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BOUND STATES IN QUANTUM FIELD THEORY

G.V.Efimov

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
141980 Dubna, Russia

The method called «the Bosonization of Nonlocal Currents» (BNC), used for calculations of bound states in a quark model, is demonstrated within the simplest relativistic quantum field model of two scalar fields with the Yukawa type interaction.

Hadronization of quarks and gluons is one of the most important and interesting problems of QCD. In papers [1] we have formulated «the Model of Induced Nonlocal Quark Currents», based on the assumption that QCD vacuum is realized by the (anti-)self-dual homogeneous vacuum field. All calculations of the meson spectrum and other characteristics of light mesons in this model were done by the method of «Bosonization of Nonlocal Currents» and quite good agreement with experimental data was obtained. Our method is quite close to the so-called $Z_2 = 0$ method (see, for example, [2–4]) and differs from the known axiomatic methods (see, for example, [5–7]). In this talk I would like to demonstrate our method on a simple but nontrivial QFT model.

Our initial idea is based on the standard physical interpretation of a Lagrangian of a system of fields (for example, ϕ). Usually the Lagrangian can be represented in the form $L = L_0[\phi] + gL_I[\phi]$. The physical particles ϕ are described by the «free Lagrangian» $L_0[\phi]$, which is quadratic over fields ϕ. The interactions of particles are described by the «interaction Lagrangian» $L_I[\phi]$, which contains the field operators ϕ in the third or more degree. This interpretation can be reasonable if the coupling constant g is small enough. The generating functional, or partition function contains amplitudes of all physical processes and can be written in the form of the functional integral

$$Z[J] = Z[J;\phi] = \int D\phi \; e^{i\int dx L_0[\phi] + ig\int dx L_I[\phi] + \int dx \phi J}. \qquad (1)$$

Usually we can perform calculations expanding the generating functional over the coupling constant g.

Let us suppose that we are able after some functional transformations and changing the functional variables to rewrite the representation (1) in the form

$$Z[J] = Z[J;B] = \int DB \; e^{i\int dx \mathrm{L}_0[B] + ig_{\mathrm{eff}}\int dx \mathrm{L}_I[B] + W[B,J]}. \qquad (2)$$

Here the «free Lagrangian» $\mathrm{L}_0[B]$ is quadratic over fields B and the «interaction Lagrangian» $\mathrm{L}_I[B]$ contains the field B in the third or more degree. The effective coupling constant g_{eff} should be small enough. Then we can say that the new Lagrangian $\mathrm{L} = \mathrm{L}_0[B] + g_{\mathrm{eff}}\mathrm{L}_I[B]$ describes the physical particles B and these paricles can be considered as bound states of the initial particles ϕ.

Let us demonstrate our method on the simple quantum field model describing the Yukawa interaction of charged scalar bosons Φ and neutral bosons ϕ. The Lagrangian density is

$$L(x) \quad = \quad \Phi^+(\Box - M^2)\Phi + \frac{1}{2}\phi(\Box - m^2)\phi + g\Phi^+\Phi\phi. \tag{3}$$

In this model it is possible to retrace all details of bound states arising in quantum field theory. Generalization to the case of the Dirac field presents no difficulties of principle and leads to technical problems connected with the algebra of γ-matrices only. This model is superrenormalizable so that the renormalization procedure has the simplest form.

1. The Initial Representation. Let us put the Lagrangian (3) into the representation (1) and integrate over the field ϕ, we get

$$Z \quad = \quad \int\int D\Phi D\Phi^+ \cdot e^{-(\Phi^+ D_M^{-1}\Phi) + \frac{g^2}{2}(\Phi^+\Phi D_m \Phi^+\Phi)}, \tag{4}$$

$$\begin{aligned}(\Phi^+ D_M^{-1}\Phi) \quad &= \quad \int dx\ \Phi^+(x)(-\Box + M^2)\Phi(x), \\ \frac{g^2}{2}(\Phi^+\Phi D_m\Phi^+\Phi) \quad &= \quad \int dx \int dy\ \Phi^+(x)\Phi(x)D_m(x-y)\Phi^+(y)\Phi(y).\end{aligned}$$

For simplicity we have omitted the term with the current J, which can be restored without any problems.

