Письма в ЭЧАЯ. 2011. Т. 8, № 2(165). С. 169–181

NEW APPROACH TO NUCLEAR BINDING ENERGY IN INTEGRATED NUCLEAR MODEL

N. Ghahramany [a], *Sh. Gharaati* [a], *M. Ghanaatian* [b]

[a] Physics Department, Shiraz University, Shiraz, Iran

[b] Physics Group, Payame Noor University, Iran

In this paper, the integrated nuclear model is introduced and a binding energy formula based on this model is presented. The binding energies of most nuclides in this model are compared with available experimental values and also with values from the liquid drop model (LDM).

В статье введена интегральная ядерная модель, в которой получена формула для вычисления энергии связи ядер. Энергии связи большинства нуклидов, рассчитанные в модели, сравниваются с доступными экспериментальными данными и с результатами модели жидкой капли.

PACS: 21.10.Dr; 21.60.-n

INTRODUCTION

One of the purposes of the nuclear physics is to introduce the proper mathematical models from which the properties and the behavior of nuclides can be explained. One of the outstanding features of the nuclides is the fact that their nuclear density is approximately constant. Therefore, the volume of nuclide is proportional to the mass number A. The same proportionality holds for liquids and one of the early fundamental nuclear models presented by Carl Friedrich Von Weizsäcker [1] and developed by Niels Bohr and John Archibald Wheeler [2] was based upon liquid drops. Nuclides are considered as incompressible liquid drops with enormous density. Based upon the fact that average binding energy per nucleon and the nuclear density are constant, Weizsäcker was able to present his liquid drop model with the following basic assumptions:

1) The nuclides are made of incompressible matter so that $R \sim A^{1/3}$ (R is the mean nuclear radius).

2) The nuclear force is the same for each nucleon.

3) The nuclear force saturates.

The liquid drop model led to the famous semi-empirical mass formula from which the dependency of nuclear mass upon A and Z is given [3]. First, the nucleus is considered as a collection of interacting particles like a liquid drop. And then, the Coulomb force, the Pauli exclusion principle effect and other details are added to the model as corrections and finally the following formula is derived for nuclear binding energy:

$$B(A,Z) = a_\nu A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a (N-Z)^2 A^{-1} \pm \delta + \eta. \qquad (1)$$

In the liquid drop model, nucleons are not described individually. They are considered as averaged values, therefore this model has been successful in describing some properties of nuclei such as average binding energy per nucleon, whereas for other nuclear properties such as nuclear excited states, magic numbers and nuclear magnetic moments it has not so much to present.

The nuclear properties can be described simply in terms of free particles behavior instead of strongly interacting particles as viewed in the liquid drop model. If nuclide is considered as a degenerate Fermi gas of nucleons (Fermi gas model), then a nuclear free particle model is obtained. In this model, it is assumed that nucleons are freely (except under the Pauli exclusion constraint) moving within a nuclide with radius $R = R_0\ A^{1/3}$. Using the quantum tunneling theory and Pauli exclusion principle, one can find the average kinetic energy of the nucleons within the nuclide as follows:

$$\langle E\,(Z,N)\rangle = \frac{3}{10\,m}\frac{\hbar^2}{R_0^2}\left(\frac{9\pi}{8}\right)^{2/3}\left\{A + \frac{5}{9}\left(\frac{Z-N}{A}\right)^2 + \dots\right\}. \tag{2}$$

In contrast to the liquid drop model and Fermi gas model in which the macroscopic properties of nuclei are presented, the nuclear shell model [4–6] deals with the microscopic properties of nuclei. The nucleons as free particles moving in a spherical potential and also the Pauli exclusion principle intensively limit the interaction between the nucleons. Such a consideration in the shell model provides orbits with approximate stability and defined energy levels. The fundamental assumption in a nuclear shell model is the independence of nucleon motions (free particles) regardless of the existence of strong attractive force between the nucleons. With these assumptions it is predictable that such a model is able to describe nuclear microscopic properties such as excited states energy, magic number and nuclear magnetic moments, but it is impotent to provide a nuclear binding energy formula.

In this paper, it is attempted to present an integrated new nuclear model and a new formula for binding energy of most nuclides based upon intuitive assumptions that will be presented in the next section.

