

NEUTRINO IN STANDARD MODEL AND BEYOND

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After discovery of the Higgs boson at CERN, the Standard Model acquired a status of the theory of elementary particles in the electroweak range (up to about 300 GeV). What general conclusions can be inferred from the Standard Model? It looks that the Standard Model teaches us that in the framework of such general principles as local gauge symmetry, unification of weak and electromagnetic interactions, and Brout–Englert–Higgs spontaneous breaking of the electroweak symmetry, nature chooses the simplest possibilities. Two-component left-handed massless neutrino fields play crucial role in the determination of the charged current structure of the Standard Model. The absence of the right-handed neutrino fields in the Standard Model is the simplest, most economical possibility. In such a scenario, Majorana mass term is the only possibility for neutrinos to be massive and mixed. Such a mass term is generated by the lepton-number violating Weinberg effective Lagrangian. In this approach, three Majorana neutrino masses are suppressed with respect to the masses of other fundamental fermions by the ratio of the electroweak scale and the scale of a lepton-number violating physics. The discovery of the neutrinoless double β -decay and the absence of transitions of flavor neutrinos into sterile states would be the evidence in favor of the minimal scenario we advocate here.

После открытия хиггсовского бозона в ЦЕРН Стандартная модель приобрела статус теории элементарных частиц в электрослабой области (до энергий приблизительно 300 ГэВ). Какие общие заключения можно сделать из Стандартной модели? Стандартная модель учит нас, что в рамках таких общих принципов, как локальная калибровочная симметрия, объединение слабого и электромагнитного взаимодействий и спонтанное нарушение электрослабой симметрии Браута–Энглера–Хиггса, природа выбирает простейшие возможности. Двухкомпонентное левое безмассовое нейтрино играет доминирующую роль в определении структуры заряженного тока Стандартной модели. Отсутствие правых нейтринных полей в Стандартной модели — простейшая, наиболее экономная возможность. В такой схеме майорановский массовый член — единственно возможный. Такой массовый член генерируется эффективным лагранжианом Вайнберга, нарушающим лептонный заряд. При этом массы нейтрино подавлены по сравнению с массами других фундаментальных фермионов отношением параметра, который характеризует электрослабый масштаб, и параметра, который характеризует масштаб новой физики. Открытие безнейтринного двойного бета-распада и отсутствие

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переходов флейворных нейтрино в стерильные состояния были бы свидетельствами в пользу рассмотренной схемы.

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INTRODUCTION

The discovery of neutrino oscillations in the atmospheric Super-Kamiokande experiment [1] in the SNO [2] and other solar neutrino experiments [3–5], and in the long-baseline reactor KamLAND experiment [6] is one of the most important recent discovery in the particle physics. The phenomenon of the neutrino oscillations was further investigated in the long-baseline accelerator K2K [7], MINOS [8] and *T2K* [9] experiments, in the reactor experiments Daya Bay [10], RENO [11] and Double Chooz [12], and in the solar BOREXINO experiment [13].

Neutrino oscillation results imply that the flavor neutrino fields $\nu_{lL}(x)$ ($l = e, \mu, \tau$) are the “mixtures” of the left-handed components of the fields of neutrinos with definite masses

$$\nu_{lL}(x) = \sum_{i=1}^3 U_{li} \nu_{iL}(x). \quad (1)$$

Here U is the unitary PMNS mixing matrix [14–16], and $\nu_i(x)$ is the field of neutrino (Majorana or Dirac) with mass m_i . Flavor fields $\nu_{lL}(x)$ enter into Standard Model charged current (CC)

$$\mathcal{L}_I^{\text{CC}}(x) = -\frac{g}{2\sqrt{2}} j_\alpha^{\text{CC}}(x) W^\alpha(x) + \text{h.c.} \quad (2)$$

and neutral current (NC) interactions

$$\mathcal{L}_I^{\text{NC}}(x) = -\frac{g}{2 \cos \theta_W} j_\alpha^{\text{NC}}(x) Z^\alpha(x). \quad (3)$$

Here

$$j_\alpha^{\text{CC}}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_\alpha l_L(x) \quad (4)$$

is the leptonic CC and

$$j_\alpha^{\text{NC}}(x) = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_\alpha \nu_{lL}(x) \quad (5)$$

is the neutrino NC; $W^\alpha(x)$ and $Z^\alpha(x)$ are the fields of W^\pm and Z^0 vector bosons; g is the electroweak interaction constant, and θ_W is the weak (Weinberg) angle.

We will consider now briefly *phenomenon of neutrino oscillations in vacuum* (see, for example, reviews [17, 18]). In the mixing relation (1) quantum fields enter. What about the states of the flavor neutrinos ν_e, ν_μ, ν_τ in the case of neutrino mixing?

The flavor neutrino ν_l is produced in CC weak decays together with l^+ or produces l^- in CC neutrino processes (for example, the muon neutrino ν_μ is produced in the decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or produces μ^- in the process $\nu_\mu + N \rightarrow \mu^- + X$, etc.).

From the Heisenberg uncertainty relation follows that in neutrino production and detection processes it is impossible to reveal small neutrino mass-squared differences. The state of the flavor neutrino ν_l is a *coherent superposition of states of neutrinos with definite masses* (see, for example, [19])

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle. \quad (6)$$

Here $|\nu_i\rangle$ is the state of neutrino with mass m_i , momentum \vec{p} and energy $E_i = \sqrt{p^2 + m_i^2} \simeq p + m_i^2/2E$.

Small neutrino mass-squared differences can be revealed in neutrino experiments with large distances between a source and a detector. For the evolution of the flavor neutrino state we have

$$|\nu_l\rangle_t = e^{-iH_0 t} |\nu_l\rangle = \sum_i |\nu_i\rangle e^{-iE_i t} U_{li}^* = \sum_{l'} |\nu_{l'}\rangle \left(\sum_i U_{l'i} e^{-iE_i t} U_{li}^* \right). \quad (7)$$

From (7) for the probability of $\nu_l \rightarrow \nu_{l'}$ transition we find the following expressions:

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \delta_{l'l} + \sum_{i \neq p} U_{l'i} (e^{-i(E_i - E_p)t} - 1) U_{li}^* \right|^2, \quad (8)$$

where p is an arbitrary fixed index.

For the ultrarelativistic neutrino we have $t \simeq L$, where L is the distance between a neutrino source and a neutrino detector. From (8) it follows that neutrino oscillations can be observed if

$$(E_i - E_p)t \simeq \frac{\Delta m_{pi}^2 L}{2E} \gtrsim 1, \quad (9)$$

where $\Delta m_{pi}^2 = m_i^2 - m_p^2$. The inequality (9) is the time-energy uncertainty relation applied to neutrino oscillations (see [19]).

In the more general case of the mixing of three-flavor neutrino fields and n_s sterile neutrino fields ν_{sL} we have

$$\nu_{\alpha L}(x) = \sum_{i=1}^{3+n_s} U_{\alpha i} \nu_{iL}(x), \quad \alpha = e, \mu, \tau, s_1, \dots, s_{n_s}. \quad (10)$$

For $\nu_\alpha \rightarrow \nu_{\alpha'}$ ($\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$) transition probability we find the following expression [20]:

$$\begin{aligned}
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) &= \delta_{\alpha'\alpha} - 4 \sum_i |U_{\alpha i}|^2 (\delta_{\alpha'\alpha} - |U_{\alpha' i}|^2) \sin^2 \Delta_{pi} + \\
&+ 8 \sum_{i>k} \text{Re } U_{\alpha' i} U_{\alpha i}^* U_{\alpha' k}^* U_{\alpha k} \cos(\Delta_{pi} - \Delta_{pk}) \sin \Delta_{pi} \sin \Delta_{pk} \pm \\
&\pm 8 \sum_{i>k} \text{Im } U_{\alpha' i} U_{\alpha i}^* U_{\alpha' k}^* U_{\alpha k} \sin(\Delta_{pi} - \Delta_{pk}) \sin \Delta_{pi} \sin \Delta_{pk}. \quad (11)
\end{aligned}$$

Here $\Delta_{pk} = \frac{\Delta m_{pk}^2 L}{4E}$, $\Delta m_{ik}^2 = m_k^2 - m_i^2$, and $\alpha, \alpha' = e, \mu, \tau, s_1, \dots, s_{n_s}$.

Existing neutrino oscillation data are perfectly described if we assume three-neutrino mixing. Two neutrino mass spectra are possible in this case:

1) normal spectrum (NS)

$$m_1 < m_2 < m_3, \quad (\Delta m_{12}^2 \equiv \Delta m_S^2) \ll (\Delta m_{23}^2 \equiv \Delta m_A^2), \quad (12)$$

2) inverted spectrum (IS)

$$m_3 < m_1 < m_2, \quad (\Delta m_{12}^2 \equiv \Delta m_S^2) \ll (|\Delta m_{13}^2| \equiv \Delta m_A^2). \quad (13)$$

For the normal neutrino mass spectrum from (11), we find the following expression:

$$\begin{aligned}
P^{\text{NS}}(\bar{\nu}_l \rightarrow \bar{\nu}_l) &= \delta_{ll} - 4 |U_{l1}|^2 (\delta_{ll} - |U_{l1}|^2) \sin^2 \Delta_S - \\
&- 4 |U_{l3}|^2 (\delta_{ll} - |U_{l3}|^2) \sin^2 \Delta_A - 8 \text{Re } U_{l3} U_{l3}^* U_{l1}^* U_{l1} \times \\
&\times \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S \mp 8 \text{Im } U_{l3} U_{l3}^* U_{l1}^* U_{l1} \times \\
&\times \sin(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S, \quad (14)
\end{aligned}$$

where solar and atmospheric neutrino mass-squared differences Δm_S^2 and Δm_A^2 are determined by the relation (12) and we choose $p = 2$.

In the case of the inverted mass spectrum, we choose $p = 1$. For the transition probability from (11) we have

$$\begin{aligned}
P^{\text{IS}}(\bar{\nu}_l \rightarrow \bar{\nu}_l) &= \delta_{ll} - 4 |U_{l2}|^2 (\delta_{ll} - |U_{l2}|^2) \sin^2 \Delta_S - \\
&- 4 |U_{l3}|^2 (\delta_{ll} - |U_{l3}|^2) \sin^2 \Delta_A - 8 \text{Re } U_{l3} U_{l3}^* U_{l2}^* U_{l2} \times \\
&\times \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S \pm 8 \text{Im } U_{l3} U_{l3}^* U_{l2}^* U_{l2} \times \\
&\times \sin(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S, \quad (15)
\end{aligned}$$

where Δ_S and Δ_A are determined by (13).

If neutrinos with definite masses ν_i are Dirac particles, the 3×3 PMNS mixing matrix is characterized by three mixing angles and one CP phase. In the standard parameterization it has the following form:

$$U^D = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}. \quad (16)$$

Here $c_{12} = \cos \theta_{12}$, $s_{12} = \sin \theta_{12}$, etc.

If neutrinos with definite masses are Majorana particles, the mixing matrix is given by the expression

$$U^M = U^D S, \quad (17)$$

where $S = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$ is a diagonal phase matrix. From (8) follows that Majorana phases $\alpha_{1,2}$ do not enter into neutrino transition probabilities [21, 22].

In Table 1, we present values of neutrino oscillation parameters obtained from the recent global analysis of the neutrino oscillation data [23].

