} \tag{51}$$

The differential cross sections have the form

$$d\sigma^{eY\to(eQ\bar{Q})Y} = \frac{2\alpha^4}{\pi}\frac{R^{Q\bar{Q}}}{(q^2)^2}d\gamma_4, \quad d\gamma_4 = \frac{dx_+dx_-}{xx_+x_-}\frac{d^2\mathbf{q}}{\pi}\frac{d^2\mathbf{q}_+}{\pi}\frac{d^2\mathbf{q}_-}{\pi}. \tag{52}$$

In the case of processes with a quark instead of a muon, we must take into account the quark color degrees of freedom

$$d\sigma^{qY\to(qQ\bar{Q})Y} = C_{\rm col}\frac{\alpha_s^2}{\alpha^2}d\sigma^{eY\to(eQ\bar{Q})Y}, \tag{53}$$

with $C_{\rm col} = (N^2-1)/(4N^2)$, where we also take into account the averaging over the color of quarks. The explicit expressions for $R_{\rm br}, R_{2\gamma}, R_{\rm odd}$ are given in Appendix B. In the double WW approximation, the contribution to the differential cross section has the form

$$\frac{d\sigma^{ep\to(Q\bar{Q}e)p}}{dx_+dx_-dz} = \frac{\alpha^4L^2}{\pi M^2}\Phi(z, x_+, x_-), \quad z = \frac{\mathbf{q}_+^2}{M^2}. \tag{54}$$

Exact formulae are given in Appendix B.
The function

$$\Phi\left(z, x_+ = x_- = \frac{1}{3}\right) = \Phi(z) = \frac{3}{8}\frac{7 + 268z + 243z^2}{(1+z)^4}$$

is presented in Fig. 4.

Table 1. **The function $A_{+-}\,(y_+, y_-)$ (defined in (55)) is presented for different values of the final quark (lepton) transverse momenta y_+ and y_- for the quark-pair (lepton-pair) production (in units of M)**

A_{+-}	0.0279	0.0295	0.0314	0.0326	0.0348	0.0377	0.0405	0.0421	0.0462	0.0513
y_+	4	3	3	2	2	2	1	1	1	1
y_-	5	4	5	3	4	5	2	3	4	5

In Table 1, the charge asymmetry defined as

$$A_{+-}(y_+, y_-, x_+, x_-) = \left[\frac{2}{q_1^2 q_2^2} R_{\mathrm{WW}}^{\mathrm{odd}}\right] \Big/ \left[\frac{1}{(q_1^2)^2} R_{\mathrm{WW}}^{\mathrm{br}} + \frac{1}{(q_2^2)^2} R_{\mathrm{WW}}^{2g}\right], \qquad y_\pm = \frac{\mathbf{q}_\pm^2}{M^2} \tag{55}$$

is presented at the symmetric point $x = x_- = x_+ = 1/3$, $\phi = \pi/2$ for several typical values $y_+ < y_-$. This quantity has a value of an order of $A_{+-} \sim 10^{-2}$. For the use of $y_+ > y_-$, the quantity A_{+-} changes the sign.

5. DOUBLE BREMSSTRAHLUNG IN ELECTRON–PROTON COLLISIONS

In the lowest order of perturbation QED theory there are 20 Feynman diagrams describing the double bremsstrahlung process (see Fig. 3)

$$e(p_1) + p(p_2) \to e(p_1') + \gamma(k_1) + \gamma(k_2) + p(p_2'), \tag{56}$$

i.e., emission of two hard photons in collisions of the high-energy electron with a charged heavy target (heavy lepton). We will restrict ourselves to the consideration of the emission from the electron line only. The set of six Feynman diagrams provides the gauge-invariant set (see Fig. 1, *a*, *b*). With respect to the exchanged photon they split into two independent subsets of Feynman amplitudes, both gauge-invariant. The relevant matrix element is

$$M^{ep\to(e\gamma\gamma)p} = \frac{2s(4\pi\alpha)^2}{q^2} N \frac{1}{s} \bar{u}(p_1')[O_{12} + O_{21}]u(p_1), \tag{57}$$

with (see details in Appendices C, D),

$$\begin{aligned} O_{12} &= \frac{1}{(1)D} N_1 - \frac{1}{(1)(2')} N_2 + \frac{1}{(2')D'} N_3, \\ N_1 &= \hat{p}_2(\hat{p}_1' - \hat{q} + m)\hat{e}_2(\hat{p}_1 - \hat{k}_1 + m)\hat{e}_1, \\ N_2 &= \hat{e}_2(\hat{p}_1' + \hat{k}_2 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_1 + m)\hat{e}_1, \\ N_3 &= \hat{e}_2(\hat{p}_1' + \hat{k}_2 + m)\hat{e}_1(\hat{p}_1 + \hat{q} + m)\hat{p}_2, \end{aligned} \tag{58}$$

and

$$\begin{aligned}
O_{21} &= \frac{1}{(2)D}N'_1 - \frac{1}{(2)(1')}N'_2 + \frac{1}{(1')D'}N'_3,\\
N'_1 &= \hat{p}_2(\hat{p}'_1 - \hat{q} + m)\hat{e}_1(\hat{p}_1 - \hat{k}_2 + m)\hat{e}_2,\\
N'_2 &= \hat{e}_1(\hat{p}'_1 + \hat{k}_1 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_2 + m)\hat{e}_2,\\
N'_3 &= \hat{e}_1(\hat{p}'_1 + \hat{k}_1 + m)\hat{e}_2(\hat{p}_1 + \hat{q} + m)\hat{p}_2,\\
N &= \frac{1}{s}\bar{Y}(p'_2)\hat{p}_1 Y(p_2).
\end{aligned} \tag{59}$$

Here $e_i = e_i(k_i)$ are the polarization vectors of hard photons. It can be checked that the expression for $M^{2Y\to(2\gamma\gamma)Y}$ turns to zero in replacing $p_2 \to q$ as well as $e_i \to k_i$ which is the consequence of gauge invariance.

We adopt below the Sudakov parameterization of the relevant 4-vectors

$$\begin{gathered}
q = \alpha p_2 + q_\perp, \quad k_i = \alpha_i p_2 + x_i p_1 + k_{i\perp},\\
p'_1 = \alpha' p_2 + x p_1 + p_\perp,
\end{gathered} \tag{60}$$

and use the notation and relations (different compared to the previous section)

$$\begin{gathered}
(1) = 2p_1k_1 = \frac{1}{x_1}[m^2x_1^2 + \mathbf{k}_1^2] = \frac{y_1}{x_1}, \;\; (2') = 2p'_1k_2 = \frac{1}{xx_2}[m^2x_2^2 + \mathbf{r}_2^2] = \frac{z_2}{xx_2},\\
(2) = 2p_1k_2 = \frac{1}{x_2}[m^2x_2^2 + \mathbf{k}_2^2] = \frac{y_2}{x_2}, \;\; (1') = 2p'_1k_1 = \frac{1}{xx_1}[m^2x_1^2 + \mathbf{r}_1^2] = \frac{z_1}{xx_1},\\
D = -[(p'_1 - q)^2 - m^2] = \frac{d}{x_1x_2}, \quad D' = (p_1 + q)^2 - m^2 = \frac{1}{xx_1x_2}d',\\
\mathbf{r}_2 = \bar{x}_1\mathbf{k}_2 + x_2(\mathbf{k}_1 + \mathbf{q}), \quad \mathbf{r}_1 = \bar{x}_2\mathbf{k}_1 + x_1(\mathbf{k}_2 + \mathbf{q}),\\
d' = d + \mathbf{q}^2\bar{x}x_1x_2 - 2x_1x_2\mathbf{q}(\mathbf{k}_1 + \mathbf{k}_2),\\
d = m^2x_1x_2\bar{x} + x_1\bar{x}_1\mathbf{k}_2^2 + x_2\bar{x}_2\mathbf{k}_1^2 + 2x_1x_2\mathbf{k}_1\mathbf{k}_2,\\
2k_1k_2 = \frac{1}{x_1x_2}(x_2\mathbf{k}_1 - x_1\mathbf{k}_2)^2, \quad 2p_1p'_1 = \frac{1}{x}[\mathbf{p}^2 + m^2x^2],\\
(1) + (2) + (1') + (2') = D + D'.
\end{gathered} \tag{61}$$

The expression for the matrix element given above can be written in a form to display the explicit gauge invariance, which is suitable especially for investigation in the (WW) [7] approximation. For this aim we note that the combinations

$$R_1 = \frac{x}{(1)D} - \frac{\bar{x}_1}{(1)(2')} + \frac{1}{(2')D'},$$
$$R_2 = \frac{x}{(2)D} - \frac{\bar{x}_2}{(2)(1')} + \frac{1}{(1')D'}, \qquad (62)$$
$$r = \frac{1}{D'} - \frac{x}{D}$$

turn to zero in the limit $\mathbf{q} \to 0$. Excluding the term containing the denominator (1), (2) we can rewrite the expression for O_{12} in the form

$$O_{12} = \frac{1}{\bar{x}_1}[R_1 \hat{e}_2(\hat{p}_1' + \hat{k}_2 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_1 + m)\hat{e}_1 + r\hat{e}_2\hat{p}_2\hat{e}_1 + \\ + c_1\hat{e}_2(\hat{p}_1' + \hat{k}_2 + m)[\bar{x}_1\hat{e}_1\hat{q}\hat{p}_2 + \hat{p}_2\hat{q}\hat{e}_1] + \\ + d_1[\bar{x}_1\hat{p}_2\hat{q}\hat{e}_2 + x\hat{e}_2\hat{q}\hat{p}_2](\hat{p}_1 - \hat{k}_1 + m)\hat{e}_1], \qquad (63)$$

with

$$r = \frac{x x_1 x_2}{dd'}\rho, \quad \rho = d - d' = x_1 x_2[-\bar{x}\mathbf{q}^2 + 2\mathbf{q}(\mathbf{k}_1 + \mathbf{k}_2)],$$
$$c_1 = \frac{x_1(x x_2)^2}{z_2 d'}, \quad d_1 = -\frac{x_2 x_1^2}{y_1 d}. \qquad (64)$$

A similar expression for the set of other Feynman diagrams (can be obtained from the first one by the replacement $k_1, e_1 \leftrightarrow k_2, e_2$) is

$$O_{21} = \frac{1}{\bar{x}_2}[R_2 \hat{e}_1(\hat{p}_1' + \hat{k}_1 + m)\hat{p}_2(\hat{p}_1 - \hat{k}_2 + m)\hat{e}_2 + r\hat{e}_1\hat{p}_2\hat{e}_2 + \\ + c_2\hat{e}_1(\hat{p}_1' + \hat{k}_1 + m)[\bar{x}_2\hat{e}_2\hat{q}\hat{p}_2 + \hat{p}_2\hat{q}\hat{e}_2] + \\ + d_2[\bar{x}_2\hat{p}_2\hat{q}\hat{e}_1 + x\hat{e}_1\hat{q}\hat{p}_2](\hat{p}_1 - \hat{k}_2 + m)\hat{e}_2], \qquad (65)$$

with

$$c_2 = \frac{x_2(x x_1)^2}{z_1 d'}, \quad d_2 = -\frac{x_1 x_2^2}{y_2 d}. \qquad (66)$$

The matrix element squared summed over spin states is

$$\sum |M^{ep\to(e\gamma\gamma)p}|^2 = \frac{32(4\pi\alpha)^4 s^2}{\pi(q^2)^2} R^{\gamma\gamma},$$
$$R^{\gamma\gamma} = (1 + \mathcal{P}_{12})\frac{1}{4s^2}\,\mathrm{Sp}\,[\hat{p}_1' O_{12}\hat{p}_1 O_{12}^+ + \hat{p}_1' O_{12}\hat{p}_1 O_{21}^+]. \qquad (67)$$

From a topological point of view, it is convenient to write down $R_{\gamma\gamma}$ as a sum of planar and nonplanar Feynman diagrams for the cross section

$$R^{\gamma\gamma} = (1 + \mathcal{P}_{12})[R_{\rm pl} + R_{\rm npl}]. \tag{68}$$

The differential cross section has the form

$$\begin{aligned} d\sigma^{ep\to(e\gamma\gamma)p} &= \frac{1}{2!}\frac{\alpha^4}{2\pi}\frac{R_{\gamma\gamma}}{(q^2)^2}d\gamma_4, \\ d\gamma_4 &= \frac{dx_1 dx_2}{xx_1x_2}\frac{d^2\mathbf{q}}{\pi}\frac{d^2\mathbf{k}_1}{\pi}\frac{d^2\mathbf{k}_2}{\pi}. \end{aligned} \tag{69}$$

Factor 1/2! takes into account the identity of photons in the final state.

In the WW approximation we have

$$\begin{aligned} 2\pi\frac{d\sigma^{ep\to(e\gamma\gamma)p}}{dx_1\, dx_2\, dy_1\, dy_2\, d\phi} &= \frac{\alpha^4(L_p-1)}{4\pi}\frac{1}{M^2}R^{\gamma\gamma}(y_1,y_2,x_1,x_2,\phi), \\ y_i &= \frac{k_i^2}{M^2}, \end{aligned} \tag{70}$$

with M being the mass of a heavy quark in the scale parameter for the values of the transverse momenta of photons (gluons).