1. NUCLEAR BINDING ENERGY IN INTEGRATED NUCLEAR MODEL

In general, the total mass of nuclei (Z, N) is less than the sum of the masses of its constituent particles, namely, protons and neutrons. This mass difference is defined as nuclear binding energy. In 1966, G. T. Garvey and I. Kelson presented a formula for the nuclear binding energy [7, 8]. Since the nuclear energy possesses saturation property, therefore it is possible for the mass difference between two neighboring nuclides of (Z_0, N_0) and (Z, N), namely, $M(Z,N) - M(Z_0,N_0)$, to be expanded as power series in terms of $\Delta Z = Z - Z_0$ and $\Delta N = N - N_0$. Consequently, it is possible to write the following formula for the nuclear binding energy [9, 10]:

$$B\,(Z,N) = B\,(Z_0,N_0) + B_{10}\Delta Z + B_{01}\Delta N + \\ + B_{20}\,(\Delta Z)^2 + B_{02}(\Delta N)^2 + B_{11}(\Delta Z)(\Delta N) + \dots \tag{3}$$

where the coefficients $B_{10}, B_{01}, \ldots$ are the partial derivatives of $B(Z,N)$ with respect to Z and N at (Z_0, N_0). A good approximation is to neglect the second and higher order derivatives in series (3). Then considering a linear relation for the binding energy, we will have the following two formulas:

$$B\left(Z,N\right) = g_1\left(Z\right) + g_2\left(N\right) + g_3\left(N+Z\right), \tag{4}$$

$$B\left(Z,N\right) = f_1\left(Z\right) + f_2\left(N\right) + f_3\left(N-Z\right). \tag{5}$$

The detailed mathematical derivation of (4) and (5) from (3) is given in references [9, 10].

Now we may use the mentioned models, namely, liquid drop, Fermi gas and shell models in addition to relations (1), (2), (4) and (5) to express our fundamental assumptions in order to present a new formula for the nuclear binding energy:

1) The nuclear binding energy is of the order of about one percent of the energy of the total rest mass of the constituent nucleons [11].

2) The nuclear binding energy is proportional to the volume of the nuclide ($B \sim A$).

3) The nuclear binding energy depends upon the asymmetry between the number of protons and neutrons (specially in heavy nuclides) and also depends upon the Coulomb repulsion force between protons.

According to formulas (4) and (5), the binding energy equation can be expressed as a function of N, Z, $N+Z$ and $N-Z$ (where f and g are arbitrary functions). We have chosen only the linear ones and ignored the higher orders. In order to present a formula consistent with the experimental binding energy data, our binding energy is expressed in a combination form, namely, Eq. (6), in such a way to observe the nucleon asymmetry and the Coulomb corrections much simpler than the way that is presented in the liquid drop model. From the conditions of relations (4) and (5), it is noticed that the binding energy is proportional to both $(N+Z)$ and $(N-Z)$. Therefore, a term $(N^2-Z^2)/3Z$ appears for the nucleon asymmetry and the Coulomb correction in the third assumption.

Based upon the above assumptions, the following formula is presented for the nuclear binding energy of all elements:

$$B\left(Z,N\right) = \left\{\left[A - \left(\frac{\left(N^2-Z^2\right)+\delta\left(N-Z\right)}{3Z}+3\right)\right]\frac{m_N c^2}{\alpha}, \quad A>5, \right. \tag{6}$$

where α is defined as an adjusting coefficient, $\alpha = 90-100$, and will be explained in the next section, and δ stands for nuclear beta-stability line condition and is defined as follows:

$$\delta\left(N-Z\right) = \begin{cases} 0 & \text{for} \quad N \neq Z, \\ 1 & \text{for} \quad N = Z. \end{cases} \tag{7}$$

In the Table the nuclear binding energies for all nuclides are given by using formula (6) and have been compared with the results of LDMs and with experimental results. The nuclear binding energies per nucleon obtained by using formula (6) are in good agreement with the existing experimental data and also with LDM for all mass numbers as shown in Figs. 1–3.