Table 1. The values of neutrino oscillation parameters

Parameter	Normal spectrum	Inverted spectrum
$\sin^2 \theta_{12}$	$0.304_{-0.012}^{+0.013}$	$0.304_{-0.012}^{+0.013}$
$\sin^2 \theta_{23}$	$0.452_{-0.028}^{+0.052}$	$0.579_{-0.037}^{+0.025}$
$\sin^2 \theta_{13}$	$0.0218_{-0.0010}^{+0.0010}$	$0.0219_{-0.0010}^{+0.0011}$
$\delta, ^\circ$	(306_{-70}^{+39})	(254_{-62}^{+63})
Δm_{S}^2	$(7.50_{-0.17}^{+0.19}) \cdot 10^{-5} \text{ eV}^2$	$(7.50_{-0.17}^{+0.19}) \cdot 10^{-5} \text{ eV}^2$
Δm_A^2	$(2.457_{-0.047}^{+0.047}) \cdot 10^{-3} \text{ eV}^2$	$(2.449_{-0.047}^{+0.048}) \cdot 10^{-3} \text{ eV}^2$

As we see from this Table, existing neutrino oscillation data do not allow one to distinguish normal and inverted neutrino mass spectra. Neutrino oscillation parameters are known at present with accuracies from about 3% ($\Delta m_{S,A}^2$) to about 10% ($\sin^2 \theta_{23}$).

Neutrino oscillation data allow one to determine only neutrino mass-squared differences. Absolute values of the neutrino masses at present are unknown. From the measurement of the high-energy part of the β -spectrum of tritium in Mainz [24] and Troitsk [25] experiments, it was found, respectively,

$$m_\beta < 2.3 \text{ eV (Mainz)}, \quad m_\beta < 2.05 \text{ eV (Troitsk)}. \quad (18)$$

Here $m_\beta = \left(\sum_i |U_{ei}|^2 m_i^2 \right)^{1/2}$.

From the recent results of the Planck and other cosmological measurements for the sum of the neutrino masses, it was obtained the following bound [26]:

$$\sum_i m_i < 0.23 \text{ eV}. \quad (19)$$

From these bounds it follows that neutrino masses are much smaller than masses of other fundamental fermions (leptons and quarks). By this reason, it is unlikely that neutrino masses are of the same Standard Model Higgs origin as masses of quarks and leptons. Small neutrino masses are commonly considered as a signature of a beyond the Standard Model physics. However, at present a mechanism of generation of neutrino masses and neutrino mixing is unknown. In this introductory section, we will briefly consider general possibilities for neutrino masses and mixing (see reviews [17, 18]).

Masses and mixing are characterized by a mass term which (in the fermion case we are interested in) is a sum of Lorentz-invariant products of left-handed and right-handed components of the fields. For charged particles, only Dirac mass term is allowed. Since electric charges of neutrinos are equal to zero, three neutrino mass terms are possible. The left-handed flavor fields $\nu_{lL}(x)$, which enter into interaction, must enter also into the neutrino mass term. The type of the neutrino mass term depends on the presence in it of right-handed fields $\nu_{lR}(x)$ and on the total lepton number conservation.

The Standard Dirac Mass Term. If in the Lagrangian there are left-handed and right-handed fields $\nu_{lL}(x)$ and $\nu_{lR}(x)$ and the total lepton number is conserved, the neutrino mass term has the form

$$\mathcal{L}^D(x) = - \sum_{l'l} \bar{\nu}_{l'L}(x) M_{l'l}^D \nu_{lR}(x) + \text{h.c.} \quad (20)$$

A complex 3×3 matrix M^D can be presented in the form

$$M^D = U m V^\dagger, \quad (21)$$

where U and V are unitary mixing matrices, and m is a diagonal matrix. From (20) and (21) we find

$$\mathcal{L}^D(x) = - \sum_{i=1}^3 m_i \bar{\nu}_i(x) \nu_i(x). \quad (22)$$

Thus, ν_i is the field of neutrino with mass m_i .

The flavor fields $\nu_{lL}(x)$ are connected with the fields $\nu_i(x)$ by the mixing relation

$$\nu_{lL}(x) = \sum_{i=1}^3 U_{li} \nu_i(x), \quad (23)$$

where U is the unitary PMNS mixing matrix which is characterized by three mixing angles and one CP phase.

In the case of the mass term (20), the invariance under the global gauge transformations

$$\begin{aligned} \nu'_{lL}(x) &= e^{i\alpha} \nu_{lL}(x), & \nu'_{lR}(x) &= e^{i\alpha} \nu_{lR}(x), \\ l'_{L,R}(x) &= e^{i\alpha} l_{L,R}(x), & l &= e, \mu, \tau \end{aligned} \quad (24)$$

takes place (α is a constant phase, the same for all flavors). The invariance under the transformations (24) means that the total lepton number L is conserved, and $\nu_i(x)$ is the Dirac field of neutrinos ($L(\nu_i) = 1$) and antineutrinos ($L(\bar{\nu}_i) = -1$). The mass term (20) is the standard Dirac mass term.

The Most Economical Majorana Mass Term. If there are only left-handed fields $\nu_{lL}(x)$ in the Lagrangian, we can build the neutrino mass term if we take into account that $(\nu_{lL})^c = C\bar{\nu}_{lL}^T$ is a right-handed component (C is the matrix of the charge conjugation which satisfies the relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$). For the mass term we have in this case

$$\mathcal{L}^L(x) = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'L}(x) M_{l'l}^L (\nu_{lL})^c(x) + \text{h.c.} \quad (25)$$

Here M^L is a complex symmetrical 3×3 matrix. The matrix M^L can be presented in the form

$$M^L = U m U^T, \quad (26)$$

where U is a unitary matrix, and $m_{ik} = m_i \delta_{ik}$, $m_i > 0$. From (25) and (26) we have

$$\mathcal{L}^L(x) = -\frac{1}{2} \sum_{i=1}^3 m_i \bar{\nu}_i(x) \nu_i(x), \quad (27)$$

where the field $\nu_i(x)$ ($i = 1, 2, 3$) satisfies the condition

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x). \quad (28)$$

Thus $\nu_i(x)$ is the field of the truly neutral

Majorana neutrino ($\nu_i \equiv \bar{\nu}_i$) with mass m_i . The flavor field $\nu_{lL}(x)$ is connected with left-handed components ν_{iL} by the mixing relation

$$\nu_{lL}(x) = \sum_{i=1}^3 U_{li} \nu_{iL}(x), \quad l = e, \mu, \tau. \quad (29)$$

The unitary mixing matrix U is characterized by three mixing angles and three CP phases.

The Lagrangian (25) is not invariant under the global gauge transformation $\nu'_{lL}(x) = e^{i\alpha} \nu_{lL}(x)$. Thus, in the case of the mass term (25), the total lepton number L is not conserved, and there is no conserved quantum number which can distinguish neutrino and antineutrino. This is the reason why the fields of neutrinos with definite masses $\nu_i(x)$ are the Majorana fields.

The Most General Dirac and Majorana Mass Term. The most general neutrino mass term has the form

$$\mathcal{L}^{D+M}(x) = \mathcal{L}^L(x) + \mathcal{L}^D(x) + \mathcal{L}^R(x). \quad (30)$$

Here

$$\mathcal{L}^R(x) = -\frac{1}{2} \sum_{l'l} \overline{(\nu_{l'R}(x))^c} M_{l'l}^R \nu_{lR}(x) + \text{h.c.} \quad (31)$$

and $\mathcal{L}^D(x)$ and $\mathcal{L}^L(x)$ are given, respectively, by (20) and (25).

The mass term (30) does not conserve the total lepton number L . After the diagonalization of the mass term we have

$$\mathcal{L}^{D+M}(x) = -\frac{1}{2} \sum_{i=1}^6 m_i \bar{\nu}_i(x) \nu_i(x), \quad (32)$$

where $\nu_i(x)$ is the Majorana field with mass m_i :

$$\nu_i(x) = \nu_i^c(x) = C \bar{\nu}_i(x)^T, \quad i = 1, 2, \dots, 6. \quad (33)$$

The flavor fields $\nu_{lL}(x)$ and the fields $(\nu_{lR}(x))^c$ are connected with the left-handed components of the Majorana fields $\nu_{iL}(x)$ by the following mixing relations:

$$\nu_{lL}(x) = \sum_{i=1}^6 U_{li} \nu_{iL}(x), \quad (\nu_{lR}(x))^c = \sum_{i=1}^6 U_{\bar{l}i} \nu_{iL}(x), \quad l = e, \mu, \tau. \quad (34)$$

Here U is a unitary 6×6 mixing matrix.

The right-handed neutrino fields $\nu_{lR}(x)$ do not enter into the SM Lagrangian and are called sterile fields. As we see from (34), in the case of the Dirac and Majorana mass term, the flavor fields ν_{lL} are mixtures of six left-handed components of the Majorana fields ν_{iL} . Sterile fields $(\nu_{lR}(x))^c$ are mixtures of the same six left-handed components.

Different possibilities can be considered in the case of the Dirac and Majorana mass term. The most popular are the following.

1. Transitions into sterile states.

If the number of light Majorana neutrinos $\nu_i(x)$ is larger than three, transitions of flavor neutrinos into sterile neutrinos become possible. For the neutrino mixing

we have in this case

$$\nu_{\alpha L}(x) = \sum_{i=1}^{3+n_s} U_{\alpha i} \nu_{iL}(x), \quad \alpha = e, \mu, \tau, s_1, \dots, \quad (35)$$

where n_s is the number of the sterile neutrinos.

There exist at present some indications in favor of transitions of flavor neutrinos into sterile states. We will discuss these indications later.

2. Seesaw mechanism of the neutrino mass generation.

If in the spectrum of masses of the Majorana particles there are three light (neutrino) masses and three heavy masses, we can explain smallness of neutrino masses with respect to the masses of leptons and quarks. This is the famous seesaw mechanism of the neutrino mass generation [27–31]. We will consider this mechanism later.

The Dirac mass term can be generated by the Standard Higgs mechanism of the mass generation. This mechanism cannot explain, however, the smallness of neutrino masses. The Majorana mass term and the Dirac and Majorana mass term can be generated only by beyond the SM mechanisms. At the moment we do not know the type of neutrino mixing: all possibilities are open. Later we will discuss the most plausible and economical possibility.

1. ON THE STANDARD MODEL OF THE ELECTROWEAK INTERACTION

1.1. Introduction. The Standard Model [32–34] is one of the greatest achievements of the physics of the XX century. It emerged as a result of numerous experiments and fundamental theoretical principles (local gauge invariance and others). After discovery of the Higgs boson at LHC, the Standard Model got the status of *the theory* of physical phenomena in the electroweak energy scale (up to about 300 GeV). We will try here to make some general conclusions which can be inferred from the Standard Model and apply them to neutrinos.

There are many questions connected with the Standard Model: why left-handed and right-handed quark, lepton and neutrino fields have different transformation properties, why in unified electroweak interaction, the weak CC part maximally violates parity and the electromagnetic part conserves parity, etc. I suggest here that the CC structure of the Standard Model and such its features are due to neutrinos.

The Standard Model is based on the following principles:

- 1) local gauge symmetry,
- 2) unification of the electromagnetic and weak interactions into one electroweak interaction,
- 3) spontaneous breaking of the electroweak symmetry.

We will demonstrate here that in the framework of these principles, nature chooses the simplest, most economical possibilities.

1.2. Two-Component Neutrino. *From my point of view, the SM started with the theory of the two-component neutrino.* First of all, some historical remark.

In 1929, soon after Dirac proposed his famous equation for four-component spinors, which describe relativistic particle with spin 1/2, Weyl published a paper [35] in which he introduced two-component spinors. For a particle with spin 1/2, Weyl wanted to build an equation for the two-component wave function, like the Pauli one, but Lorentz-invariant. He came to a conclusion that this is impossible if mass of the particle is not equal to zero. For a massless particle, he found the equations

$$i\gamma^\alpha\partial_\alpha\psi_L(x) = 0, \quad i\gamma^\alpha\partial_\alpha\psi_R(x) = 0, \quad (36)$$

where $\psi_L(x)$ and $\psi_R(x)$ are left-handed and right-handed two-component spinors which satisfy the conditions

$$\gamma_5\psi_{L,R}(x) = \mp\psi_{L,R}(x). \quad (37)$$

Under the inversion of the coordinates, the left-handed (right-handed) spinor is transformed into right-handed (left-handed) spinor:

$$\psi'_{R,L}(x') = \eta\gamma^0\psi_{L,R}(x), \quad x' = (x^0, -\mathbf{x}). \quad (38)$$

Here η is a phase factor. Thus, Weyl equations (36) are not invariant under the inversion (do not conserve parity).

