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RENORMALIZATIONS IN SOFTLY BROKEN SUSY GAUGE THEORIES AND GRASSMANNIAN EXPANSION

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It is demonstrated that renormalizations in a softly broken SUSY gauge theory follow those of an unbroken one with the modification of the coupling constants. This gives an explicit relation between the soft and rigid coupling renormalizations. Substituting the modified couplings into renormalization constants, RG equations, solutions to these equations, approximate solutions, fixed points, etc., one can get corresponding relations for the soft terms by a simple Taylor expansion over the Grassmannian variables. Some new examples including the MSSM in high $\tan\beta$ regime and softly broken N=2 SUSY Seiberg–Witten model are given.

1. INTRODUCTION

In a recent paper [1], which is based on the previous publications [2, 3] we have shown that renormalizations in a softly broken SUSY theory follow from those of an unbroken SUSY theory in a simple way.

The main idea is that a softly broken supersymmetric gauge theory can be considered as a rigid SUSY theory imbedded into external space-time independent superfield, so that all couplings and masses become external superfields. The crucial statement is that the *singular* part of effective action depends on external superfield, but not on its derivatives, so that one can calculate it when the external field is a constant, i.e., in a rigid theory. This approach to a softly broken sypersymmetric theory allows us to use remarkable mathematical properties of $N = 1$ SUSY theories such as nonrenormalization theorems, cancellation of quadratic divergences, etc. The renormalization procedure in a softly broken SUSY gauge theory can be performed in the following way:

One takes renormalization constants of a rigid theory, calculated in some massless scheme, substitutes instead of the rigid couplings (gauge and Yukawa) their modified expressions, which depend on a Grassmannian variable, and expand over this variable.

This gives renormalization constants for the soft terms. Differentiating them with respect to a scale one can find corresponding renormalization group equations.

In fact as it has been shown in [4] this procedure works at all stages. One can make the above-mentioned substitution on the level of the renormalization constants, RG equations, solutions to these equations, approximate solutions, fixed points, finiteness conditions, etc. Expanding then over a Grassmannian variable one obtains corresponding expressions for the soft terms. This way one can get new solutions of the RG equations and explore their asymptotics, or approximate solutions, or find their stability properties, starting from the known expressions for a rigid theory.

Below we give some examples and in particular consider the MSSM within high $\tan\beta$ regime, where analytical solutions are known in iterative form and obtain iterative solutions to the RG equations for the soft mass terms. Another example is the N=2 SUSY model, where the exact (nonperturbative) Seiberg–Witten solution is known. Here one can extend the S–W solution to the soft terms.

2. SOFT SUSY BREAKING AND RENORMALIZATION

Consider an arbitrary $N=1$ SUSY gauge theory with unbroken SUSY. The Lagrangian of a rigid theory is given by

$$\begin{aligned}\mathcal{L}_{\text{rigid}} &= \int d^2\theta\, \frac{1}{4g^2}\,\mathrm{Tr}W^\alpha W_\alpha + \int d^2\bar\theta\, \frac{1}{4\bar g^2}\,\mathrm{Tr}\bar W^\alpha \bar W_\alpha \\ &+ \int d^2\theta d^2\bar\theta\ \ \bar\Phi^i (e^V)^j_i \Phi_j + \int d^2\theta\ \ \mathcal{W} + \int d^2\bar\theta\ \ \bar{\mathcal{W}}, \qquad (1)\end{aligned}$$

where W^α is the field strength chiral superfield and the superpotential $\mathcal{W}$ has the form

$$\mathcal{W} = \frac{1}{6}\lambda^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}M^{ij}\Phi_i\Phi_j. \qquad (2)$$

To perform the SUSY breaking, which satisfies the requirement of «softness», one can introduce a gaugino mass term as well as cubic and quadratic interactions of scalar superpartners of the matter fields [2]

$$-\mathcal{L}_{\text{soft-breaking}} = \left[\frac{M}{2}\lambda\lambda + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}B^{ij}\phi_i\phi_j + \text{h.c.}\right] + (m^2)^i_j\phi^*_i\phi^j, \qquad (3)$$

where λ is the gaugino field and ϕ_i is the lower component of the chiral matter superfield.