Nuclear binding energy per nucleon for most of the stable nuclei in our model, LDM and experimental values

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
1	H	1	0	0	0	–26.461	–26.461	0	0
1	H	2	1	2.225	1.1125	–5.22551	–2.61276	2.22539	1.11269
1	H	3	2	8.482	2.82733	1.8315	0.6105	8.45045	2.81682
2	He	3	1	7.718	2.57267	0.35259	0.11753	7.71382	2.57127
2	He	4	2	28.296	7.074	21.9452	5.4863	28.27305	7.06826
3	Li	6	3	31.994	5.33233	27.64	4.60667	30.10864	5.01811
3	Li	7	4	39.244	5.60629	38.3835	5.48336	33.58272	4.79753
4	Be	9	5	58.165	6.46278	56.6316	6.2924	54.71667	6.07963
5	B	10	5	64.751	6.4751	63.0939	6.30939	65.03467	6.50347
5	B	11	6	76.205	6.92773	75.0627	6.82388	75.73481	6.88498
6	C	12	6	92.162	7.68017	87.749	7.31242	92.19658	7.68305
6	C	13	7	97.108	7.46985	93.629	7.20223	96.69506	7.43808
7	N	14	7	104.659	7.47564	99.6605	7.11861	104.82993	7.48785
7	N	15	8	115.492	7.69947	112.2803	7.48535	115.06522	7.67101
8	O	16	8	127.619	7.97619	123.7138	7.73211	127.94649	7.99666
8	O	17	9	131.763	7.75076	130.9744	7.70438	131.23772	7.71987
8	O	18	10	139.807	7.76706	141.24997	7.84722	139.15385	7.73077
9	F	19	10	147.801	7.779	149.6775	7.87776	147.91676	7.78509
10	Ne	20	10	160.645	8.03225	160.15493	8.00775	160.75488	8.03774
10	Ne	21	11	167.406	7.97171	168.363	8.01729	167.29278	7.96632
10	Ne	22	12	177.77	8.08045	179.44476	8.15658	178.76377	8.12563
11	Na	23	12	186.564	8.11148	188.0092	8.17431	186.66229	8.11575
12	Mg	24	12	198.257	8.26071	196.68558	8.19523	198.70651	8.27944
12	Mg	25	13	205.588	8.22352	205.5993	8.22397	206.02692	8.24108
12	Mg	26	14	216.681	8.33388	217.2668	8.35642	217.4098	8.36192
13	Al	27	14	224.952	8.33156	224.1192	8.30071	225.38779	8.3477
14	Si	28	14	236.537	8.44775	233.089	8.32461	236.6431	8.45154
14	Si	29	15	245.011	8.44866	242.5576	8.36406	244.7457	8.43951
14	Si	30	16	255.62	8.52067	254.6751	8.48917	255.17021	8.50567
15	P	31	16	262.917	8.48119	260.9052	8.4163	261.40635	8.43246
16	S	32	16	271.781	8.49316	269.23215	8.4135	271.82458	8.49452
16	S	33	17	280.422	8.49764	279.1541	8.45922	280.5625	8.50189
16	S	34	18	291.839	8.5835	291.6321	8.57741	292.09649	8.59107
16	S	36	20	308.714	8.57539	309.13735	8.58715	309.23077	8.58974
17	Cl	35	18	298.21	8.52029	297.29795	8.49423	299.71709	8.56335
17	Cl	37	20	317.101	8.5703	317.675	8.58581	317.5582	8.58265
18	Ar	36	18	306.717	8.51992	305.02832	8.47301	309.3663	8.59351
18	Ar	38	20	327.343	8.61429	328.10722	8.6344	328.22762	8.63757
18	Ar	40	22	343.811	8.59528	346.7388	8.66847	343.29829	8.58246
19	K	39	20	333.724	8.55703	333.24958	8.54486	334.60819	8.5797
19	K	40	21	341.524	8.5381	342.62116	8.56553	340.70927	8.51773
19	K	41	22	351.619	8.57607	354.4661	8.64551	350.20724	8.54164
20	Ca	40	20	342.052	8.5513	340.41858	8.51046	346.90367	8.67259

Table (continued)