At the time when Weyl proposed equations (36) (and many years later), physicists believed that the conservation of the parity is the law of the nature. So, the Weyl theory was rejected*.

After it was discovered [37, 38] (1957) that parity is not conserved in the β -decay and other weak processes, Landau [39], Lee and Yang [40], and Salam [41] proposed the theory of the two-component neutrino. These authors had different arguments in favor of such a theory. Landau built CP-invariant neutrino theory, Salam considered γ_5 invariant theory and Lee and Yang applied to neutrino the Weyl theory.

The authors of the two-component neutrino theory assumed that neutrino mass is equal to zero (which was compatible with the data existed at that time) and that neutrino field was $\nu_L(x)$ or $\nu_R(x)$. Such fields satisfy the Weyl equations

$$i\gamma^\alpha\partial_\alpha\nu_L(x) = 0, \quad i\gamma^\alpha\partial_\alpha\nu_R(x) = 0. \quad (39)$$

*Pauli in his book on Quantum Mechanics [36] wrote: "... because the equation for $\psi_L(x)$ ($\psi_R(x)$) is not invariant under space reflection, it is not applicable to the physical reality". Notice, however, the following statement which belongs to Weyl: "My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually choose the beautiful".

If neutrino is the two-component particle, in this case:

1. Large violation of the parity in the β -decay, μ -decay and other weak processes must be observed (in agreement with the results of the Wu *et al.* and other experiments [37,38]).

2. Neutrino (antineutrino) helicity is equal to -1 ($+1$) in the case of the field $\nu_L(x)$ and is equal to $+1$ (-1) in the case of the field $\nu_R(x)$.

The point 1 is obvious from (38). In order to see that two-component neutrino is a particle with definite helicity, let us consider the spinor $u^r(p)$ which describes a massless particle with the momentum p and helicity r . We have

$$\gamma \cdot pu^r(p) = 0, \quad \Sigma \cdot \mathbf{n}u^r(p) = ru^r(p), \quad r = \pm 1. \quad (40)$$

Here $\Sigma = \gamma_5 \gamma^0 \boldsymbol{\gamma}$ is the operator of the spin, and $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$ is the unit vector in the direction of the neutrino momentum. From (40) it follows that

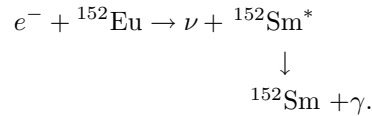
$$\gamma_5 u^r(p) = ru^r(p). \quad (41)$$

Thus, we have

$$\frac{1}{2}(1 \mp \gamma_5)u^r(p) = \frac{1}{2}(1 \mp r)u^r(p). \quad (42)$$

From this relation it follows that $r = -1$ ($r = +1$) if neutrino field is $\nu_L(x)$ ($\nu_R(x)$). Analogously, it is easy to show that antineutrino helicity is equal to $+1$ (-1) in the case if neutrino field is $\nu_L(x)$ ($\nu_R(x)$).

The neutrino helicity was measured in the spectacular Goldhaber *et al.* experiment [42]. In this experiment, the neutrino helicity was obtained from the measurement of the circular polarization of γ -quanta produced in the chain of reactions



The authors of the paper [42] concluded: "... our result is compatible with 100% negative helicity of neutrino emitted in orbital electron capture".

Thus, the Goldhaber *et al.* experiment confirmed the two-component neutrino theory. It was shown that from two possibilities ($\nu_L(x)$ or $\nu_R(x)$) the nature chooses the first one.

Let us notice that at the time when the two-component neutrino theory was proposed, it was unknown whether there exist three types of neutrino. In 1962, in the Brookhaven experiment [43] it was shown that muon and electron neutrinos ν_e and ν_μ are different particles. In 2000, the third neutrino ν_τ was discovered in the DONUT experiment [44].

The number of degrees of freedom of the two-component Weyl field is two times smaller than the number of the degrees of freedom of the four-component Dirac field. It looks plausible that *for neutrino the nature chooses this simplest and most economical possibility.*

1.3. Local Gauge Symmetry. The local gauge symmetry is a natural symmetry for the Quantum Field Theory with quantum fields which depend on x . In accordance with the two-component neutrino theory we will assume that the fields of electron, muon, and tau neutrinos are left-handed two-component Weyl fields. We will denote them $\nu'_{eL}, \nu'_{\mu L}, \nu'_{\tau L}$. Neutrinos ν_e, ν_μ, ν_τ take part in the CC weak interaction together with, correspondingly, e, μ, τ . The requirements of the symmetry can be satisfied if electron, muon, and tau fields, like neutrino fields, are also left-handed two-component Weyl fields (e'_L, μ'_L, τ'_L). The simplest symmetry group is $SU_L(2)$ and the simplest possibility for neutrino and lepton fields is to be, correspondingly, up and down components of the doublets*:

$$\psi_{eL}^{\text{lep}} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, \quad \psi_{\mu L}^{\text{lep}} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}, \quad \psi_{\tau L}^{\text{lep}} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}. \quad (43)$$

In order to insure the invariance under the local gauge transformations

$$(\psi_l^{\text{lep}})'(x) = \exp\left(i \frac{1}{2} \boldsymbol{\tau} \cdot \boldsymbol{\Lambda}(x)\right) \psi_l^{\text{lep}}(x) \quad (l = e, \mu, \tau), \quad (44)$$

($\boldsymbol{\tau} \cdot \boldsymbol{\Lambda}(x) = \sum_{i=1}^3 \tau^i \Lambda^i(x)$), τ^i are Pauli matrices, and $\Lambda^i(x)$ are *arbitrary functions of x*), we need to assume that neutrino–lepton fields interact with massless vector fields $\mathbf{A}_\alpha(x)$ and in the free Lagrangian derivatives of the fermion, fields are changed by the covariant derivatives

$$\partial_\alpha \psi_{lL}^{\text{lep}}(x) \rightarrow \left(\partial_\alpha + i g \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{A}_\alpha(x) \right) \psi_{lL}^{\text{lep}}(x), \quad (45)$$

where g is a dimensionless constant, and the field $\mathbf{A}_\alpha(x)$ is transferred as follows:

$$\mathbf{A}'_\alpha(x) = \mathbf{A}_\alpha(x) - \frac{1}{g} \partial_\alpha \boldsymbol{\Lambda}(x) - \boldsymbol{\Lambda}(x) \times \mathbf{A}_\alpha(x). \quad (46)$$

With the change (45), we generate the following Lagrangian of the interaction of the lepton and vector $\mathbf{A}_\alpha(x)$ fields:

$$\mathcal{L}_I(x) = -g \mathbf{j}_\alpha(x) \mathbf{A}^\alpha(x). \quad (47)$$

*We will consider only leptons. Notice also that meaning of primes will be clear later.

Here

$$\mathbf{j}_\alpha = \sum_{l=e,\mu,\tau} \bar{\psi}_{lL}^{\text{lep}} \gamma_\alpha \frac{1}{2} \boldsymbol{\tau} \psi_{lL}^{\text{lep}} \quad (48)$$

is the isovector current.

The expression (47) can be written in the form

$$\mathcal{L}_I(x) = \left(-\frac{g}{2\sqrt{2}} j_\alpha^{\text{CC}}(x) W^\alpha(x) + \text{h.c.} \right) - g j_\alpha^3(x) A^{3\alpha}(x). \quad (49)$$

Here

$$j_\alpha^{\text{CC}} = 2(j_\alpha^1 + i j_\alpha^2) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}'_{lL} \gamma_\alpha l'_L \quad (50)$$

is the lepton charged current, and $W_\alpha = \frac{A_\alpha^1 - i A_\alpha^2}{\sqrt{2}}$ is the field of charged, vector W^\pm bosons.

The following remarks are in order:

1. The local gauge invariance *requires* existence of the vector field $\mathbf{A}^\alpha(x)$. This field is called gauge vector field.

2. The interaction (47) is *the minimal interaction* compatible with local gauge invariance.

3. From (46) it follows that the strength tensor of the vector field $\mathbf{A}^\alpha(x)$ is given by the expression

$$\mathbf{F}_{\alpha\beta}(x) = \partial_\alpha \mathbf{A}_\beta(x) - \partial_\beta \mathbf{A}_\alpha(x) - g \mathbf{A}_\alpha(x) \times \mathbf{A}_\beta(x), \quad (51)$$

where the last term is due to the fact that $SU(2)_L$ is a non-Abelian group. Because interaction constant g enters into expression for the strength tensor, it must be the same for all doublets $\psi_{lL}^{\text{lep}}(x)$ ($l = e, \mu, \tau$). As a result, we came to $e - \mu - \tau$ universal charged current weak interaction (49).

1.4. Unification of the Weak and Electromagnetic Interactions. *The Standard Model is the unified theory of the weak and electromagnetic interactions.* In the electromagnetic current of the charged leptons, the left-handed and right-handed fields enter:

$$j_\alpha^{\text{em}} = \sum_l (-1) \bar{l}' \gamma_\alpha l' = \sum_l (-1) \bar{l}'_L \gamma_\alpha l'_L + \sum_l (-1) \bar{l}'_R \gamma_\alpha l'_R. \quad (52)$$

Thus, in order to unify weak and electromagnetic interactions, we must enlarge a symmetry group. A new symmetry group must include not only transformations of the left-handed fields, but also transformations of the right-handed fields *of charged leptons*. There is a fundamental difference between neutrinos and other fermions: neutrinos electric charges are equal to zero, there is no electromagnetic

current of neutrinos. The unification of the weak and electromagnetic interactions does not require right-handed neutrino fields. *A minimal possibility is to assume that there are no right-handed neutrino fields in the Standard Model.*

The minimal enlargement of the $SU_L(2)$ group is a direct product $SU_L(2) \times U_Y(1)$. In order to ensure local gauge $SU_L(2) \times U_Y(1)$ invariance, we need to change the free Lagrangian derivatives of the left-handed and right-handed fields by the covariant derivatives

$$\begin{aligned}\partial_\alpha \psi_{lL}^{\text{lep}} &\rightarrow \left(\partial_\alpha + ig \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{A}_\alpha + ig' \frac{1}{2} Y_L^{\text{lep}} B_\alpha \right) \psi_{lL}^{\text{lep}}, \\ \partial_\alpha l_R &\rightarrow \left(\partial_\alpha + ig' \frac{1}{2} Y_R^{\text{lep}} B_\alpha \right) l'_R,\end{aligned}\tag{53}$$

where B_α is the vector gauge field of the $U_Y(1)$ group.

There are no constraints on the interaction constants of the Abelian $U_Y(1)$ local group. In order to unify the weak and electromagnetic interactions, we assume that the interaction constants for lepton doublets and charged lepton singlets have the form

$$g' \frac{1}{2} Y_L^{\text{lep}}, \quad g' \frac{1}{2} Y_R^{\text{lep}}.\tag{54}$$

Here g' is a constant, and hypercharges of left-handed and right-handed fields Y_L^{lep} and Y_R^{lep} are determined by the Gell-Mann–Nishijima relation

$$Q = T_3 + \frac{1}{2} Y,\tag{55}$$

where Q is the electric charge and T_3 is the third projection of the isotopic spin.

For the Lagrangian of the minimal interaction of the lepton fields and the fields A_α^3 and B_α of neutral vector bosons, we obtain the following expression:

$$\mathcal{L}_I^0 = -gj_\alpha^3 A^{3\alpha} - g' \frac{1}{2} j_\alpha^Y B^\alpha.\tag{56}$$

Here

$$\frac{1}{2} j_\alpha^Y = j_\alpha^{\text{em}} - j_\alpha^3,\tag{57}$$

where j_α^{em} is the electromagnetic current of the leptons.

