«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА» 1999, ТОМ 30, ВЫП. 5

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BEYOND THE BLACK-DISK LIMIT: FROM SHADOW TO ANTISHADOW SCATTERING MODE S.M. Troshin, N.E. Tyurin

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Предсказывается появление новой моды – антитеневой – в рассеянии адронов при энергиях выше $\sqrt{s} \simeq 2$ ТэВ. Эксперименты на LHC и VLHC по изучению адронных процессов могут обнаружить эту моду. Возможность появления данной моды при сверхвысоких энергиях рассматривается на основе унитарности и геометрических представлений об адронном рассеянии. Рассмотрена связь с непертурбативными моделями в рамках КХД.

New mode in the hadron scattering is predicted to appear at the energies beyond $\sqrt{s} \simeq 2$ TeV: the antishadow scattering mode, and the experiments at LHC and VLHC in hadronic reactions will be able to reveal it. The appearance of the antishadow scattering mode at these energies is considered on the basis of unitarity and geometrical notions of hadron interactions. Connections with the nonperturbative-QCD models are discussed.

INTRODUCTION

One of the most fundamental discoveries in hadron interactions at high energies is the rise of total cross sections with energy. It is accompanied by the rise of elastic and inelastic cross sections as well as of the ratio of elastic to the total cross section.

For the first time the total cross section increase was observed in K^+p interactions at the Serpukhov accelerator in 1970 [1] and it was discovered later in pp interactions at CERN ISR [2] and at Fermilab [3] in other nucleon- and mesonproton interactions. Recent HERA data [4] demonstrated the rising behavior of the virtual photon-proton total cross sections. Since then a great progress in the experimental and theoretical studies of hadronic reactions was achieved. Quantum Chromodynamics appeared as a theory of strong interactions and gave an explanation for the behavior of the observables in the hard hadronic reactions, i.e., the reactions with high momentum transfers. However, the dynamics of long distance interactions (soft processes) is rather far from its understanding despite much work has been done in this field. The problems are directly related to the problems of confinement and chiral symmetry breaking.

The approaches to soft hadronic processes are widely varied: Regge-type, geometrical or QCD-inspired models consider aspects of such processes from the

different points of view and use various ideas on hadron structure and interaction dynamics. The major part of the models consider the global characteristics of hadron interactions such as σ_{tot} , σ_{inel} , and σ_{diff} related to large distance interaction dynamics as reflecting gross features of hadron structure [5], [6]. Despite the difficulties in application of perturbative QCD for the description of long-distance interactions and their obvious nonperturbative character, it is often possible to represent the high-energy amplitude in the various model approaches as an expansion over a small parameter which depends on the kinematics of the process, e.g., for the case of nonincreasing total cross section the general form of the amplitude is

$$F(s,t) = s \sum_{n} [\tau(s)]^n \exp\left[\frac{a(s)t}{n}\right],$$

where $\tau(s) \sim 1/\ln s$ is a small parameter at $s \to \infty$.

Since the expansion is not valid for the rising total cross sections, it is possible to find another representation for that case with the t-dependent expansion parameter [7]:

$$F(s,t) = s \sum_{m=1}^{\infty} [\tau(\sqrt{-t})]^m \Phi_m[R(s), \sqrt{-t}], \qquad t \neq 0.$$

where

$$\tau(\sqrt{-t}) = \exp\left(-\sqrt{-t}/\mu_0\right),$$

and $\Phi_m[R(s), \sqrt{-t}]$ is an oscillating function of transferred momentum. The above formulas as well as some other representations may be successfully used for the phenomenological analysis of the scattering amplitude at high energies.

Thus, by now the theoretical treatment of soft hadronic reactions involves substantial piece of phenomenology and uses various model approaches. They are often based on divergent postulates, but their phenomenological parts are similar. In particular, an amplitude V(s,t) is considered as an input for the subsequent unitarization procedure:

$$F(s,t) = \Phi[V(s,t)].$$

To reproduce the total cross section rise the input amplitude V(s,t) is usually considered as a power function of energy. This function being taken as an amplitude itself violates unitarity in the direct channel. To obey unitarity in the direct channel a unitarization procedure should be used.

There are several ways to restore unitarity of the scattering matrix. We consider two schemes: based on the use of eikonal and generalized reaction matrix, respectively. There are also combined methods, but those are not often used. As was mentioned various models for V(s,t) may be successfully used to provide phenomenological description of high-energy hadron scattering. However, in the

particular model approaches the important dynamical aspects of interaction could be significantly obscured due to a large number of free parameters.

In this paper we discuss some general properties of hadron scattering, the implications of unitarity and analyticity, in particular, manifestations of the antishadow scattering mode and respective model predictions for the observables in elastic scattering and diffraction dissociation. Our main goal is to draw attention to the existence of the antishadow scattering mode at the energies of LHC and VLHC. It might provide a new insight into the dynamics of diffraction and head-on hadronic collisions at superhigh energies.

1. GEOMETRICAL PICTURE

In the collisions of two high energy particles the de Broglie wavelength can be short compared to the typical hadronic size and hence optical concepts may be used as useful guidelines. Thus, the hadron scattering can be considered as a collision of two relativistically contracted objects of finite size.

The relevant mathematical tool for description of high-energy hadronic scattering is based on the impact parameter representation for the scattering amplitude. In the case of spinless particle scattering this representation has the following form:

$$F(s,t) = \frac{s}{\pi^2} \int_0^\infty bdb f(s,b) J_0(b\sqrt{-t}).$$

$$\tag{1}$$

Note that for the scattering of particles with nonzero spin the impact parameter representation for the helicity amplitudes has a similar form with substitution $J_0 \rightarrow J_{\Delta\lambda}$, where $\Delta\lambda$ is the net helicity change between the final and initial states. The impact parameter representation as was shown in [8] is valid for all physical energies and scattering angles. This representation provides simple semiclassical picture of hadron scattering.