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
20	Ca	42	22	361.896	8.61657	364.07726	8.66851	363.59588	8.65704
20	Ca	43	23	369.829	8.60067	372.6376	8.66599	369.82604	8.60061
20	Ca	44	24	380.96	8.65818	383.66084	8.71956	379.85674	8.63311
20	Ca	46	26	398.769	8.66889	399.70627	8.68927	400.21333	8.70029
21	Sc	45	24	387.848	8.61884	390.6604	8.68134	389.4375	8.65417
22	Ti	46	24	398.193	8.65637	399.52474	8.68532	398.22944	8.65716
22	Ti	47	25	407.073	8.66113	408.53494	8.69223	409.04261	8.70303
22	Ti	48	26	418.7	8.72292	419.9271	8.74848	420.01354	8.75028
22	Ti	49	27	426.842	8.71106	427.26614	8.71972	426.51645	8.70442
22	Ti	50	28	437.781	8.75562	437.02429	8.74049	437.60839	8.75217
23	V	50	27	434.794	8.69588	434.6673	8.69335	435.44378	8.70888
23	V	51	28	445.845	8.74206	445.416	8.73365	446.8546	8.76185
24	Cr	50	26	435.049	8.70098	434.43674	8.68873	436.56349	8.73127
24	Cr	52	28	456.349	8.77594	455.5536	8.76065	455.28655	8.75551
24	Cr	53	29	464.289	8.76017	463.39155	8.74324	462.20892	8.72092
24	Cr	54	30	474.008	8.77793	473.57011	8.76982	474.09783	8.77959
25	Mn	55	30	482.075	8.765	481.1957	8.74901	482.30496	8.76918
26	Fe	54	28	471.763	8.73635	468.80385	8.68155	470.09324	8.70543
26	Fe	56	30	492.258	8.79032	490.5518	8.75985	494.95007	8.83839
26	Fe	57	31	499.905	8.77026	498.831	8.75142	497.10202	8.72109
26	Fe	58	32	509.949	8.79222	509.38082	8.78243	509.73201	8.78848
27	Co	59	32	517.313	8.76802	516.2997	8.75084	516.9666	8.76215
28	Ni	58	30	506.459	8.73205	502.61932	8.66585	508.02694	8.75909
28	Ni	60	32	526.846	8.78077	524.93075	8.74885	529.02083	8.81701
28	Ni	61	33	534.666	8.76502	533.6038	8.7476	536.82281	8.80037
28	Ni	62	34	545.262	8.79455	544.48462	8.78201	544.55319	8.78312
28	Ni	64	36	561.758	8.77747	561.5415	8.77409	559.78986	8.74672
29	Cu	63	34	551.385	8.75214	550.7451	8.74199	550.87284	8.74401
29	Cu	65	36	569.212	8.75711	569.4077	8.76012	572.5846	8.80899
30	Zn	64	34	559.098	8.73591	558.6975	8.72965	562.37022	8.78703
30	Zn	66	36	578.136	8.75964	576.4769	8.7345	578.59789	8.76663
30	Zn	67	37	585.189	8.73416	586.73452	8.75723	586.63806	8.75579
30	Zn	68	38	595.387	8.75569	596.72888	8.77542	594.627	8.74451
30	Zn	70	40	611.087	8.72981	612.37915	8.74827	610.44444	8.72063
31	Ga	69	38	601.996	8.72458	603.8851	8.75196	600.3837	8.70121
31	Ga	71	40	618.951	8.71762	620.99784	8.74645	616.54873	8.68378
32	Ge	70	38	610.521	8.72173	612.65537	8.75222	611.89844	8.74141
32	Ge	72	40	628.686	8.73175	631.17806	8.76636	628.65957	8.73138
32	Ge	73	41	635.469	8.70505	638.33681	8.74434	636.9953	8.72596
32	Ge	74	42	645.665	8.7252	647.61204	8.75151	645.29982	8.72027
32	Ge	76	44	661.598	8.70524	662.12347	8.71215	661.81111	8.70804
33	As	75	42	652.564	8.70085	655.4886	8.73985	650.43133	8.67242
34	Se	74	40	642.891	8.68772	645.75477	8.72642	644.48393	8.70924
34	Se	76	42	662.073	8.71149	664.9141	8.74887	661.92405	8.70953

Table (continued)