Notice that the electromagnetic current appeared in (57) due to the fact that electric charges of left-handed components l'_L (coming from doublets) and right-handed components l'_R (coming from singlets) are the same. Thus, if we choose coupling constants of the $U_Y(1)$ local gauge group in accordance with the Gell-Mann–Nishijima relation, we can combine the electromagnetic interaction, which conserves parity, and the weak interaction, which violates parity, into one electroweak interaction.

In order to separate in (56) the Lagrangian of electromagnetic interaction of leptons with the electromagnetic field,

- instead of the fields $A^{3\alpha}$ and B^α , we introduce “mixed” fields

$$Z^\alpha = \cos \theta_W A^{3\alpha} - \sin \theta_W B^\alpha, \quad A^\alpha = \sin \theta_W A^{3\alpha} + \cos \theta_W B^\alpha, \quad (58)$$

where angle θ_W is determined by the relation

$$\frac{g'}{g} = \tan \theta_W; \quad (59)$$

- we assume that the following relation holds:

$$g \sin \theta_W = e. \quad (60)$$

Here e is the proton charge. The relation (60) is called the *unification condition*. Finally, the interaction Lagrangian takes the form

$$\mathcal{L}_I = \mathcal{L}_I^{\text{CC}} + \mathcal{L}_I^{\text{NC}} + \mathcal{L}_I^{\text{em}}. \quad (61)$$

Here

$$\mathcal{L}_I^{\text{CC}} = \left(-\frac{g}{2\sqrt{2}} j_\alpha^{\text{CC}} W^\alpha + \text{h.c.} \right) \quad (62)$$

is the charged current Lagrangian,

$$\mathcal{L}_I^{\text{NC}} = -\frac{g}{2 \cos \theta_W} j_\alpha^{\text{NC}} Z^\alpha \quad (63)$$

is the neutral current Lagrangian,

$$\mathcal{L}_I^{\text{em}} = -e j_\alpha^{\text{em}} A^\alpha \quad (64)$$

is the electromagnetic Lagrangian.

We considered up to now only neutrinos and charged leptons. If we include also quarks, the total charged, neutral and electromagnetic currents are given by the following expressions:

$$j_\alpha^{\text{CC}} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}'_{lL} \gamma_\alpha l'_L + 2(\bar{u}'_L \gamma_\alpha d'_L + \bar{c}'_L \gamma_\alpha s'_L + \bar{t}'_L \gamma_\alpha b'_L), \quad (65)$$

$$j_\alpha^{\text{NC}} = 2j_\alpha^3 - 2 \sin^2 \theta_W j_\alpha^{\text{em}}, \quad (66)$$

where

$$j_\alpha^3 = \frac{1}{2} \sum_{l=e,\mu,\tau} \bar{\nu}'_{lL} \gamma_\alpha \nu'_{lL} - \frac{1}{2} \sum_{l=e,\mu,\tau} \bar{l}'_L \gamma_\alpha l'_L + \frac{1}{2} \sum_{q=u,c,t} \bar{q}'_L \gamma_\alpha q'_L - \frac{1}{2} \sum_{q=d,s,b} \bar{q}'_L \gamma_\alpha q'_L \quad (67)$$

and

$$j_{\alpha}^{\text{em}} = (-1) \sum_{l=e,\mu,\tau} \bar{l}' \gamma_{\alpha} l' + \left(\frac{2}{3}\right) \sum_{q=u,c,t} \bar{q}' \gamma_{\alpha} q' + \left(-\frac{1}{3}\right) \sum_{q=d,s,b} \bar{q}' \gamma_{\alpha} q'. \quad (68)$$

Let us stress that the structure of the CC term is determined by the two-component neutrinos. The structure of the NC term is determined by the unification of the CC and EM interactions on the basis of the $SU_L(2) \times U_Y(1)$ group.

The Lagrangian of interaction of fundamental fermions and gauge vector bosons is *the minimal, simplest Lagrangian*. However, due to requirements of the local gauge $SU_L(2) \times U_Y(1)$ symmetry, there are no mass terms of all fermions and gauge vector bosons in the Lagrangian.

In order to build a realistic theory of the electroweak interaction, we need to violate local gauge symmetry and generate masses of W^{\pm} and Z^0 bosons and mass terms of quarks and charged leptons. The photon must remain massless. Neutrino masses is a special subject. We will discuss it later.

1.5. Brout–Englert–Higgs Spontaneous Symmetry Breaking. *The Standard Model mechanism of the mass generation is the Brout–Englert–Higgs mechanism* [45–47]. It is based on the phenomenon of the spontaneous symmetry breaking. The spontaneous symmetry breaking takes place in the ferromagnetism and other many-body phenomena. It happens if the Hamiltonian of the system has some symmetry, and vacuum states are degenerated. It was suggested [48–50] that the phenomenon of the spontaneous symmetry breaking takes place also in the Quantum Field Theory.

In order to ensure the spontaneous symmetry breaking in addition to the fields of fundamental fermions and gauge vector bosons, *we must include also the scalar Higgs field in the system*.

We will assume that the Higgs field

$$\phi(x) = \begin{pmatrix} \phi_+(x) \\ \phi_0(x) \end{pmatrix} \quad (69)$$

is transformed as $SU_L(2)$ doublet. Here $\phi_+(x)$ and $\phi_0(x)$ are complex charged and neutral scalar fields. According to the Gell-Mann–Nishijima relation the hypercharge of the doublet $\phi(x)$ is equal to one. We will see later that *this assumption gives us the most economical possibility to generate masses of W^{\pm} and Z^0 vector bosons*.

The part of $SU_L(2) \times U_Y(1)$ invariant Lagrangian, in which the Higgs field enters, has the form

$$\mathcal{L} = \left(\left(\partial_{\alpha} + ig \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{A}_{\alpha} + ig' \frac{1}{2} B_{\alpha} \right) \phi \right)^{\dagger} \times \\ \times \left(\partial^{\alpha} + ig \frac{1}{2} \boldsymbol{\tau} \cdot \mathbf{A}^{\alpha} + ig' \frac{1}{2} B^{\alpha} \right) \phi - V(\phi^{\dagger} \phi), \quad (70)$$

where potential $V(\phi^\dagger \phi)$ is given by the expression

$$V(\phi^\dagger \phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \quad (71)$$

Here μ^2 and λ are positive constants. The constant μ has dimension M , and the constant λ is dimensionless constant.

Existence of the Higgs field fundamentally changes the properties of the system: the energy of the system reaches minimum *at nonzero values of the Higgs field*. In fact, the energy reaches the minimum at such values of Higgs field which minimize the potential. We can rewrite the potential in the form

$$V(\phi^\dagger \phi) = \lambda \left(\phi^\dagger \phi - \frac{\mu^2}{2\lambda} \right)^2 - \frac{\mu^4}{4\lambda}. \quad (72)$$

From this expression, it is obvious that the potential reaches minimum at

$$(\phi^\dagger \phi)_0 = \frac{v^2}{2}, \quad (73)$$

where

$$v^2 = \frac{\mu^2}{\lambda}. \quad (74)$$

Taking into account the conservation of the electric charge, for the vacuum values of the Higgs field, we have

$$\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\alpha}, \quad (75)$$

where α is an arbitrary phase. It is obvious that this freedom is due to the gauge symmetry of the Lagrangian. We can choose

$$\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}. \quad (76)$$

With this choice we break the symmetry. Notice that in the quantum case the constant v , having the dimension M , is the vacuum expectation value (vev) of the Higgs field.

The doublet $\phi(x)$ can be presented in the form

$$\phi(x) = \exp \left(i \frac{1}{v} \frac{1}{2} \boldsymbol{\tau} \cdot \boldsymbol{\theta}(x) \right) \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix}. \quad (77)$$

Here $\theta_i(x)$ ($i = 1, 2, 3$) and $H(x)$ are real functions which have dimension of the scalar field (M). Vacuum values of these functions are equal to zero.

The Lagrangian (70) is invariant under $SU_L(2) \times U_Y(1)$ local gauge transformations. We can choose the arbitrary gauge in such a way that

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix}. \quad (78)$$

Such a gauge is called the unitary gauge. From (78) it follows that the Lagrangian (70) takes the form

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\alpha H \partial^\alpha H + \frac{1}{4} (v + H)^2 g^2 W_\alpha^\dagger W^\alpha + \\ & + \frac{1}{4} (v + H)^2 (g^2 + g'^2) \frac{1}{2} Z_\alpha Z^\alpha - \frac{\lambda}{4} (2vH + H^2)^2. \end{aligned} \quad (79)$$

The mass terms of W^\pm and Z^0 vector bosons and the scalar Higgs boson are given by the expression

$$\mathcal{L}^m = m_W^2 W_\alpha^\dagger W^\alpha + \frac{1}{2} m_Z^2 Z_\alpha Z^\alpha - \frac{1}{2} m_H^2 H^2, \quad (80)$$

where m_W , m_Z , and m_H are the masses of W^\pm , Z^0 , and Higgs bosons. From (79) and (80) we find

$$m_W = \frac{1}{2} g v, \quad m_Z = \frac{1}{2} \sqrt{(g^2 + g'^2)} v, \quad m_H = \sqrt{2\lambda} v. \quad (81)$$

Thus, after the spontaneous symmetry breaking, $W^\alpha(x)$ becomes the field of the charged vector W^\pm bosons with the mass $(1/2)g v$, $Z^\alpha(x)$ is the field of neutral vector Z^0 bosons with the mass $(1/2)\sqrt{(g^2 + g'^2)} v$, $A_\alpha(x)$ remains the field of massless photons.

Three (Goldstone) degrees of freedom are necessary to provide longitudinal components of massive W^\pm and Z^0 bosons. The Higgs doublet (two complex scalar fields, 4 degrees of freedom) is a *minimal possibility*. One remaining degree of freedom is the neutral Higgs field $H(x)$ of scalar particles with the mass $\sqrt{2\lambda} v$.

The Brout–Englert–Higgs mechanism of the generation of masses of W^\pm and Z^0 bosons predicts existence of the massive scalar boson. Recent discovery of the scalar boson at LHC [51, 52] is an impressive confirmation of this prediction of the Standard Model.

The expressions (81) for masses of the W^\pm and Z^0 bosons are characteristic expressions for masses of vector bosons in a theory with spontaneous symmetry

breaking (and covariant derivative of the Higgs field in the Lagrangian). In fact, it is evident from (70) that masses of the vector bosons must have a form of a product of the constant part of the Higgs field (v) and interaction constants.

The first relation (81) allows one to connect the constant v with the Fermi constant G_F . In fact, the Fermi constant, which can be determined from the measurement of time of life of muon and from other CC data, is given by the expression

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}. \quad (82)$$

From (81) and (82) we obviously have

$$v^2 = \frac{1}{\sqrt{2}G_F}. \quad (83)$$

Thus, we find

$$v = (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}. \quad (84)$$

The interaction constant g is connected with the electric charge e and the parameter $\sin \theta_W$ by the unification condition (60). From (60), (81), and (84) for the mass of the W boson we find the following expression:

$$m_W = \left(\frac{\pi\alpha}{\sqrt{2}G_F} \right)^{1/2} \frac{1}{\sin \theta_W}, \quad (85)$$

where $\alpha \simeq \frac{1}{137.036}$ is the fine-structure constant. For the mass of the Z^0 boson, we have

$$m_Z = \frac{m_W}{\cos \theta_W} = \left(\frac{\pi\alpha}{\sqrt{2}G_F} \right)^{1/2} \frac{1}{\sin \theta_W \cos \theta_W}. \quad (86)$$

The parameter $\sin^2 \theta_W$ can be determined from the data on the investigation of NC weak processes. From existing data it was found the value $\sin^2 \theta_W = 0.23116(12)$ [53].

Thus, the Standard Model allows one to connect masses of W^\pm and Z^0 bosons with constants G_F , α , and $\sin^2 \theta_W$.