6. QUARK–PROTON COLLISION: EMISSION OF TWO-GLUON JETS

The matrix element of the process of two-gluon jets production in a peripheral quark-colorless fermion target collision (see Fig. 2),

$$q(p_1) + Y(p_2) \to q(p_1') + Y(p_2') + g(k_1) + g(k_2), \tag{71}$$

has the form

$$\begin{aligned} M &= \frac{32s\alpha\alpha_s}{q^2}J^qN, \quad J^q = \bar{u}(p_1')Ru(p_1), \quad N = \frac{1}{s}\bar{u}(p_2')\Gamma_\mu u(p_2)p_1^\mu, \\ R &= O_{12}R_2 + O_{21}R_1 + (R_2 - R_1)O_3, \quad \Gamma_\mu = F_1\gamma_\mu + \sigma_{\mu\nu}q_\nu F_2, \end{aligned} \tag{72}$$

where $R_1 = (t^at^b)_{r_2r_1}$, $R_2 = (t^bt^a)_{r_2r_1}$, with $r_2(r_1)$ describing the color states of the scattered (initial) quark. Here the quantities O_{12}, O_{21} were obtained above (see (59) and (65)), with the replacement $k_1 \to q_+$, $k_2 \to q_-$, where we imply $e_1 \to e^a$, $e_2 \to e^b$ and

$$\begin{aligned} O_3 &= -\frac{2}{q_1^2}\left[-\frac{1}{D}\hat{p}_2(\hat{p}_1' - \hat{q} + m)\hat{V}^{ab} + \frac{1}{D'}\hat{V}^{ab}(\hat{p}_1 + \hat{q} + m)\hat{p}_2\right], \\ \hat{V}^{ab} &= (k_1e^b)e^a - \hat{e}^b(k_2e^a) + \hat{k}_2(e^ae^b), \quad q_1^2 = (k_1+k_2)^2. \end{aligned} \tag{73}$$

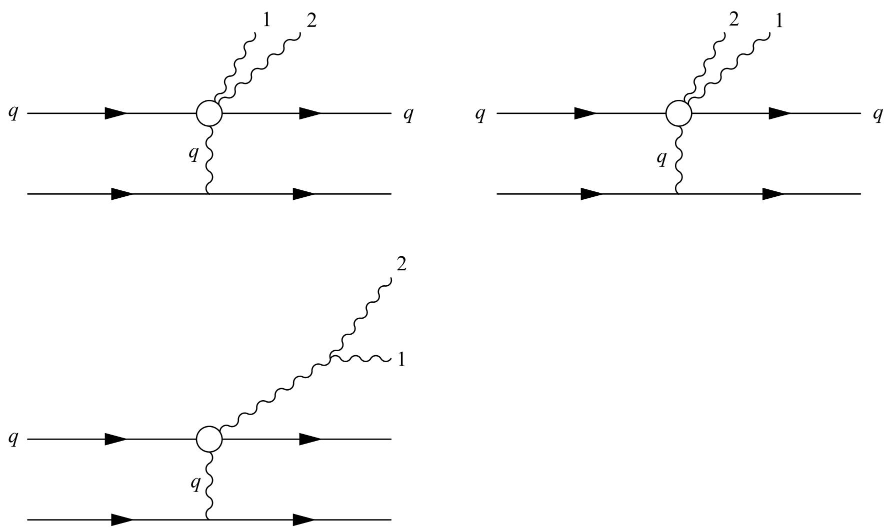

Fig. 2. Emission of two gluons

The quantity q_1^2 is presented (26) and (35). It can be checked that the matrix element obeys gauge invariance, namely, it turns to zero if one replaces $p_2 \to q$ and $e_i(k_i) \to k_i$. The expression for the matrix element at $R_1 = R_2 = 1$, coincides with the QED result [21]. Below we will use the expression for O_3 in the form

$$\begin{gathered} O_3 = -\frac{2}{q_1^2}\frac{x_1 x_2}{dd'}[xs\rho\hat{V} + d'\hat{p}_2\hat{q}\hat{V} + xd\hat{V}\hat{q}\hat{p}_2], \\ \hat{V} = \hat{e}^a(k_1 e^b) + \hat{k}_2(e^a e^b) - \hat{e}^b(k_2 e^a), \quad \rho = 2x_1 x_2 \mathbf{qQ}. \end{gathered} \tag{74}$$

To work with the irreducible color states, we use the projectors in color space

$$\begin{aligned} C_1 &= \frac{1}{\sqrt{N(N^2-1)}}\delta^{ab}\delta_{r_2 r_1}, \\ C_2 &= \sqrt{\frac{2N}{(N^2-1)(N^2-4)}}d^{abc}(t^c)_{r_2 r_1}, \\ C_3 &= i\sqrt{\frac{2}{N(N^2-1)}}f^{abc}(t^c)_{r_2 r_1}. \end{aligned} \tag{75}$$

These projectors obey the equations

$$C_i\tilde{C}_j = \begin{pmatrix} (c_i^{ab})_{r_1r_2} \\ ((c_i^{ab})_{r_1r_2})^+ = (c_i^{ab})_{r_2r_1} \end{pmatrix}, \tag{76}$$

$$C_i\tilde{C}_j = \delta_{ij}, \quad i,j = 1,2,3. \tag{77}$$

Here $(\tilde{A})_{r_1r_2} = (A)_{r_2r_1}$ and summation over a, b is implied.

In our case*

$$\begin{aligned} R_1 &= \sqrt{\frac{N^2-1}{4N}}\left[C_1 + \sqrt{\frac{N^2-4}{2}}C_2 + \frac{N}{\sqrt{2}}C_3\right], \\ R_2 &= \sqrt{\frac{N^2-1}{4N}}\left[C_1 + \sqrt{\frac{N^2-4}{2}}C_2 - \frac{N}{\sqrt{2}}C_3\right]. \end{aligned} \tag{78}$$

The expansion on irreducible color representations is

$$\begin{aligned} R = C_1(R\tilde{C}_1) + C_2(R\tilde{C}_2) + C_3(R\tilde{C}_3) = \sqrt{\frac{N^2-1}{4N}}\times \\ \times\left[\left(C_1 + \sqrt{\frac{N^2-4}{2}}C_2\right)(O_{12}+O_{21}) + \frac{N}{\sqrt{2}}(O_{21}-O_{12}-2O_3)\right]. \end{aligned} \tag{79}$$

So the matrix element squared summed over color and spin states can be written as

$$\begin{gathered} \sum |M|^2 = \frac{32s^2(16\pi^2\alpha\alpha_s)^2}{(q^2)^2}\frac{N^2-1}{4N}F, \\ F = F^{\text{Abel}} + F^{\text{non-Abel}}, \\ F^{\text{Abel}} = \frac{N^2-2}{2}(1+\mathcal{P}_{12})(R_{\text{pl}}+R_{\text{npl}}) + \frac{N^2}{2}[(1+\mathcal{P}_{12})(R_{\text{pl}}-R_{\text{npl}})], \\ F^{\text{non-Abel}} = 2N^2[R_{33} - R_{321} + R_{312}], \end{gathered} \tag{80}$$

*As a check we have $(R_1\tilde{R}_1) = (R_2\tilde{R}_2) = \mathrm{Tr}\, t^at^bt^bt^a = NC_F^2$; $(R_1\tilde{R}_2) = \mathrm{Tr}\, t^at^bt^at^b = -(1/2)C_F$. These relations are fulfilled.

with

$$R_{\rm pl} = \frac{1}{4s^2}\,{\rm Sp}\,\hat{p}_1'O_{12}\hat{p}_1O_{12}^+,$$
$$R_{\rm npl} = \frac{1}{4s^2}\,{\rm Sp}\,\hat{p}_1'O_{12}\hat{p}_1O_{21}^+,$$
$$R_{33} = \frac{1}{4s^2}\,{\rm Sp}\,\hat{p}_1'O_3\hat{p}_1O_3^+, \tag{81}$$
$$R_{312} = \frac{1}{4s^2}\,{\rm Sp}\,\hat{p}_1'O_3\hat{p}_1O_{12}^+,$$
$$R_{321} = \frac{1}{4s^2}\,{\rm Sp}\,\hat{p}_1'O_3\hat{p}_1O_{21}^+.$$

In the case $x_1 = x_2 = x = 1/3$ and $\phi = \pi/2$, $F^{\text{non-Abel}}$ and F are presented (see Table 2) for typical values of y_1, y_2.

Table 2. **The functions $F^{\text{non-Abel}}(y_1, y_2)$ and $F(y_1, y_2)$ (defined in (80)) are presented for different values of the final gluon transverse momenta y_1 and y_2**

$F^{\text{non-Abel}}$	0.0398	0.0845	0.0561	0.2309	0.1315	0.0812	1.0521	0.4257	0.2074	0.1135
F	0.0415	0.0882	0.0591	0.2422	0.1396	0.0872	1.117	0.4625	0.2306	0.1294
y_1	4	3	3	2	2	2	1	1	1	1
y_2	5	4	5	3	4	5	2	3	4	5

The differential cross section is

$$d\sigma^{qY\to(qgg)Y} = \frac{(\alpha\alpha_s)^2}{8\pi}\frac{N^2-1}{N^2}\frac{F}{(q^2)^2}d\gamma_4. \tag{82}$$

The explicit expressions for F as well as for $R_{\gamma\gamma}$ are too cumbersome. Nevertheless, they are suitable for further analytic and numerical integration when obtaining different distributions.

As some probe of QCD, the quantity

$$A^{gg} = \frac{F^{\text{non-Abel}}}{F} \tag{83}$$

can be considered as a specific for QCD deviation from the QED process of the double bremsstrahlung. It is presented in Table 3 in the WW approximation for

Table 3. **The function $A^{gg}(y_1, y_2)$ (defined in (83)) is presented for different values of the final gluon transverse momenta y_1 and y_2**

A^{gg}	0.8396	0.8318	0.8024	0.8167	0.7755	0.7382	0.7755	0.7034	0.6393	0.5805
y_1	4	3	3	2	2	2	1	1	1	1
y_2	5	4	5	3	4	5	2	3	4	5

$x = x_1 = x_2 = 1/3$, $\phi = \pi/2$, for different values y_1, y_2. For values $q_1^2 \sim M^2$, the non-Abelian contribution dominates in A^{gg} (see Table 3). For large q_1^2, $A^{gg} \sim (M^2/q_1^2)$.

7. THREE-JET STRUCTURE IN PERIPHERAL COLLISIONS ON A FIXED TARGET

In the case of jet production with a projectile on a fixed target (nucleon), an additional wide-angle jet can be created by the recoil target particle.

Consider for simplicity the photoproduction of a pair $q\bar{q}$ process on a nucleon,

$$\gamma(k) + p(p) \to q(q_-) + \bar{q}(q_+) + p'(p'), \tag{84}$$

$s = 2pk$, $k^2 = 0$, $p = M(1, 0, 0, 0)$.

As the Sudakov expansion basis we use $k = \omega(1, 1, 0, 0)$ and $p_1 = p - (M^2/s)k = (M/2)(1, -1, 0, 0)$. For the transferred 4-momentum $q = p - p'$ we have

$$q = \alpha k + \beta p_1 + q_\perp. \tag{85}$$

Solving the on-mass shell condition of a recoil proton we find

$$p_1'^2 - M^2 = s\alpha\beta - \mathbf{q}^2 - M^2\beta - s\alpha = 0, \;\; s\alpha = \mathbf{q}^2. \tag{86}$$

Considering its longitudinal and transverse component we have

$$\mathbf{p}'^2 = \mathbf{q}^2 + \frac{1}{4M^2}|\mathbf{q}^2|^2. \tag{87}$$

So we have

$$\sin^2\theta = \frac{\mathbf{q}^2}{\mathbf{p}'^2}, \tag{88}$$

with $\theta(\mathbf{p}', \mathbf{k})$, and

$$\left|\frac{\mathbf{p}'}{M}\right| = \frac{2\cos\theta}{\sin^2\theta}. \tag{89}$$

The recoil jet penetrates in a rather wide cone $\theta \sim 60^\circ$. Relation (72) was first obtained in [28].

8. THE PROCESS $gP \to (g\bar{Q}Q)P$

Another process where the non-Abelian structure of QCD manifests itself is the crossing process to one considered in Sec. 3

$$g(k) + P(p_2) \to (\bar{Q}(q_+)Q(q_-)g(k_1))P(p_2'). \tag{90}$$

The matrix element can be written as

$$M_1 = (M_1^{(1)} + M_1^{(2)} + M_1^{(3)} + M_1^{(4)} + M_1^{(5)} + M_1^{(6)} + M_1^{(7)} + M_1^{(8)})_{\mu\nu},$$
$$M_2 = (M_2^{(1)} + M_2^{(2)} + M_2^{(3)} + M_2^{(4)} + M_2^{(5)} + M_2^{(6)} + M_2^{(7)} + M_2^{(8)})_{\mu\nu}, \tag{91}$$

$$\begin{gathered}
M = \bar{u}(q_-)[t^c t^a M^{(1)} + t^a t^c M^{(2)}]_{\mu\nu} v(q_+) e_\mu^a(k) e_\nu^b(k_1), \\
M_1^{(1)} = \frac{\gamma_\nu(q_- + k_1 + m)\gamma_\mu(-q_+ + q + m)p_2}{[(q_- + k_1)^2 - m^2][(-q_+ + q)^2 - m^2]} = R_1, \\
M_2^{(1)} = \frac{\gamma_\nu(q_- + k_1 + m)p_2(-q_+ + k + m)\gamma_\mu}{[(q_- + k_1)^2 - m^2][(-q_+ + k)^2 - m^2]} = R_2, \\
M_3^{(1)} = \frac{p_2(q_- - q + m)\gamma_\nu(-q_+ + k + m)\gamma_\mu}{[(-q_+ + k)^2 - m^2][(q_- - q)^2 - m^2]} = R_3, \\
M_7^{(1)} + M_8^{(1)} = \frac{V_{\mu\lambda\nu}}{(k_1 - k)^2}\left[\frac{\gamma_\lambda(-q_+ + q + m)p_2}{(-q_+ + q)^2 - m^2} + \frac{p_2(q_+ - q + m)\gamma_\lambda}{(q_+ - q)^2 - m^2}\right] = R_4,
\end{gathered} \tag{92}$$

$$\begin{gathered}
M_4^{(2)} = \frac{\gamma_\mu(q_- - k_1 + m)\gamma_\nu(-q_+ + q + m)p_2}{[(-q_- - k)^2 - m^2][(-q_+ + q)^2 - m^2]} = Q_1, \\
M_5^{(2)} = \frac{\gamma_\mu(q_- - k + m)p_2(-q_+ - k_1 + m)\gamma_\nu}{[(q_- - k)^2 - m^2][(-q_+ - k_1)^2 - m^2]} = Q_2, \\
M_6^{(2)} = \frac{p_2(q_- - q + m)\gamma_\mu(-q_+ - k_1 + m)\gamma_\nu}{[(q_- - q)^2 - m^2][(-q_+ - k_1)^2 - m^2]} = Q_3, \\
M_7^{(2)} + M_8^{(2)} = \frac{-V_{\mu\lambda\nu}}{(k_1 - k)^2}\left[\frac{\gamma_\lambda(-q_+ + q + m)p_2}{(-q_+ + q)^2 - m^2} + \frac{p_2(q_- - q + m)\gamma_\lambda}{(q_- - q)^2 - m^2}\right] = Q_4, \\
V_{\mu\lambda\nu} = -(k_1 + k)_\lambda g_{\mu\nu} + (2k - k_1)_\nu g_{\mu\lambda} + (2k_1 - k)_\mu g_{\lambda\nu}, \\
M_{\mu\nu}^{(1,2)} k_1^\nu = M_{\mu\nu}^{(1,2)} k^\mu = 0.
\end{gathered} \tag{93}$$

It can be checked that both contributions $M^{(1)}$ and $M^{(2)}$ obey the gauge condition

$$M \sim t^c t^a M^{(1)} + t^a t^c M^{(2)} = \sqrt{\frac{N^2 - 1}{4N}}\left\{\left[c_1 + \sqrt{\frac{N^2 - 1}{2}} c_2\right] \times \right.$$
$$\left. \times (M^{(1)} + M^{(2)}) + \frac{N}{\sqrt{2}} c_3 \left[M^{(1)} - M^{(2)}\right]\right\}, \tag{94}$$

$$\begin{gathered}
M^{(1)} + M^{(2)} = R_1 + R_2 + R_3 + R_4 + Q_1 + Q_2 + Q_3 + Q_4, \\
M^{(1)} - M^{(2)} = (R_1 + R_2 + R_3) - (Q_1 + Q_2 + Q_3) + 2R_4,
\end{gathered} \tag{95}$$