Let us introduce the bilocal current:

$$J(y_1, y_2) = \sqrt{D_m(y_1 - y_2)}(\Phi^+(y_1)\Phi(y_2)),$$

and use the Gaussian representation

$$e^{\frac{g^2}{2}(\Phi^+\Phi D_m\Phi^+\Phi)} = e^{\frac{g^2}{2}(J^+J)} = \int DA\ e^{-\frac{1}{2}(A^+A) - g\ (A^+J)}.$$

Here $A = A(x_1, x_2)$ is a bilocal field. Now we can calculate in (4) the Gaussian integral over Φ and Φ^+:

$$Z = \int DA\ e^{-\frac{1}{2}(A^+A) - \mathrm{tr}\ln\left[1 + g(A^+\sqrt{D_m})D_M\right]}. \tag{5}$$

2. Linear Term. Our problem is to give the standard particle interpretation to the action $S[A]$ in (5). For this aim this action should be represented in the form

$$S[A] = -\frac{1}{2}(A^+ R^{-1} A) + I_{int}[A], \qquad I_{int}[A] = O(A^3).$$

It means that we have to remove the term linear in A and extract the quadratic term out of $S[A]$. Let us introduce the displacement

$$A(y_1, y_2) = A_1(y_1, y_2) + \frac{a(x_1 - x_2)}{g\sqrt{D_m(x_1 - x_2)}}.$$

The term linear in A will be equal to zero if

$$a(x_1 - x_2) = -g^2 D_m(x_1 - x_2)\mathcal{D}(x_1 - x_2),$$

where

$$\mathcal{D} = D_M \cdot \frac{1}{1 + aD_M}, \qquad \tilde{\mathcal{D}}(k^2) = \frac{1}{M^2 + k^2 + \tilde{a}(k^2)},$$

which is the Schwinger–Dyson equation. In the momentum representation this Schwinger–Dyson equation contains the logarithmic ultraviolet divergence which can be removed by the renormalization of the mass M. It means that we should put

$$M^2 + \tilde{a}(k^2) = M_r^2 + \tilde{a}_r(k^2), \qquad \tilde{a}_r(k^2) = \tilde{a}(k^2) - \tilde{a}(-M_r^2),$$

where M_r is the «physical» mass of the constituent particle Φ and the renormalized function $a_r(k^2) = \tilde{a}(k^2) - \tilde{a}(-M_r^2)$ satisfies the equation

$$\begin{aligned} a_r(k^2) &= g^2 \int \frac{dp}{(2\pi)^4} \cdot \left[\frac{1}{(m^2 + (q-p)^2)(M_r^2 + p^2 + a_r(p^2))}\bigg|_{q^2 = -M_r^2} \right. \\ &- \left. \frac{1}{(m^2 + (k-p)^2)(M_r^2 + p^2 + a_r(p^2))} \right]. \end{aligned} \tag{6}$$

This functional equation is of the type $a_r(k^2) = F[a_r, k^2]$ and can be solved by the fixed point method, i.e., we choose the initial «point» $a_r^{(0)}(k^2)$ and calculate

$$a_r^{(n+1)}(k^2) = F[a_r^{(n)}, k^2], \quad \text{and} \quad \lim_{n\to\infty} a_r^{(n)}(k^2) = a_r(k^2).$$

In subsequent calculations we use the zeroth approximation

$$\tilde{\mathcal{D}}(k^2) = \tilde{D}_r = \frac{1}{M_r^2 + k^2}$$

which gives quite acceptable qualitative semiquantitative estimations.

3. From Bilocal to Local Fields. After removing the linear term we have

$$\begin{aligned} S[A] &= -\frac{1}{2}(A^+A) - \mathrm{tr}\ln_1\left[1 + g(A^+\sqrt{D_m})D_r\right], \\ &\ln_1(1+s) = \ln(1+s) - s. \end{aligned}$$

Let a system of functions $\{U_Q(y)\}$ with quantum numbers $Q = (nl\{\mu\})$, where n, l and $\{\mu\}$ are radial, orbital and magnetic quantum numbers, be orthonormal, i.e.,

$$\begin{aligned} &(U_Q U^*_{Q'}) = \int d^4y\ U_Q(y)U^*_{Q'}(y) = \delta_{QQ'} = \delta_{nn'}\delta_{ll'}\delta_{\{\mu\}\{\mu'\}}, \\ &\sum_Q U_Q(y)U^*_Q(y') = \delta(y-y'). \end{aligned} \tag{7}$$