It is often assumed, after the Chou – Yang model was proposed, that the driving mechanism of hadron scattering is due to overlapping of the two matter distributions of colliding hadrons. It could be understood by analogy with Glauber theory of nuclear interactions: one assumes that the matter density comes from the spatial distribution of hadron constituents and also assumes a zero-range interaction between those constituents. Such contact interaction might result from the effective QCD, e.g., based on the Nambu – Jona-Lasinio Lagrangian.

The important role in the geometrical approach belongs to the notion of the interaction radius. The general definition of the interaction radius which is in agreement with the above geometrical picture was given in [9]:

$$R(s) = l_0(s)/k,$$
(2)

where $k = \sqrt{s}/2$ is the particle momentum in the c.m.s. The value for $l_0(s)$ is chosen provided the contributions of the partial amplitudes from the angular momenta $l > l_0(s)$ are vanishingly small.

As a first approximation one can consider the energy independent interaction intensity and describe the elastic scattering amplitude in terms of the black disk model where it has the form:

$$F(s,t) \propto iR^2(s) \frac{J_1(R(s)\sqrt{-t})}{R(s)\sqrt{-t}}.$$
(3)

Here $R \sim 1f$ is the interaction radius. The model is consistent with the observed structure in the differential cross sections of pp and $\bar{p}p$ scattering at t close to 1 (GeV/c)².

In the simplest case, neglecting the real part and spin, the impact parameter amplitude f(s, b) can be obtained as an inverse transformation according to Eq.(1) with

$$F(s,t) \propto \sqrt{s \frac{d\sigma}{dt}(s,t)}.$$

Thus, one can extract information on the geometrical properties of interaction from the experimental data. The analysis of the experimental data on highenergy diffractive scattering shows that the effective interaction area expands with energy and the interaction intensity — opacity — increases with energy at fixed impact parameter b. Such analysis used to be carried out every time as the new experimental data become available. For example analysis of the data at the ISR energies (the most precise data set on differential cross section for wide t range available for $\sqrt{s} = 53$ GeV) shows that one can observe a central impact parameter profile with a tail from the higher partial waves and some suppression (compared to Gaussian) of low partial waves. The scattering picture at such energies is close to gray disk with smooth edge which is getting darker in its centre with energy.

Beside the above simple geometrical observations it is useful to keep in mind the rigorous bounds for the experimental observables.

2. BOUNDS FOR OBSERVABLES AND THE EXPERIMENTAL DATA

Bounds for the observables obtained on the firm ground of general principles such as unitarity and analyticity are very important for any phenomenological analysis of soft interactions. However, there are only few results obtained on the basis of the axiomatic field theory. First of all it is the Froissart–Martin bound that gives the upper limit for the total cross section:

$$\sigma_{\rm tot} \le C \ln^2 s,\tag{4}$$

where $C = \pi/m_{\pi}^2$ (= 60 mb) and m_{π} is the pion mass.

Saturation of this bound, as it is suggested by the existing experimental data, implies the dominance of long-distance dynamics. It also leads to a number of important consequences for the other observables. For instance, unitarity leads to the following bound for elastic cross section:

$$\sigma_{\rm el}(s) \ge c \frac{\sigma_{\rm tot}^2(s)}{\ln^2 s}.$$
(5)

Therefore, when the total cross section asymptotically increases as $\ln^2 s$, elastic cross section also must rise like $\ln^2 s$. It is important to note here that there is no similar bound for the inelastic cross section and as we will see further the absence of such bound allows for appearance of the antishadow scattering mode at very high energies.

If one considers a more general case when $\sigma_{tot} \propto \ln^{\gamma} s$, then at asymptotic energies one should have

$$\frac{\operatorname{Re}\,F(s,0)}{\operatorname{Im}\,F(s,0)} \simeq \frac{\gamma\pi}{2\ln s} \tag{6}$$

and

$$\frac{\sigma_{\text{tot}}^{\bar{a}}(s) - \sigma_{\text{tot}}^{a}(s)}{\sigma_{\text{tot}}^{\bar{a}}(s) + \sigma_{\text{tot}}^{a}(s)} \le \ln^{-\gamma/2}(s),\tag{7}$$

where $\sigma_{\text{tot}}^{\bar{a}}(s)$ and $\sigma_{\text{tot}}^{a}(s)$ are the total cross sections of the processes $\bar{a} + b \to X$ and $a + b \to X$, correspondingly. In the case of $\gamma = 2$ the total cross section difference of antiparticle and particle interactions should obey the following inequality

$$\Delta \sigma_{\rm tot}(s) \le \ln s. \tag{8}$$

Contrary to the total cross section behavior, the existing experimental data seem to prefer decreasing $\Delta \sigma_{tot}(s)$. Possible deviations from such behavior could be expected on the basis of perturbative QCD [10] and it was one of the reasons for the recent discussions on the Pomeron counterpart — the Odderon. However, the recent measurements of the real-to-imaginary part ratio for forward $\bar{p}p$ scattering provide little support for the Odderon. We will not discuss more thoroughly the interesting problem of Re F/Im F ratio and will consider for simplicity the case of pure imaginary amplitude.

For the slope of diffraction cone at t = 0 in the case of a pure imaginary scattering amplitude the following inequality takes place:

$$B(s) \ge \frac{\sigma_{\text{tot}}^2(s)}{18\pi\sigma_{el}(s)} \tag{9}$$

which means that when the total cross section increases as $\ln^2 s$, the same dependence is mandatory for the slope of diffraction cone. It is the stronger shrinkage than the Regge model predicts: $B(s) \sim \alpha' \ln s$.