Table 4. **The function $A\,(y_+, y_-)$ (defined in (98)) is presented for different values of the final quark (lepton) transverse momenta y_+ and y_- for the quark-pair (lepton-pair) production (in units of M)**

A	0.0428	0.117	0.0652	0.4489	0.193	0.0925	0.0574	0.84	0.275	0.1167
y_+	4	3	3	2	2	2	1	1	1	1
y_-	5	4	5	3	4	5	6	3	4	5

$$|M^{gP\to(gQ\bar{Q})P}|^2 \approx \frac{N^2-2}{2}\,\mathrm{Sp}\,\hat{q}_-(M_1+M_2)\hat{q}_+(M_1+M_2)^* + \\ + \frac{N^2}{2}\,\mathrm{Sp}\,\hat{q}_-(M_1-M_2)\hat{q}_+(M_1-M_2)^* = A^{\mathrm{tot}}, \quad (96)$$

$$\begin{aligned} M_1+M_2 &= R_1+R_2+R_3+Q_1+Q_2+Q_3, \\ M_1-M_2 &= R_1+R_2+R_3-Q_1-Q_2-Q_3+2R_4. \end{aligned} \quad (97)$$

Again one can define the asymmetry A, which appears because of non-Abelian nature of QCD. This asymmetry is defined as (see Table 4):

$$A = \frac{A^{\text{non-Abel}}}{A^{\mathrm{tot}}}, \quad (98)$$

$$A^{\text{non-Abel}} = 2N^2\big\{\mathrm{Sp}\,\hat{q}_-R_4\hat{q}_+R_4^+ + \\ + \mathrm{Sp}\,\hat{q}_-(R_1+R_2+R_3-Q_1-Q_2-Q_3)\hat{q}_+R_4^+\big\}, \quad (99)$$

$$\begin{aligned} R_1 &= \frac{\gamma_\nu(\hat{q}_-+\hat{k}_1)\gamma_\mu(-\hat{q}_++\hat{q})\hat{p}_2}{d_{-1}d_{+q}}, \quad R_2 = \frac{\gamma_\nu(\hat{q}_-+\hat{k}_1)\hat{p}_2(-\hat{q}_++\hat{k})\gamma_\mu}{d_{-1}d_{+k}}, \\ R_3 &= \frac{\hat{p}_2(\hat{q}_--\hat{q})\gamma_\nu(-\hat{q}_++\hat{k})\gamma_\mu}{d_{-q}d_{+k}}, \quad Q_1 = \frac{\gamma_\mu(\hat{q}_--\hat{k})\gamma_\nu(-\hat{q}_++\hat{q})\hat{p}_2}{d_{-k}d_{-q}}, \\ Q_2 &= \frac{\gamma_\mu(\hat{q}_--\hat{k})\hat{p}_2(-\hat{q}_+-\hat{k}_1)\gamma_\nu}{d_{-k}d_{+(-k_1)}}, \quad Q_3 = \frac{\hat{p}_2(\hat{q}_--\hat{q})\gamma_\mu(-\hat{q}_+-\hat{k}_1)\gamma_\nu}{d_{-q}d_{+(-k_1)}}, \\ R_4 &= \frac{V_{\mu\lambda\nu}}{d}\left[\frac{\gamma_\lambda(-\hat{q}_++\hat{q})\hat{p}_2}{d_{+q}} + \frac{\hat{p}_2(\hat{q}_--\hat{q})\gamma_\lambda}{d_{-q}}\right]. \end{aligned} \quad (100)$$

CONCLUSION

In conclusion, we remind a remarkable property of the kinematics of processes in the fragmentation region. It is known as a «cumulation» phenomenon (see Fig. 3). It consists of events with production of a heavy quark–antiquark

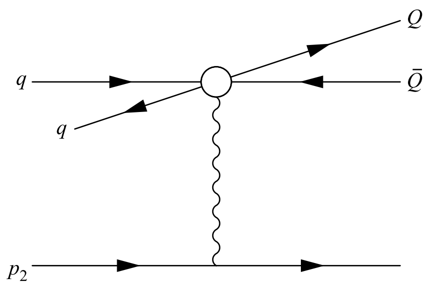

Fig. 3. The «cumulation effect»

pair, accompanied by the «reflected» scattered parent light particle. It was known in the processes of production of a muon–antimuon pair in the fragmentation region of an electron in electron–positron collisions [26]. It turns out that the electron «accompanying» the pair created in the kinematic region near the threshold moves in the direction opposite to the initial electron direction. In the case of production of a heavy quark–antiquark pair by one of the valence quarks from the initial proton, the parent (light) quark is effectively reflected. So the jet created by this quark corresponds in fact to two jets, one consists of the pair created and two spectator quarks from the initial proton and the other, moving in the opposite direction, created by the «reflected» quark. To see it, let us consider the kinematics of a peripheral process $q(p_1)+q(p_2)\to Q(p_a)+\bar{Q}(p_b)+q(p_1')+q(p_2')$. Using the Sudakov parameterization (3) with

$$\tilde{p}_1 = E(1,1,0,0), \quad \tilde{p}_2 = E(1,-1,0,0), \quad p_\perp = (0,0,\mathbf{p}), \tag{101}$$

we obtain for 4-momentum of the scattered quark

$$\frac{1}{E}p_1' = \frac{m^2+\mathbf{p}^2}{xs}(1,-1,0,0) + x(1,1,0,0) + (0,0,\mathbf{p}). \tag{102}$$

Comparing its component along the z axis from the first and second terms we find that for

$$x - \frac{m^2+\mathbf{p}^2}{4E^2x} < 0, \quad \mathbf{p}^2 = (\mathbf{p}_a+\mathbf{p}_b)^2, \tag{103}$$

the «reflection» phenomenon takes place. For instance, assuming $\mathbf{p}^2 \sim M^2 \gg m^2$, we have $x < (M/2E) \sim 1$. This situation can be realized near the threshold of the heavy-pair production.

The expressions for the differential cross section of emission of two hard photons were obtained in the WW approximation in [24] by using the explicit expression of the double Compton scattering cross section obtained in [25].

Using the formulae given above, the energy–energy correlations of the jets in the final state can be constructed. It consists in the construction of an average of the product of the energy fractions of the heavy quarks. As well, the azimuthal angle correlation, which is the average of $2(\mathbf{q}_{+i})(\mathbf{q}_{-j})/\sqrt{s}$, can be investigated. Energy spectra, total cross sections, and sum rules for different processes in the fragmentation region can be investigated in full analogy with the QED program for colliding $eY = (eX)Y$ [13].

The approach developed here can be used for description of jets in the fragmentation region with creation of $K, \bar{K}$ states when the heavy strange quark and antiquark are created. The jets originated from $D, \bar{D}$ and $B, \bar{B}$ can be considered as well.

In the plot (see Fig. 4), the dependence of the $Q\bar{Q}$-pair production cross section from the so-called «two-photon» mechanism is presented. It has rather a large cross section and can be measured in experiment.

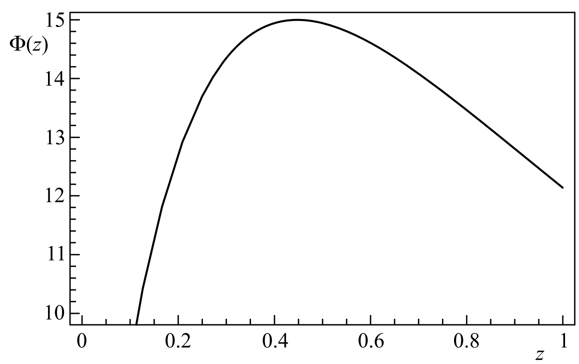

Fig. 4. The $\Phi(z)$ (defined in (54)) as a function of z

In Table 1, the charge-asymmetry effect due to interference of the «bremsstrahlung» and «two-photon» mechanisms is presented as a ratio of the corresponding contributions to the differential cross section. This quantity can also be measured in spite of its rather small value $|A_{+-}| \sim 0.02{-}0.03$.

In Table 2, the contributions from the so-called «planar» and «nonplanar» contributions to the differential cross section of the double bremsstrahlung process in electron–target collisions are presented.

Table 3 gives the ratio of contributions with the QED-type gluon splitting contribution to the process of two-gluon jet production in the quark–target collisions. Its rather large expected value A^{gg} can also be measured in experiment.

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Appendix A
HEAVY-QUARK PAIR PRODUCTION BEYOND THE WW APPROXIMATION

The explicit expression for $R_{\rm br}$ is

$$\begin{aligned}R_{\rm br} = \left(\frac{x_+x_-}{dd'}\right)^2 [dd'[\mathbf{q}^2[2xx_+x_-(p_1p_1') - xx_+(p_1'q_-) - xx_-(p_1'q_+)-\\
- 2xx_+(\mathbf{q}\mathbf{q}_-) - 2xx_-(\mathbf{q}\mathbf{q}_+) - x^2x_+(p_1q_-) - x^2x_-(p_1q_+) + 2x^2(q_+q_-)]+\\
+ 2x(1+x)(\mathbf{q}\mathbf{q}_+)(\mathbf{q}\mathbf{q}_-) + 2xx_+(\mathbf{q}\mathbf{q}_-)^2 + 2xx_-(\mathbf{q}\mathbf{q}_+)^2]]+\\
+ 2\rho^2x^2[M^2(p_1p_1') + (p_1q_+)(p_1'q_-) + (p_1q_-)(p_1'q_+)]+\\
+xd'^2\mathbf{q}^2[M^2+x_+(p_1q_-)+x_-(p_1q_+)]+x^2d^2\mathbf{q}^2[M^2x+x_+(p_1'q_-)+x_-(p_1'q_+)]+\\
+ 2x^2d\rho[-M^2\mathbf{q}^2 + (\mathbf{q}\mathbf{q}_+)(M^2 - 2(p_1'q_-)) + (\mathbf{q}\mathbf{q}_-)(M^2 - (p_1'q_+))]+\\
+2xd'\rho[(M^2+x_+(p_1q_-)+x_-(p_1q_+))(\mathbf{q}\mathbf{p})-x((p_1q_+)(\mathbf{q}\mathbf{q}_-)+(p_1q_-)(\mathbf{q}\mathbf{q}_+))]],\end{aligned} \tag{104}$$

with the notation given above (see (25), (26), (32)).

The quantity R_{2g} enters into the differential cross section in combination $R_{2g}/((q_2^2)^2)$ with $q_2^2 = -(m^2\bar{x}^2 + \mathbf{p}^2)/x$. To see rather delicate compensations in the region of small $q_2^2, \mathbf{p}^2$, we must rearrange the electron tensor as (here we use the gauge condition $q_{2\mu}\bar{u}(q_-)Q_\mu v(q_+) = 0$ and $q_2 \approx \bar{x}p_1 - p_\perp$)

$$\frac{1}{4}\,\mathrm{Sp}\,(\hat{p}_1'+m)\gamma_\mu(\hat{p}_1+m)\gamma_\nu = 2p_{1\mu}p_{1\nu} + \frac{q_2^2}{2}g_{\mu\nu} = \frac{2}{\bar{x}^2}p_{\perp\mu}p_{\perp\nu} + \frac{1}{2}q_2^2g_{\mu\nu}. \tag{105}$$

In this form, the compensation is clearly seen. So we have

$$R_{2g} = (1+\mathcal{P}_{+-})\left(\frac{x}{d_+d_-}\right)^2\left[\frac{2}{\bar{x}^2}R_{2ga} + \frac{q_2^2}{2}R_{2gb}\right], \tag{106}$$

with

$$\begin{aligned}R_{2ga} = \frac{1}{2}(x_+x_-\rho_1)^2\left[2(\mathbf{p}\mathbf{q}_-)(\mathbf{p}\mathbf{q}_+) + \frac{1}{2}q_1^2\mathbf{p}^2\right] -\\
- x_+x_-^2\rho_1d_-[2x_+(\mathbf{p}\mathbf{q}_-)(\mathbf{p}\mathbf{q}) + \mathbf{p}^2(\mathbf{q}\mathbf{r})]+\\
+ \frac{1}{2}x_-^3x_+d_-^2\mathbf{p}^2\mathbf{q}^2 - \frac{1}{4}(x_+x_-)^2d_+d_-(2(\mathbf{p}\mathbf{q})^2 - \mathbf{q}^2\mathbf{p}^2),\end{aligned} \tag{107}$$

$$R_{2gb} = -\rho_1^2q_1^2(x_+x_-)^2 + 2x_+x_-^2\rho_1d_-(\mathbf{q}\mathbf{r}) - x_-^3x_+d_-^2\mathbf{q}^2,$$

where we remind

$$\begin{gathered}\rho_1 = -2x(\mathbf{q}\mathbf{r}),\quad \mathbf{r} = x_-\mathbf{q}_+ - x_+\mathbf{q}_-,\quad \mathbf{p} = \mathbf{q} - \mathbf{Q},\quad \mathbf{Q} = \mathbf{q}_+ + \mathbf{q}_-,\\
q_1^2 = \frac{1}{x_+x_-}[M^2\bar{x}^2 + \mathbf{r}^2].\end{gathered} \tag{108}$$

At least

$$
\begin{aligned}
R_{\mathrm{odd}} = \frac{xx_+x_-}{dd'd_+d_-}\Bigg[(1-\mathcal{P}_{+-})\times \\
\times\Bigg[xx_+d_+\rho[x_-(p_1q_+)(\mathbf{q}\mathbf{q}_+) + (\mathbf{q}\mathbf{q}_-)(\bar{x}_+(p_1q_+) + (p_1'q_+)) - x_-(p_1q_+)\mathbf{q}^2] + \\
+ x_+xd_+d\Bigg[\frac{1}{2}\mathbf{q}^2[-M^2x - x_-(p_1q_+) + 2x_-(\mathbf{q}\mathbf{q}_+) - x_+x_-(p_1p_1') + \\
+ xx_+(p_1q_-) - x(\mathbf{q}\mathbf{q}_-)] - \bar{x}_+(\mathbf{q}\mathbf{q}_-)(\mathbf{q}\mathbf{q}_+) - x_-(\mathbf{q}\mathbf{q}_+)^2\Bigg] + \\
+ x_+d_+d'\Bigg[\frac{1}{2}\mathbf{q}^2[-M^2x - xx_-(p_1q_+) - x(q_+q_-) + 2x_+(\mathbf{q}\mathbf{q}_-) + \\
+ x_+(p_1q_-) - x_+x_-(p_1p_1')] - \bar{x}_-(\mathbf{q}\mathbf{q}_-)(\mathbf{q}\mathbf{q}_+) - x_+(\mathbf{q}\mathbf{q}_-)^2\Bigg]\Bigg] + \\
+ 2xx_+x_-\rho\rho_1[M^2(p_1p_1') + (p_1q_+)(p_1'q_-) + (p_1q_-)(p_1'q_+)] + \\
+ xx_+x_-d\rho_1[(\mathbf{q}\mathbf{q}_+)(M^2 - (p_1'q_-)) + (\mathbf{q}\mathbf{q}_-)(M^2 - (p_1'q_+)) - \mathbf{q}^2M^2] + \\
+ d'\rho_1x_+x_-[-(\mathbf{q}\mathbf{q}_+)(M^2 + \bar{x}_-(p_1q_-) + x_-(p_1q_+)) - (\mathbf{q}\mathbf{q}_-)\times \\
\times(M^2 + \bar{x}_+(p_1q_+) + x_- + x_+(p_1q_-)) + \mathbf{q}^2(M^2 + x_-(p_1q_+) + x_+(p_1q_-))]\Bigg],
\end{aligned}
\tag{109}
$$

with $\rho = x_+x_-[-\bar{x}\mathbf{q}^2 + 2(\mathbf{q}\mathbf{Q})]$. The operator $\mathcal{P}_{+-}$ acts as $\mathcal{P}_{+-}f(x_+, \mathbf{q}_+; x_-, \mathbf{q}_-) \to f(x_-, \mathbf{q}_-; x_+, \mathbf{q}_+)$.