For the average of the measured values of m_W and m_Z we have [53]

$$m_W = (80.420 \pm 0.031) \text{ GeV}, \quad m_Z = (91.1876 \pm 0.0021) \text{ GeV}. \quad (87)$$

Using the values of G_F , α , and $\sin^2 \theta_W$ (and taking into account radiative corrections) for predicted by the SM values of m_W and m_Z , we have

$$m_W = (80.381 \pm 0.014) \text{ GeV}, \quad m_Z = (91.1874 \pm 0.0021) \text{ GeV}. \quad (88)$$

The agreement of the experimental data with one of the basic predictions of the SM is an important confirmation of the idea of the spontaneous breaking of the electroweak symmetry.

We will consider now the Higgs mechanism of generation of masses of leptons and quarks. The fermion mass terms can be generated by a $SU_L(2) \times U_Y(1)$ -invariant Yukawa Lagrangians. We will consider first the charged leptons. The most general Yukawa Lagrangian, which can generate the mass term of the charged leptons, has the following form:

$$\mathcal{L}_Y^{\text{lep}} = -\sqrt{2} \sum_{l_1, l_2} \bar{\psi}_{l_1 L}^{\text{lep}} Y_{l_1 l_2} l'_{2R} \phi + \text{h.c.}, \quad (89)$$

where Y is a 3×3 complex nondiagonal matrix. The Standard Model does not predict elements of the matrix Y : they are parameters of the SM.

After the spontaneous breaking of the symmetry from (43), (76), and (89), we have

$$\mathcal{L}_Y^{\text{lep}} = - \sum_{l_1, l_2} \bar{l}_{1L} Y_{l_1 l_2} l'_{2R} (v + H) + \text{h.c.} \quad (90)$$

The term proportional to v is the mass term of charged leptons. In order to present it in the canonical form, we need to diagonalize matrix Y . The general complex matrix Y can be diagonalized by the biunitary transformation

$$Y = V_L y V_R^\dagger, \quad (91)$$

where V_L and V_R are unitary matrices and y is a diagonal matrix with positive diagonal elements. From (90) and (91) we find

$$\mathcal{L}_Y^{\text{lep}} = - \sum_{l=e, \mu, \tau} \bar{l}_L m_l l_R \left(1 + \frac{1}{v} H\right) + \text{h.c.} = - \sum_{l=e, \mu, \tau} m_l \bar{l} l \left(1 + \frac{1}{v} H\right). \quad (92)$$

Here

$$l_L = \sum_{l_1} (V_L^\dagger)_{l_1 l} l'_{1L}, \quad l_R = \sum_{l_1} (V_R^\dagger)_{l_1 l} l'_{1R}, \quad l = l_L + l_R \quad (93)$$

and

$$m_l = y_l v. \quad (94)$$

From (93) it follows that $l(x)$ is the field of the charged leptons l with mass m_l ($l = e, \mu, \tau$). Left-handed and right-handed components of the fields of leptons with definite masses are connected with primed left-handed fields, components of the doublets $\psi_{lL}^{\text{lep}}(x)$, and primed singlets right-handed fields l'_R by the unitary transformations (93).

The second term of (92) is the Lagrangian of interaction of leptons and the Higgs boson

$$\mathcal{L}_Y = - \sum_{l=e,\mu,\tau} f_l \bar{l} l H, \quad (95)$$

where dimensionless interaction constants f_l are given by the relation

$$f_l = \frac{1}{v} m_l = (\sqrt{2} G_F)^{1/2} m_l \simeq 4.06 \cdot 10^{-3} \frac{m_l}{\text{GeV}}. \quad (96)$$

Let us express leptonic electromagnetic, charged, and neutral currents in terms of the fields of leptons with definite masses $l(x)$. Taking into account the unitarity of the matrices V_L and V_R for the EM current, we have

$$\begin{aligned} j_\alpha^{\text{em}} &= \sum_l (-1) \bar{l}'_L \gamma_\alpha l'_L + \sum_l (-1) \bar{l}'_R \gamma_\alpha l'_R = \\ &= \sum_l (-1) \bar{l}_L \gamma_\alpha l_L + \sum_l (-1) \bar{l}_R \gamma_\alpha l_R = \sum_l (-1) \bar{l} \gamma_\alpha l. \end{aligned} \quad (97)$$

For the leptonic charged current we find

$$j_\alpha^{\text{CC}} = 2 \sum_l \bar{\nu}'_{lL} \gamma_\alpha l'_L = 2 \sum_l \bar{\nu}_{lL} \gamma_\alpha l_L, \quad (98)$$

where

$$\nu_{lL} = \sum_{l_1} (V_L^\dagger)_{l_1 l} \nu'_{l_1 L}. \quad (99)$$

The field ν_l is called flavor neutrino field.

Finally, for the leptonic NC we obtain the following expression:

$$j_\alpha^{\text{NC}} = \sum_l \bar{\nu}'_{lL} \gamma_\alpha \nu'_{lL} - \sum_l \bar{l}'_L \gamma_\alpha l'_L - 2 \sin^2 \theta_W j_\alpha^{\text{em}}, \quad (100)$$

$$= \sum_l \bar{\nu}_{lL} \gamma_\alpha \nu_{lL} - \sum_l \bar{l}_L \gamma_\alpha l_L - 2 \sin^2 \theta_W j_\alpha^{\text{em}}. \quad (101)$$

We will consider now briefly the Brout–Englert–Higgs mechanism of the generation of masses of quarks. Let us assume that in the total Lagrangian enter the following $SU_L(2) \times U_Y(1)$ invariant Lagrangian of the Yukawa interaction of quark and Higgs fields

$$\mathcal{L}_Y^{\text{quark}} = -\sqrt{2} \sum_{k,q_1=d,s,b} \bar{\psi}_{kL} Y_{kq_1}^{\text{down}} q'_{1R} \phi - \sqrt{2} \sum_{k,q_1=u,c,t} \bar{\psi}_{kL} Y_{kq_1}^{\text{up}} q'_{1R} \tilde{\phi} + \text{h.c.} \quad (102)$$

Here

$$\psi_{1L} = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \quad \psi_{2L} = \begin{pmatrix} c'_L \\ s'_L \end{pmatrix}, \quad \psi_{3L} = \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \quad (103)$$

are quark doublets,

$$\tilde{\phi} = i\tau_2 \phi^* \quad (104)$$

is the conjugated Higgs doublet, and $Y_{kq_1}^{\text{down}}, Y_{kq_1}^{\text{up}}$ are 3×3 complex nondiagonal matrices.

After the spontaneous breaking of the symmetry in the unitary gauge we have

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{pmatrix}, \quad \tilde{\phi}(x) = \begin{pmatrix} \frac{v + H(x)}{\sqrt{2}} \\ 0 \end{pmatrix}. \quad (105)$$

From (102) and (105) we find

$$\begin{aligned} \mathcal{L}_Y^{\text{quark}} = & - \sum_{q_1, q_2=d, s, b} \bar{q}'_{1L} Y_{q_1 q_2}^{\text{down}} q'_{2R} (v + H) - \\ & - \sum_{q_1, q_2=u, c, t} \bar{q}'_{1L} Y_{q_1 q_2}^{\text{up}} q'_{2R} (v + H) + \text{h.c.} \end{aligned} \quad (106)$$

For the complex matrices Y^{down} and Y^{up} we have

$$Y^{\text{down}} = V_L^{\text{down}} y^{\text{down}} V_R^{\text{down}\dagger}, \quad Y^{\text{up}} = V_L^{\text{up}} y^{\text{up}} V_R^{\text{up}\dagger}. \quad (107)$$

Here $V_{L,R}^{\text{down}}$ and $V_{L,R}^{\text{up}}$ are unitary matrices, and $y^{\text{down}}, y^{\text{up}}$ are diagonal matrices with positive diagonal elements.

Using (107) for the Lagrangian $\mathcal{L}_Y^{\text{quark}}$, we find

$$\mathcal{L}_Y^{\text{quark}} = - \sum_{q=u, d, c, s, t, b} m_q \bar{q} q \left(1 + \frac{1}{v} H \right). \quad (108)$$

Here

$$m_q = y_q v, \quad q = u, d, c, s, t, b \quad (109)$$

are masses of the quarks,

$$\begin{aligned} q_L &= \sum_{q_1=d, s, b} (V_L^{\text{down}\dagger})_{qq_1} q'_{1L} \quad (q = d, s, b), \\ q_L &= \sum_{q_1=u, c, t} (V_L^{\text{up}\dagger})_{qq_1} q'_{1L} \quad (q = u, c, t) \end{aligned} \quad (110)$$

and

$$\begin{aligned}
 q_R &= \sum_{q_1=d,s,b} (V_R^{\text{down}\dagger})_{qq_1} q'_{1R} \quad (q = d, s, b), \\
 q_R &= \sum_{q_1=u,c,t} (V_R^{\text{up}\dagger})_{qq_1} q'_{1R} \quad (q = u, c, t).
 \end{aligned} \tag{111}$$

The first terms in the r.h.s. of Eq.(108) is the mass term of the quark

$$\mathcal{L}_m^{\text{quark}} = - \sum_{q=u,d,\dots} m_q \bar{q}q. \tag{112}$$

The second term

$$\mathcal{L}_H^{\text{quark}} = - \sum_{q=u,d,\dots} f_q \bar{q}qH \tag{113}$$

is the Lagrangian of the interaction of quarks and the scalar Higgs boson. The interaction constants f_q are given by the relation

$$f_q = \frac{m_q}{v} = m_q (\sqrt{2}G_F)^{1/2} \simeq 4.06 \cdot 10^{-3} \frac{m_q}{\text{GeV}}. \tag{114}$$

Let us express the electromagnetic current, neutral current, and charged current of quarks in terms of the fields of quarks with definite masses. Taking into account the unitarity of the matrices $V_{L,R}^{\text{up}}$ and $V_{L,R}^{\text{down}}$ for the electromagnetic current of quarks, we have the following expression:

$$j_\alpha^{\text{em}} = \frac{2}{3} \sum_{q=u,c,t} \bar{q}' \gamma_\alpha q' + \left(-\frac{1}{3}\right) \sum_{q=d,s,b} \bar{q}' \gamma_\alpha q' = \sum_{q=u,d,\dots} e_q \bar{q} \gamma_\alpha q, \tag{115}$$

where $e_q = (2/3)$ for $q = u, c, t$ and $e_q = -(1/3)$ for $q = d, s, b$.

Analogously, for the neutral current of quarks we find

$$\begin{aligned}
 j_\alpha^{\text{NC}} &= \sum_{q=u,c,t} \bar{q}'_L \gamma_\alpha q'_L - \sum_{q=d,s,b} \bar{q}'_L \gamma_\alpha q'_L - 2 \sin^2 \theta_W j_\alpha^{\text{em}}, \\
 &= \sum_{q=u,c,t} \bar{q}_L \gamma_\alpha q_L - \sum_{q=d,s,b} \bar{q}_L \gamma_\alpha q_L - 2 \sin^2 \theta_W j_\alpha^{\text{em}}.
 \end{aligned} \tag{116}$$

Thus, NC of the Standard Model is diagonal over quark fields (conserves quark flavor).

Finally, for the charged current of quarks we have

$$j_\alpha^{\text{CC}} = \bar{u}'_L \gamma_\alpha d'_L + \bar{c}'_L \gamma_\alpha s'_L + \bar{t}'_L \gamma_\alpha b'_L = \bar{u}_L \gamma_\alpha d_L^{\text{mix}} + \bar{c}_L \gamma_\alpha s_L^{\text{mix}} + \bar{t}_L \gamma_\alpha b_L^{\text{mix}}. \tag{117}$$

Here

$$d_L^{\text{mix}} = \sum_{q=d,s,b} V_{uq} q_L, \quad s_L^{\text{mix}} = \sum_{q=d,s,b} V_{cq} q_L, \quad b_L^{\text{mix}} = \sum_{q=d,s,b} V_{tq} q_L. \quad (118)$$

The matrix $V = V_L^{\text{up}} V_L^{\text{down}\dagger}$ is a unitary 3×3 Cabibbo–Kobayashi–Maskawa (CKM) matrix. Thus, the fields of down quarks enter into CC in the mixed form. The mixing is connected with the fact that the unitary matrices V_L^{up} and V_L^{down} are different.