There is also bound [11] for the total cross section of single diffractive processes. It was obtained by Pumplin in approach, where inelastic diffraction as well as elastic scattering are assumed to arise in the form of a shadow of inelastic processes, c and has the form

$$\sigma_{\rm diff}(s,b) \le \frac{1}{2} \,\sigma_{\rm tot}(s,b) - \sigma_{\rm el}(s,b). \tag{10}$$

The most significant assumption was that the diffractive eigenamplitudes in the Good–Walker [12] picture do not exceed the black disk limit.

At this point some details of the experimental situation have to be mentioned. At the highest energies the experimental data for the total and elastic cross sections, slope parameter of diffraction cone and cross section of single inelastic diffraction dissociation have been obtained in $\bar{p}p$ collisions at Fermilab. In particular, those measurements show that

- the rise of the total cross section of $p\bar{p}$ interactions is consistent with $\ln^2 s$ dependence, however other dependences are not ruled out;
- elastic cross section rises faster than the inelastic and total cross sections and has a magnitude about 1/4 of the total cross section.

Comparing the value of the elastic-to-total cross section ratio with the lower energy data one can conclude that the higher the energy, the higher both absolute and relative probabilities of elastic collisions.

Impact parameter analysis [13] of the data shows that the scattering amplitude is probably beyond the black disk limit |f(s,b)| = 1/2 in head-on collisions. The Pumplin bound (Eq.(10)) is also violated in such collisions and this is not surprising if one recollects the original ad hoc assumption on the shadow scattering mode.

3. ANTISHADOW SCATTERING MODE

The basic role in our consideration belongs to unitarity of the scattering matrix $SS^+ = 1$ which is a reformulation of the probability conservation. In the impact parameter representation the unitarity equation rewritten for the elastic scattering amplitude f(s, b) at high energies has the form

$$Im f(s,b) = |f(s,b)|^2 + \eta(s,b)$$
(11)

where the inelastic overlap function $\eta(s, b)$ is the sum of all inelastic channel contributions. It can be expressed as a sum of *n*-particle production cross sections at the given impact parameter

$$\eta(s,b) = \sum_{n} \sigma_n(s,b).$$
(12)

As was mentioned assumption of a pure imaginary amplitude is a rather common approximation at high energies and is adequate for our qualitative analysis. Then the unitarity Eq.(11) points out that the elastic scattering amplitude at given impact parameter value is determined by the inelastic processes. Equation (11) implies the constraint

$$|f(s,b)| \le 1$$

while the black-disk limit presumes inequality

$$|f(s,b)| \le 1/2.$$

The equality |f(s,b)| = 1/2 corresponds to maximal absorption in the partial wave with angular momentum $l \simeq b\sqrt{s}/2$.

The maximal absorption limit is chosen *a priori* in the eikonal method of unitarization when the scattering amplitude is written in the form:

$$f(s,b) = \frac{i}{2} (1 - \exp[i\omega(s,b)]),$$
(13)

and imaginary eikonal $\omega(s, b) = i\Omega(s, b)$ is considered. The function $\Omega(s, b)$ is called opacity. Eikonal unitarization automatically satisfies the unitarity Eq.(11) and in the case of pure imaginary eikonal leads to the amplitude which always obeys the black-disk limit.

However, unitarity equation has two solutions for the case of pure imaginary amplitude:

$$f(s,b) = \frac{i}{2} \left[1 \pm \sqrt{1 - 4\eta(s,b)} \right].$$
(14)

Eikonal unitarization with pure imaginary eikonal corresponds to the choice of the particular solution with sign minus.

Several models have been proposed for the eikonal function. For instance, Regge-type models lead to the Gaussian dependence of $\Omega(s, b)$ on impact parameter. To provide rising total cross sections, opacity should have a power dependence on energy

$$\Omega(s,b) \propto s^{\Delta} \exp\left[-b^2/a(s)\right],\tag{15}$$

where $a(s) \sim \ln s$. In the framework of perturbative QCD-based models the driving contribution to the opacity is due to jet production in gluon-gluon interactions, when

$$\Omega(s,b) \propto \sigma_{\rm jet} \exp\left[-\mu b\right],\tag{16}$$

where $\sigma_{jet} \sim (s/s_0)^{\Delta}$. Such parameterizations lead to the rising total and elastic cross sections and slope parameter:

$$\sigma_{\rm tot}(s) \sim \sigma_{\rm el}(s) \sim B(s) \sim \ln^2 s$$
 (17)

and the ratio

$$\frac{\sigma_{\rm el}(s)}{\sigma_{\rm tot}(s)} \to \frac{1}{2}.$$
(18)

To include the mode, where the scattering amplitude exceeds the black-disk limit, one should consider the eikonal functions with nonzero real parts. To ensure the transition from shadow to antishadow mode the real part of eikonal should gain an abrupt increase equal to π at some $s = s_0$. The conventional models do not foresee such a critical behavior for real part of the eikonal.

However, it does not mean that the eikonal model itself is in trouble. In particular, the account for fluctuations of the eikonal [14] strongly modifies the structure of the amplitude and reduces it to algebraic form which is similar to that used in the unitarization scheme based on the generalized reaction matrix.