Appendix B
DISTRIBUTIONS IN THE WW APPROXIMATION, HEAVY-FERMION PAIR PRODUCTION

Differential distributions in the WW approximation are

$$
\begin{aligned}
d\sigma^{ep\to(eQ\bar{Q})p} &= \frac{2\alpha^4}{\pi}(L_q - 1)R_{\mathrm{WW}}^{Q\bar{Q}}\frac{dx_+\,dx_-}{xx_+x_-}\,dq_+^2\,dq_-^2\,\frac{d\phi}{2\pi}, \\
R_{\mathrm{WW}}^{Q\bar{Q}} &= \frac{1}{(q_1^2)^2}R_{\mathrm{WW}}^{\mathrm{br}} + \frac{1}{(q_2^2)^2}R_{\mathrm{WW}}^{2g} + \frac{2}{q_1^2q_2^2}R_{\mathrm{WW}}^{\mathrm{odd}},
\end{aligned}
\tag{110}
$$

where we imply $q_{\pm}^2 = \mathbf{q}_{\pm}^2$; ϕ is the azimuthal angle between two-dimensional vectors $\mathbf{q}_+, \mathbf{q}_-$,

$$\begin{gathered} q_1^2 = \frac{1}{x_+x_-}[\bar{x}^2M^2 + r^2], \quad q_2^2 = -\frac{1}{x}[m^2\bar{x}^2 + Q^2], \\ r^2 = \mathbf{r}^2,\ \mathbf{r} = x_-\mathbf{q}_+ - x_+\mathbf{q}_-, \quad Q^2 = \mathbf{Q}^2,\ \mathbf{Q} = \mathbf{q}_+ + \mathbf{q}_-, \end{gathered} \tag{111}$$

and

$$\begin{aligned} R_{\mathrm{WW}}^{\mathrm{br}} &= (1+\mathcal{P}_{+-})\left(\frac{x_+x_-}{d^2}\right)^2 \times \\ &\times \Bigg[d^2\Bigg[x_+x_-Q^2 - x_+(p_1'q_-) - x^2x_+(p_1q_-) + x^2(q_+q_-) + \\ &+ \frac{1}{2}x(1+x)(\mathbf{q}_+\mathbf{q}_-) + x_+xq_+^2 + \frac{1}{2}x(1+x^2)M^2 + xx_+(p_1q_-) + 2x^2x_+(p_1'q_-)\Bigg] + \\ &+ x^2Q^2[M^2(p_1p_1') + 4(p_1q_+)(p_1'q_-)] + 2x^2x_+x_-d(\mathbf{Q}\mathbf{q}_+)[M^2 - 2(p_1'q_-)] - \\ &- x_+x_-xdQ^2[M^2 + 2x_+(p_1q_-) + 2(p_1q_+)] - 2x^2x_+x_-d(\mathbf{Q}\mathbf{q}_+)(p_1q_-)\Bigg], \end{aligned} \tag{112}$$

$$\begin{gathered} R_{\mathrm{WW}}^{2g} = (1+\mathcal{P}_{+-})\left(\frac{x}{d^2}\right)^2\left[\frac{2}{\bar{x}^2}p^2R_a + \frac{1}{2}q_2^2R_b\right], \\ R_a = (xx_+x_-)^2r^2\left((\mathbf{q}_+\mathbf{q}_-) + \frac{1}{2}q_1^2\right) + xx_+x_-^2d[x_+(\mathbf{r}\mathbf{q}_-) + r^2] + \frac{1}{2}d^2x_-^3x_+, \\ R_b = -2(xx_+x_-)^2r^2q_1^2 - 2xx_+x_-^2\,dr^2 - x_-^3x_+d^2, \end{gathered} \tag{113}$$

$$\begin{aligned} R_{\mathrm{WW}}^{\mathrm{odd}} &= (1-\mathcal{P}_{+-})\frac{xx_+x_-}{d^4}\Bigg[\frac{1}{2}d^2\big[-M^2(1+x)xx_+ - xx_+x_-(p_1'q_-) + x_+^2(p_1'q_+) - \\ &- x[x_+\bar{x}_+ + \bar{x}_-](\mathbf{q}_+\mathbf{q}_-) - xx_+x_-\mathbf{q}_+^2 - x_+^2\mathbf{q}_-^2 - x_+^2x_-(1+x)(p_1p_1') - \\ &- xx_+(1+x)(q_+q_-) + (xx_+)^2(p_1q_-) - xx_+x_-(p_1q_+)\big] + \\ &+ x_+x_-d\mathbf{Q}\big[xx_+x_-(p_1q_+)\mathbf{q}_+ + \mathbf{q}_-\big[xx_+\bar{x}_+(p_1q_+) + xx_+(p_1'q_+)\big]\big] - \\ &- \frac{1}{2}xd\mathbf{r}\big[-\mathbf{q}_+\big[M^2x_+x_-\bar{x} + xx_+x_-(p_1'q_-) + x_+x_-\bar{x}_-(p_1q_-) + x_+x_-^2(p_1q_+)\big] - \\ &- \mathbf{q}_-\big[M^2x_+x_-\bar{x} + xx_+x_-(p_1'q_+) + x_+x_-\bar{x}_+(p_1q_+) + x_-x_+^2(p_1q_-)\big]\big] - \\ &- 2(x_+x_-x)^2(\mathbf{Q}\mathbf{r})\big[M^2(p_1p_1') + (p_1q_-)(p_1'q_+) + (p_1q_+)(p_1'q_-)\big]\Bigg]. \end{aligned} \tag{114}$$

Here we use the notation

$$
\begin{gathered}
(p_1 p_1') = \frac{1}{2x} Q^2, \ (p_1 q_+) = \frac{1}{2x_+}[M^2 + q_+^2], \quad (p_1 q_-) = \frac{1}{2x_-}[M^2 + q_-^2], \\
(p_1' q_+) = \frac{1}{2xx_+}[M^2 + r_+^2], \quad (p_1' q_-) = \frac{1}{2xx_+}[M^2 + r_-^2], \\
(q_+ q_-) = \frac{1}{2x_+ x_-}[M^2(x_+^2 + x_-^2) + r^2], \quad r^2 = (x_- \mathbf{q}_+ - x_+ \mathbf{q}_-)^2, \\
q_1^2 = \frac{1}{x_+ x_-}[M^2 \bar{x}^2 + r^2], \ q_2^2 = -\frac{1}{x}[m^2 \bar{x}^2 + Q^2], \qquad (115) \\
d = m^2 x_+ x_- \bar{x} + M^2 x \bar{x} + q_+^2 x_- \bar{x}_- + q_-^2 x_+ \bar{x}_+ + 2x_+ x_- q_+ q_- \cos\phi, \\
r_-^2 = (\bar{x}_+ \mathbf{q}_- + x_- \mathbf{q}_+)^2, \quad r_+^2 = (\bar{x}_- \mathbf{q}_+ + x_+ \mathbf{q}_-)^2, \\
(p_1 p_1') = \frac{1}{2x} Q^2, \quad (\mathbf{q}_+ \mathbf{q}_-) = q_+ q_- \cos\phi.
\end{gathered}
$$

The differential distribution in the WW approximation is

$$
\begin{aligned}
d\sigma^{ep \to (eQ\bar{Q})p} &= \frac{2\alpha^4}{\pi}(L_q - 1) \times \\
&\times \left[\frac{1}{(q_1^2)^2} R_{\mathrm{WW}}^{\mathrm{br}} + \frac{1}{(q_2^2)^2} R_{\mathrm{WW}}^{2g} + \frac{2}{q_1^2 q_2^2} R_{\mathrm{WW}}^{\mathrm{odd}} \right] \frac{dx_+ \, dx_-}{xx_+ x_-} dq_+^2 \, dq_-^2 \frac{d\phi}{2\pi}. \qquad (116)
\end{aligned}
$$

We put as well the contribution to the differential cross section from two gamma mechanisms integrated over both virtual photon transfer momentum variables

$$
\begin{aligned}
\frac{d\sigma_{2g}^{ep \to (eQ\bar{Q})p}}{dx_+ dx_- dq_+^2} &= \frac{2\alpha^4}{\pi} \frac{L_q - 1}{xx_+ x_-}(1 + \mathcal{P}_{+-}) \frac{x^2}{d_0^4} \left[\frac{2x^2}{\bar{x}^2}(L_p - 1) R_{a0} + \frac{1}{2} x L_p R_{b0} \right], \\
R_{a0} &= (xx_+ x_-)^2 q_+^2 \bar{x}^2 [q_{10}^2 - q_+^2] + \bar{x} x_+ x_- \, d_0 q_+^2 + \frac{1}{2} d_0^2 x_+ x_-^3, \qquad (117) \\
R_{b0} &= 2(xx_+ x_- \bar{x})^2 q_+^2 q_{10}^2 + 2(\bar{x} x_-)^2 xx_+ \, d_0 q_+^2 + d_0^2 x_+ x_-^3,
\end{aligned}
$$

where

$$
\begin{gathered}
q_{10}^2 = \frac{\bar{x}^2}{x_+ x_-}[M^2 + \mathbf{q}_+^2], \quad d_0 = \bar{x}[m^2 x_+ x_- + x(M^2 + \mathbf{q}_+^2)], \\
L_p = \ln \frac{M^2}{m^2 \bar{x}^2}, \quad L_q = \ln \frac{M^2 s^2}{M_\Upsilon^2 d_0^2}. \qquad (118)
\end{gathered}
$$

The restrictions

$$x_+ + x_- = \bar{x} = 1 - x, \quad \frac{2M}{\sqrt{s}} < x_\pm < 1 - \frac{2M}{\sqrt{s}},$$
$$\frac{\bar{x}^2}{x_+ x_-} > \frac{4M^2}{M^2 + \mathbf{q}_+^2} \tag{119}$$

are implied.

The distribution as a function of the two-gluon invariant mass square was considered in paper [27].

Appendix C
DISTRIBUTIONS IN THE WW APPROXIMATION, TWO-PHOTON AND TWO-GLUON EMISSION

Differential distribution for the process $eY \to (e\gamma\gamma)Y$ in the WW approximation is

$$d\sigma_{\mathrm{WW}}^{ep\to(e\gamma\gamma)p} = \frac{2\alpha^4}{\pi}(L_q - 1)(1 + \mathcal{P}_{12})[R_{\mathrm{pl}} + R_{\mathrm{npl}}]\, dk_1^2\, dk_2^2 \frac{d\phi}{2\pi},$$
$$d\sigma_{\mathrm{WW}}^{qp\to(qgg)p} = \frac{\alpha^2\alpha_s^2}{\pi}\frac{N^2-1}{N^3}(L_q - 1)[(1 + \mathcal{P}_{12})[R_{\mathrm{pl}} + R_{\mathrm{npl}}]\frac{N^2-2}{2} + \\ + \frac{N^2}{2}[(1+\mathcal{P}_{12})(R_{\mathrm{pl}} - R_{\mathrm{npl}}) + 4(R_{33} + R_{321} - R_{312})]\, dk_1^2\, dk_2^2 \frac{d\phi}{2\pi}. \tag{120}$$

The expressions for R_i are

$$R_{\mathrm{pl}} = \frac{1}{\bar{x}_1^2} I_{\mathrm{pl}}, \quad R_{\mathrm{npl}} = \frac{1}{\bar{x}_1 \bar{x}_2} I_{\mathrm{npl}},$$
$$R_{33} = I_{33}, \quad R_{321} = -\frac{1}{\bar{x}_2} I_{321}, \quad R_{312} = -\frac{1}{\bar{x}_1} I_{312}, \tag{121}$$

and

$$I_{\mathrm{pl}} = \frac{1}{4s^2}\,\mathrm{Sp}\,\hat{p}_1' O_{12} \hat{p}_1 O_{12}^+,$$
$$I_{\mathrm{npl}} = \frac{1}{4s^2}\,\mathrm{Sp}\,\hat{p}_1' O_{12} \hat{p}_1 O_{21}^+,$$
$$I_{33} = \frac{1}{4s^2}\,\mathrm{Sp}\,\hat{p}_1' O_3 \hat{p}_1 O_3^+, \tag{122}$$
$$I_{321} = \frac{1}{4s^2}\,\mathrm{Sp}\,\hat{p}_1' O_3 \hat{p}_1 O_{21}^+,$$
$$I_{312} = \frac{1}{4s^2}\,\mathrm{Sp}\,\hat{p}_1' O_3 \hat{p}_1 O_{12}^+.$$

