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FLUCTUATIONS OF CASCADES INDUCED BY GAMMA QUANTA FROM 200 TO 3375 MeV IN DENSE AMORPHOUS MATERIALS

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We present the results of investigation of both the longitudinal and transverse fluctuation of electromagnetic cascades (EC) produced in liquid xenon by gamma quanta of energy from 200 to 3375 MeV at four different values of cut-off energy (for all cascade particles) 0.6, 1.25, 2 and 3 MeV. The work has been performed with the EGS modeling code [3]. The ultimate objective of this investigation is to obtain exhaustive and concise information in the form of simple formulas suitable for experiments (in particular, for PANDA detector, GSI, Darmstadt).

Представлены результаты исследования продольной и поперечной флуктуаций электромагнитных каскадов (ЭК), возникающих в жидком ксеноне под воздействием гамма-квантов с энергиями от 200 до 3375 МэВ при четырех разных значениях энергии обрезания (для всех частиц каскада) 0,6, 1,25, 2 и 3 МэВ. Расчеты проведены с помощью EGS-кода [3]. Главной целью данного исследования было получение исчерпывающей информации в форме простых формул, пригодных при описании экспериментов (в частности, для детектора PANDA, GSI, Дармштадт).

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INTRODUCTION

Longitudinal and lateral profiles of electromagnetic cascades (EC) produced in heavy amorphous media by high enough energy gamma quanta and electrons (or positrons) are the basic characteristics of the phenomenon both from cognitive and application viewpoints (for example, [1]). Such is also the corresponding fluctuation of these profiles since the process of EC is of strongly expressed stochastic nature that it is especially manifested at intermediate energies (i.e., from hundredths MeV to several GeV) when in the cascade process not too much particles are involved. It is customary to assume that average longitudinal EC profiles (LP) may be satisfactorily parameterized by a gamma function, whereas in view of practical usage the longitudinal fluctuation (LF) may be described as the distribution of a definite part A (later on called also as threshold) of the total EC energy loss deposited up to the absorbent

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depth t, at which the average part of released cascade energy is equal to $\langle A \rangle$ (for electrons and positrons), on condition that the absorbent is sufficiently large in the lateral direction [1]. An alternative approach to the problem, depending, in particular, on specific experimental conditions, starts from the distribution of the depth t at which a certain part A of total EC energy is released. Similarly one can define transverse fluctuations (TF) of EC [1]. Such an approach to the problem of fluctuation in EC is adequate to the real experimental situation when high enough energy gamma quanta or/and electrons are registered by using a detector of limited dimensions.

Our current knowledge about fluctuation is mainly limited to the estimation of its integral characteristics as energy resolution (for example, [2]). Nevertheless, it should be mentioned that it has been shown experimentally that both longitudinal and transverse fluctuation scale satisfactorily with energy of primary gamma quanta within large enough energy interval, from 200 MeV to 3.5 GeV, in liquid xenon [1].

In the work we study both the longitudinal and transverse fluctuation in EC produced in liquid xenon by gamma quanta of energy $E_\gamma = 200$, 550, 2375 and 3375 MeV at four different cut-off energies $E_{\rm c.o} = 0.6$, 1.25, 2.0 and 3.0 MeV (for all cascade particles), and three values of threshold $A = 0.5$, 0.7 and 0.9. The work has been performed using the EGS code [3]. In total 48 000 events of cascades are modeled. The general objective of this investigation is to obtain exhaustive and concise information about the EC fluctuation in the form of simple analytic approximations suitable for experiment (in particular, for PANDA detector, GSI, Darmstadt).

1. LONGITUDINAL PROFILES AND FLUCTUATIONS

It is commonly agreed that the average longitudinal profile of EC initiated by gamma quanta in dense enough amorphous absorber may be parameterized acceptably by the gamma-like function (although some inconsistency appears at the cascade beginning and at large depths too) [1]:

$$f(t) = at^b \exp(-t/c). \tag{1}$$

Here a, b and c are free parameters to be determined by the fit to experiment or modeled data. They depend both on initial E_γ and cut-off $E_{\rm c.o}$ energies. The depth t is usually expressed in unit of radiation length (r. l.). The normalization parameter a may be chosen in such a way that $f(t)\Delta t$ is the probability density of cascade energy release (SER) inside an absorber layer of Δt at its depth t. Then we can define the part $A(t)$ of average cascade energy deposition as

$$A(t) = \int_0^t f(\tau)\, d\tau. \tag{2}$$

From the practical point of view of significant importance is the fluctuation of SER at some fixed depth t at which the part $A(t)$ of the average shower energy deposition (called later on as threshold energy) is equal to: 0.5, 0.7 and 0.9 [1]. It corresponds to the actual experimental situation when a detector of limited thickness of its active medium is destined to analyze a specific gamma quanta energy spectrum. An example of such a fluctuation is given in Fig. 1