The latter method is based on the relativistic generalization of the Heitler equation of radiation damping [15]. In this approach the elastic scattering amplitude satisfies unitarity equation since it is constructed as a solution of the following equation [15]

$$F = U + iUDF \tag{19}$$

presented here in the operator form. Eq.(19) allows one to satisfy unitarity provided the inequality

$$\operatorname{Im} U(s,b) \ge 0 \tag{20}$$

is fulfilled. The form of the amplitude in the impact parameter representation is the following:

$$f(s,b) = \frac{U(s,b)}{1 - iU(s,b)},$$
(21)

where U(s,b) is the generalized reaction matrix, which is considered as an input dynamical quantity similar to eikonal function. Similar form for the scattering amplitude was obtained by Feynman in his parton model of diffractive scattering [16]. Inelastic overlap function is connected with U(s,b) by the relation

$$\eta(s,b) = \frac{\mathrm{Im}\,U(s,b)}{|1 - iU(s,b)|^2}.$$
(22)

Construction of particular models in the framework of the U-matrix approach proceeds with the same steps as it does for the eikonal function, i.e., the basic dynamics as well as the notions on hadron structure are used to obtain a particular

form for the U matrix. For example, the Regge-pole approach [17] provides the following form for the U matrix:

$$U(s,b) \propto is^{\Delta} \exp\left[-b^2/a(s)\right], \quad a(s) \sim \alpha' \ln s, \tag{23}$$

while the chiral quark model which will be discussed below gives the exponential b dependence

$$U(s,b) \propto is^{\Delta} \exp\left[-\mu b\right],\tag{24}$$

where μ is the constant proportional to the masses of the constituent quarks. We have mentioned here only the gross features of those model parameterizations without going into details.

Both the parameterizations lead to $\ln^2 s$ rise of the total and elastic cross sections and slope parameter B(s):

$$\sigma_{\rm tot}(s) \sim \sigma_{\rm el}(s) \sim B(s) \sim \ln^2 s \tag{25}$$

at $s \to \infty$. The above results are similar to conclusions of eikonal unitarization.

However, these two unitarization schemes lead to different predictions for the inelastic cross sections and for the ratio of elastic to total cross section. This ratio in the U-matrix unitarization scheme reaches its maximal possible value at $s \rightarrow \infty$, i.e.,

$$\frac{\sigma_{\rm el}(s)}{\sigma_{\rm tot}(s)} \to 1,$$
 (26)

which reflects in fact that the bound for the partial-wave amplitude in the U-matrix approach is $|f| \leq 1$, while the bound for the case of imaginary eikonal is (black-disk limit): $|f| \leq 1/2$.

When the amplitude exceeds the black-disk limit (in central collisions at high energies), then the scattering at such impact parameters turns out to be of an antishadow nature. It corresponds to the solution of unitarity equation (11) with plus sign. In this antishadow scattering mode the elastic amplitude increases with the decrease of the inelastic channels contribution.

The shadow scattering mode is considered usually as the only possible one. But the two solutions of the unitarity equation have an equal meaning and the antishadow scattering mode could also appear in central collisions first as the energy becomes higher. Both the scattering modes are realized in a natural way in the U-matrix approach despite the two modes are described by the two different solutions of unitarity Eq.(14).

Let us consider the transition to the antishadow scattering mode [18]. With conventional parameterizations of the U-matrix in the form of Eq. (23) or Eq.(24) the inelastic overlap function increases with energies at modest values of s. It reaches its maximum value $\eta(s, b = 0) = 1/4$ at some energy $s = s_0$, and beyond this energy the antishadow scattering mode appears at small values of b.

The region of energies and impact parameters corresponding to the antishadow scattering mode is determined by the conditions Im f(s,b) > 1/2 and $\eta(s,b) < 1/4$. The quantitative analysis of the experimental data [19] gives the threshold value of energy: $\sqrt{s_0} \simeq 2$ TeV.

Thus, the function $\eta(s, b)$ becomes peripheral when energy is increasing. At such energies the inelastic overlap function reaches its maximum value at b = R(s), where R(s) is the interaction radius. So, beyond the transition threshold there are two regions in impact parameter space: the central region of antishadow scattering at b < R(s) and the peripheral region of shadow scattering at b > R(s). At b = R(s) the maximal absorption (black ring) takes place (Fig. 1).



Fig. 1. Shadow and antishadow scattering regions

The transition to the antishadow scattering at small impact parameters at high energies results also in a relatively slow rise of inelastic cross section:

$$\sigma_{\rm inel}(s) = 8\pi \int_0^\infty \frac{bdb \operatorname{Im} U(s, b)}{|1 - iU(s, b)|^2} \sim \ln s$$
(27)

at $s \to \infty$.

It should be noted that appearance of the antishadow scattering mode does not contradict the basic idea that the particle production is the driving force for elastic scattering. Indeed, the imaginary part of the generalized reaction matrix is the sum of inelastic channel contributions:

$$\operatorname{Im} U(s,b) = \sum_{n} \bar{U}_{n}(s,b), \qquad (28)$$

where n runs over all inelastic states, and

$$\bar{U}_n(s,b) = \int d\Gamma_n |U_n(s,b,\{\xi_n\})|^2,$$
(29)

and $d\Gamma_n$ is the *n*-particle element of the phase space volume. The functions $U_n(s, b, \{\xi_n\})$ are determined by the dynamics of $2 \rightarrow n$ processes. Thus, the quantity Im U(s, b) itself is a shadow of the inelastic processes. However, unitarity leads to self-damping of the inelastic channels [20] and increase of the function Im U(s, b) results in decrease of the inelastic overlap function $\eta(s, b)$, when Im U(s, b) exceeds unity.

At the energies when the antishadow mode starts to develop (it presumably could already occur at the energies of the Tevatron-Collider) the Pumplin bound Eq.(10) for inelastic diffraction dissociation cannot be applied since the main assumption used under its derivation is not valid any more.

4. THE TWO MODES OF HADRON SCATTERING AND THE PREASYMPTOTIC EFFECTS

In this section we give a specific analysis of the hadron scattering on the basis of particular model. In Refs.21,22 the notion of an effective chiral quark model was used for the description of elastic scattering at small and large angles. Hadron dynamics is considered in the framework of effective Lagrangian approach.