The simplified expressions for effective vertices are

$$
\begin{aligned}
&O_{12} = R_1\hat{e}_2(\hat{p}_1' + \hat{k}_2)\hat{p}_2(\hat{p}_1 - \hat{k}_1)\hat{e}_1 + r\hat{e}_2\hat{p}_2\hat{e}_1 + \\
&+ c_1\hat{e}_2(\hat{p}_1' + \hat{k}_2)[\bar{x}_1\hat{e}_1\hat{q}\hat{p}_2 + \hat{p}_2\hat{q}\hat{e}_1] + d_1[\bar{x}_1\hat{p}_2\hat{q}\hat{e}_2 + x\hat{e}_2\hat{q}\hat{p}_2](\hat{p}_1 - \hat{k}_1)\hat{e}_1
\end{aligned}
\tag{123}
$$

and

$$
\begin{aligned}
&O_{21} = R_2\hat{e}_1(\hat{p}_1' + \hat{k}_1)\hat{p}_2(\hat{p}_1 - \hat{k}_2)\hat{e}_2 + r\hat{e}_1\hat{p}_2\hat{e}_2 + \\
&+ c_2\hat{e}_1(\hat{p}_1' + \hat{k}_1)[\bar{x}_2\hat{e}_2\hat{q}\hat{p}_2 + \hat{p}_2\hat{q}\hat{e}_2] + d_2[\bar{x}_2\hat{p}_2\hat{q}\hat{e}_1 + x\hat{e}_1\hat{q}\hat{p}_2](\hat{p}_1 - \hat{k}_2)\hat{e}_2],
\end{aligned}
\tag{124}
$$

$$
\begin{aligned}
O_3 &= -\frac{2}{q_1^2}\frac{x_1x_2}{d^2}[xs\rho\hat{V} + d'\hat{p}_2\hat{q}\hat{V} + x\,d\hat{V}\hat{q}\hat{p}_2], \\
\hat{V} &= \hat{e}_1(k_1e_2) + \hat{k}_2(e_1e_2) - \hat{e}_2(k_2e_1),
\end{aligned}
\tag{125}
$$

with

$$
\begin{aligned}
R_1 &= \frac{2(xx_1^2x_2^2)^2}{k_1^2r_2^2d^2}\mathbf{q}[d\mathbf{r}_2 - xx_2k_1^2\mathbf{Q}], \quad R_2 = \frac{2(xx_1^2x_2^2)^2}{k_2^2r_1^2d^2}\mathbf{q}[d\mathbf{r}_1 - xx_1k_2^2\mathbf{Q}], \\
r &= \frac{2xx_1^2x_2^2}{d^2}\mathbf{q}\mathbf{Q}, \quad d = m^2\bar{x}x_1x_2 + k_1^2x_2\bar{x}_2 + k_2^2x_1\bar{x}_1 + 2x_1x_2k_{12}, \\
&\rho = 2x_1x_2\mathbf{q}\mathbf{Q},
\end{aligned}
\tag{126}
$$

where we use the notation $k_i^2 = \mathbf{k}_i^2$, $k_{12} = \mathbf{k}_1\mathbf{k}_2$,

$$
\begin{aligned}
\mathbf{r} &= x_2\mathbf{k}_1 - x_1\mathbf{k}_2, \mathbf{Q} = \mathbf{k}_1 + \mathbf{k}_2, \\
\mathbf{r}_1 &= \bar{x}_2\mathbf{k}_1 + x_1\mathbf{k}_2, \mathbf{r}_2 = \bar{x}_1\mathbf{k}_2 + x_2\mathbf{k}_1
\end{aligned}
\tag{127}
$$

and use, besides,

$$
\begin{aligned}
c_1 &= \frac{x_1(xx_2)^2}{r_2^2d}, \quad d_1 = -\frac{x_2x_1^2}{k_1^2d}, \\
c_2 &= \frac{x_2(xx_1)^2}{r_1^2d}, \quad d_2 = -\frac{x_1x_2^2}{k_2^2d}, \\
(p_1p_1') &= \frac{1}{2x}\mathbf{Q}^2, \quad (p_1k_1) = \frac{1}{x_1}k_1^2, \quad (p_1k_2) = \frac{1}{x_2}k_2^2, \\
(p_1'k_1) &= \frac{1}{xx_1}r_1^2, \quad (p_1'k_2) = \frac{1}{xx_2}r_2^2, \quad (k_1k_2) = \frac{1}{2x_1x_2}r^2.
\end{aligned}
\tag{128}
$$

Appendix D
REPRODUCE NOW THE LANDAU–LIFSHITZ (LL) RESULT

In the case of not too large invariant mass square of the subject F created by two photons, cross section is

$$\sigma_{\rm tot}(s) = \frac{\alpha^2}{\pi^2}\int\limits_{4M^2}^{\infty}\frac{ds_1}{s_1}\sigma^{\gamma\gamma\to F}(s_1)\int\limits_{s_1/s}^{1}\frac{d\beta_1}{\beta_1}N(\beta_1)N(\alpha_2), \quad \alpha_2=\frac{s_1}{s\beta_1},$$
$$N(\beta_1)=\int\limits_0^{\sigma}\frac{z_1\,dz_1}{(z_1+\beta_1^2)^2}=\ln\frac{\sigma+\beta_1^2}{\beta_1^2}-\frac{\sigma}{\sigma+\beta_1^2}, \quad \sigma=\left\langle\frac{M}{m_e}\right|^2\gg 1. \tag{129}$$

Here we choose the upper limits of transverse momentum σ to be large compared with electron mass and do not exceed the mass of the created pair. Performing the integration on β_1 we obtain

$$\sigma_{\rm tot}(s)=\frac{\alpha^2}{\pi^2}\int\limits_{4M^2}^{\infty}\frac{ds_1}{s_1}\sigma^{\gamma\gamma\to F}(s)\left[\frac{2}{3}L_\sigma^3-2L_\sigma^2+\left(1+\frac{\pi^2}{3}\right)L_\sigma-\frac{\pi^2}{3}\right],$$
$$L_\sigma=\ln\frac{s\sigma}{s_1}. \tag{130}$$

This region gives the leading contribution to the cross section. The regions when one or both transverse momenta exceed the mass of the created system produce lower orders of L_σ and as well the terms proportional to powers of $\ln\sigma$. The total contribution of all kinematical regions does not depend on the auxiliary parameter σ. It is cited above. To restore the coefficient of the cubic term, we remind the explicit form of the total cross section of production of lepton pair by two teal photons

$$\sigma^{\gamma\gamma\to F}(s)=\sigma_p(s)=\frac{\pi\alpha^2}{xm^2}\left[\left(2+\frac{2}{x^2}-\frac{1}{x^4}\right)\ln\left(x+\sqrt{x^2-1}\right)-\right.$$
$$\left.-\left(1+\frac{1}{x^2}\right)\sqrt{1-\frac{1}{x^2}}, \quad s=4x^2m^2. \tag{131}$$

Using the result

$$\int\limits_{4m^2}^{\infty}\frac{ds_1}{s_1}\sigma(s_1)=\frac{\pi\alpha^2}{m^2}\frac{14}{9}, \tag{132}$$

we arrive to LL result

$$\sigma_{\rm tot}=\frac{28}{27}\frac{\alpha^4}{\pi m^2}\ln^3\frac{s}{m^2}. \tag{133}$$

Appendix E
REPRODUCE NOW THE BRODSKY–KINOSHITA–TERAZAWA (BKT) RESULT

In the case when the energies of the scattered electrons are essentially less than the energies of the initial ones, the formulae for total cross sections must be modified. We start from the usual expression for the matrix element

$$M = \frac{4\pi\alpha}{q_1^2 q_2^2}\bar{u}(p_1')\,\gamma_\mu u(p_1)\,\bar{u}(p_2')\,\gamma_\nu u(p_2) T^{\mu\nu}. \tag{134}$$

First we will use the 4-momenta of virtual photons instead of the momenta of the scattered electrons, besides, we accept the Sudakov parameterization of 4-momenta of the problem, for the phase volume of the scattered electron moving in direction close to the momentum of electron p_1

$$\begin{gathered}\frac{d^3p_1'}{2\epsilon'} = d^4q_1\delta^4(p_1 - p_1' - q_1)\,d^4p_1'\delta((p_1 - q_1)^2 - m^2) = \frac{s}{2}d^2\mathbf{q}_1\,d\alpha_1\,d\beta_1, \\ \delta(-s\alpha_1(1-\beta_1) - m^2\beta_1 - \mathbf{q}_1^2).\end{gathered} \tag{135}$$

Applying the Sudakov parameterization

$$\begin{gathered}q_1 = \alpha_1 p_2 + \beta_1\tilde{p}_1 + q_{1\perp}, \quad q_{1\perp}p_2 = q_{1\perp}p_1 = 0, \\ \tilde{p}_1 = p_1 - p_2\frac{m^2}{s}, \quad p_1^2 = m^2, \quad 2p_1\tilde{p}_1 = m^2, \quad q_{1\perp}^2 = -\mathbf{q}_1^2 < 0, \\ q_1^2 = -\frac{\mathbf{q}_1^2 + m^2\beta_1^2}{1-\beta_1},\end{gathered} \tag{136}$$

we obtain

$$\frac{d^3p_1'}{2\epsilon'} = \frac{d\beta_1}{1-\beta_1}\frac{1}{2}d\mathbf{q}_1^2\frac{d\phi_1}{2\pi}. \tag{137}$$

The square of current, associated with electron $e(p_1)$, summed on spin states and averaged on the azimuthal angle ϕ_1, is

$$\left\langle\sum \bar{u}(p_1')\gamma_\mu u(p_1)\bar{u}(p_1')\gamma_{\mu_1}u(p_1))^*\right\rangle = 4\left\langle\left[2p_{1\mu}p_{1\mu_1} + \frac{1}{2}q_1^2 g_{\mu\nu}\right]\right\rangle. \tag{138}$$

More convenient formulae can be obtained if one uses the gauge condition $q_1^\mu T_{\mu\nu} = (\beta_1 p_1 + q_{1\perp})T_{\mu\nu} = 0$. In such a way we obtain

$$\left\langle\sum \bar{u}(p_1')\gamma_\mu u(p_1)\bar{u}(p_1')\gamma_{\mu_1}u(p_1))^*\right\rangle = -\frac{4\mathbf{q}_1^2 g_{\mu\nu}}{1-\beta_1}\left[\frac{1-\beta_1}{\beta_1^2} + \frac{1}{2}\right]. \tag{139}$$

In the similar way we obtain for the current associated with electron $e(p_2)$:

$$\left\langle \sum \bar{u}(p_2')\gamma_\nu u(p_2)(\bar{u}(p_2')\gamma_{\nu_1} u(p_2))^* \right\rangle = -\frac{4\mathbf{q}_2^2 g_{\nu\nu_1}}{1-\alpha_2}\left[\frac{1-\alpha_2}{\alpha_2^2}+\frac{1}{2}\right]. \quad (140)$$

Here we use the similar Sudakov parameterization $q_2 = \beta_2\tilde{p}_2 + \alpha_2 p_1 + q_{2\perp}$, writing the phase volume as

$$\begin{gathered} d\Gamma = d\gamma \frac{1}{(2\pi)^6}\frac{\pi\, d\mathbf{q}_1^2\, d\beta_1}{2(1-\beta_1)}\frac{\pi\, d\mathbf{q}_2^2\, d\alpha_2}{2(1-\alpha_2)}, \\ d\gamma = \frac{(2\pi)^4}{(2\pi)^6}\frac{d^3q_+}{2E_+}\frac{d^3q_-}{2E_-}\delta^4(q_1+q_2-q_+-q_-). \end{gathered} \quad (141)$$

Let us introduce, as a new variable, the invariant mass square of the created system $s_1 = (q_+ + q_-)^2 \approx s\alpha_2\beta_1$

$$\int d\alpha_2\, d\beta_1 \theta(s\alpha_2\beta_1 - 4M^2) = \frac{1}{s}\int\limits_{4M^2}^{\infty} ds_1 \int\limits_{s_1/s}^{1} \frac{d\beta_1}{\beta_1}. \quad (142)$$

Note now that the quantity

$$\int \frac{1}{8s_1} T_{\mu\nu}(T_{\mu_1\nu_1})^* g^{\mu\mu_1} g^{\nu\nu_1}\, d\gamma = \sigma^{\gamma\gamma\to F}(s_1) \quad (143)$$

coincides with the total cross section of production of the system F by two photons.

For the differential cross section we have

$$d\sigma = \frac{\alpha^2}{\pi^2}\frac{\mathbf{q}_1^2\, d\mathbf{q}_1^2 \mathbf{q}_2^2\, d\mathbf{q}_2^2}{(\mathbf{q}_1^2 + m^2\beta_1^2)^2(\mathbf{q}_2^2 + m^2\alpha_2^2)^2}\int\limits_{4M^2}^{\infty} ds_1 \sigma^{\gamma\gamma}(s_1)\frac{s_1}{s^2} I, \quad (144)$$

with

$$I = \int\limits_{s_1/s}^{1} \frac{d\beta_1}{\beta_1}\left[\frac{1-\beta_1}{\beta_1^2}+\frac{1}{2}\right]\left[\frac{1-\alpha_2}{\alpha_2^2}+\frac{1}{2}\right], \quad \alpha_2 = \frac{s_1}{s\beta_1}. \quad (145)$$

The calculation leads to

$$I = \frac{s^2}{4s_1^2} f(z), \quad f(z) = (2+z)^2 \ln\frac{1}{z} - 2(1-z)(3+z), \quad z = \frac{s_1}{s}. \quad (146)$$

Integration on the transversal momenta of virtual photons in the region $0 < \mathbf{q}_{1,2}^2 < E^2$ leads to the famous formulae of BKT

$$\sigma(s)^{ee\to eeF} = \left(\frac{\alpha}{\pi}\right)^2 \left(\ln\frac{E}{m_e}\right)^2 \int\limits_{4M^2}^{\infty} \frac{ds_1}{s_1}\sigma^{\gamma\gamma\to F}(s_1) f\left(\frac{s_1}{s}\right). \quad (147)$$

Appendix F
CREATION OF A HEAVY PARTICLE IN THE FRAGMENTATION REGION OF ONE ELECTRON

For definiteness, we consider the process $ee \to (eF)e$

$$e(p_1) + e(p_2) \to [e(p_1')Q^+(q_+)Q^-(q_-)]e(p_2'). \tag{148}$$

For azimuthal averaged squares of currents, summed on the spin states, we have

$$\begin{gathered}\left\langle \sum \bar{u}(p_1')\gamma_\mu u u(p_1)(\bar{u}(p_1')\gamma_{\mu_1}u(p_1))^* \right\rangle = -\frac{4\mathbf{q}_1^2 g_{\mu\mu_1}}{1-\beta_1}\left[\frac{1-\beta_1}{\beta_1^2} + \frac{1}{2}\right], \\ \left\langle \sum \bar{u}(p_2')\gamma_\nu u(p_2)(\bar{u}(p_2')\gamma_{\nu_1}u(p_2))^* \right\rangle = -\frac{4\mathbf{q}_2^2 g_{\nu\nu_1}}{\alpha_2^2}.\end{gathered} \tag{149}$$

In the last expression, we use the gauge condition $q_2^\nu T_{\mu\nu} = (\alpha_2 p_2 + q_{2\perp})^\nu T_{\mu\nu} = 0$. Expressing the phase volume of the scattered electrons as

$$\begin{gathered}\frac{d^3p_1'}{2E_1'} = \frac{d\beta_1}{1-\beta_1}\frac{\pi}{2}d\mathbf{q}_1^2\left(\frac{d\phi_1}{2\pi}\right), \\ \frac{d^3p_2'}{2E_2'} = d\alpha_2\frac{\pi}{2}d\mathbf{q}_2^2\left(\frac{d\phi_1}{2\pi}\right),\end{gathered} \tag{150}$$

we remind that the quantity β_1 is of an order of unity, whereas α_2 is small. The threshold condition must be fulfilled

$$s_1 = s\alpha_2\beta_1 > s_{\text{th}} = 4M^2. \tag{151}$$

For the contribution to the total cross section we obtain

$$\begin{gathered}\sigma(s)^{ee\to(eF)e} = \left(\frac{\alpha}{\pi}\right)^2\left(\ln\frac{E}{m_e}\right)^2 \int\limits_{4M^2}^{\infty}\frac{ds_1}{s_1}\sigma^{\gamma\gamma\to F}(s_1)\phi\left(\frac{s_1}{s}\right), \\ \phi(z) = 4\ln\frac{1}{z} - (1-z)(3-z).\end{gathered} \tag{152}$$

Appendix G
TRANSFER OF CIRCULAR POLARIZATION OF THE INITIAL ELECTRON TO THE POSITRON IN THE FRAGMENTATION REGION

This phenomenon is similar to «handedness» when the initial polarized particle causes polarization of the fermionic fragments of the jet created by this projectile, or reveals itself in kinematical correlations of momenta of different pions from the jet.