Let us introduce in the «$\mathrm{tr}\ln_1\left[1 + g(A^+\sqrt{D_m})D_r\right]$» new variables

$$x_j = z_j + \frac{y_j}{2}, \qquad x_{j+1} = z_j - \frac{y_j}{2},$$

and represent the bilocal functions in the form:

$$A(x_j, x_{j+1}) = \sum_Q W_Q(z_j)U_Q(-y_j). \tag{8}$$

Then we have

$$\begin{aligned} &(A^+\sqrt{D_m}) = (WV) = \sum_Q W_Q(z)V_Q\left(\overleftrightarrow{p}_x\right), \qquad \overleftrightarrow{p}_x = \frac{1}{i}\left(\overleftarrow{\partial}_x - \overrightarrow{\partial}_x\right), \\ &V_Q\left(\overleftrightarrow{p}_x\right) = \int dy\sqrt{D_m(y)}U_Q(y)e^{-i\frac{y}{2}\overleftrightarrow{p}_x}. \end{aligned} \tag{9}$$

The basic representation for the partition function gets the form

$$Z = \int\prod_Q DW_Q \cdot e^{-\frac{1}{2}(WW) - \mathrm{tr}\ln_1[1+g_r(WV)D_r]}. \tag{10}$$

4. Particle Interpretation of the Quadratic Term. Let us extract the quadratic form from $\mathcal{S}[W]$

$$\begin{aligned} \mathcal{S}[W] &= -\frac{1}{2}(W[I - g_r^2\Pi]W) - \mathrm{tr}\ln_2[1 + g_r(WV)D_r], \\ &\ln_2(1+s) = \ln(1+s) - s + \frac{s^2}{2}. \end{aligned} \tag{11}$$

Here

$$(Wg_r^2\Pi W) = \sum_{QQ'}\int\int dxdx'\ W_Q(x)g_r^2\Pi_{QQ'}(x-x')W_{Q'}(x').$$

The polarization operator $g_r^2\tilde{\Pi}_{QQ'}$ looks

$$\begin{aligned} g_r^2\Pi_{QQ'}(x-x') &= g_r^2\iint dydy'\ U_Q(y)P(x-x';y,y')U^*_{Q'}(y'),\\ P(x;y,y') &= \sqrt{D_m(y)}D_r\left(x-\frac{y-y'}{2}\right)D_r\left(x+\frac{y-y'}{2}\right)\sqrt{D_m(y')},\\ \tilde{P}(p;y,y') &= \int dx\ e^{ipx}P(x;y,y'). \end{aligned}$$

In the momentum space we get

$$g_r^2\tilde{\Pi}_{QQ'}(p) = g_r^2\int\frac{dk}{(2\pi)^4}\cdot\frac{V_Q(k)V_{Q'}(k)}{\left(M_r^2+\left(k+\frac{p}{2}\right)^2\right)\left(M_r^2+\left(k-\frac{p}{2}\right)^2\right)}. \quad (12)$$

The orthonormal system $\{U_Q(x)\}$ should be chosen so that the polarization operator $\tilde{\Pi}_{QQ'}(p)$ should be diagonal in radial (n,n') and orbital (l,l') quantum numbers. The index structure of the diagonal polarization operator $\tilde{\Pi}^{(nl)}_{\{\mu\}\{\mu'\}}(p)$ looks like

$$\tilde{\Pi}^{(nl)}_{\{\mu\}\{\mu'\}}(p) = \tilde{\Pi}^{(nl)}(p^2)\cdot\delta_{\{\mu\}\{\mu'\}} + \sum_j\tilde{\Pi}^{(nl)}_j(p^2)\cdot t^j_{\{\mu\}\{\mu'\}}(p), \quad (13)$$

where the tensors $t^j_{\{\mu\}\{\mu'\}}(p)$ contain combinations of the vectors $p_\mu p_{\mu'}$.