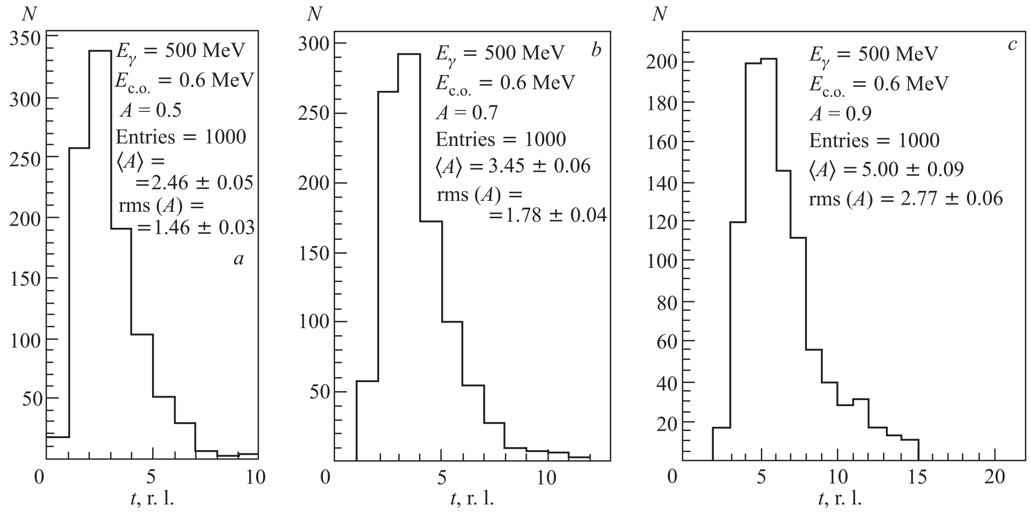

Fig. 1. Distribution of the cascade depth t at which fixed parts $A = 0.5$ (a), 0.7 (b) and 0.9 (c) of the total cascade energy is deposited in liquid xenon when the cascades are produced by gamma quanta of $E_\gamma = 500$ MeV energy and the cut-off energy is equal to $E_{\rm c.o} = 0.6$ MeV. Pointed are also relevant average and rms values. Simulation is performed by using EGS code [3]

for the case of $E_\gamma = 500$ MeV at $E_{\rm c.o} = 0.6$ MeV and three values of threshold energies $A = 0.5$, 0.7 and 0.9.

It turned out that the distribution of t_A may be approximated as well by the gamma-like function

$$P(t_A) = \alpha t_A^\beta \exp\left(-t_A/\gamma\right), \tag{3}$$

where the parameters β and γ depending on E_γ and $E_{\rm c.o}$ have been determined as the best fit to modeled data and α is simply a normalization parameter. In Fig. 2 such a fit is demonstrated, for example, at $E_\gamma = 555$ MeV and all investigated values of $E_{\rm c.o}$ and A.

The calculation has been done by using EGS code [3]. The number of events is equal to 10000 for every set of parameters (E_γ, $E_{\rm c.o}$, A).

Table 1 contains, as an example, the obtained values of parameters β and γ of fitting function (3) for $A = 0.5$. Similar results have been calculated also for other values of A: 0.7 and 0.9. One can notice that these parameters perceptibly depend on energy E_γ and $E_{\rm c.o}$. This dependence is illustrated in Fig. 3 and Fig. 4, respectively.

The E_γ energy dependence of coefficients β and γ has been approximated by the function

$$p(E_\gamma) = a_p \ln\left(E_\gamma\right) - b_p. \tag{4}$$

The relevant numerical data are collected in Table 2 for $A = 0.5$. Nevertheless, it should be mentioned that for this so-called second step fitting procedure the usual Pearson's χ^2 statistics is completely unsuitable because every observable value is now characterized not only by its error but also by its probability and the corresponding minimization functional should be properly generalized taking it into account. Therefore, our numerical values obtained using the standard χ^2 statistics have only an approximate character.