The matrix element of the process $e(p_1, \lambda)\bar{e}(p_2) \to [e(p_1')e(q_-)\bar{e}(q_+, \lambda_1)] \times \bar{e}(p_2')$ has the form

$$\frac{4\pi\alpha^2}{q_1^2 q^2}\frac{1}{2}\bar{u}(p_1')\gamma_\mu(1+\lambda\gamma_5)u(p_1)\frac{1}{2}\bar{u}(q_-)O_{\mu\nu}(1-\gamma_5)v(q_+)\bar{u}(p_2')\gamma_\nu u(p_2). \quad (153)$$

For the summed on the spin states square of the matrix element, we have

$$\sum |M|^2 = (4\pi\alpha)^4 8\left\{\frac{1}{4}\,\mathrm{Sp}\,(\hat{q}_- + m)O_\mu(\hat{q}_+ + m)O_\nu^* \frac{\mathbf{q}_1^2 g_{\mu\nu}}{1-\beta_1}\left(\frac{1-\beta_1}{\beta_1^2} + \frac{1}{2}\right) + \right.$$
$$\left. + \lambda\frac{1}{4}\,\mathrm{Sp}\,\hat{q}_- Q_\mu \hat{q}_+ Q_\nu^+ \gamma_5 \frac{1}{4}\,\mathrm{Sp}\,\hat{p}_1\hat{q}_1\gamma_\mu\gamma_\nu\gamma_5\right\}\frac{q_1^2 q^2 (1-\beta_1)^2}{(\mathbf{q}_1^2 + \beta_1^2 m^2)^2(\mathbf{q}^2 + m^2\alpha_2^2)^2}, \quad (154)$$

with

$$Q_\mu = \frac{1}{s_1^2 x_+ x_-}[2\mathbf{qr}\gamma_\mu - s_1 x_- \gamma_\mu \hat{q}\hat{p}_2 + s_1 x_+ \hat{p}_2 \hat{q}\gamma_\mu],$$
$$s_1 = \frac{\bar{x}}{x_+ x_-}[\mathbf{q}_-^2 + am^2], \quad (155)$$

with $a = (\bar{x}_+\bar{x}_-)/x$, $\bar{x} = 1 - x = x_+ + x_-$.

Averaging over the azimuthal angle $d^2\mathbf{q}$, permits one to extract the general factor $s^2\mathbf{q}^2$, which will be absorbed in the total expression for spectral distributions on the energy fractions of fermions in a jet. Using the expression for phase volume in the fragmentation region

$$d\Gamma_4 = \frac{d^3p_1'\, d^3p_2'\, d^3q_+\, d^3q_-}{2\varepsilon_1' 2\varepsilon_2' 2\varepsilon_+ 2\varepsilon_-}\frac{(2\pi)^4}{(2\pi)^2}\delta^4(p_1 + p_2 - p_1' - p_2' - q_+ - q_-) =$$
$$= \pi^3(2\pi)^{-8}\frac{dx_-\, d\beta_1}{8sxx_+x_-}\frac{d^2q}{\pi}\frac{d^2q_1}{\pi}\frac{d^2q_-}{\pi}, \quad (156)$$

we first extract the leading logarithmic factor $L_1 L_q$ with

$$L_1 = \int_0^s \frac{d\mathbf{q}_1^2 \mathbf{q}_1^2}{(\mathbf{q}_1^2 + m_e^2\beta_1^2)^2}, \quad L_q = \int_0^s \frac{d\mathbf{q}^2\mathbf{q}^2}{(\mathbf{q}^2 + m_e^2\alpha_2^2)^2}.$$

With the logarithmic accuracy, we have

$$L_1 = L_q = L = \ln\frac{s}{m_e^2}. \quad (157)$$

For the unpolarized part of the cross section we obtain

$$d\sigma_{\rm unp} = \frac{\alpha^4}{2\pi}L^2\frac{d\mathbf{q}_-^2}{s_1^4(x_+x_-)^2}\frac{dx_-dx(1+x^2)}{2\bar{x}^2}\biggl[x_+x_-(x_+^2+x_-^2)s_1^2- \\ -2\bar{x}^3s_1^2\mathbf{q}_-^2+2\frac{\bar{x}^2}{x_+x_-}\mathbf{q}_-^2[\bar{x}^2\mathbf{q}^2+m^2(\bar{x}^2+2x_+x_-)]\biggr]. \quad (158)$$

Integration over $d\mathbf{q}_-^2$ leads to

$$d\sigma_{\rm unp} = \frac{\alpha^4}{2\pi m_e^2}L^2\frac{xdx_-\,dx(1+x^2)}{\bar{x}^4\bar{x}_+\bar{x}_-}\times \\ \times\left[-2x_+x_-+\frac{2}{3}\bar{x}^2+\frac{x}{3\bar{x}_+\bar{x}_-}[\bar{x}^2+2x_+x_-]\right]. \quad (159)$$

Here we imply the threshold restriction $(4m_e^2/s < x_+ + x_-)$.

Consider now the contribution to the cross section associated with the polarized part of the matrix element. Performing the extraction of factor $\mathbf{q}_1^2$ and the relevant azimuthal averaging procedure, we must do a shift transformation $\mathbf{q}_- = \tilde{\mathbf{q}}_- + (x_-/\bar{x})\mathbf{q}_1$ and $\mathbf{q}_+ = -\tilde{\mathbf{q}}_- + (x_-/\bar{x})\mathbf{q}_1$.

In terms of the shifted variables, the quadratic form s_1 is $s_1 = (\bar{x}/(x_+x_-)\times [\tilde{\mathbf{q}}_-^2 + am^2]$. In a similar way, we obtain

$$d\sigma_{\rm pol} = \frac{\alpha^4}{2\pi}L^2\frac{d\mathbf{q}_-^2}{s_1^4(x_+x_-)^3}x\,dx_-\,dx\lambda(x_+-x_-)\times \\ \times\left[-x_+x_-s_1^2+\frac{1}{\bar{x}}(2\bar{x}^2-x_+x_-)s_1\mathbf{q}_-^2-\frac{2(x_+-x_-)^2}{x_+x_-}(\mathbf{q}_-^2)^2\right]. \quad (160)$$

We note that compared with the unpolarized case, the terms proportional to the electron mass squared do not contribute. Further integration leads to the final result

$$d\sigma_{\rm pol} = \lambda\frac{\alpha^4}{12\pi m_e^2}L^2\frac{x^2dx_-dx}{\bar{x}^4\bar{x}_+\bar{x}_-x_+x_-}(x_+-x_-)[4\bar{x}^2-5x_+x_-]. \quad (161)$$

The degree of polarization transferred from the initial electron to the final positron can be found as (see Table 5)

$$\frac{\langle\lambda\rangle_{\rm positron}}{\lambda} = \frac{d\sigma_{\rm pol}}{d\sigma_{\rm unp}} = F(x_-,x), \\ F(x_-,x) = (x_+-x_-)\frac{4\bar{x}^2-5x_+x_-}{(x^2+1)x_+x_-\left[-6x_+x_-+2\bar{x}^2+\frac{x}{\bar{x}_+\bar{x}_-}(\bar{x}^2+2x_+x_-)\right]}. \quad (162)$$

Table 5. **The function $F(x_-, x)$ (defined in (162)) is presented for different values of the final lepton transverse momenta x_+ and x_- for the lepton-pair production (in units of M)**

x_-	x_+					
	0.25	0.3	0.35	0.4	0.45	0.5
0.25		0.853				
0.3	–0.853		0.705			
0.35		–0.705		0.6315		
0.4			–0.6315		0.6308	
0.45				–0.6303		0.786
0.5					–0.786	

Appendix H
PROCESSES OF TYPE $2 \to 2$ IN QCD. HIGH-ENERGY LIMIT

Processes with interaction of quark, antiquarks, and gluon of type $2 \to 2$ are investigated in the approximation of high energies and finite scattering angles. All particles assumed to be massless. The summed on spin and color states of matrix elements square and the relevant cross sections are presented. Chiral amplitudes method and projections on the definite color states are used.

Below we will consider, in some detail (with the pedagogical aim), calculation of matrix elements and the differential cross sections of the simplest processes of type $2 \to 2$ in the frames of Quantum ChromoDynamics (QCD) with gluons and quarks taking part. We imply the acquaintance of the reader with basic knowledge of Quantum Field Theory [6].

H.1. Process $2 \to 2$ in QCD. We will consider the processes

$$a(p_1, l_a) + b(p_2, l_b) \to c(p_3, l_c) + d(p_4, l_d) \tag{163}$$

with p_i-4-momenta of particles, $p_1 + p_2 = p_3 + p_4$ and l_j incorporate the information on the color and spin states of particles. We imply the center of mass of the initial particles reference frame $\mathbf{p_1} + \mathbf{p_2} = 0$ with the kinematic invariants

$$\begin{gathered} s = 2p_1p_2 = 2p_3p_4, \quad t = -2p_1p_3 = -2p_2p_4, \quad u = -2p_1p_4 = -2p_2p_3, \\ s + t + u = 0, \quad s = 4E^2, \quad t = -s(1-c)/2, \quad u = -s(1+c)/2, \\ p_i^2 = 0, \quad i = 1, 2, 3, 4, \end{gathered} \tag{164}$$

with E being the energy of one of initial particles, $c = \cos\theta$, $\theta = (\mathbf{p}_1, \mathbf{p}_3)$ being the angle between the direction of 3-momenta of initial and the scattered particles.

We will calculate below the summed on spin and color states of matrix elements squares of the typical processes $2 \to 2$

$$\sum |M|^2 = g^4 \sum_{abcd} \sum_{l_1 l_2 l_3 l_4} |M_{abcd}^{l_1 l_2 l_3 l_4}|^2, \tag{165}$$

with $g^2 = 4\pi\alpha_s$-strong coupling constant.

H.2. Process $qq \to qq$**.** Consider first the process of scattering of quarks of the same flavor

$$q(p_1, \lambda_1, i_1) + q(p_2, \lambda_2, i_2) \to q(p_3, \lambda_3, i_3) + q(p_4, \lambda_4, i_4), \tag{166}$$

with λ_j and $\imath_j$ being the chirality $\lambda_i = \pm 1$ and color of quarks $i = 1, 2, 3$ — yellow, green and red states of quarks. Matrix element has the form

$$\begin{aligned} M_{i_1 i_2 i_3 i_4}^{\lambda_1, \lambda_2, \lambda_3, \lambda_4} &= (t^a)_{31} (t^a)_{42} M_1 - (t^b)_{41} (t^b)_{32} M_2, \\ M_1 &= \frac{1}{t} \bar{u}^{\lambda_3}(p_3) \gamma_\mu u^{\lambda_1}(p_1) \bar{u}^{\lambda_4}(p_4) \gamma_\mu u^{\lambda_2}(p_2), \\ M_2 &= \frac{1}{u} \bar{u}^{\lambda_4}(p_4) \gamma_\mu u^{\lambda_1}(p_1) \bar{u}^{\lambda_3}(p_3) \gamma_\mu u^{\lambda_2}(p_2), \end{aligned} \tag{167}$$

and $(t^a)_{31} = \bar{e}_3 t^a e_1, (t^a)_{42} = \bar{e}_4 t^a e_2$, t^a is the generator of the color group $SU(N)$; and e_2, the color spinor. Chiral states are defined as [19]

$$\begin{gathered} u^\pm = \omega_\pm u, \quad \bar{u}^\pm = \bar{u}\omega_\mp, \quad v^\pm = \omega_\mp v, \quad \bar{v}^\pm = \bar{v}\omega_\pm, \\ \omega_\pm = \frac{1}{2}(1 \pm \gamma_5), \quad \omega_\pm \omega_\mp = 0, \quad \omega_\pm + \omega_\mp = 1, \quad \omega_\pm \omega_\pm = \omega_\pm. \end{gathered} \tag{168}$$

The completeness relations take place

$$u(p)^\pm \bar{u}(p)^\pm = \omega_\pm \hat{p}, \quad v(p)^\pm \bar{v}(p)^\pm = \omega_\mp \hat{p}. \tag{169}$$

Using this relation, we calculate the definite chiral amplitudes in such a way (we use here the short-hand notations $u_3 = u(p_3)^{\lambda_3}$ and the similar ones):

$$\begin{gathered} M_1^{++++} = \frac{1}{t R_{14} R_{23}} \bar{u}_3 \gamma_\mu \omega_+ u_1 R_{14} \bar{u}_4 \gamma_\mu \omega_+ u_2 R_{23}, \\ R_{14} = \bar{u}_1 \hat{p}_2 \omega_+ u_4, \quad R_{23} = \bar{u}_2 \hat{p}_1 \omega_+ u_3. \end{gathered} \tag{170}$$

Using the completeness relations and Dirac equations $\hat{p}_i u(p_i) = 0$, we obtain

$$M_1^{++++} = \frac{1}{t R_{14} R_{23}} \operatorname{Sp} \hat{p}_3 \gamma_\mu \hat{p}_1 \hat{p}_2 \hat{p}_4 \gamma_\mu \hat{p}_2 \hat{p}_1 \omega_+ = \frac{2s^2 t}{R_{14} R_{23}}. \tag{171}$$

In a similar way, we obtain

$$\begin{gathered}
M_2^{++++} = \frac{2s^2 u}{uR_{13}R_{24}}, \\
R_{13} = \bar{u}_1 \hat{p}_2 \omega_+ u_3, \quad R_{24} = \bar{u}_2 \hat{p}_1 \omega_+ u_4.
\end{gathered} \tag{172}$$