A common feature of the chiral models [23] is the representation of a baryon as an inner core carrying the baryonic charge and anouter condensate surrounding this core [24]. Following these observations it is natural to represent a hadron as consisting of the inner region where valence quarks are located and the outer region filled with quark condensate [22]. Such a picture for the hadron structure implies that overlapping and interaction of peripheral condensates in hadron collision occurs at the first stage. In the overlapping region the condensates interact and as a result virtual massive quarks appear. Being released part of hadron energy carried by the peripheral condensates goes to a generation of massive quarks. Besides mass, quark acquires an internal structure and a finite size. Quark radii are determined by the radii of the clouds. Strong interaction radius of quark Q is determined by its Compton wavelength:

$$r_Q = \xi/m_Q,\tag{30}$$

where constant ξ is universal for different flavors. In the model valence quarks located in the central part of a hadron are supposed to scatter in a quasi-independent way by the produced virtual massive quarks at given impact parameter and by the other valence quarks.

The function U(s, b) (generalized reaction matrix) [15] — the basic dynamical quantity of this approach — is chosen as a product of the averaged quark amplitudes

$$U(s,b) = \prod_{Q=1}^{N} \langle f_Q(s,b) \rangle \tag{31}$$

in accordance with assumed quasi-independent nature of valence quark scattering. The b dependence of the function $\langle f_Q \rangle$ related to the quark form factor $F_Q(q)$ has a simple form $\langle f_Q \rangle \propto \exp(-m_Q b/\xi)$.

Thus, the generalized reaction matrix (in a pure imaginary case) gets the following form

$$U(s,b) = ig \left[1 + \alpha \frac{\sqrt{s}}{m_Q}\right]^N \exp\left(-Mb/\xi\right),\tag{32}$$

where $M = \sum_{q=1}^{N} m_Q$. At moderate energies $s \ll s_0$ (where $\sqrt{s_0} \equiv m_Q/\alpha$) the function U(s, b) can be represented in the form

$$U(s,b) = ig\left[1 + N\alpha \frac{\sqrt{s}}{m_Q}\right] \exp\left(-Mb/\xi\right).$$
(33)

At very high energies $s \gg s_0$ we could neglect the energy independent term in (32) and rewrite the expression for U(s, b) as

$$U(s,b) = ig \left(s/m_Q^2 \right)^{N/2} \exp\left(-Mb/\xi \right).$$
(34)

Calculation of the scattering amplitude is based on the impact parameter representation and the analysis of singularities of $F(s,\beta)$ in complex β plane [7].

Besides the energy dependence of these observables we will emphasize its dependence on geometrical characteristics of nonperturbative quark interactions.

The total cross section has the following energy and quark-mass dependences

$$\sigma_{\rm tot}(s) = \frac{\pi \xi^2}{\langle m_Q \rangle^2} \Phi(s, N), \tag{35}$$

where $\langle m_Q \rangle = \frac{1}{N} \sum_{Q=1}^{N} m_Q$ is the mean value of the constituent quark masses in the colliding hadrons. The function Φ has the following behavior:

$$\Phi(s,N) = \begin{cases} \left(8g/N^2 \right) \left[1 + N\alpha\sqrt{s}/m_Q \right], & s \ll s_0, \\ \ln^2 s, & s \gg s_0. \end{cases}$$
(36)

Thus, at asymptotically high energies the model provides

$$\lim_{s \to \infty} \frac{\sigma_{\text{tot}}(\bar{a}b)}{\sigma_{\text{tot}}(ab)} = 1.$$

Linear with \sqrt{s} preasymptotic rise of the total cross sections is in agreement with the experimental data up to $\sqrt{s} \sim 0.5$ TeV [19].

The inelastic cross section can be calculated in the model explicitly, viz:

$$\sigma_{|rminel}(s) = \frac{8\pi\xi^2}{N^2 \langle m_Q \rangle^2} \ln\left[1 + g(1 + \frac{\alpha\sqrt{s}}{m_Q})^N\right].$$
(37)

At asymptotically high energies the inelastic cross section rise is as follows

$$\sigma_{\rm inel}(s) = \frac{4\pi\xi^2}{N\langle m_Q \rangle^2} \ln s.$$
(38)

At $s \gg s_0$ the dependence of the hadron interaction radius R(s) and the ratio $\sigma_{\rm el}/\sigma_{\rm tot}$ on $\langle m_Q \rangle$ is provided by the following equations:

$$R(s) = \frac{\xi}{2\langle m_Q \rangle} \ln s, \qquad (39)$$

$$\frac{\sigma_{\rm el}(s)}{\sigma_{\rm tot}(s)} = 1 - \frac{4}{N \ln s}.$$
(40)

It is important to note here that such a behavior of the ratio $\sigma_{\rm el}/\sigma_{\rm tot}$ and $\sigma_{\rm inel}(s)$ results from self-damping of inelastic channels [20] at small impact distances. Numerical estimates [19] show that the ratio $\sigma_{\rm el}(s)/\sigma_{\rm tot}(s)$ becomes close to the asymptotic value 1 at extremely high energies $\sqrt{s} = 500$ TeV.

Thus, unitarization drastically changes the scattering picture: at lower energies inelastic channels provide dominant contribution and scattering amplitude has a shadow origin, while at high energies elastic scattering dominates over inelasic contribution and the scattering picture corresponds to the antishadow mode. The functional s dependences of observables also differ significantly. For example, s dependence of total cross section at $s \ll s_0$ is described by a simple linear function of \sqrt{s} . It has been shown that such dependence does not contradict the experimental data for hadron total cross sections up to $\sqrt{s} \sim 0.5$ TeV. Such dependence corresponds to that of the hard Pomeron with $\Delta = 0.5$, however, it was obtained in different approach [22]. This is a preasymptotic dependence and it has nothing to do with the true asymptotics of the total cross sections. In the model such behavior of the hadronic cross sections reflects the energy dependence of a number of virtual quarks generated under condensate collisions in the intermediate transient stage of hadronic interaction.