The CKM matrix is characterized by three mixing angles θ_{12} , θ_{23} , θ_{13} and one phase δ responsible for the CP violation in the quark sector. It can be presented in the same form as the neutrino mixing matrix (see (16)). Existing data allows one to determine all matrix elements of CKM matrix. From the global fit of the data of numerous experiments it was found [53]

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351 \pm 0.00015 \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}. \quad (119)$$

From (96) and (114) for the masses of charged leptons and quarks we have

$$m_l = f_l v, \quad m_q = f_q v. \quad (120)$$

Thus, masses of leptons (quarks) have the form of the product of constant v (coming from the Higgs field) and the constants of interaction of leptons (quarks) and the Higgs bosons. Notice that masses of W^\pm and Z^0 vector bosons have the same form (see (81)).

Masses of leptons and quarks are known. From (120) it follows that *the SM predicts the constants of interaction of leptons and quarks with the Higgs boson*. The first LHC measurements of the constants f_τ and f_b are in agreement with the SM prediction (see [54,55]).

Up to now, we considered the Standard Model Brout–Englert–Higgs mechanism of generation of masses of charged leptons and quarks. What about neutrinos? As we discussed earlier, in the minimal Standard Model there are no right-handed neutrino fields. Thus, in the minimal SM there is no Yukawa interaction which can generate the neutrino mass term. This means that *after spontaneous breaking of the electroweak symmetry, neutrino fields in the SM remain two-component Weyl fields and neutrino mass term can be generated only by a beyond the Standard Model mechanism*.

In conclusion, we will present some additional arguments in favor of a beyond the SM origin of the neutrino masses. Let us assume that not only ν'_{iL} but also ν'_{iR} are Standard Model fields. In this case we have the following $SU_L(2) \times U_Y(1)$

invariant Yukawa interaction of lepton and Higgs fields

$$\mathcal{L}_Y^\nu = -\sqrt{2} \sum_{\nu l} \bar{\psi}_{l_1 L}^{\text{lep}} Y_{l_1 l_2}^\nu \nu'_{l_2 R} \tilde{\phi} + \text{h.c.} \quad (121)$$

After spontaneous breaking of the electroweak symmetry from (121), we obtain the Dirac neutrino mass term

$$\mathcal{L}^D = -v \sum_{\nu, l} \bar{\nu}'_{\nu L} Y_{\nu l}^\nu \nu'_{l R} + \text{h.c.} = - \sum_{\nu, l} \bar{\nu}_{\nu L} M_{\nu l}^D \nu_{l R} + \text{h.c.} \quad (122)$$

Here $M^D = v V_L^\dagger Y^\nu$, where the matrix V_L connects fields ν'_{lL} and flavor neutrino fields ν_{lL} (see (99)). After the standard diagonalization of the matrix $V_L^\dagger Y^\nu$, we find

$$\mathcal{L}^D = \sum_{i=1}^3 m_i \bar{\nu}_i \nu_i, \quad \nu_{lL} = \sum_i U_{li} \nu_{iL}, \quad (123)$$

where U is a unitary mixing matrix, and ν_i is a field of the Dirac neutrinos with mass m_i . For neutrino mass we have

$$m_i = v y_i, \quad (124)$$

where y_i is the Yukawa coupling constant.

In order to estimate y_i , we need to know neutrino masses. Values of neutrino masses are determined by the lightest neutrino mass m_0 which is unknown at present. We will consider two extreme cases:

1) normal mass hierarchy $m_1 \ll m_2 \ll m_3$, $m_1 \ll \sqrt{\Delta m_S^2} \simeq 9 \cdot 10^{-3}$ eV,

$$y_1 \ll y_2 \ll y_3 \simeq \frac{\sqrt{\Delta m_A^2}}{v} \simeq 2 \cdot 10^{-13}.$$

2) inverted mass hierarchy $m_3 \ll m_1 < m_2$, $m_3 \ll \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV,

$$y_3 \ll y_1 \lesssim y_2 \simeq \frac{\sqrt{\Delta m_A^2}}{v} \simeq 2 \cdot 10^{-13}.$$

3) quasi-degenerate mass spectrum $m_{1,3} \gg \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV.

In this case $m_1 \lesssim m_2 \lesssim m_3 \simeq \sum m_i / 3$ or $m_1 \lesssim m_2 \lesssim m_3 \simeq m_\beta$.

Using (18) and (19) from the cosmological data and tritium β -decay data we find, respectively,

$$y_1 \lesssim y_2 \lesssim y_3 \simeq 3 \cdot 10^{-13}, \quad y_1 \lesssim y_2 \lesssim y_3 \simeq 10^{-11}.$$

Values of the quarks and leptons Yukawa coupling constants depend on generation. For the particles of the first, second, and third generation they are of

the order of 10^{-6} – 10^{-5} , 10^{-3} – 10^{-2} and 10^{-2} – 1 , respectively. Thus, neutrino Yukawa coupling constants are many orders of magnitude smaller than Yukawa constants of quarks and leptons. Extremely small neutrino masses and, correspondingly, neutrino Yukawa coupling constants are an evidence that *masses of quarks, leptons, and neutrinos are not of the same SM origin.*

2. BEYOND THE STANDARD MODEL NEUTRINO MASSES

In the Standard Model with left-handed, two-component Weyl fields ν_{lL} , the neutrino mass term cannot be generated. The neutrino mass term can be generated only by a beyond the SM mechanism. There are many approaches to neutrino masses (see [56]). *The most economical possibility of generation of neutrino masses and mixing is provided by an effective Lagrangian.*

The effective Lagrangian method [57, 58] is a general, powerful method which allows one to describe effects of a beyond the SM physics in the electroweak region. The effective Lagrangian is a $SU_L(2) \times U_Y(1)$ -invariant, non-renormalizable Lagrangian built from SM fields (*including the Higgs field*). It has the following form:

$$\mathcal{L}_{4+n}^{\text{eff}} = \sum_{n=1,2,\dots} \frac{O_{4+n}}{\Lambda^n} + \text{h.c.} \quad (125)$$

Here O_{4+n} is a $SU_L(2) \times U_Y(1)$ invariant operator which has dimension M^{4+n} , and Λ is a constant of the dimension M . The constant Λ characterizes a scale of a new, beyond the SM physics.

In order to generate the neutrino mass term we need to build the effective Lagrangian which is quadratic in the lepton fields. The terms $\bar{\psi}_{lL}^{\text{lep}} \tilde{\phi}$ and $\tilde{\phi}^\dagger \psi_{lL}^{\text{lep}}$ ($l = e, \mu, \tau$) are $SU_L(2) \times U_Y(1)$ invariants which have dimensions $M^{5/2}$. After spontaneous breaking of the symmetry they contain, correspondingly, $v\bar{\nu}'_{lL}$ and $v\nu'_{lL}$. *The effective Lagrangian which generates the neutrino mass term has the following lepton-number violating form [57]:*

$$\mathcal{L}_5^{\text{eff}} = -\frac{1}{\Lambda} \sum_{l_1, l_2} (\bar{\psi}_{l_1 L}^{\text{lep}} \tilde{\phi}) Y'_{l_1 l_2} (\tilde{\phi}^T (\psi_{l_2 L}^{\text{lep}})^c) + \text{h.c.} \quad (126)$$

Here $Y' = (Y')^T$ is a symmetric dimensionless 3×3 matrix, and Λ is a parameter which characterizes a scale of a beyond the SM lepton-number violating physics.

After spontaneous breaking of the electroweak symmetry, from (105) and (126) we find

$$\mathcal{L}_I^{\text{eff}} = -\frac{1}{2\Lambda} \sum_{l_1, l_2} \bar{\nu}'_{l_1 L} Y'_{l_1 l_2} (\nu'_{l_2 L})^c (v + H)^2 + \text{h.c.} \quad (127)$$

The term proportional to v^2 is the neutrino mass term.

The flavor neutrino fields ν_{iL} , which enter into the leptonic charged and neutral currents, are connected with fields ν'_{iL} by the relation (99). In terms of the flavor neutrino fields from (127) we obtain the Majorana mass term (25) in which the matrix M^M is given by the following expression:

$$M^M = \frac{v^2}{\Lambda} Y, \quad (128)$$

where

$$Y = V_L^\dagger Y' (V_L^\dagger)^T \quad (129)$$

is a symmetrical 3×3 matrix. We have

$$Y = U y U^T, \quad (130)$$

where $U^\dagger U = 1$ and $y_{ik} = y_i \delta_{ik}$, $y_i > 0$.

From (128) and (130) for the Majorana neutrino mass we find the following expression:

$$m_i = \frac{v}{\Lambda} (y_i v), \quad i = 1, 2, 3. \quad (131)$$

Majorana neutrino mass m_i , generated by the effective Lagrangian (126), is a product of a “typical fermion mass” $v y_i$ and a suppression factor which is given by the ratio of the electroweak scale v and a scale Λ of a lepton-number violating physics ($\Lambda \gg v$). Thus, effective Lagrangian approach provides a natural framework for generation of neutrino masses which are much smaller than the masses of leptons and quarks. Let us stress that such a scheme does not put any constraints of the mixing matrix U .

In order to estimate the parameter Λ , we need to know neutrino masses m_i and Yukawa coupling constant y_i . Let us assume hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$. For the mass of the heaviest neutrino we have in this case $m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}$ eV. Assuming also that y_3 is of the order of one, we find the following estimate $\Lambda \simeq 10^{15}$ GeV. Thus, small Majorana neutrino masses could be a signature of a very large lepton-number violating scale in physics*.

Effective Lagrangian (126) could be a result of exchange of virtual super-heavy Majorana leptons between lepton-Higgs pairs [59]**.

*Let us stress that for the dimensional arguments we used, it is important that Higgs is not composite particle and there *exists the scalar Higgs field having dimension M*. Discovery of the Higgs boson at CERN [51, 52] confirms this assumption.

**An example of the effective Lagrangian is the Fermi Lagrangian which describes β -decay and other low-energy processes. This effective Lagrangian is generated by the exchange of the virtual W -boson between $e - \nu$ and $p - n$ pairs. It is a product of the Fermi constant which has dimension M^{-2} and dimension six four-fermion operator.

In fact, let us assume that there exist heavy Majorana leptons N_i ($i = 1, 2, \dots, N$), singlets of $SU_L(2) \times U_Y(1)$ group, which have the following Yukawa lepton-number violating interaction:

$$\mathcal{L}_I^Y = -\sqrt{2} \sum_{l,i} \bar{\psi}_{lL}^{\text{lep}} \tilde{\phi} y'_{li} N_{iR} + \text{h.c.} \quad (132)$$

Here y'_{li} are dimensionless Yukawa coupling constants, and $N_i = N_i^c$ is the Majorana field with mass M_i ($M_i \gg v$).

In the second order of the perturbation theory with virtual N_i at the electroweak energies ($Q^2 \ll M_i^2$), the interaction (132) generates the following effective Lagrangian:

$$\mathcal{L}^{\text{eff}} = - \sum_{l',l} (\bar{\psi}_{l'L}^{\text{lep}} \tilde{\phi}) \left(\sum_i y'_{l'i} \frac{1}{M_i} y'_{li} \right) (\tilde{\phi}^T (\psi_{lL}^{\text{lep}})^c) + \text{h.c.} \quad (133)$$

After spontaneous breaking of the electroweak symmetry from (133) we obtain Majorana neutrino mass term

$$\mathcal{L}^L = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'L} \left(\sum_i y'_{l'i} \frac{v^2}{M_i} y'_{li} \right) (\nu_{lL})^c + \text{h.c.} \quad (134)$$

In terms of flavor neutrino fields ν_{lL} from (134) we find

$$\mathcal{L}^L = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'L} M_{l'l}^L (\nu_{lL})^c + \text{h.c.} \quad (135)$$

Here

$$M^L = y \frac{v^2}{M} y^T, \quad (136)$$

where $y = V_L^\dagger y'$. From (26) and (136) for the Majorana neutrino mass m_i ($i = 1, 2, 3$), we find the following expression:

$$m_i = \sum_{k=1}^N (U^\dagger y)_{ik}^2 \frac{v^2}{M_k}. \quad (137)$$

The scale of a new lepton-number violating physics is determined by masses of heavy Majorana leptons N_i . It follows from (137) that Majorana neutrino masses are suppressed with respect to the masses of other fundamental fermions by the factors $v/M_k \ll 1$.