One can rewrite the Lagrangian (3) in terms of N=1 superfields introducing the external spurion superfields [2] $\eta = \theta^2$ and $\bar\eta = \bar\theta^2$, where θ and $\bar\theta$ are

Grassmannian parameters, as [3]

$$\mathcal{L}_{\rm soft}=\int d^2\theta\ \frac{1}{4g^2}(1-2M\theta^2){\rm Tr}W^\alpha W_\alpha+\int d^2\bar\theta\ \frac{1}{4g^2}(1-2\bar M\bar\theta^2){\rm Tr}\bar W^\alpha \bar W_\alpha$$

$$+\int d^2\theta d^2\bar\theta\ \ \bar\Phi^i(\delta_i^k-(m^2)_i^k\eta\bar\eta)(e^V)_k^j\Phi_j$$

$$+\int d^2\theta\left[\frac{1}{6}(\lambda^{ijk}-A^{ijk}\eta)\Phi_i\Phi_j\Phi_k+\frac{1}{2}(M^{ij}-B^{ij}\eta)\Phi_i\Phi_j\right]+{\rm h.c.} \tag{4}$$

Comparing Eqs. (1) and (4) one can see that Eq. (4) is equivalent to Eq. (1) with modification of the rigid couplings g^2, λ^{ijk}, and M^{ij}, so that they become external superfields dependent on Grassmannian parameters θ^2 and $\bar\theta^2$. The scalar mass term $m^2\eta\bar\eta$ modifies fields Φ and $\bar\Phi$, however, finally it can be rewritten as a modification of the Yukawa couplings ($\eta=\theta^2,\ \bar\eta=\bar\theta^2$).

$$\begin{aligned}
\tilde g_i^2 &= g_i^2(1+M_i\eta+\bar M_i\bar\eta+2M_i\bar M_i\eta\bar\eta), &(5)\\
\tilde\lambda^{ijk} &= \lambda^{ijk}-A^{ijk}\eta+\frac{1}{2}\left(\lambda^{njk}(m^2)_n^i+\lambda^{ink}(m^2)_n^j+\lambda^{ijn}(m^2)_n^k\right)\eta\bar\eta, &(6)\\
\tilde{\bar\lambda}_{ijk} &= \bar\lambda_{ijk}-\bar A_{ijk}\bar\eta+\frac{1}{2}\left(\bar\lambda_{njk}(m^2)_i^n+\bar\lambda_{ink}(m^2)_j^n+\bar\lambda_{ijn}(m^2)_k^n\right)\eta\bar\eta. &(7)
\end{aligned}$$

These modifications of the couplings and fields are valid not only for the classical Lagrangian but also for the quantum one.*

In what follows we would like to simplify the notations and consider numerical rather than tensorial couplings. When group structure and field content of the model are fixed, one has a set of gauge $\{g_i\}$ and Yukawa $\{\lambda_k\}$ couplings. It is useful to consider the following rigid parameters $\alpha_i\equiv g_i^2/16\pi^2,\ Y_k\equiv \lambda_k\bar\lambda_k/16\pi^2$, then Eqs.(5)–(7) look like

$$\begin{aligned}
\tilde\alpha_i &= \alpha_i(1+M_i\eta+\bar M_i\bar\eta+(M_i\bar M_i+\Sigma_{\alpha_i})\eta\bar\eta),\\
\tilde Y_k &= Y_k(1+A_k\eta+\bar A_k\bar\eta+(A_k\bar A_k+\Sigma_k)\eta\bar\eta),
\end{aligned} \tag{8}$$

where to standardize the notations we have redefined parameter A: $A\to A\lambda$ in a usual way and have changed the sign of A to match it with the gauge soft terms. Here Σ_k stands for a sum of m^2 soft terms, one for each leg in the Yukawa vertex, and

$$\Sigma_{\alpha_i}=M_i\bar M_i+\tilde m^2_{gh_i}, \tag{9}$$

*Throughout the paper we use a supergraph technique and assume the existence of some SUSY invariant regularization.

where $\tilde{m}^2_{gh_i}$ is the soft ghost mass, which is eliminated by solving the RG equation and in one-loop $\tilde{m}^2_{gh_i} = 0$.

One can make the expansion for any analytic solution in a rigid theory. Below we consider two particular examples, namely the MSSM in high $\tan\beta$ regime and the Seiberg–Witten N=2 SUSY model.

3. EXAMPLES

The MSSM in High $\tan\beta$ Regime

Consider the MSSM in high $\tan\beta$ regime. One has three gauge and three Yukawa couplings. The one-loop RG equations are [6]

$$\dot{\alpha}_i = -b_i\alpha_i^2, \qquad \dot{Y}_k = Y_k(\sum_i c_{ki}\alpha_i - \sum_l a_{kl}Y_l), \tag{10}$$

where $\cdot \equiv d/dt,\ t = \log M^2_{GUT}/Q^2$ and

$$\begin{aligned} b_i &= \{33/5, 1, -3\}, \quad a_{tl} = \{6,1,0\}, \quad a_{bl} = \{1,6,1\}, \quad a_{\tau l} = \{0,3,4\}, \\ c_{ti} &= \{13/15, 3, 16/3\}, \quad c_{bi} = \{7/15, 3, 16/3\}, \quad c_{\tau i} = \{9/5, 3, 0\}. \end{aligned}$$