5. ANTISHADOW SCATTERING MODE AND INELASTIC DIFFRACTIVE PROCESSES

Inelastic diffractive production as well as elastic scattering at low transferred momenta are the two basic processes which would lead to understanding of large distance dynamics and hadron structure. Concerning inelastic diffractive processes this statement can be traced back to the seminal paper [12], where such processes were considered as a result of a difference in absorption of various proton states. Later on, these states have got a parton-like interpretation. New data were obtained for single diffraction production process

$$h_1 + h_2 \to h_1 + h_2^*,$$
 (41)

when the hadron h_2 is excited to the state h_2^* with invariant mass M and the same quantum numbers. Its subsequent decay results in the multiparticle final state. The inclusive differential cross section shows a simple dependence on the invariant mass

$$\frac{d\sigma_{\rm diff}}{dM^2} \propto \frac{1}{M^2}.$$
(42)

However, energy dependence of the diffractive production cross section $\sigma_{\text{diff}}(s)$ is not so evident from the data. This ambiguity is partly due to difficulties in the experimental definition of the inelastic diffractive cross section.

The particular experimental regularities observed in diffractive production can be described in the framework of different approaches. $1/M^2$ dependence is naturally described by the triple-pomeron diagrams in the framework of Reggemodel. The proposed in Ref.25 similarity between the Pomeron and photon exchanges allowed one to calculate diffractive dissociation cross section in terms of structure function νW_2 measured in deep inelastic lepton scattering. Several models use optical picture for the description of diffractive production [26] but these models in large extent concern the angular distribution of diffractive cross section and M^2 dependence is left beyond their scope. The attempt to explain M^2 dependence in the framework of optical model, considering diffractive dissociation as a bremsstrahlung where virtual quanta are released from a strong field was made in Ref.27.

In this section for description of single diffractive processes we use model approach described in section 4.

To obtain the cross section of the diffractive dissociation process we should single out among the final states in Eq.(28) those corresponding to the process (41). Let us for simplicity consider again the case of pure imaginary U matrix. Then we can represent $d\sigma_{\text{diff}}/dM^2$ in the following form

$$\frac{d\sigma_{\text{diff}}}{dM^2} = 8\pi \int_0^\infty b db \frac{U_{\text{diff}}(s, b, M)}{[1 + U(s, b)]^2},\tag{43}$$

where expression for $U_{\text{diff}}(s, b, M)$ includes contributions from all the final states $|n\rangle_{\text{diff}}$ which results from the decay of the excited hadron h_2^* of mass $M: h_2^* \rightarrow |n\rangle_{\text{diff}}$.

For consideration of the diffractive production at the quark level we extend the picture for hadron interaction for elastic scattering, described in section 4. Since the constituent quark is an extended object there is a nonzero probability of its excitation at the first stage of hadron collision during the interaction of peripheral condensates. Therefore it is natural to assume that the origin of diffractive production process is the excitation of one of the valence quarks in colliding hadron: $Q \rightarrow Q^*$, its subsequent scattering and decay into the final state. The excited constituent quark is scattered similar to other valence quarks in a quasi-independent way. The function $U_{\text{diff}}(s, b, M)$ can be represented then as a product

$$U_{\text{diff}}(s,b,M) = \langle f_{Q^*}(s,b,M_{Q^*}) \rangle \prod_{Q=1}^{N-1} \langle f_Q(s,b) \rangle, \tag{44}$$

where M_{Q^*} is the mass of excited constituent quark, which is proportional to the mass M of excited hadron h_2^* for large values of M. The last statement presumes the additivity of constituent quark masses. The b dependence of the amplitude $\langle f_{Q^*} \rangle$ is related to the form factor of excited quark whose radius is determined by its mass M_{Q^*} ($r_{Q^*} = \xi/M_{Q^*}$). The expression for $U_{\text{diff}}(s, b, M)$ can be rewritten then in the following form:

$$U_{\text{diff}}(s, b, M) = g^* U(s, b) \exp\left[-(M_{Q*} - m_Q)b/\xi\right],$$
(45)

where constant g^* is proportional to the relative probability of excitation of the constituent quark. The value of g^* is a nonzero one, however, $g^* < 1$ since we expect that the excitation of any constituent quark has lower probability compared to probability for this quark to stay unexcited. The excited quark is not stable and its subsequent decay is associated with the decay of excited hadron h_2^* into the multiparticle final state $|n\rangle_{\text{diff}}$.

The cross section of diffractive dissociation process is given by expression (43) and has the following s- and M^2 dependences

$$\frac{d\sigma_{\text{diff}}}{dM^2} \simeq \frac{8\pi g^* \xi^2}{(M_{Q^*} - m_Q^2)^2} \eta(s, 0) \simeq \frac{8\pi g^* \xi^2}{M^2} \eta(s, 0).$$
(46)

Thus, we obtained the familiar $1/M^2$ dependence of the diffraction cross section which is related in this model to the geometrical size of excited constituent quark.

The double dissociation processes

$$h_1 + h_2 \to h_1^* + h_2^*$$
 (47)

can be considered on the grounds of previous approach to the single diffractive dissociation. Here one of the constituent quarks in each of the colliding hadrons should be excited. The cross section of double diffraction process has similar

 M^2 and s dependences and is to be suppressed in comparison with the single diffractive cross section by an extra factor $g^* < 1$.