The diagonal quadratic form of (11) gives the equation of motion for the field $W_Q(x) = W^{(nl)}_{\{\nu\mu_2\ldots\mu_l\}}(x)$

$$\left[\delta_{QQ'} - g_r^2\tilde{\Pi}_{QQ'}\left(\frac{\partial}{i\partial x}\right)\right]W_{Q'}(x) = 0, \quad \left[\delta_{QQ'} - g_r^2\tilde{\Pi}_{QQ'}(p)\right]\tilde{W}_{Q'}(p) = 0.$$

The requirement that this equation on the mass shell should be the Klein–Gordon equation gives the constraint

$$\frac{\partial}{\partial x_\nu}W^{(nl)}_{\nu\mu_2\ldots\mu_l}(x) = 0 \qquad \text{or} \qquad p_\nu\tilde{W}^{(nl)}_{\{\nu\mu_2\ldots\mu_l\}}(p) = 0$$

on the mass shell. Thus, the function $\tilde{W}^{(nl)}_{\{\mu\}}(p)$ satisfies the equation

$$\left[1 - g_r^2\tilde{\Pi}^{(nl)}(p^2)\right]\tilde{W}^{(nl)}_{\{\mu_1\ldots\mu_l\}}(p) = 0. \quad (14)$$

The mass of the state with quantum numbers $Q = (nl)$ is defined by

$$1 - g_r^2 \tilde{\Pi}^{(nl)}(-M^2_{(nl)}) = 0. \tag{15}$$

Let us write

$$-1 + g_r^2 \tilde{\Pi}^{(nl)}(p^2) = -Z_{(nl)}(p^2 + M^2_{(nl)}) + \Sigma^{(nl)}(p^2),$$
$$Z_{(nl)} = g_r^2 \left[-\tilde{\Pi}'_{(nl)}(-M^2_{(nl)}) \right], \quad \Sigma^{(nl)}(p^2) = O((p^2 + M^2_{(nl)})^2).$$

The constant $Z_{(nl)}$ is positive.

New field variables can be introduced as follows:

$$W_Q(x) = \frac{\varphi_Q(x)}{\sqrt{Z_{(nl)}}}. \tag{16}$$

The representation (10) assumes the form

$$Z = \int \prod_Q D\varphi_Q \; e^{-\frac{1}{2}(\varphi \mathcal{D}^{-1} \varphi) - \mathcal{I}_{\rm int}[\varphi]}. \tag{17}$$

Here the kinetic term is

$$\begin{aligned} (\varphi \mathcal{D}^{-1} \varphi) &= (\varphi \left[-\Box + M_b^2 + \Sigma_b \right] \varphi) \\ &= \int dp \sum_{\mathcal{Q}} \tilde{\varphi}^+_Q(p) \left[p^2 + M^2_{(nl)} + \Sigma^{(nl)}(p^2) \right] \tilde{\varphi}_Q(p) \end{aligned} \tag{18}$$

and the interaction term is

$$\mathcal{I}_{\rm int}[\varphi] = \mathrm{tr} \ln_2 \left[1 + (h\varphi V)\mathcal{D} \right], \tag{19}$$
$$(h\varphi V) = \sum_Q h_Q \varphi_Q V_Q, \qquad h_Q = \frac{1}{\sqrt{-\Pi'_Q(-M^2_Q)}}.$$

The effective dimensionless coupling constants are defined as

$$\lambda_Q^{(\rm eff)} = \frac{h_Q^2}{16\pi M_r^2} = \frac{1}{16\pi[-M_r^2 \tilde{\Pi}'_{(nl)}(-M^2_{(nl)})]}. \tag{20}$$

As a result, the final representation (17) can be interpreted as a partition function of the quantum field system of bosonic fields $\{\phi_Q\}$ which have masses M_Q and interact by means of the nonlocal interaction Lagrangian (19).

We would like to stress that the resulting representation for the generating functional does not contain the initial coupling constant g.