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
34	Se	77	43	669.492	8.6947	672.45666	8.7332	670.62891	8.70947
34	Se	78	44	679.99	8.71782	682.06496	8.74442	679.32321	8.70927
34	Se	80	46	696.866	8.71082	697.35903	8.71699	696.67873	8.70848
34	Se	82	48	712.843	8.69321	710.93318	8.66992	706.05447	8.61042
35	Br	79	44	686.321	8.68761	689.2439	8.72461	683.54105	8.65242
35	Br	81	46	704.37	8.69593	705.81675	8.71379	701.12258	8.65583
36	Kr	78	42	675.578	8.66126	678.21414	8.69505	676.38095	8.67155
36	Kr	80	44	695.434	8.69293	697.9578	8.72447	694.45293	8.68066
36	Kr	82	46	714.274	8.71066	715.7677	8.72887	712.55477	8.68969
36	Kr	83	47	721.737	8.69563	722.76424	8.708	721.61748	8.69419
36	Kr	84	48	732.258	8.71736	731.78274	8.7117	730.68841	8.69867
36	Kr	86	50	749.235	8.71203	746.12903	8.67592	748.85597	8.70763
37	Rb	85	48	739.283	8.69745	739.5233	8.70027	742.09493	8.73053
37	Rb	87	50	757.856	8.71099	755.05569	8.6788	760.81913	8.74505
38	Sr	84	46	728.906	8.67745	730.32687	8.69437	726.27564	8.64614
38	Sr	86	48	748.928	8.70847	748.7453	8.70634	745.0301	8.66314
38	Sr	87	49	757.356	8.70524	756.09715	8.69077	754.44401	8.67177
38	Sr	88	50	768.469	8.7326	765.42723	8.69804	772.1865	8.77485
39	Y	89	50	775.538	8.71391	772.4887	8.67965	774.67303	8.70419
40	Zr	90	50	783.893	8.70992	781.019	8.67799	784.95789	8.72175
40	Zr	91	51	791.087	8.69326	788.70082	8.66704	794.88848	8.73504
40	Zr	92	52	799.722	8.69263	798.32021	8.67739	796.30213	8.65546
40	Zr	94	54	814.677	8.66678	814.04797	8.66008	815.99203	8.68077
40	Zr	96	56	828.996	8.63537	828.30119	8.62814	826.67692	8.61122
41	Nb	93	52	805.765	8.66414	804.7359	8.65307	806.51142	8.67217
42	Mo	92	50	796.508	8.6577	793.09932	8.62064	795.94785	8.65161
42	Mo	94	52	814.256	8.6623	812.60714	8.64476	816.25231	8.68354
42	Mo	95	53	821.625	8.64868	820.5965	8.63786	817.88079	8.60927
42	Mo	96	54	830.779	8.65395	830.48545	8.65089	827.97895	8.62478
42	Mo	97	55	837.6	8.63505	837.71003	8.63619	838.13357	8.64055
42	Mo	98	56	846.243	8.63513	846.8345	8.64117	848.34648	8.6566
42	Mo	100	58	860.458	8.60458	861.74656	8.61747	859.50966	8.5951
44	Ru	96	52	826.496	8.60933	823.52747	8.57841	826.02571	8.60443
44	Ru	98	54	844.79	8.62031	843.52579	8.60741	846.8666	8.6415
44	Ru	99	55	852.255	8.60864	851.80259	8.60407	857.39063	8.66051
44	Ru	100	56	861.928	8.61928	861.94355	8.61944	858.94508	8.58945
44	Ru	101	57	868.73	8.60129	869.4767	8.60868	869.40782	8.608
44	Ru	102	58	874.844	8.5769	878.87415	8.61641	870.67943	8.53607
44	Ru	104	60	893.083	8.58734	894.40381	8.60004	891.54317	8.57253
45	Rh	103	58	884.163	8.58411	885.0934	8.59314	880.17114	8.54535
46	Pd	102	56	875.212	8.58051	873.78898	8.56656	876.82609	8.59633
46	Pd	104	58	892.82	8.58481	892.71256	8.58377	889.22905	8.55028
46	Pd	105	59	899.914	8.57061	900.53543	8.57653	899.97871	8.57123
46	Pd	106	60	909.474	8.57994	910.189	8.58669	910.81159	8.59256