Let us summarize our discussion of the generation of the neutrino masses by the Weinberg effective Lagrangian.

1. There is one possible lepton-number violating effective Lagrangian. After spontaneous breaking of the symmetry it leads to the Majorana neutrino mass term

which is the only possible (in the case of the left-handed fields ν_{iL}) neutrino mass term (see [60]). Neutrino masses in this approach are suppressed with respect to the masses of lepton and quarks by the ratio of the electroweak scale v and the scale Λ of a new lepton-number violating physics ($\Lambda \gg v$). The Lagrangian (126) is the only effective Lagrangian of the dimension 5 (proportional to $1/\Lambda$). This means that *neutrino masses are the most sensitive probe of a new physics at a scale which is much larger than the electroweak scale.*

2. Number of Majorana neutrinos with definite masses is determined by the number of lepton flavors and is equal to three.

3. Heavy Majorana leptons with masses much larger than v could exist.

Alternative mechanism of generation of small Majorana neutrino masses is the famous seesaw mechanism [27–31]. This mechanism is based on GUT models (like $SO(10)$) with multiplets which contain not only left-handed neutrino fields ν_{iL} but also right-handed fields. In such models, the most general lepton-number violating Dirac and Majorana mass term (30) is generated. If we assume that

1) $M^L = 0$,

2) the elements of the matrix M^D are proportional to v (the Dirac term M^D is generated by the standard Higgs mechanism),

3) the right-handed Majorana term M^R (which can be always diagonalized) is given by $M_{ik}^R = M_k \delta_{ik}$, $M_k \gg v$,

then we come to the Majorana neutrino mass term

$$\mathcal{L}^L = -\frac{1}{2} \sum_{l', l} \bar{\nu}_{l'L} (M_{l'l}^L)_{\text{seesaw}} (\nu_{lL})^c + \text{h.c.}, \quad (138)$$

where

$$(M^L)_{\text{seesaw}} = -M^D (M^R)^{-1} (M^D)^T. \quad (139)$$

In the seesaw case, in the mass spectrum there are three light (neutrino) Majorana masses m_i and heavy lepton Majorana masses M_k . From (139) it follows that the scale of neutrino masses is determined by the factor $v^2/M_k \ll v$.

The seesaw mechanism of generation of the neutrino masses is equivalent to the effective Lagrangian mechanism considered before. Let us notice that the mechanism based on the interaction (132) is called type I seesaw. The effective Lagrangian (126) can also be generated by the Lagrangian of interaction of lepton-Higgs doublets with the heavy triplet leptons (type III seesaw) and by the Lagrangian of interaction of lepton doublets and Higgs doublets with heavy triplet scalar bosons (type II seesaw).

3. IMPLICATIONS OF THE STANDARD SEESAW MECHANISMS OF NEUTRINO MASS GENERATION

In this section we will briefly discuss practical implications of the effective Lagrangian (seesaw) mechanism of the neutrino mass generation.

3.1. Neutrinoless Double β -Decay. The search for neutrinoless double β -decay ($0\nu\beta\beta$ -decay)

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- \quad (140)$$

of ^{76}Ge , ^{130}Te , ^{136}Xe and other even–even nuclei is the most practical way which allows one to reveal the nature of neutrinos with definite masses (Majorana or Dirac?) (see [17, 61–63]).

The expected half-life of this process is extremely large (many orders of magnitude larger than the time of life of the Universe). There are two main reasons for that.

1. The process (140) is the second order of the perturbation theory process with the exchange of the virtual neutrinos between $n \rightarrow pe^-$ vertexes. The matrix element of the process is proportional to G_F^2 .

2. Because in the Hamiltonian of the standard weak interaction there enter left-handed neutrino fields

$$\nu_{eL} = \sum_i U_{ei} \nu_{iL}, \quad (141)$$

neutrino propagator has the form

$$\sum_i U_{ei}^2 \frac{1 - \gamma_5}{2} \frac{\gamma \cdot q + m_i}{q^2 - m_i^2} \frac{1 - \gamma_5}{2} \simeq \frac{m_{\beta\beta}}{q^2} \frac{1 - \gamma_5}{2}. \quad (142)$$

Here

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \quad (143)$$

is the effective Majorana mass, and q is the momentum of virtual neutrinos. From neutrino data it follows that $|m_{\beta\beta}| \lesssim 1$ eV. An average momentum of the virtual neutrino is about 100 MeV [17, 61]. Thus, the factor $m_{\beta\beta}/q^2$ gives strong suppression of the matrix element of $0\nu\beta\beta$ -decay*.

In the case of Majorana neutrino mixing (141), the half-life of the $0\nu\beta\beta$ -decay $T_{1/2}^{0\nu}(A, Z)$ has the following general form (see [17, 61]):

$$\frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{\beta\beta}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(Q, Z). \quad (144)$$

Here $M^{0\nu}(A, Z)$ is the nuclear matrix element (NME), which is determined by the nuclear properties and does not depend on elements of the neutrino mixing matrix and small neutrino masses, and $G^{0\nu}(Q, Z)$ is the known phase space factor

*It follows from (142) that for massless neutrinos $0\nu\beta\beta$ -decay is forbidden. This corresponds to the theorem on the equivalence of theories with massless Dirac and Majorana neutrinos [64, 65].

which includes the Fermi function describing final state Coulomb interaction of two electrons and nuclei.

The calculation of NME is a very complicated many-body nuclear problem. At present, NME for the $0\nu\beta\beta$ -decay of ^{76}Ge , ^{130}Te , ^{136}Xe and other nuclei were calculated in the framework of NSM, QRPA, IBM, EDF, and PHFB many-body approximate schemes (see review [63] and references thereby). Results of these calculations are significantly different. In Table 2, we present ranges of NME for ^{76}Ge and other nuclei, ratios of maximal and minimal values of NME and ranges of half-lives calculated under the assumption that $|m_{\beta\beta}| = 0.1$ eV (see [63] for details).

Table 2. Ranges of calculated values of $|M^{0\nu}|$, ratios $|M^{0\nu}|_{\max}/|M^{0\nu}|_{\min}$ and ranges of half-lives (calculated for $m_{\beta\beta} = 0.1$ eV) for the neutrinoless double β -decay of several nuclei of experimental interest

$0\nu\beta\beta$ -decay	$ M^{0\nu} $	$\frac{ M^{0\nu} _{\max}}{ M^{0\nu} _{\min}}$	$T_{1/2}^{0\nu}(m_{\beta\beta} = 0.1 \text{ eV}),$ 10^{26} y
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	3.59–10.39	2.9	1.0–8.6
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	4.39–12.13	2.8	0.1–0.8
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2.06–8.00	3.9	0.3–4.3
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	1.85–6.38	3.4	0.4–5.2

Up to now, $0\nu\beta\beta$ -decay was not observed and rather stringent lower bounds on half-life of the $0\nu\beta\beta$ -decay of different nuclei were obtained. We will present here some recent results.

In the EXO-200 experiment [66], the $0\nu\beta\beta$ -decay of ^{136}Xe (with 80.6% enrichment in ^{136}Xe) was searched for in the liquid time-projection chamber. After 100 kg · y exposure, the following lower bound was obtained:

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.1 \cdot 10^{25} \text{ y} \quad (90\% \text{ C.L.}). \quad (145)$$

Using different calculations of NME from this result for the effective Majorana mass the following upper bounds were found:

$$|m_{\beta\beta}| < (1.9\text{--}4.5) \cdot 10^{-1} \text{ eV}. \quad (146)$$

In the KamLAND-Zen experiment [67], 383 kg of liquid ^{136}Xe (enriched to 90.77%) was loaded in the liquid scintillator. After 115 days of exposure, for the half-life of ^{136}Xe the following lower bound was inferred:

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 1.3 \cdot 10^{25} \text{ y} \quad (90\% \text{ C.L.}). \quad (147)$$

Combining this result with the result of the previous run, for the half-life of ^{136}Xe there was obtained

$$T_{1/2}^{0\nu}(^{136}\text{Xe}) > 2.6 \cdot 10^{25} \text{ y} \quad (90\% \text{ C.L.}). \quad (148)$$

From this bound, for the effective Majorana mass it was found

$$|m_{\beta\beta}| < (1.4-2.8) \cdot 10^{-1} \text{ eV}. \quad (149)$$

In the germanium GERDA experiment [68], the $0\nu\beta\beta$ -decay of ^{76}Ge was studied. In the Phase-I of the experiment, the germanium target mass was 21.6 kg (86% enriched in ^{76}Ge). Very low background (10^{-2} cts/keV · kg · y) was reached. For the lower bound of the half-life of ^{76}Ge there was obtained the value*

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) > 2.1 \cdot 10^{25} \text{ y} \quad (90\% \text{ C.L.}). \quad (150)$$

Combining (150) with the results of Heidelberg–Moscow [70] and IGEX [71] experiments we found

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) > 3.0 \cdot 10^{25} \text{ y} \quad (90\% \text{ C.L.}). \quad (151)$$

From this bound, for the effective Majorana mass there was obtained the following bound:

$$|m_{\beta\beta}| < (2-4) \cdot 10^{-1} \text{ eV}. \quad (152)$$

The value of the effective Majorana mass strongly depends on the character of neutrino mass spectrum. Two mass spectra are of special interest.

1. Normal hierarchy of neutrino masses ($m_1 \ll m_2 \ll m_3$).

In this case

$$m_2 \simeq \sqrt{\Delta m_S^2}, \quad m_3 \simeq \sqrt{\Delta m_A^2}, \quad m_1 \ll \sqrt{\Delta m_S^2} \simeq 8.7 \cdot 10^{-3} \text{ eV} \quad (153)$$

and for the effective Majorana mass we find

$$|m_{\beta\beta}| = \left| \sin^2 \theta_{12} e^{2i\alpha} \sqrt{\Delta m_S^2} + \sin^2 \theta_{13} \sqrt{\Delta m_A^2} \right|, \quad (154)$$

where 2α is the relative phase. Using best-fit values of the parameters, we find

$$\sin^2 \theta_{12} \sqrt{\Delta m_S^2} \simeq 3 \cdot 10^{-3} \text{ eV}, \quad \sin^2 \theta_{13} \sqrt{\Delta m_A^2} \simeq 1 \cdot 10^{-3} \text{ eV}. \quad (155)$$

*This result allowed one to refute the claim of the observation of the $0\nu\beta\beta$ -decay of ^{76}Ge made in [69].

From (154) and (155) we find the following upper bound:

$$|m_{\beta\beta}| \lesssim 4 \cdot 10^{-3} \text{ eV}. \quad (156)$$

This bound is too small to be reached in the next generation of experiments on the search for $0\nu\beta\beta$ -decay.

2. Inverted hierarchy of neutrino masses ($m_3 \ll m_1 < m_2$).

In this case for the neutrino masses we have

$$m_1 \simeq m_2 \simeq \sqrt{\Delta m_A^2}, \quad m_3 \ll \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2} \text{ eV}, \quad (157)$$

and the effective Majorana mass is equal to

$$|m_{\beta\beta}| = \sqrt{\Delta m_A^2 (1 - \sin^2 2\theta_{12} \sin^2 \alpha)^{1/2}}. \quad (158)$$

Thus, we have

$$\sqrt{\Delta m_A^2} \cos 2\theta_{12} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m_A^2}. \quad (159)$$

From this inequality it follows that in the case of the inverted hierarchy of the neutrino masses the value of the effective Majorana mass lies in the range

$$2 \cdot 10^{-2} \lesssim |m_{\beta\beta}| \lesssim 5 \cdot 10^{-2} \text{ eV}. \quad (160)$$

More detailed calculations (see, for example, [63]) show that this result is valid for the inverted mass spectrum at $m_3 \leq 1 \cdot 10^{-2} \text{ eV}$.