For two remaining nonzero amplitudes with $\lambda_1 = +$, M_1^{+-+-}, M_2^{+--+}, we have

$$\begin{gathered}
M_1^{+-+-} = \frac{1}{tP_{14}P_{23}} \bar{u}_3 \gamma_\mu \omega_+ u_1 P_{14} \bar{u}_4 \gamma_\mu \omega_+ u_2 P_{23} = \frac{2u^2}{P_{14}P_{23}}, \\
P_{14} = \bar{u}_1 \omega_- u_4, \quad P_{23} = \bar{u}_2 \omega_- u_3, \\
M_2^{+--+} = \frac{1}{uP_{13}P_{24}} \bar{u}_4 \gamma_\mu \omega_+ u_1 P_{13} \bar{u}_4 \gamma_\mu \omega_+ u_2 P_{24} = \frac{2t^2}{P_{13}P_{24}}, \\
P_{13} = \bar{u}_1 \omega_- u_3, \quad P_{24} = \bar{u}_2 \omega_+ u_4.
\end{gathered} \tag{173}$$

In such a way we have

$$\begin{aligned}
M^{++++} &= \frac{1}{t}(t^a)_{31}(t^a)_{42} \frac{2s^2 t}{R_{14}R_{23}} - \frac{1}{u}(t^b)_{41}(t^b)_{32} \frac{2u^2 t}{R_{13}R_{24}}, \\
M^{+-+-} &= \frac{1}{t}(t^a)_{31}(t^a)_{42} \frac{2u^2}{P_{14}P_{23}}, \\
M^{+--+} &= -\frac{1}{u}(t^b)_{41}(t^b)_{32} \frac{2t^2}{P_{13}P_{24}}.
\end{aligned} \tag{174}$$

We use the relation

$$\begin{gathered}
|R_{14}|^2 = \mathrm{Sp}\, \hat{p}_1 \hat{p}_2 \hat{p}_4 \hat{p}_2 \omega_+ = -st, \quad |R_{23}|^2 = -st, \quad |R_{13}|^2 = |R_{24}|^2 = -su, \\
|P_{14}|^2 = |P_{23}|^2 = -u, \quad |P_{13}|^2 = |P_{24}|^2 = -t, \\
R_{14}R_{23}[R_{13}R_{24}]^* = -s^2 tu.
\end{gathered} \tag{175}$$

For the color structures, we use

$$\begin{gathered}
(t^a)_{31}(t^a)_{42}[(t^b)_{31}(t^b)_{42}]^* = (\mathrm{Tr}\, t^a t^b)^2 = \frac{1}{4}\delta_{aa} = \frac{1}{4}(N^2 - 1), \\
(t^a)_{31}(t^a)_{42}[(t^b)_{41}(t^b)_{32}]^* = \mathrm{Tr}\, t^a t^b t^a t^b = -\frac{1}{2}C_F, \quad C_F = \frac{1}{2N}(N^2 - 1).
\end{gathered} \tag{176}$$

As a result, we have

$$|M^{++++}|^2 = (N^2 - 1)\left[\frac{s^2}{t^2} + \frac{s^2}{u^2} - \frac{2}{N}\frac{s^2}{tu}\right], \tag{177}$$

$$|M^{+-+-}|^2 = (N^2 - 1)\frac{u^2}{t^2}, \quad |M^{+--+}|^2 = (N^1 - 1)\frac{t^2}{u^2}. \tag{178}$$

And for the total sum ($\sum |M^{\lambda_1\lambda_2\lambda_3\lambda_4}|^2 = 2\sum |M^{+\lambda_2\lambda_3\lambda_4}|^2$), we have

$$\sum |M|^2_{qq\to qq} = 2(N^2-1)\left[\frac{s^2}{t^2} + \frac{s^2}{u^2} - \frac{2s^2}{Ntu} + \frac{u^2}{t^2} + \frac{t^2}{u^2}\right]. \tag{179}$$

For differential cross section, we have ($N = 3$)

$$\frac{d\sigma^{qq\to qq}}{dO_3} = \frac{\alpha_s^2}{9s}\left[\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3}\frac{s^2}{tu}\right]. \tag{180}$$

H.3. Scattering of Quarks of Different Flavors. Only one Feynman diagram (scattering channel type) contributes

$$M = (t^a)_{31}(t^a)_{42}\frac{1}{t}\bar{u}_3\gamma_\mu u_1\bar{u}_4\gamma_\mu u_2. \tag{181}$$

Two relevant chiral amplitudes are

$$\begin{aligned} M^{++++} &= (t^a)_{31}(t^a)_{42}\frac{1}{t}\frac{2s^2t}{R_{14}R_{23}}, \\ M^{+-+-} &= (t^a)_{31}(t^a)_{42}\frac{1}{t}\frac{2u^2}{P_{14}P_{23}}. \end{aligned} \tag{182}$$

We have for the summed matrix element squared

$$\sum |M|^2 = \frac{2(N^2-1)(s^2+u^2)}{t^2}, \tag{183}$$

and for the cross section

$$\frac{d\sigma}{dO_3} = \frac{\alpha_s^2(N^2-1)}{8N^2 s}\frac{s^2+u^2}{t^2}. \tag{184}$$

H.4. Process $q\bar{q} \to q\bar{q}$. Matrix elements have the form

$$M = (t^a)_{31}(t^a)_{24}M_1 - (t^b)_{21}(t^b)_{34}M_2, \tag{185}$$

with

$$\begin{aligned} M_1 &= \frac{1}{t}\bar{u}_3\gamma_\mu u_1\bar{v}_2\gamma_\mu v_4, \\ M_2 &= \frac{1}{s}\bar{v}_2\gamma_\lambda u_1\bar{u}_3\gamma_\lambda v_4. \end{aligned} \tag{186}$$

For chiral amplitudes, we have

$$\begin{aligned}
M_1^{++++} &= \frac{1}{tN_{12}N_{43}}\,\mathrm{Sp}\,\hat{p}_3\gamma_\mu\hat{p}_1\hat{p}_2\gamma_\mu\hat{p}_4\omega_+ = \frac{2s^2}{tN_{12}N_{43}}, \\
& N_{12} = \bar{u}_1\omega_- v_2, \quad N_{43} = \bar{v}_4\omega_+ u_3, \\
M_2^{+--+} &= \frac{2t^2}{sN_{13}N_{42}}, \quad N_{13} = \bar{u}_1\omega_- u_3, \quad N_{42} = \bar{v}_4\omega_+ v_2, \\
M_1^{+-+-} &= \frac{1}{tK_{12}K_{43}}\,\mathrm{Sp}\,\hat{p}_3\gamma_\mu\hat{p}_1\hat{p}_3\hat{p}_2\gamma_\mu\hat{p}_4\hat{p}_2\omega_+ = \frac{-2t^2u}{tK_{12}K_{43}}, \\
M_2^{+-+-} &= \frac{-2u^2s}{sK_{13}K_{42}}, \quad K_{12} = \bar{u}_1\hat{p}_3\omega_- v_2, \quad K_{43} = \bar{v}_4\hat{p}_2\omega_+ u_3, \\
& K_{13} = \bar{u}_4\hat{p}_2\omega_+ u_3, \quad K_{42} = \bar{v}_4\hat{p}_1\omega_+ v_3.
\end{aligned} \tag{187}$$

Keeping in mind the relations

$$\begin{aligned}
&|N_{12}|^2 = |N_{43}|^2 = s, \quad |N_{13}|^2 = |N_{42}|^2 = -t, \quad |K_{12}|^2 = |K_{43}|^2 = tu, \\
&|K_{13}|^2 = |K_{42}|^2 = -us, \quad K_{12}K_{43}[K_{13}K_{42}]^* = -u^2ts,
\end{aligned} \tag{188}$$

and taking into account the color factors, we obtain

$$\begin{aligned}
&|M^{+-+-}|^2 = (N^2-1)\left[\frac{u^2}{t^2} + \frac{u^2}{s^2} - \frac{2u^2}{Nts}\right], \\
&|M^{++++}|^2 = (N^2-1)\frac{s^2}{t^2}, \quad |M^{+--+}|^2 = (N^2-1)\frac{t^2}{s^2}.
\end{aligned} \tag{189}$$

The differential cross section is (the averaging on the color states of initial quarks factor $1/N^2$ is included)

$$\frac{d\sigma}{dO_3} = \frac{(N^2-1)\alpha_s^2}{8N^2}\left[\frac{t^2+u^2}{s^2} + \frac{s^2+u^2}{t^2} - \frac{2u^2}{Nts}\right]. \tag{190}$$

H.5. Subprocess $q\bar{q} \to q'\bar{q}'$. Only annihilation channel Feynman diagram is relevant. We have for the summed matrix element square

$$\sum |M|^2 = 2(N^2-1)\frac{u^2+t^2}{s^2} \tag{191}$$

and for the cross section (averaging color factor $1/N^2$ is included), $N = 3$

$$\frac{d\sigma}{dO_3} = \frac{\alpha_s^2}{9s}\frac{u^2+t^2}{s^2}. \tag{192}$$

H.6. Process $q\bar{q} \to gg$**.** Matrix element of the process

$$q(q_1) + \bar{q}(q_2) \to g(q_3, e^a, a) + g(q_4, e^b, b) \tag{193}$$

is described by three Feynman diagrams. It has the form $M = \bar{v}_2 O u_1$ with

$$O = t^b t^a \frac{1}{t} \hat{e}^b (\hat{q}_1 - \hat{q}_3) \hat{e}^a + t^a t^b \frac{1}{u} \hat{e}^a (\hat{q}_1 - \hat{q}_4) \hat{e}^b - \\ - \frac{2}{s} (t^a t^b - t^b t^a) [-\hat{e}^a (q_3 e^b) + \hat{e}^b (q_4 e^a) + \hat{q}_3 (e^a e^b)]. \tag{194}$$

One can be convinced that the gauge condition is fulfilled. Namely, with replacement $e^a \to q_3$, the expression for matrix element turns to zero. We obtain

$$\sum |M|^2 = \operatorname{Sp} \hat{p}_2 O \hat{p}_1 O^* + = 8NC_F \left[N^2 \frac{t^2 + u^2}{s^2} + 2C_F \frac{t^2 + u^2}{tu} \right]. \tag{195}$$

For the differential cross section, we obtain (averaging color factor $1/N^2$ is included, but identity factor $1/2!$ for the final 2-gluon state is not included)

$$\frac{d\sigma}{dO_3} = \frac{8\alpha_s^2}{27s} \frac{t^2 + u^2}{tu} \left[1 - \frac{9}{4} \frac{tu}{s^2} \right]. \tag{196}$$

H.7. Process $qg \to qg$**.** Matrix element of process $g(p_1) + q(p_2) \to g(p_3) + q(p_4)$ has the form $M = \bar{u}_4 O u_2$ with

$$O = \frac{1}{s} t^3 t^1 \hat{e}_3 (\hat{p_3} + \hat{p}_4) \hat{e}_1 + \frac{1}{u} t^1 t^3 \hat{e}_1 (\hat{p_2} - \hat{p}_3) \hat{e}_3 - \\ - \frac{2}{t} (t^1 t^3 - t^3 t^1) [-\hat{p}_3 (e_1 e_3) + \hat{e}_1 (e_3 p_1) + \hat{e}_3 (e_1 p_3)]. \tag{197}$$

For the summed on the spin and color states of matrix element square, we obtain

$$\sum |M|^2 = -8NC_F^2 \frac{s^2 + u^2}{su} \left[1 - \frac{2N^2 su}{(N^2 - 1) t^2} \right]. \tag{198}$$

For the differential cross section (color averaging factor is $1/(N(N^2 - 1))$, we obtain ($N = 3$)

$$\frac{d\sigma}{dO_3} = -\frac{\alpha_s^2}{27s} \frac{s^2 + u^2}{tu} \left[1 - \frac{9}{4} \frac{su}{t^2} \right]. \tag{199}$$

H.8. QCD Process $gg \to gg$**.** Matrix element of process

$$g(p_1, e_1^a) + g(p_2, e_2^b) \to g(p_3, e_3^c) + g(p_4, e_4^d) \tag{200}$$

has the form

$$M = [I(abcd)T + J(abcd)K + L(abcd)Q]_{\mu\nu\lambda\rho} e^a_{1\mu} e^b_{2\nu} e^c_{3\lambda} e^d_{4\rho}, \tag{201}$$

with color structures

$$I(abcd) = f^{abe} f^{cde}, \quad J(abcd) = f^{ace} f^{dbe}, \quad L(abcd) = f^{ade} f^{bce}, \tag{202}$$

Lorentz structures

$$\begin{aligned} T_{\mu\nu\lambda\rho} &= g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda} - \frac{4}{s} V^{11}_{\mu\nu\eta} V^{12}_{\rho\lambda\eta}, \\ K_{\mu\nu\lambda\rho} &= g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\rho\lambda} + \frac{4}{t} V^{21}_{\lambda\mu\eta} V^{22}_{\nu\rho\eta}, \\ Q_{\mu\nu\lambda\rho} &= g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho} - \frac{4}{u} V^{31}_{\rho\mu\eta} V^{32}_{\nu\lambda\eta}, \end{aligned} \tag{203}$$

and

$$\begin{aligned} V^{11}_{\mu\nu\eta} &= p_{1\eta} g_{\mu\nu} + p_{2\mu} g_{\nu\eta} - p_{1\nu} g_{\eta\mu}, \\ V^{12}_{\rho\lambda\eta} &= \frac{1}{2}(p_3 - p_4)_\eta g_{\rho\lambda} - p_{3\rho} g_{\lambda\eta} + p_{4\lambda} g_{\eta\rho}, \\ V^{21}_{\lambda\mu\eta} &= -p_{1\eta} g_{\lambda\mu} + p_{1\lambda} g_{\mu\eta} + p_{3\mu} g_{\eta\lambda}, \\ V^{22}_{\nu\rho\eta} &= \frac{1}{2}(p_2 + p_4)_\eta g_{\nu\rho} - p_{2\rho} g_{\eta\nu} - p_{4\nu} g_{\rho\eta}, \\ V^{31}_{\rho\mu\eta} &= -p_{1\eta} g_{\rho\mu} + p_{1\rho} g_{\mu\eta} + p_{4\mu} g_{\eta\rho}, \\ V^{32}_{\nu\lambda\eta} &= \frac{1}{2}(p_2 + p_3)_\eta g_{\nu\lambda} - p_{2\lambda} g_{\eta\nu} - p_{3\nu} g_{\lambda\eta}. \end{aligned} \tag{204}$$

One can be convinced in fulfillment of gauge condition: when replacing $e_1(p_1)$ by p_1, matrix element turns to zero. We must use Lorentz conditions $(e_i p_i) = 0, i = 2, 3, 4$ and Jacobi identity $I + J + L = 0$. Matrix element obeys the Bose symmetry: it is invariant over the simultaneous replacement of four momenta, Lorentz indices and color indices of any two gluons. Using the completeness equations $\sum\limits_\lambda e^a_{\mu\lambda} e^{b*}_{\nu\lambda} = -g_{\mu\nu}\delta^{ab}$ and

$$\begin{gathered} \sum_{abcd} I^2(abcd) = \sum_{abcd} J^2(abcd) = \sum_{abcd} L^2(abcd) = N^2(N^2-1), \\ \sum_{abcd} I(abcd) J_{abcd} J = -\frac{1}{2} N^2(N^2-1), \end{gathered} \tag{205}$$

after some algebra, one obtains

$$\sum |M|^2 = 16N^2(N^2-1)\left[3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right]. \tag{206}$$

For differential cross section, we obtain (color averaging factor is $1/(N^2-1)^2$)

$$\frac{d\sigma}{dO} = \frac{9\alpha_s^2}{32s}\frac{1}{(stu)^2}[s^4+t^4+u^4][s^2+t^2+u^2], \tag{207}$$

where we use the alternative form of the factor in the square brackets of the previous formula [19]. In conclusion we note that the expressions for all the processes considered above are in agreement with ones given in the Particle Data Group compilation [29].