Table (continued)

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
46	Pd	108	62	925.239	8.56703	926.29915	8.57684	922.81499	8.54458
46	Pd	110	64	940.207	8.54734	941.11778	8.55562	944.64934	8.58772
47	Ag	107	60	915.263	8.55386	915.80492	8.55892	919.77497	8.59603
47	Ag	109	62	931.727	8.54795	932.86781	8.55842	932.11781	8.55154
48	Cd	106	58	905.14	8.53906	903.40923	8.52273	906.15432	8.54863
48	Cd	108	60	923.402	8.55002	922.8084	8.54452	928.3299	8.59565
48	Cd	110	62	940.646	8.55133	940.7986	8.55271	940.98553	8.55441
48	Cd	111	63	947.622	8.53714	948.22807	8.5426	952.19342	8.57832
48	Cd	112	64	957.016	8.54479	957.45559	8.54871	953.35906	8.51213
48	Cd	113	65	963.556	8.52704	964.25463	8.53323	964.54063	8.53576
48	Cd	114	66	972.599	8.53157	972.84985	8.53377	975.82258	8.55985
48	Cd	116	68	987.44	8.51241	987.04697	8.50903	987.84541	8.51591
49	In	113	64	963.094	8.52296	963.41231	8.52577	962.12798	8.51441
49	In	115	66	979.404	8.51656	979.7077	8.5192	984.90679	8.56441
50	Sn	112	62	953.532	8.51368	952.24527	8.50219	957.52571	8.54934
50	Sn	114	64	971.574	8.52258	970.72011	8.51509	970.49155	8.51308
50	Sn	115	65	979.121	8.5141	978.4227	8.50802	981.96875	8.53886
50	Sn	116	66	988.684	8.52314	987.89353	8.51632	983.20639	8.47592
50	Sn	117	67	995.627	8.50963	994.97954	8.5041	994.67495	8.5015
50	Sn	118	68	1004.955	8.51657	1003.8319	8.50705	1006.25447	8.52758
50	Sn	119	69	1011.438	8.49948	1010.3326	8.49019	1007.11929	8.46319
50	Sn	120	70	1020.546	8.50455	1018.5973	8.48831	1018.68817	8.48907
50	Sn	122	72	1035.53	8.48795	1032.2476	8.46105	1030.84841	8.44958
50	Sn	124	74	1049.963	8.46744	1044.837	8.4261	1054.312	8.50252
51	Sb	121	70	1026.325	8.48202	1025.6927	8.4768	1027.54763	8.49213
51	Sb	123	72	1042.097	8.47233	1040.1801	8.45675	1040.04681	8.45567
52	Te	120	68	1017.282	8.47735	1017.6311	8.48026	1012.38541	8.43655
52	Te	122	70	1034.333	8.47814	1034.0849	8.47611	1035.97733	8.49162
52	Te	123	71	1041.263	8.46555	1040.8692	8.46235	1047.95786	8.51998
52	Te	124	72	1050.686	8.47327	1049.3892	8.46282	1048.78942	8.45798
52	Te	125	73	1057.256	8.45805	1055.6276	8.44502	1060.77792	8.48622
52	Te	126	74	1066.369	8.46325	1063.5988	8.44126	1061.3598	8.42349
52	Te	128	76	1081.439	8.44874	1076.7653	8.41223	1085.47929	8.48031
52	Te	130	78	1095.941	8.43032	1088.937	8.37644	1097.80741	8.44467
53	I	127	74	1072.577	8.44549	1070.8771	8.4321	1069.98274	8.42506
54	Xe	124	70	1046.257	8.43756	1046.6845	8.441	1040.9224	8.39454
54	Xe	126	72	1063.909	8.44372	1063.6276	8.44149	1065.02083	8.45255
54	Xe	128	74	1080.743	8.4433	1079.4437	8.43315	1078.18194	8.4233
54	Xe	129	75	1087.651	8.4314	1085.9613	8.4183	1090.45272	8.45312
54	Xe	130	76	1096.907	8.43775	1094.1852	8.41681	1091.13029	8.39331
54	Xe	131	77	1103.512	8.42376	1100.1915	8.39841	1103.42321	8.42308
54	Xe	132	78	1112.448	8.42764	1107.9009	8.39319	1115.85749	8.45347
54	Xe	134	80	1127.435	8.41369	1120.6371	8.36296	1128.62868	8.4226
54	Xe	136	82	1141.878	8.39616	1132.437	8.32674	1141.169	8.39095