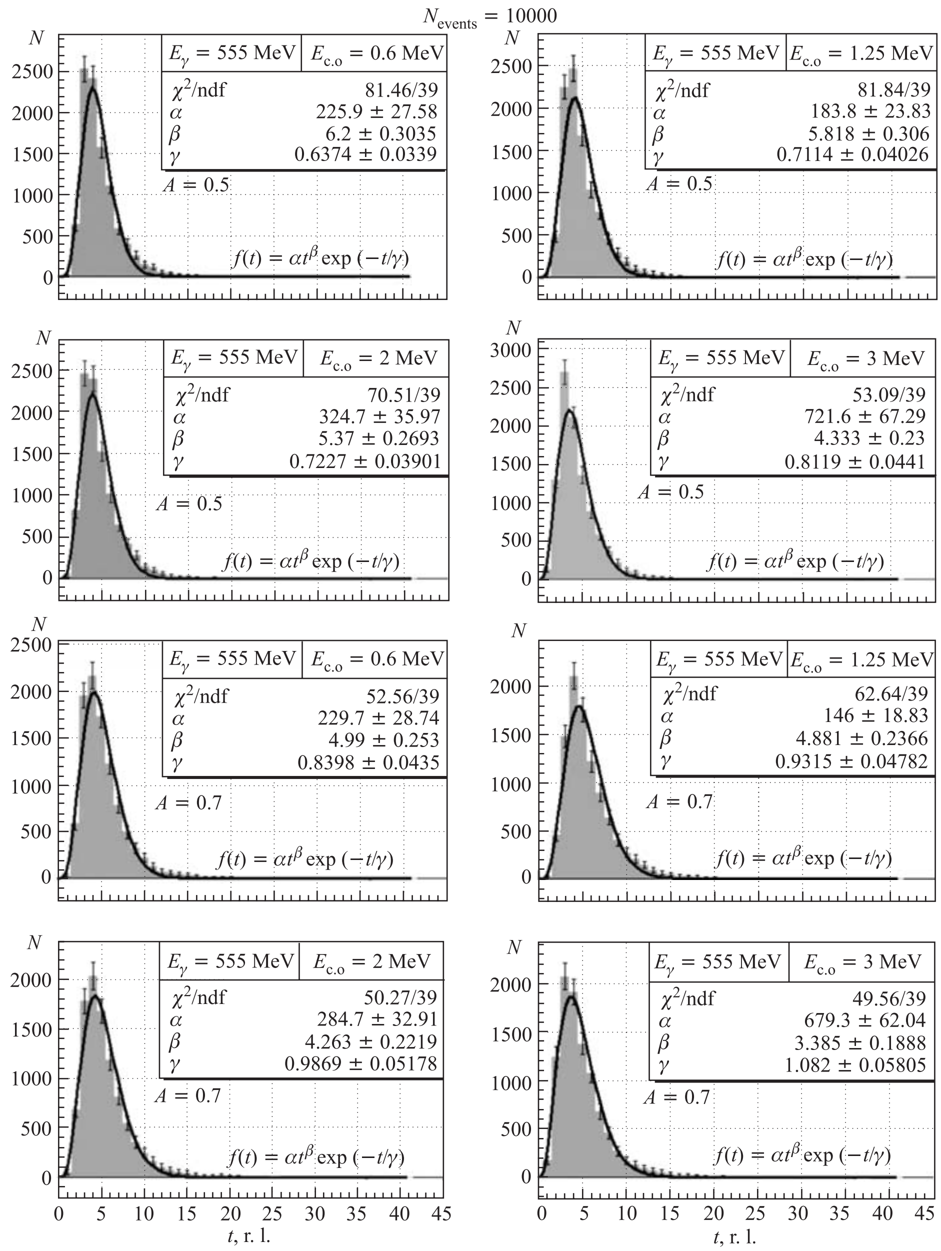

Fig. 2. Distributions of the shower depth t_A at which a fixed part A of average cascade energy is released when the cascade is initiated in liquid xenon by gamma quanta of energy $E_\gamma = 555$ MeV and detected with the cut-off energies $E_{\text{c.o}} = 0.6$, 1.25, 2.0 and 3.0 MeV (histograms). Smooth curves demonstrate the fitting function (3) with the corresponding values of the parameters α, β and γ and test statistics $\chi^2/_{\text{ndf}}$