The energy dependence of single diffractive cross section has the following form

$$\sigma_{\rm diff}(s) = 8\pi g^* \xi^2 \eta(s,0) \int_{M_0^2}^{M_1^2} \frac{dM^2}{M^2} = 8\pi g^* \xi^2 \eta(s,0) \ln \frac{s(1-x_1)}{M_0^2}, \qquad (48)$$

where x_1 is the lower limit of the relative momentum of hadron $h_1(x_1 \simeq 0.8-0.9)$ which corresponds to the experimental constraint on diffractive process. Eq. (48) shows that the total cross section of diffractive dissociation has a nontrivial energy dependence which is determined by the contribution of inelastic channels into unitarity equation at zero value of impact parameter. The dependence of $\eta(s, 0)$ is determined by Eq. (22), where expression for U(s, b) is given by Eq. (32). At $s \leq s_0$, (s_0 is determined by equation $|U(s_0, 0)| = 1$) $\eta(s, 0)$ increases with energy. This increase as it follows from Eq. (32) and from the experimental data [28] is rather slow one. However at $s \geq s_0$, $\eta(s, 0)$ reaches its maximum value $\eta(s, 0) = 1/4$ and at $s > s_0$, the function $\eta(s, 0)$ decreases with energy. At $s \to \infty$:

$$\sigma_{\rm diff}(s) \propto \left(\frac{1}{\sqrt{s}}\right)^N \ln s$$
 (49)

since $\eta(s,0) \propto (1/\sqrt{s})^N$ in this limit.

Thus at asymptotical energies the inelastic diffraction cross section drops to zero. Decrease of diffractive production cross section at high energies $(s > s_0)$ is due to the fact that $\eta(s, b)$ becomes peripheral at $s > s_0$ and the whole picture corresponds to the antishadow scattering at b < R(s) and to the shadow scattering at b > R(s), where R(s) is the interaction radius. The quitative behavior of $\sigma_{\text{diff}}(s)$ is shown in Fig.2.



Fig. 2. Energy dependence of diffractive cross section

The development of the antishadow mode in head-on pp- and $p\bar{p}$ collisions could be associated with new phenomena in the central hadronic collisions, where

the temperatures are high and the energy density can be up to several GeV/fm^3 . In such collisions the constituent quarks have noticeable probability to be excited. Due to its high mass and small transverse size the excited state has low probability of interactions with other particles. It may be also related to an interesting phenomena in cosmic ray experiments where particles with abnormal persistency in lead chambers were observed [29].

Of course, there might be different reasons for decrease of $\sigma_{diff}(s)$. The decreasing energy dependence of $\sigma_{diff}(s)$ was also predicted in Refs.30,31. As was pointed out in Ref.12 in the limit of complete absorption the diffractive dissociation should vanish. It was advocated in Ref.14 that this situation will occur at superhigh energies and it is the reason for decrease of inelastic diffractive cross section. This is completely the same behavior as is predicted by the model presented, however in our case the reason for that is the transition to the antishadow scattering mode in head-on collisions in the multi-TeV energy range. It should be noted, however, that the diffractive cross section at preasymptotic energies has a similar to total and elastic cross-section energy dependence and it will be discussed in the concluding part of this paper.

6. UNIVERSAL PREASYMPTOTICS

The straitforward interpretation of the recent HERA data on the deep-inelastic scattering together with the analysis of the data on hadron-hadron scattering in terms of the Regge model could lead to the unexpected conclusion on the existence of the various Pomerons [32] or the various manifestations of unique Pomeron in the different processes depending on the typical scale of the process [33]. The approaches [34, 35] contending the dominance of the soft Pomeron do not rule out existence of the hard Pomeron either.

Indeed, soft hadronic reactions imply that Pomeron's intercept $\alpha_{\mathcal{P}} = 1.08$ [32], and small-x dependence of the structure function $F_2(x, Q^2)$ leads to $\alpha_{\mathcal{P}} = 1.4$ -1.5 [36, 37] and the measurements of the diffractive cross section in the deep-inelastic scattering provide $\alpha_{\mathcal{P}} = 1.23$ [38]. So, does this mean that we have few Pomerons or we have few different manifestations of the same Pomeron depending on the particular process? Probably both options are not to be considered as the firm ones, since the experimental data used to advocate these statements were obtained at not high enough energies where, in fact, the preasymptotic regime of interactions does take place. The above conclusions are based on the presumed dominance of the Pomeron contribution already in the preasymptotic energy region. What is called a Pomeron is to be interpreted as a true asymptotical contribution of the driving mechanism.

In this section we argue that all the three classes of the processes described above are related to the similar mechanisms, and the corresponding energy dependence of the cross sections can be well described by the universal functional energy dependence of the type $a + b\sqrt{s}$. Such dependence is valid for the preasymptotic energy region only and beyond this region unitarity changes the picture drastically. We consider for illustration the unitarized chiral quark model (section 4).

Fit to the total hp cross sections gives small values for the parameters g and $\alpha(g, \alpha \ll 1)$ [19]. It means that at $s \ll s_0$ the second term in the square brackets in Eqs. (21) and (22) is small and we can expand over it. The numerical value of s_0 is determined by the equation |U(s, 0)| = 1 and is [19] $\sqrt{s_0} \simeq 2 \text{ TeV}$. At this energy the amplitude has the value $|f(s_0, 0)| = 1/2$. The value of s_0 is on the verge of the preasymptotic energy region, i.e., the Tevatron energy is at the beginning of the road to the asymptotics. Evidently the HERA energy range $W(=\sqrt{s_{\gamma p}}) \leq 300 \text{ GeV}$ is in a preasymptotic domain.