All calculations with the generating functional (17) can be performed by perturbation expansions in coupling constants h_Q. We can trust these calculations if and only if the effective coupling constants (20) are small enough:

$$\lambda_Q^{(\mathrm{eff})} \ll 1.$$

5. The Orthonormal System. The next step is to determine the orthonormal system (7). The problem is to find the spectrum and eigenfunctions of the operator $\tilde{P}(p; y, y')$ in (12), i.e.,

$$\int dy' \tilde{\mathcal{P}}(p; y, y') U_Q(y', p) = E_Q(p) U_Q(y, p), \qquad Q = (n, l, \{\mu\}). \quad (21)$$

This equation can be represented in a standard form of the Bethe–Salpeter equation in the one-boson exchange approximation. Using the relation

$$K_+ K_- \cdot \int dx\ e^{ipx} D_r \left(x - \frac{y - y'}{2} \right) D_r \left(x + \frac{y - y'}{2} \right) = \delta(y - y')$$

with

$$K_\pm = \left[M_r^2 + \left(i \frac{\partial}{\partial y} \pm \frac{p}{2} \right)^2 \right]$$

and introducing the functions

$$\Psi_Q(y, p) = \frac{1}{\sqrt{D_m(y)}} \cdot U_Q(y, p)$$

we get the standard form of the Bethe–Salpeter equation (see, for example, [10])

$$\left[M_r^2 + \left(i \frac{\partial}{\partial y} + \frac{p}{2} \right)^2 \right] \cdot \left[M_r^2 + \left(i \frac{\partial}{\partial y} - \frac{p}{2} \right)^2 \right] \Psi_Q(y, p) = g_r^2 D_m(y) \Psi_Q(y, p),$$

where the spectrum is defined by the equation

$$g_r^2 E_Q(-M_Q^2) = 1.$$

Thus the diagonalization of the operator $\tilde{P}(p; y, y')$ is equivalent to the solution of the Bethe–Salpeter equation in one-boson exchange approximation.

Now we would like to remark the following. Our mathematical task is to diagonalize an operator and we are not able to do it analytically. There exist two ways to overcome this difficulty and these ways are defined by physical problems under consideration. If we calculate corrections to precision experiments (for example, quantum electrodynamics phenomena), which require quite high

accuracy, we have to get solutions, accuracy of which should be around $10^{-5} \div 10^{-6}$ %. This accuracy can be obtained by numerical methods using quite powerful computers only. If we consider particle physics phenomena where required accuracy is not so high, then the mathematical method with accuracy $1 \div 5$ % is completely acceptable. Our method formulated in the given paper is relative to the second point of view.

The main problem to use the Bethe–Salpeter basis is that the Bethe–Salpeter equation can only be solved by numerical methods. Even the solution obtained by Wick and Cutkosky [8,10] is reduced to the differential equation which should be numerically computed. Our aim is to continue analytic calculations as long as possible in order to get a visible general picture of arising bound states in the system under consideration. Therefore we choose a more practical way, namely, we use an orthonormal basis that is simple enough from an analytic point of view and is directly connected with the problem under consideration. In this case the operators $g_r^2\tilde{\Pi}_{QQ'}$ are not diagonal so that we should diagonalize them. The idea consists in finding an effective basis for diagonalization of $g_r^2\tilde{\Pi}_{QQ'}$ such that its lowest function would provide a good qualitative description for the eigenvalues $E_{(nl)}$ and the next two or three functions only give a good quantitative description for those eigenvalues.

This effective basis $\{U_Q(x)\}$ can be constructed using the standard boson Green function $D_a(u)$ with a mass a as a weight function inducing uniquely the system of orthonormal polynomials in the space R^4. Thus, the full orthonormal system of functions (7) can be chosen in the form

$$U_Q(x,a) = i^l\sqrt{D_a(x)}aP_Q(ax). \tag{22}$$

Here $P_Q(u)$ are real polynomials. The mass parameter a that enters into the orthonormal system is fixed by a variation condition formulated below.

The construction of this basis is presented in the Appendix A in [11].

6. The Polarization Operator. The problem is that our basis does not diagonalize the polarization operators (12) for states $Q = (nl\{\mu\})$ and $Q' = (n'l\{\mu'\})$ and does not have the form (13), so that the diagonalization procedure should be performed. In the momentum space the nondiagonal polarization operators (12) for states $Q = (nl\{\mu\})$ and $Q' = (n'l\{\mu'\})$ according to the representation (13) for $p^2 = -M_b^2$ look like

$$g_r^2\tilde{\Pi}^{(nn',l)}(p^2|a) = \frac{32\lambda_r M_r^2}{(2\pi)^2}\cdot\frac{2^l(l!)^2}{(2l+1)!}\cdot\int\limits_0^\infty dk\ k^{3+2l}\cdot V_{(nl)}(k^2|a)V_{(n'l)}(k^2|a)\times$$

$$\times\int\limits_0^1\frac{dt\ (1-t^2)^{\frac{1}{2}+l}}{\left(M_r^2-\frac{M_b^2}{4}+k^2\right)^2+k^2M_b^2t^2},$$

where $\lambda_r = \frac{g_r^2}{16\pi M_r^2}$ and the vertex looks

$$V_{(nl)}(k^2|m,a) \quad = \quad \int dx \ \sqrt{D_m(x)D_a(x)}a^{1+l}P_{(nl)}(a^2x^2)e^{ixp}. \qquad (23)$$