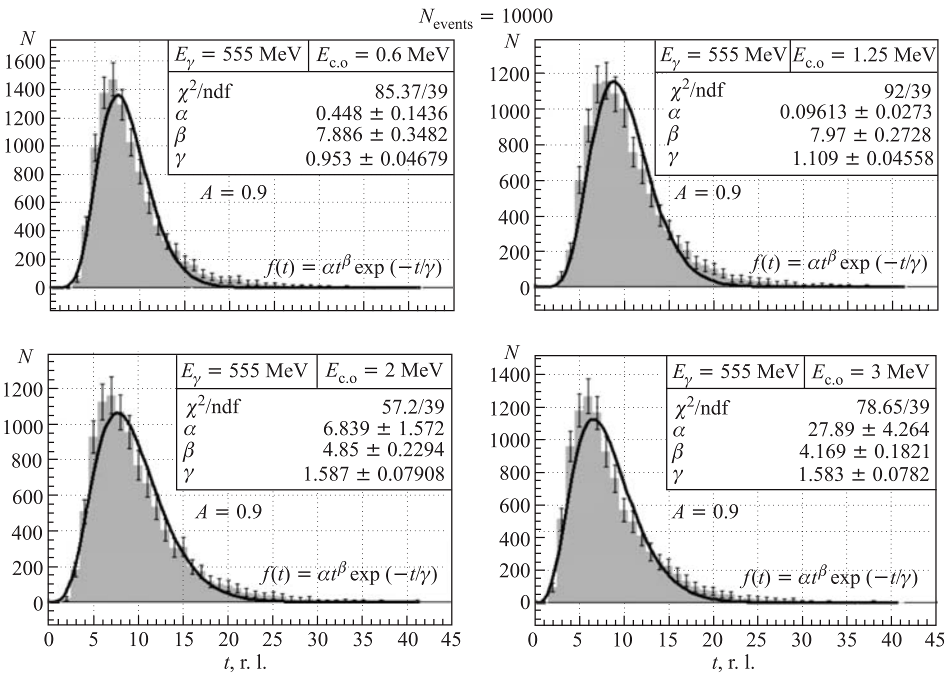

Fig. 2. Continuation

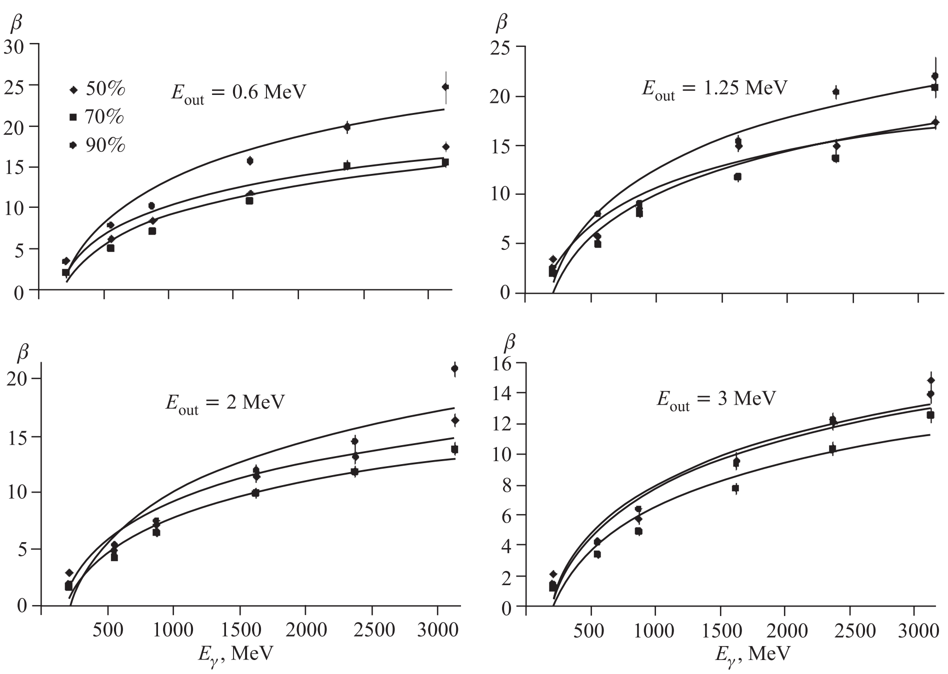

Fig. 3. Energy dependence of the coefficient β of formula (3) at different values of $E_{\text{c.o}}$ and thresholds A. Solid curves represent the approximation function (4)