The above model gives the linear with \sqrt{s} dependence for the total cross sections according to Eqs. (21) and (22):

$$\sigma_{\rm tot}^{hp,\gamma p} = a + b\sqrt{s},\tag{50}$$

where parameters a and b are different for different processes and the same is true for the scale s_0 . It was shown [19] that Eq.(50) is in good agreement with the experimental data.

The same dependence for the total cross section of $\gamma^* p$ scattering is assumed by the small-x behavior of the structure function $F_2(x, Q^2)$ [36,37] and obtained in [39]:

$$F_2(x,Q^2) = a(Q^2) + b(Q^2)/\sqrt{x}.$$
(51)

The experimental data also indicate the critical behavior of the function $b(Q^2)$ at $Q^2 \simeq 1 \text{ (GeV/c)}^2$. This scale could be related to the radius of a constituent quark and its structure.

The third value for the Pomeron intercept $\alpha_{\mathcal{P}} = 1.23$ has been obtained from the analysis of the experimental data on the diffractive cross section in deepinelastic scattering [38] where the dependence of $d\sigma_{\gamma^*p\to XN}^{\text{diff}}/dM_X^2$ on W was parameterized according to the Regge model and the Pomeron dominance has been assumed:

$$d\sigma_{\gamma^* p \to XN}^{\text{diff}} / dM_X^2 \propto (W^2)^{2\alpha_{\mathcal{P}} - 2}.$$
(52)

The data demonstrate linear rise of the differential cross section $d\sigma_{\gamma^*p\to XN}^{\text{diff}}/dM_X^2$ with W, i.e., we observe here just the same functional dependence on the c.m.s. energy as for $\sigma_{\text{tot}}^{hp,\gamma p,\gamma^*p}$. Regarding the preasymptotic nature of the interaction mode we arrive to the universal c.m.s. energy dependence in the framework of the used model.

Indeed, in the framework of this model the hadron inelastic diffractive cross section is given by the following expression [40]:

$$\frac{d\sigma_{hp\to XN}^{\text{diff}}}{dM_X^2} \simeq \frac{8\pi g^* \xi^2}{M_X^2} \eta(s,0),\tag{53}$$

where

$$\eta(s,b) = \operatorname{Im} U(s,b) / [1-iU(s,b)]^2$$

is the inelastic overlap function.

At the preasymptotic energies $s \ll s_0$ the energy dependence of inelastic diffractive cross section resulting from Eq. (22) is again determined by the generic form

$$\frac{d\sigma_{hp\to XN}^{\text{diff}}}{dM_X^2} \propto a + b\sqrt{s}.$$
(54)

Inelastic diffractive cross section for the $\gamma^* p$ interactions can be obtained using for example VMD model, i.e.,

$$\frac{d\sigma_{\gamma^*p \to XN}^{\text{diff}}}{dM_X^2} \propto a(Q^2) + b(Q^2)W.$$
(55)

The same functional dependence can be obtained using the "aligned jet" model [41] along with the unitarized chiral quark model [42].

The above linear dependences for the cross sections of different processes is the generic feature associated with the preasymptotic nature of the interaction dynamics at $s \ll s_0$. As one goes above this energy range, the function |U(s,b)|is rising and when $|U(s,0)| \ge 1$, the unitarity starts to play the major role and provides the $\ln^2 s$ rise of the total cross sections at $s \gg s_0$ [42] and also the following behavior of the structure function $F_2(x, Q^2)$

$$F_2(x, Q^2) \propto \ln^2(1/x)$$
 (56)

at $x \to 0$ [39]. At the same time unitarity leads to the decreasing dependence of the inelastic diffractive cross section at $s \to \infty$

$$\frac{d\sigma^{\text{diff}}}{dM_X^2} \propto \left(\frac{1}{\sqrt{s}}\right)^N,\tag{57}$$

for the *hp*-, γp - and $\gamma^* p$ processes [40]. Eq. (57) is associated with the antishadow scattering mode which develops at small impact parameters at $s > s_0$.

Thus, we might expect the different asymptotic and universal preasymptotic behaviors for the different classes of the diffraction processes.

To summarize, we would like to emphasize that the unified description of the processes of hp-, γp - and $\gamma^* p$ - diffraction scattering with the universal cross

section dependence on the c.m.s. interaction energy is possible. For illustration we used the unitarized chiral quark model which has a nonperturbative origin and leads to the linear c.m.s. energy dependence of the cross sections in the preasymptotic energy region for the above processes. Universality of such preasymptotic behavior agrees with the experiment.

The assumption on the existence of the different Pomerons results from the use of the asymptotic formulas in the preasymptotic energy region and the neglect of the unitarity at higher energies beyond this preasymptotic region. It should be taken with certain caution.

CONCLUSION

Studies of soft interactions at the highest energies can lead to the discoveries of fundamental importance. The genesis of hadron scattering with rising energy can be described as transition from the grey to black-disk and eventually to black ring with the antishadow scattering mode in the centre. Such transitions are under control of unitarity of the scattering matrix.

The appearance of antishadow scattering mode could be revealed performing impact parameter analysis of elastic scattering and directly in the measurements of the inelastic diffractive cross section (cf. Figs. 1,2).

It would be interesting to speculate on the particular physical origin of the antishadow scattering mode. Its existence can be correlated with the new phenomena expected at high energies in the central hadronic collisions. Such collisions are usually associated with the formation of quark–gluon plasma and disoriented chiral condensate in the inner part of the interaction region. What are the particular correlations between those phenomena and the antishadow scattering? The answer can be obtained in the nonperturbative QCD studies and in the experiments devoted to studies of soft processes at LHC and VLHC. It seems that the anomalies observed in cosmic ray experiments [29] might also be correlated with development of the antishadow scattering mode in the central hadron collisions.

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