Table (continued)

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
55	Cs	133	78	1118.528	8.40998	1115.3434	8.38604	1124.19407	8.45259
56	Ba	130	74	1092.722	8.40555	1092.4768	8.40367	1093.41237	8.41086
56	Ba	132	76	1110.038	8.40938	1108.78	8.39985	1106.89583	8.38557
56	Ba	134	78	1126.696	8.40818	1124.028	8.38827	1120.19298	8.35965
56	Ba	135	79	1133.668	8.39754	1130.3087	8.37266	1132.7633	8.39084
56	Ba	136	80	1142.775	8.40276	1138.2674	8.36961	1145.48387	8.42268
56	Ba	137	81	1149.681	8.39183	1144.0675	8.35086	1145.90412	8.36426
56	Ba	138	82	1158.293	8.39343	1151.5421	8.34451	1158.66304	8.39611
57	La	138	81	1155.774	8.37517	1151.2653	8.3425	1153.85554	8.36127
57	La	139	82	1164.551	8.37806	1159.105	8.33888	1166.73445	8.39377
58	Ce	136	78	1138.792	8.37347	1137.4151	8.36335	1134.95888	8.34529
58	Ce	138	80	1156.035	8.37707	1153.1463	8.35613	1160.6686	8.41064
58	Ce	140	82	1172.692	8.37637	1167.8856	8.34204	1174.39252	8.38852
58	Ce	142	84	1185.29	8.34711	1181.675	8.32165	1187.94784	8.36583
59	Pr	141	82	1177.919	8.35404	1174.1085	8.32701	1181.66244	8.38058
60	Nd	142	82	1185.142	8.34607	1181.5574	8.32083	1188.56759	8.37019
60	Nd	143	83	1191.266	8.33053	1188.3505	8.31014	1189.38183	8.31736
60	Nd	144	84	1199.083	8.32697	1196.7746	8.31093	1202.61474	8.35149
60	Nd	145	85	1204.838	8.30923	1203.1016	8.29725	1203.21813	8.29806
60	Nd	146	86	1212.403	8.30413	1211.0565	8.29491	1216.51726	8.33231
60	Nd	148	88	1225.028	8.27722	1224.4411	8.27325	1230.27049	8.31264
60	Nd	150	90	1237.448	8.24965	1236.9644	8.24643	1243.86957	8.29246
62	Sm	144	82	1195.737	8.30373	1192.643	8.28224	1201.36866	8.34284
62	Sm	147	85	1217.251	8.28062	1216.3099	8.27422	1216.71733	8.27699
62	Sm	148	86	1225.392	8.27968	1224.9518	8.2767	1230.17944	8.31202
62	Sm	149	87	1231.263	8.26351	1231.5256	8.26527	1230.86234	8.26082
62	Sm	150	88	1239.25	8.26167	1239.7055	8.2647	1244.40272	8.29602
62	Sm	152	90	1253.104	8.24411	1253.5745	8.2472	1258.49943	8.2796
62	Sm	154	92	1266.94	8.22688	1266.5934	8.22463	1272.46549	8.26276
63	Eu	151	88	1244.141	8.23934	1245.4718	8.24816	1250.92515	8.28427
63	Eu	153	90	1258.998	8.22875	1260.0164	8.2354	1252.0625	8.18342
64	Gd	152	88	1251.485	8.23345	1252.4331	8.23969	1257.1134	8.27048
64	Gd	154	90	1266.627	8.22485	1267.6392	8.23142	1271.63325	8.25736
64	Gd	155	91	1273.062	8.2133	1274.015	8.21945	1272.19303	8.2077
64	Gd	156	92	1281.598	8.21537	1281.9731	8.21778	1286.04737	8.24389
64	Gd	157	93	1287.958	8.20355	1287.9312	8.20338	1286.40735	8.19368
64	Gd	158	94	1295.896	8.20187	1295.468	8.19916	1300.35239	8.23008
64	Gd	160	96	1309.29	8.18306	1308.1556	8.17597	1314.5448	8.21591
65	Tb	159	94	1302.027	8.18885	1301.9809	8.18856	1306.82008	8.21899
66	Dy	156	90	1278.021	8.19244	1279.233	8.20021	1283.44156	8.22719
66	Dy	158	92	1294.046	8.19016	1294.8735	8.1954	1298.23576	8.21668
66	Dy	160	94	1309.455	8.18409	1309.6541	8.18534	1312.94339	8.2059
66	Dy	161	95	1315.909	8.17335	1315.8509	8.17299	1313.38752	8.15769
66	Dy	162	96	1324.106	8.17349	1323.6067	8.17041	1327.56172	8.19483