The aim of the future experiments on the search for $0\nu\beta\beta$ -decay is to probe the predicted by the inverted hierarchy of neutrino masses range (160).

3.2. On the Search for Transitions into Sterile Neutrinos. All data on atmospheric, solar, reactor and accelerator neutrino oscillation experiments are perfectly described by three-neutrino mixing with two neutrino mass-squared differences $\Delta m_S^2 \simeq 7.5 \cdot 10^{-5} \text{ eV}^2$ and $\Delta m_A^2 \simeq 2.4 \cdot 10^{-5} \text{ eV}^2$. There exist, however, indications in favor of neutrino oscillations with much larger neutrino mass-squared difference(s) about 1 eV^2 . These indications were obtained in the following short baseline neutrino experiments.

1. The LSND [72] and MiniBooNE [73, 74] experiments. In the LSND experiment, neutrinos are produced in decays at rest of π^+ 's and μ^+ 's. Electron antineutrinos, presumably produced in the transition $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, were detected. In the MiniBooNE experiment, low energy excess of ν_e ($\bar{\nu}_e$) was observed in the ν_μ ($\bar{\nu}_\mu$) experiments.

2. Reactor neutrino experiments. Indications in favor of disappearance of the reactor $\bar{\nu}_e$'s were obtained from the new analysis of the data of old reactor neutrino experiments [75] in which recent calculations of the reactor neutrino flux [76, 77] were used.

3. Radiative source experiments. In the calibration experiments, performed with radiative sources by the GALLEX [78] and SAGE [79] collaborations, a deficit of ν_e 's was observed.

In order to interpret these data in terms of neutrino oscillations, we must assume that there exist more than three neutrinos with definite masses and in addition to the flavor ν_e, ν_μ, ν_τ sterile neutrinos exist also.

In the case of the simplest 3 + 1 scheme with three light neutrinos and one neutrino with mass about 1 eV, for short baseline experiments, sensitive to large Δm_{14}^2 , from (14) we find the following expression for $\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}$ transition probability:

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_{\alpha'}) = \delta_{\alpha,\alpha'} - 4(\delta_{\alpha,\alpha'} - |U_{\alpha'4}|^2)|U_{\alpha'4}|^2 \sin^2 \frac{\Delta m_{14}^2 L}{4E}, \quad (161)$$

where $\Delta m_{14}^2 = m_4^2 - m_1^2$.

From this expression for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ appearance probability and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ disappearance probabilities, we have, respectively, the following expressions:

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \sin^2 2\theta_{e\mu} \sin^2 \frac{\Delta m_{14}^2 L}{4E}, \quad (162)$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{ee} \sin^2 \frac{\Delta m_{14}^2 L}{4E}, \quad (163)$$

and

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) = 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \frac{\Delta m_{14}^2 L}{4E}. \quad (164)$$

Here

$$\begin{aligned} \sin^2 2\theta_{e\mu} &= 4|U_{e4}|^2|U_{\mu4}|^2, \\ \sin^2 2\theta_{ee} &= 4|U_{e4}|^2(1 - |U_{e4}|^2), \\ \sin^2 2\theta_{\mu\mu} &= 4|U_{\mu4}|^2(1 - |U_{\mu4}|^2). \end{aligned} \quad (165)$$

The Global analysis of all existing short baseline neutrino data was performed recently in [80, 81]. These analyses reveal inconsistency (tension) of existing short baseline data. The reason for this tension is connected with the fact that the amplitudes of the oscillations are constrained by the relation

$$\sin^2 2\theta_{e\mu} \simeq \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}, \quad (166)$$

which can be easily obtained from (165) if we take into account that $|U_{e4}|^2 \ll 1$ and $|U_{\mu4}|^2 \ll 1$.

Allowed regions of the parameters $\sin^2 2\theta_{e\mu}$ and $\sin^2 2\theta_{ee}$, determined by $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ data, require disappearance of $\bar{\nu}_\mu$ (due to the constraint (166)). However, there are no indications in favor of $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ disappearance in short baseline experiments [82–84].

Notice that in more complicated neutrino mixing and oscillation schemes with five neutrinos, this problem of tension between data still exists.

Many new neutrino oscillation experiments designed to check existing indications in favor of short baseline neutrino oscillations are proposed or in preparation at present (see recent review [85]). Proposed radioactive source experiments will be based on existing large detectors: Borexino [86], KamLAND [87], Daya Bay [88, 89]. Important feature of these new experiments is a possibility of studying the L/E dependence of $\bar{\nu}_e$ survival probability. Indications in favor of the disappearance of reactor $\bar{\nu}_e$'s will be checked in several future reactor neutrino experiments [85, 90, 91]. In these experiments, spectral distortion as a function of the distance from the reactor core will be studied. Anomaly, observed in the LSND experiment, will be investigated in the future MiniBooNE + experiment [92], in FermiLab experiment on the measurement of ν_μ disappearance [93], in ICARUS/NESSiE experiment [94, 95] with two LAr detectors. Direct test of the LSND anomaly is planned to be performed at the Spallation Neutron Source of the Oak Ridge Laboratory [96, 97]. There exist proposals to use, for the search of sterile neutrinos, the muon storage ring, a source of ν_e and $\bar{\nu}_\mu$ (or $\bar{\nu}_e$ and ν_μ) [98]. There is no doubt that in a few years the problem of the existence of light sterile neutrinos will be fully clarified.

3.3. On the Baryogenesis through Leptogenesis. Indirect indications in favor of existence of heavy Majorana leptons can be obtained from the cosmological data. From existing cosmological data it follows that our Universe predominantly consists of matter. For the baryon–antibaryon asymmetry we have

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = (6.11 \pm 0.19) \cdot 10^{-10}. \quad (167)$$

Here n_B , $n_{\bar{B}}$, and n_γ are baryon, antibaryon and photon number densities, respectively.

In the standard Big Bang scenario, initial numbers of baryons and antibaryons are equal. The observed baryon–antibaryon asymmetry has to be generated during the evolution of the Universe. A mechanism of the generation of the baryon–antibaryon asymmetry must satisfy the following Sakharov criteria [99]:

1. The baryon number has to be violated at some stage of the evolution.
2. C and CP must be violated.
3. Departure from thermal equilibrium must take place.

The interaction (132) with complex Yukawa couplings is a source of the CP violation. Out of equilibrium, CP violating lepton-Higgs decays of heavy

Majorana leptons, produced in the hot expanding Universe, could create lepton–antilepton asymmetry. This asymmetry, due to the Standard Model nonperturbative sphaleron transitions in which B and L are violated, could be converted into baryon–antibaryon asymmetry (see reviews [100–103]).

There are many models based on this general scenario of baryogenesis through leptogenesis. *Existence of heavy Majorana leptons is their common feature.*

CONCLUSION

The Standard Model successfully describes all observed physical phenomena in a wide range of energies up to a few hundreds of GeV. After the discovery of the Higgs boson at LHC, the Standard Model was established as *a theory of physical phenomena at the electroweak scale*. We suggest here that neutrinos play exceptional role in the Standard Model. Neutrinos apparently are crucial in the determination of symmetry properties of the Standard Model.

The Standard Model is based on

- the local gauge symmetry;
- the unification of the weak and electromagnetic interactions;
- Brout–Englert–Higgs mechanism of the spontaneous breaking of the symmetry.

The Standard Model teaches us that in the framework of these general principles, nature chooses the simplest possibilities. The simplest, most economical possibility for neutrinos is to be two-component Weyl particles (Landau–Lee–Yang–Salam two-component neutrinos). The experiment showed that from two possibilities (left-handed or right-handed) the nature chooses the left-handed possibility.

In order to ensure symmetry, fields of quarks and leptons must be also two-component, left-handed and the symmetry group must be non-Abelian. This allows one to include charged particles and ensure the universality of the minimal CC interaction of the fundamental fermions and the gauge fields. The simplest possibility is $SU_L(2)$ with doublets of the left-handed fields.

The unification of weak and electromagnetic interactions requires enlargement of the symmetry group. The simplest possibility is the $SU_L(2) \times U_Y(1)$ group. Since the electromagnetic current includes left-handed and right-handed fields of the *charged particles*, charged right-handed fields must be SM fields (singlets of the $SU_L(2)$ group). Electric charges of neutrinos are equal to zero. The unification of the weak and electromagnetic interactions does not require right-handed neutrino fields. Minimal possibility is that *there are no right-handed neutrino fields in the SM*. Nonconservation of P and C in the weak interaction apparently is connected with that. Since there are no right-handed SM neutrino fields, there is no Yukawa interaction which can generate neutrino mass term: neutrinos are

the only particles which after spontaneous breaking of the electroweak symmetry remain two-component left-handed ones.

With two-component left-handed neutrino fields ν_{iL} , only a beyond the Standard Model, lepton-number violating Majorana mass term can be built. *This is the most economical possibility.* It is generated by the unique, beyond the Standard Model dimension five Weinberg effective Lagrangian. Due to a suppression factor which is a ratio of the electroweak vacuum expectation value v and the parameter Λ , which characterizes the scale of a new lepton-number violating physics, such approach naturally explains the smallness of neutrino masses.

In the framework of the effective Lagrangian values of neutrino masses, mixing angles and CP phases cannot be predicted. The same is true for leptons and quarks: the Higgs mechanism of generation of masses and mixing of leptons and quarks does not predict the values of masses, mixing angles and CP phase. However, there are three general consequences of this mechanism of the neutrino mass generation.

1. Neutrino with definite masses ν_i are Majorana particles.
2. Number of neutrinos with definite masses is equal to the number of the flavor neutrinos (three).

The neutrino nature (Majorana or Dirac?) can be inferred from the experiments on the search for neutrinoless double β -decay of ^{76}Ge , ^{136}Xe and other nuclei. If this process will be observed, it will be a proof that neutrinos with definite masses are Majorana particles, i.e., that neutrino masses have a beyond the SM origin. Future experiments will probe inverted neutrino mass spectrum region ($m_{\beta\beta} \simeq$ a few 10^{-2} eV). In the case of normal mass hierarchy, the probability of the neutrinoless double β -decay will be so small that new methods of the detection of the process must be developed (see [104]).

A possibility that the number of the neutrinos with definite masses is more than three, will be tested in future reactor, radioactive source and accelerator experiments on the search for sterile neutrinos.

The effective Lagrangian, responsible for the Majorana neutrino mass term, can be a result of the exchange of virtual heavy Majorana leptons between lepton-Higgs pairs. The CP violating, out of equilibrium decays of heavy Majorana leptons in the early Universe, could be the origin of the baryon-antibaryon asymmetry of the Universe.

The value of the parameter Λ , which characterizes the scale of a new lepton-number violating physics, is an open problem. It is natural to assume that the Yukawa coupling constant is of the order of one. In this case $\Lambda \simeq 10^{15}$ GeV. However, much smaller values of Λ cannot be excluded. If Λ is of the order of TeV, lepton-number violating decays of Majorana leptons can be observed at LHC (see, for example, [105–109]).

The Standard Model teaches us that the simplest possibilities are more likely to be correct. Two-component left-handed Weyl neutrinos and absence of the

right-handed neutrino fields in the Standard Model is the simplest, most elegant and most economical possibility. In this case, generated by the effective, dimension five Lagrangian (or by the standard seesaw mechanism) *Majorana mass term* (three Majorana neutrinos with definite masses, absence of sterile neutrinos), *is the simplest, most economical possibility*. Future experiments will show whether this possibility is realized in nature.

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