Table 1. **Values of parameters β and γ of approximating function (3) for the threshold energy part $A = 0.5$. $\Delta\beta$ and $\Delta\gamma$ are corresponding errors**

E, MeV	$E_{c.o}$, MeV	β	$\Delta\beta$	γ	$\Delta\gamma$	χ^2/n
210	0.6	3.57	0.21	0.86	0.05	63.12/39
555	0.6	6.20	0.30	0.64	0.03	81.47/39
875	0.6	8.39	0.48	0.52	0.03	94.85/39
1625	0.6	11.68	0.57	0.43	0.02	103.09/39
2375	0.6	15.27	0.65	0.35	0.02	126.55/39
3125	0.6	17.46	0.69	0.32	0.02	133.06/39
210	1.25	3.50	0.20	0.90	0.05	62.60/39
555	1.25	5.80	0.30	0.71	0.04	81.84/39
875	1.25	8.66	0.47	0.54	0.03	101.66/39
1625	1.25	14.90	0.60	0.36	0.02	142.89/39
2375	1.25	14.90	0.73	0.38	0.02	125.87/39
3125	1.25	17.32	0.82	0.35	0.02	143.81/39
210	2.0	2.89	0.12	0.98	0.06	40.07/39
555	2.0	5.37	0.27	0.72	0.04	70.51/39
875	2.0	7.10	0.40	0.62	0.04	74.37/39
1625	2.0	11.39	0.52	0.45	0.02	103.96/39
2375	2.0	13.10	0.62	0.42	0.02	109.71/39
3125	2.0	16.39	0.66	0.35	0.02	136.32/39
210	3.0	2.10	0.17	1.10	0.07	21.82/39
555	3.0	4.33	0.23	0.81	0.04	53.09/39
875	3.0	5.72	0.34	0.71	0.04	59.93/39
1625	3.0	9.50	0.50	0.50	0.03	99.17/39
2375	3.0	12.06	0.57	0.43	0.02	113.85/39
3125	3.0	14.81	0.63	0.37	0.02	124.62/39

Table 2. **Values of parameters a and b of the approximating function (4) for $A = 0.5$. Δa and Δb mean the relevant errors**

Parameter	$E_{c.o}$, MeV	a	Δa	b	Δb	χ^2/n
β	0.6	4.41	0.17	20.49	1.04	42.92/4
β	1.25	4.63	0.18	21.77	1.08	62.50/4
β	2.0	4.12	0.15	19.57	0.94	51.61/4
β	3.0	3.73	0.14	18.33	0.87	67.39/4
γ	0.6	− 0.19	0.01	− 1.81	0.11	2.50/4
γ	1.25	− 0.19	0.02	− 1.84	0.12	31.12/4
γ	2.0	− 0.22	0.02	− 2.10	0.12	4.01/4
γ	3.0	−0.26	0.02	−2.48	0.14	1.48/4