We shall use the approximation

$$\sqrt{D_m(x)D_a(x)} \approx D_{\frac{m+a}{2}}(x), \qquad (24)$$

the accuracy of which is quite acceptable for our consideration. In particular

$$V_{(0l)}(k^2|m,a) = \frac{a^{1+l}\,2^{l/2}}{M_r^{2+2l}\left(\left(\frac{m+a}{2M_r}\right)^2 + \frac{k^2}{M_r^2}\right)^{1+l}}. \qquad (25)$$

Now we formulate the variational principle which defines the parameter a. The mass $M_{(nl)}$ of the bound state with quantum numbers (nl) should be defined by the equation

$$1 = \lambda_r \tilde{\Pi}_{(nn,l)}(M_{(nl)}|m,a).$$

The function $\tilde{\Pi}_{(nl)} = \tilde{\Pi}_{(nn,l)}$ is the largest eigenvalue of the matrix $\tilde{\Pi}^{(n_1n_2,l)}$ for $n_1, n_2 \geq n$, therefore the parameter $a = a_n$ can be defined by the variation requirement

$$\tilde{\Pi}_{(nl)}(b,\xi) = \max_a \ \tilde{\Pi}_{(nn,l)}(M_b|m,a), \qquad (26)$$

which gives $a = a_{(nl)}(b,\xi)$ where the notions $b = \left(\frac{M_b}{2M_r}\right)^2$, $\xi = \frac{m}{M_r}$ are used. Thus, the parameter $a_{(nl)}(b,\xi)$ is a function of m and M_b. The mass

$$M_{(nl)} = M_{(nl)}(\lambda_r,\xi) = 2M_r \cdot b_{(nl)}(\lambda_r,\xi)$$

is defined by the equation

$$1 = \lambda_r \tilde{\Pi}_{(n,l)}(b_{(nl)},\xi). \qquad (27)$$

In order to show that this orthonormal functions with the parameter $\eta = \frac{a}{M_r}$ give quite good approximation for the eigenvalues of the matrix $\tilde{\Pi}_{(nn')}(p^2)$ we have calculated the matrix

$$\mathcal{P}^{(N)} = \left\{\tilde{\Pi}_{(n_1n_2)}(b,\xi,\eta), \quad (n_1,n_2 = 0,1,2,...,N)\right\}$$

and their eigenvalues

$$\mathcal{E}_n^{(N)} = \mathrm{diag}\left\{\mathcal{P}^{(N)}\right\} = \mathrm{diag}\left\{E_0^{(N)}, E_1^{(N)}, ..., E_N^{(N)}\right\}.$$

Then we have to compare $E_n^{(N)}$ for different $N = 0, 1, ...$

The numerical results are given in the Table: the first case for $\xi = .5,\ b = .25,\ \eta = 2.451$ and the second case for $\xi = .2,\ b = .9,\ \eta = 1.22$. One can see that for the lowest eigenvalue practically the first lowest eigenfunction can be used, i.e., our choice of the orthonormal system gives quite a good accuracy.

Table. Diagonalization of the matrix $\mathcal{P}_N$

N	E_0	E_1	E_2	E_3	E_0	E_1	E_2	E_3
0	.04165				.1239			
1	.04166	.009941			.1262	.03564		
2	.04173	.010279	.002755		.1262	.03616	.01162	
3	.04175	.010368	.003295	.0007482	.1263	.03645	.01298	.003789
4	.04175	.010402	.003546	.0010336	.1263	.03655	.01373	.004710

In conclusion we can say that the representations (4) and (17) are equivalent and the representation (17) contains the bound states of the initial system (3) of particles Φ and ϕ.

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