The general solution to Eqs.(10) can be written as [7]

$$\alpha_i = \frac{\alpha_i^0}{1+b_i\alpha_i^0 t}, \qquad Y_k = \frac{Y_k^0 u_k}{1 + a_{kk}Y_k^0\int_0^t u_k}, \tag{11}$$

where the functions $\{u_k\}$ obey the integral system of equations

$$\begin{aligned} u_t &= \frac{E_t}{(1+6Y_b^0\int_0^t u_b)^{1/6}}, \qquad u_\tau = \frac{E_\tau}{(1+6Y_b^0\int_0^t u_b)^{1/2}}, \\ u_b &= \frac{E_b}{(1+6Y_t^0\int_0^t u_t)^{1/6}(1+4Y_\tau^0\int_0^t u_\tau)^{1/4}}, \end{aligned} \tag{12}$$

and the functions E_k are given by $E_k = \prod_{i=1}^3 (1+b_i\alpha_i^0 t)^{c_{ki}/b_i}$.

Let us stress that Eqs. (11) give the exact solution to Eqs. (10), while the u_k's in Eqs. (12), although solved formally in terms of the E_k's and Y_k^0's as continued integrated fractions, should in practice be solved iteratively.

To get the solutions for the soft terms it is enough to perform substitution $\alpha_i \to \tilde{\alpha}_i$ and $Y_k \to \tilde{Y}_k$ from Eqs. (8) and expand over η and $\bar{\eta}$. One has [8]:

$$\begin{aligned} m_i &= \frac{m_i^0}{1+b_i\alpha_i^0 t}, \qquad A_k = -e_k + \frac{A_k^0/Y_k^0 + a_{kk}\int u_k e_k}{1/Y_k^0 + a_{kk}\int u_k}, \\ \Sigma_k &= \xi_k + A_k^2 + 2e_kA_k - \frac{(A_k^0)^2/Y_k^0 - \Sigma_k^0/Y_k^0 + a_{kk}\int u_k\xi_k}{1/Y_k^0 + a_{kk}\int u_k}, \end{aligned} \tag{13}$$

where the new functions e_k and ξ_k have been introduced which obey the iteration equations. For illustration we present below the corresponding expressions for e_t and ξ_t

$$
\begin{aligned}
e_t &= \frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta} + \frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b}, \\
\xi_t &= \frac{1}{E_t}\frac{d^2\tilde{E}_t}{d\eta d\bar{\eta}} + 2\frac{1}{E_t}\frac{d\tilde{E}_t}{d\eta}\frac{A_b^0\int u_b - \int u_b e_b}{1/Y_b^0 + 6\int u_b} + 7\frac{\left(A_b^0\int u_b - \int u_b e_b\right)^2}{\left(1/Y_b^0 + 6\int u_b\right)^2} - \\
&\quad - \frac{(\Sigma_b^0 + (A_b^0)^2)\int u_b - 2A_b^0\int u_b e_b + \int u_b \xi_b}{1/Y_b^0 + 6\int u_b},
\end{aligned}
\tag{14}
$$

where the variations of $\tilde{E}_k$ should be taken at $\eta = \bar{\eta} = 0$ and are given by

$$
\left.\frac{1}{E_k}\frac{d\tilde{E}_k}{d\eta}\right|_{\eta,\bar{\eta}=0} = t\sum_{i=1}^{3} c_{ki}\alpha_i m_i^0, \tag{15}
$$

$$
\begin{aligned}
\left.\frac{1}{E_k}\frac{d^2\tilde{E}_k}{d\eta d\bar{\eta}}\right|_{\eta,\bar{\eta}=0} &= t^2\left(\sum_{i=1}^{3} c_{ki}\alpha_i m_i^0\right)^2 + \\
&+2t\sum_{i=1}^{3} c_{ki}\alpha_i (m_i^0)^2 - t^2\sum_{i=1}^{3} c_{ki} b_i \alpha_i^2 (m_i^0)^2.
\end{aligned}
\tag{16}
$$

When solving Eqs. (12) and (15) in the nth iteration one has to substitute in the r.h.s. the $(n-1)$-th iterative solution for all the corresponding functions.

The same procedure works for the approximate solutions [9]. Once one gets an approximate solution for the Yukawa couplings, one immediately has those for the soft terms as well [9].