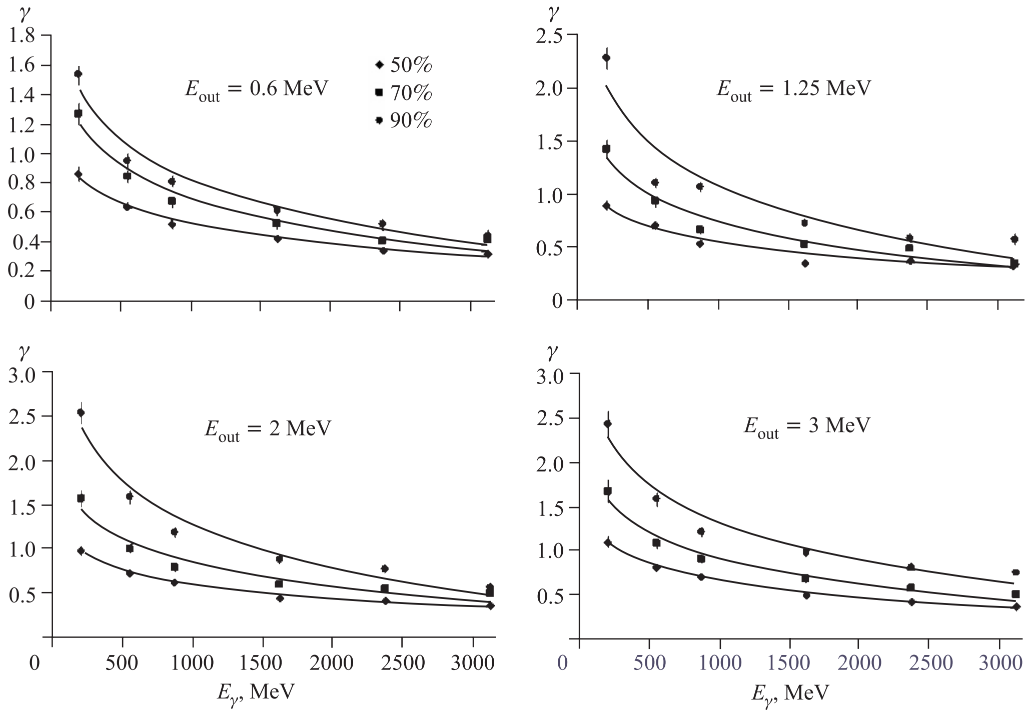

Fig. 4. Same as in Fig. 3 but for the coefficient γ

2. LATERAL PROFILES AND FLUCTUATIONS

The analysis of fluctuation of the lateral cascade development has been performed in the similar as previously way. As an example, in Fig. 5 distributions of lateral fluctuation in cascades initiated in liquid xenon by gamma quanta of $E_\gamma = 555$ MeV energy at cut-off energies $E_{\rm c.\,o} = 0.6$ and 1.25 MeV, and with two transverse thresholds $A = 0.7$ and 0.9 are depicted. At the same time it is assumed that there is no limitation in the longitudinal development of cascades.

One can see that, unfortunately, in the case of transverse fluctuation the gamma-like function in the form of Eq. (2)

$$P(r_A) = \alpha r_A^{\beta_T} \exp\left(-r_A/\gamma_T\right) \quad (5)$$

is not so good approximation as above and may be used for estimation only. Here r_A is a distance from the shower axis, i. e., the radius of a cylinder inside which rate A of the total cascade energy is released. As an example, in Fig. 6 the dependence on energy E_γ of parameters β_T and γ_T of the approximating function (5) at the threshold value $A = 0.9$ is shown. Solid lines represent the fitting function of the form (4).