H.9. Using of Color Basis. Projectors. 1) For scattering of quarks of different flavors, matrix element can be written as

$$M_t = \bar{u}_3\gamma_\mu u_1 \bar{u}_4\gamma_\mu u_2, \tag{208}$$

with $(t^a)_{j_2j_1} = \chi_3^{j_2+}(t^a)_{j_2j_1}\chi_1^{j_1}$, χ^j describes color state of quark. The system of two quarks has two basis color projectors $3\times 3 = \bar{3}+6$ antisymmetric and symmetric ones

$$\begin{aligned} P^{\bar{3}}_{i_2i_1;j_2j_1} &= N_3[\delta_{i_1i_2}\delta_{j_1j_2} - \delta_{i_1j_2}\delta_{i_2j_1}], \\ P^{6}_{i_2i_1;j_2j_1} &= N_6[\delta_{i_1i_2}\delta_{j_1j_2} - \delta_{i_1j_2}\delta_{i_2j_1}], \\ N_3 = \frac{1}{\sqrt{2N(N-1)}}&, \quad N_6 = \frac{1}{\sqrt{2N(N+1)}}. \end{aligned} \tag{209}$$

Schematically their properties can be written as

$$P^j_{a,a'}P^k_{a',b} = P^j_{a,b}\delta_{jk}. \tag{210}$$

Performing the convolutions, we obtain

$$\begin{gathered} (R_t\tilde{P}^{\bar{3}}) = -N_3NC_F; (R_t\tilde{P}^6) = N_6NC_F, \\ \tilde{P}^j_{i_1i_2,j_1j_2} = P^j_{j_1j_2,i_1i_2}, \end{gathered} \tag{211}$$

and

$$R_t = NC_F[-N_3P_{\bar{3}} + N_6P_6], \quad \sum_{\text{color}}|R_t|^2 = \frac{N^2-1}{4}, \tag{212}$$

using as well

$$\sum_{\text{spins}}\left|\frac{1}{t}M_t\right|^2 = 8\frac{s^2+u^2}{t^2}. \tag{213}$$

For the differential cross section, we obtain

$$\frac{d\sigma^{q_1q_2\to q_1q_2}}{dO_3} = \frac{\alpha_s^2(N^2-1)}{8N^2s}\frac{s^2+u^2}{t^2}. \tag{214}$$

2) For the matrix element of scattering of different quark and antiquark, we have the form

$$M^{q_1\bar{q}_2\to q_1\bar{q}_2} = \frac{1}{t}M_t Z_t, \quad Z_t = (t^a)_{j_1 j_2}(t^a)_{i_2 i_1}, \tag{215}$$

Using the projection operators $P^{(0)}, P^{(8)}$

$$P^{(0)} = \frac{1}{N}\delta_{i_1 j_1}\delta_{i_2 j_2}, \quad P^{(8)} = \frac{2}{\sqrt{N^2-1}}(t^c)_{j_1 i_1}(t^c)_{i_2 j_2} \tag{216}$$

and the relevant convolutions

$$(Z_t\tilde{P}^{(0)}) = C_F, \quad (Z_t\tilde{P}^{(8)}) = -\frac{c_F}{\sqrt{N^2-1}},$$

we obtain

$$\begin{gathered} Z = c_F P^{(0)} - P^{(8)}\frac{c_F}{\sqrt{N^2-1}}, \\ \sum_{\text{color}} |Z|^2 = \frac{1}{4}(N^2-1). \end{gathered} \tag{217}$$

The differential cross section is

$$\frac{d\sigma^{q_1 q_2\to q_1 q_2}}{dO_3} = \frac{\alpha_s^2(N^2-1)}{8N^2 s}\frac{s^2+u^2}{t^2}. \tag{218}$$

3) For scattering of quarks of the same flavor, matrix element can be written as

$$M^{q_1 q_2\to q_1 q_2} = \frac{1}{t}M_t R_t - \frac{1}{u}M_u R_u, R_u = (t^b)_{j_2 i_1}(t^b)_{i_2 j_1}, \tag{219}$$

using the previous results, and

$$(R_u\tilde{P}^{\bar{3}}) = N_3 N C_F, \quad (R_u\tilde{P}^6) = N_6 N C_F. \tag{220}$$

Matrix element has the form

$$M = NC_F\left[-\left(\frac{M_t}{t}+\frac{M_u}{u}\right)N_3 P^{\bar{3}} + \left(\frac{M_t}{t}-\frac{M_u}{u}\right)N_6 P^6\right]. \tag{221}$$

Using $\sum\limits_{\text{spins}} M_t M_u^* = -8s^2$, we obtain

$$\sum |M|^2 = 16NC_F\left[\frac{s^2+u^2}{t^2}\frac{s^2+t^2}{u^2} - \frac{2s^2}{Ntu}\right] \tag{222}$$

and the cross section cited above. The similar calculation can be done for the case of scattering of quark and antiquark of the same flavor.

4) Consider at least the process of annihilation of quark and antiquark to two gluons. Matrix element (three Feynman amplitudes contribute) has the form

$$M=\bar{v}_2\left[R_2\frac{1}{t}\hat{e}_4(\hat{q}_1-\hat{q}_3)\hat{e}_3+R_1\frac{1}{u}\hat{e}_3(\hat{q}_1-\hat{q}_4)\hat{e}_4-\right.$$
$$\left.-\frac{2}{s}(R_1-R_2)[-(q_3e_4)\hat{e}_3+(q_4e_3)\hat{e}_4+(e_3e_4)\hat{q}_3]\right]u_1, \quad (223)$$

with $R_1=(t^bt^a)_{r_2r_1}, R_2=(t^at^b)_{r_2r_1}$. We use the color basis

$$\begin{aligned} c_1&=N_1\delta_{ab}\delta_{r_1r_2}, N_1=\frac{1}{\sqrt{N(N^2-1)}},\\ c_2&=N_2D^{abc}(t^c)_{r_1r_2}, N_2=\frac{1}{\sqrt{(N^2-4)(N^2-1)/(2N)}},\\ c_3&=N_3if^{abc}(t^c)_{r_1r_2}, N_3=\frac{1}{\sqrt{N(N^2-1)/2}}, \end{aligned} \quad (224)$$

which satisfies the normalization condition $(c_i\tilde{c}_i)=1, i=1,2,3$. The convolution necessary to us is

$$\begin{aligned} &(R_1\tilde{c}_1)=(R_2\tilde{c}_1)=N_1NC_F, \quad (R_1\tilde{c}_2)=(R_2\tilde{c}_))=N_2\frac{(N^2-4)(N^2-1)}{4N},\\ &(R_1\tilde{c}_3)=-(R_2\tilde{c}_3)=N_3\frac{N(N^2-1)}{4}. \end{aligned} \quad (225)$$

So we have

$$\begin{aligned} R_1&=\sqrt{\frac{N^2-1}{4N}}\left[c_1+\sqrt{\frac{N^2-4}{2}}c_2+\frac{N}{\sqrt{2}}c_3\right],\\ R_2&=\sqrt{\frac{N^2-1}{4N}}\left[c_1+\sqrt{\frac{N^2-4}{2}}c_2-\frac{N}{\sqrt{2}}c_3\right]. \end{aligned} \quad (226)$$

As a check, we use

$$(R_1R_1^+)=\mathrm{Tr}\,t^at^bt^bt^a=NC_F^2, \quad (R_1R_2^+)=\mathrm{Tr}\,t^at^bt^at^b=-\frac{1}{2}C_F. \quad (227)$$

Both relations are fulfilled. Matrix element can be written as

$$M=\sqrt{\frac{N^2-1}{4N}}\bar{v}_2\left[O_1\left(c_1+\sqrt{\frac{N^2-4}{2}}c_2\right)+O_2\sqrt{\frac{N^2}{2}}c_3\right]u_1, \quad (228)$$

with

$$O_1 = \frac{1}{t}\hat{e}_4(\hat{q}_1 - \hat{q}_3)\hat{e}_3 + \frac{1}{u}\hat{e}_3(\hat{q}_1 - \hat{q}_4)\hat{e}_4,$$
$$O_2 = -\frac{1}{t}\hat{e}_4(\hat{q}_1 - \hat{q}_3)\hat{e}_3 + \frac{1}{u}\hat{e}_3(\hat{q}_1 - \hat{q}_4)\hat{e}_4 - \frac{4}{s}[-(q_3e_4)\hat{e}_3+(q_4e_3)\hat{e}_4+(e_3e_4)\hat{q}_3]. \tag{229}$$

Using the relations

$$\frac{1}{4}\,\mathrm{Sp}\,\hat{q}_2O_1\hat{q}_1O_1^+ = 2\frac{t^2+u^2}{tu},$$
$$\frac{1}{4}\,\mathrm{Sp}\,\hat{q}_2O_2\hat{q}_1O_2^+ = 2\left[\frac{t^2+u^2}{tu} - 4\frac{t^2+u^2}{s^2}\right], \tag{230}$$

we obtain for the summed on color and spin states matrix element square:

$$\sum|M^{q\bar{q}\to gg}|^2 = 2\frac{(N^2-1)^2}{N}(t^2+u^2)\left[\frac{1}{tu} - \frac{2N^2}{(N^2-1)s^2}\right]. \tag{231}$$

Cross section is (for $N=3$ it is in agreement with [29])

$$\frac{d\sigma^{q\bar{q}\to gg}}{dO_3} = \frac{8\alpha_s^2}{27s}(t^2+u^2)\left[\frac{1}{tu} - \frac{9}{4s^2}\right]. \tag{232}$$

H.10. Color Projectors for $gg \to gg$. For the process

$$g(p_a, r_a) + g(p_b, r_b) \to g(p_1, r_1) + g(p_2, r_2), \tag{233}$$

we use the color basis [30]

$$R^{r_a,r_1}_{i,r_b,r_2}, i = 1,2,3,4,5, \tag{234}$$

obeying the conditions

$$R^{r_a,r_1}_{i,r_c,r_3}R^{r_c,r_3}_{j,r_b,r_2} = \delta_{ij}R^{r_a,r_1}_{i,r_b,r_2}. \tag{235}$$

Explicit form of them is

$$R^{r_a,r_1}_{1,r_b,r_2} = \frac{1}{N^2-1}\delta_{r_ar_1}\delta_{r_br_2}, \quad R^{r_a,r_1}_{2,r_b,r_2} = \frac{N}{N^2-4}d_{r_ar_1c}d_{r_br_2c},$$
$$R^{r_a,r_1}_{3,r_b,r_2} = \frac{1}{N}f_{r_ar_1c}f_{r_br_2c},$$
$$R^{r_a,r_1}_{4,r_b,r_2} = \frac{1}{2}[\delta_{r_ar_b}\delta_{r_1r_2} - \delta_{r_ar_2}\delta_{r_br_1}] - \frac{1}{N}f_{r_ar_1c}f_{r_br_2c},$$
$$R^{r_a,r_1}_{5,r_b,r_2} = \frac{1}{2}[\delta_{r_ar_b}\delta_{r_1r_2} + \delta_{r_ar_2}\delta_{r_br_1}] - \frac{1}{N^2-1}\delta_{r_ar_1}\delta_{r_br_2} - \frac{N}{N^2-4}d_{r_ar_1c}d_{r_br_2c}, \tag{236}$$

with f_{abc}, d_{abc} — structure constants. We need as well $K_i = R^{a,b}_{i,a,b}$. They are $K_1 = 1$, $K_2 = K_3 = N^2 - 1$, $K_4 = (N^2-1)(N^2-4)/2$, $K_5 = (N^2-1)(N^2-2)/2 - 1$. For the case $N = 3$, we have $K_i = 1, 8, 8, 20, 27; i = 1, 2, 3, 4, 5$. We remind here the expansion of the product $8x8 = 1 + 8 + 8 + (10 + \bar{10}) + 27$ to the irreducible representation of color $SU(3)$ group [31].

Performing the conversion of color matrix element given above with the color projectors, we obtain for the matrix element squared $\sum |M|^2 = 256H$,

$$H = H_{11} + 8H_{22} + 8H_{33} + 20H_{44} + 27H_{55} = \\ = \frac{27}{2} - \frac{9}{2}\left[\frac{su}{t^2} + \frac{st}{u^2} + \frac{tu}{s^2}\right], \quad (237)$$

where we use [30]

$$\begin{gathered} H_{11} = \frac{9}{16}\left[1 - \frac{tu}{s^2} - \frac{st}{u^2} + \frac{t^2}{su}\right], \\ H_{22} = \frac{1}{4}H_{11}, \quad H_{44} = 0, \quad H_{55} = \frac{1}{9}H_{11}, \\ H_{33} = \frac{27}{64} - \frac{9}{16}\left[\frac{su}{t^2} + \frac{ut}{4s^2} + \frac{st}{4u^2}\right] + \frac{9}{32}\left[\frac{u^2}{st} + \frac{s^2}{tu} - \frac{t^2}{2su}\right]. \end{gathered} \quad (238)$$

The same result can be obtained in terms of matrix element given above $M = IT + JK + LQ$. Using the relations [31]

$$\begin{aligned} d_{piq} f_{qjr} f_{rkp} &= -\frac{N}{2} d_{ijk}, \\ d_{piq} d_{qjr} f_{rkp} &= \frac{N^2-4}{2N} f_{ijk}, \\ f_{piq} f_{qjr} f_{rkp} &= -\frac{N}{2} f_{ijk}, \end{aligned} \quad (239)$$

we obtain

$$\begin{gathered} M = (T - Q)[a_1 R_1 + a_2 R_2 + a_5 R_5] + (a_3 T + b_3 Q + c_3 K) R_3, \\ a_1 = N, \quad a_2 = \frac{1}{2}N(N^2-1), \quad a_3 = a_2, \quad b_3 = -\frac{1}{2}N(N^2+1), \\ a_5 = -N^3, \quad c_3 = N. \end{gathered} \quad (240)$$

Note that the structure R_4 drops out. The further squaring of the matrix element leads to the expression for the differential cross section given above.

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