N=2 SUSY

Consider now the $N=2$ supersymmetric gauge theory. The Lagrangian written in terms of $N=2$ superfields is [10]:

$$
\mathcal{L} = \frac{1}{4\pi}\mathcal{I}m Tr\int d^2\theta_1 d^2\theta_2 \frac{1}{2}\tau\Psi^2, \tag{17}
$$

where $N = 2$ chiral superfield $\Psi(y, \theta_1, \theta_2)$ is defined by constraints $\bar{D}_{\dot\alpha}\Psi = 0$ and $\bar{\tilde{D}}_{\dot\alpha}\Psi = 0$ and

$$\tau = i\frac{4\pi}{g^2} + \frac{\theta^{\text{topological}}}{2\pi}. \tag{18}$$

The expansion of Ψ in terms of θ_2 can be written as

$$\Psi(y, \theta_1, \theta_2) = \Psi^{(1)}(y, \theta_1) + \sqrt{2}\theta_2^\alpha \Psi_\alpha^{(2)}(y, \theta_1) + \theta_2^\alpha \theta_{2\alpha} \Psi^{(3)}(y, \theta_1),$$

where $y^\mu = x^\mu + i\theta_1\sigma^\mu\bar\theta_1 + i\theta_2\sigma^\mu\bar\theta_2$ and $\Psi^{(k)}(y, \theta_1)$ are $N = 1$ chiral superfields.

The soft breaking of $N = 2$ SUSY down to $N = 0$ can be achieved by shifting the imaginary part of τ:

$$\mathcal{I}m\tau \to \mathcal{I}m\tilde\tau = \mathcal{I}m\tau(1 + M_1\theta_1^2 + M_2\theta_2^2 + M_3\theta_1^2\theta_2^2). \tag{19}$$

This leads to

$$\Delta\mathcal{L} \quad = \quad \left[-\frac{M_1}{4}\lambda\lambda - \frac{M_2}{4}\psi\psi - (\frac{M_1M_2}{4} - \frac{M_3}{4})\phi\phi + \text{h.c.}\right] - (\frac{M_1^2}{4} + \frac{M_2^2}{4})\bar\phi\phi,$$

where the fields λ are the gauginos, ψ and ϕ are the spinor and scalar matter fields, respectively.

Now one can use the power of duality in $N = 2$ SUSY theory and take the Seiberg–Witten solution [11]

$$\tau = \frac{da_D}{du}/\frac{da}{du}, \tag{20}$$

where

$$a_D(u) = \frac{i}{2}(u-1)F(1/2, 1/2, 2; \frac{1-u}{2}),$$
$$a(u) = \sqrt{2(1+u)}F(-1/2, 1/2, 1; \frac{2}{1+u}).$$

In perturbative domain when $u \sim Q^2/\Lambda^2 \to \infty$, $a = \sqrt{2u}$, $\quad a_D = \frac{i}{\pi}a(2\ln a + 1)$ one reproduces the well-known one-loop result

$$\frac{4\pi}{g^2} = \frac{1}{\pi}[\ln\frac{Q^2}{\Lambda^2} + 3]. \tag{21}$$

Assuming that renormalizations in $N = 2$ SUSY theory follow the properties of those in $N = 0$ one can try to apply the same expansion procedure for a nonperturbative solution. Substituting Eq. (19) into (20) with

$$u \to \tilde u = u(1 + M_1^0\theta_1^2 + M_2^0\theta_2^2 + M_3^0\theta_1^2\theta_2^2)$$

and expanding over θ_1^2 and θ_2^2, one gets an analog of S-W solution for the mass terms:

$$M_1 = M_1^0 \frac{\mathcal{I}m \left[u \left(\frac{a_D''}{a_D'} - \frac{a''}{a'} \right) \tau \right]}{\mathcal{I}m\ \tau}, \qquad M_2 = M_2^0 \frac{\mathcal{I}m \left[u \left(\frac{a_D''}{a_D'} - \frac{a''}{a'} \right) \tau \right]}{\mathcal{I}m\ \tau},$$

$$M_3 = \frac{\mathcal{I}m \left[M_3^0 u \left(\frac{a_D''}{a_D'} - \frac{a''}{a'} \right) \tau + M_1^0 M_2^0 u^2 \left(\frac{a_D'''}{a_D'} - \frac{a'''}{a'} + \left(\frac{a_D''}{a_D'} - \frac{a''}{a'} \right)^2 + \left(\frac{a_D''}{a_D'} \right)^2 - \left(\frac{a''}{a'} \right)^2 \right) \tau \right]}{\mathcal{I}m\ \tau}.$$

In perturbative regime one has

$$M_1 \;=\; \frac{M_1^0}{\ln Q^2/\Lambda^2 + 3}, \qquad M_2 = \frac{M_2^0}{\ln Q^2/\Lambda^2 + 3}, \qquad M_3 = \frac{M_3^0 - M_1^0 M_2^0}{\ln Q^2/\Lambda^2 + 3}.$$

4. CONCLUSION

We conclude that the Grassmannian expansion in softly broken SUSY theories happens to be a very efficient and powerful method which can be applied in various cases where the renormalization procedure is concerned. It demonstrates once more that softly broken SUSY theories are contained in rigid ones and inherit their renormalization properties.

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