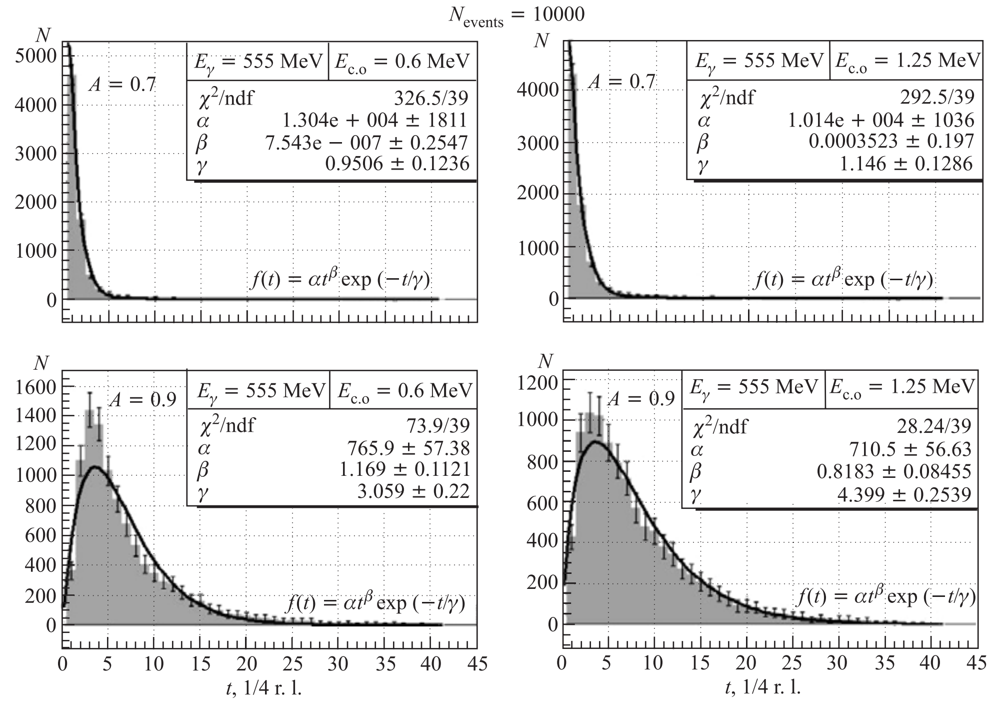

Fig. 5. Similar as Fig. 2 but for transverse fluctuation

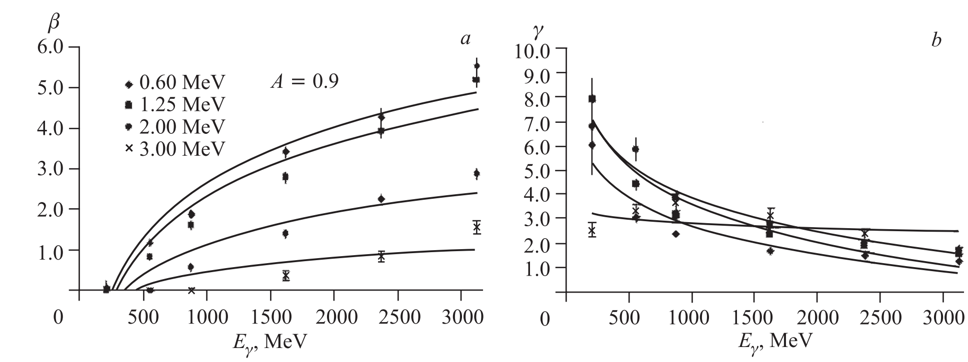

Fig. 6. Dependence on energy E_γ of parameters of the approximating function (5): β_T (a) and γ_T (b) at the threshold value $A = 0.9$. Solid lines represent the fitting function of the form (4)

3. SUMMARY AND CONCLUSION

In the work we studied for the first time the fluctuation of energy deposition in electromagnetic cascades initiated in liquid xenon by gamma quanta of energy E_γ from 200 to 3375 MeV at four specified cut-off energies $E_{\mathrm{c.o}}$ coinciding with typical experimental conditions, i.e., in the range of 0.6–3.0 MeV. The fluctuation, both longitudinal and transverse, has been investigated at three values of threshold $A = 0.5$, 0.7 and 0.9. In our definition [1], the fluctuation at the threshold A means that the depth (in the case of longitudinal development), at which part A of total shower energy is released, as a random number obeys some distribution to be found either from experiment or from simulation. The parameters of this distribution depend on all cascade characteristics: E_γ, $E_{\rm c.o}$ and A, and absorbent properties as well. In the case of transverse fluctuation the threshold A corresponds to the radius of a cylinder inside which the part A of the total shower energy is deposited.

It has been found that both longitudinal and transverse fluctuation may be parameterized by the gamma-like function (3) where the values of parameters β and γ are determined using the EGS code [3] with corresponding χ^2 as a test statistics. The relevant numerical data are tabularized in Table 1. Other approximation functions have also been investigated, in particular, Weibull function $f(t) = \alpha t^\beta \exp(-\gamma t^2)$ and logarithmic form of gamma function $f(t) = \ln(\alpha) + \beta \ln(t) - t/\gamma$, but function (3) turned out to be the best one. Next, the dependence of these parameters on E_γ, $E_{\rm c.o}$ and A was parameterized by function (4) using the same standard χ^2 test statistics. Nevertheless, this time the reliability of such a fit is not so successful than earlier because for the so-called second step fit the test statistics must also include information about the probability of the first one. Therefore, for this purpose the standard χ^2 test or any other test statistics should be adequately generalized by taking into account the probability of each individual observable. Note finally, that the obtained approximations suggest the possibility of describing the fluctuation in a universal form, at least in not too dense absorbents, such as BGO and PWO.

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