Table (continued)

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
66	Dy	163	97	1330.377	8.16182	1329.4064	8.15587	1327.81106	8.14608
66	Dy	164	98	1338.035	8.15875	1336.7617	8.15099	1342.0879	8.18346
67	Ho	165	98	1344.256	8.14701	1343.3234	8.14135	1348.27368	8.17136
68	Er	162	94	1320.698	8.15246	1321.4229	8.15693	1324.23529	8.17429
68	Er	164	96	1336.447	8.14907	1336.6331	8.1502	1339.21447	8.16594
68	Er	166	98	1351.572	8.142	1351.0262	8.13871	1354.12255	8.15736
68	Er	167	99	1358.008	8.13178	1357.0604	8.12611	1354.45782	8.11053
68	Er	168	100	1365.779	8.12964	1364.6314	8.12281	1368.95728	8.14856
68	Er	170	102	1379.04	8.112	1377.4766	8.1028	1383.71631	8.13951
69	Tm	169	100	1371.352	8.11451	1370.6194	8.11017	1374.66657	8.13412
70	Yb	168	98	1362.793	8.11186	1362.9247	8.11265	1364.88571	8.12432
70	Yb	170	100	1378.13	8.10665	1377.7421	8.10437	1380.06186	8.11801
70	Yb	171	101	1384.744	8.09792	1384.0023	8.09358	1380.47629	8.07296
70	Yb	172	102	1392.764	8.09747	1391.7813	8.09175	1395.18194	8.11152
70	Yb	173	103	1399.131	8.08746	1397.6674	8.079	1395.41458	8.06598
70	Yb	174	104	1406.595	8.08388	1405.069	8.07511	1410.24421	8.10485
70	Yb	176	106	1419.283	8.06411	1417.6309	8.05472	1425.24681	8.09799
71	Lu	175	104	1412.106	8.06918	1411.0893	8.06337	1415.66989	8.08954
71	Lu	176	105	1418.394	8.05906	1416.9087	8.05062	1415.85338	8.04462
72	Hf	174	102	1403.928	8.06855	1403.7688	8.06764	1405.40476	8.07704
72	Hf	176	104	1418.801	8.06137	1418.2265	8.05811	1420.78885	8.07266
72	Hf	177	105	1425.185	8.05189	1424.3349	8.04709	1421.10223	8.02883
72	Hf	178	106	1432.811	8.0495	1431.9414	8.04461	1436.13156	8.06815
72	Hf	179	107	1438.91	8.0386	1437.6924	8.0318	1436.26727	8.02384
72	Hf	180	108	1446.297	8.03498	1444.9382	8.02743	1451.43158	8.06351
73	Ta	180	107	1444.663	8.02591	1443.4458	8.01914	1441.37523	8.00764
73	Ta	181	108	1452.24	8.02343	1450.9735	8.01643	1456.56802	8.04734
74	W	180	106	1444.588	8.02549	1443.9814	8.02212	1445.80309	8.03224
74	W	182	108	1459.335	8.01832	1458.1089	8.01159	1461.40522	8.0297
74	W	183	109	1465.525	8.00833	1464.0784	8.00043	1461.62301	7.98701
74	W	184	110	1472.937	8.00509	1471.526	7.99742	1476.98029	8.02707
74	W	186	112	1485.882	7.98861	1484.2557	7.97987	1492.52745	8.02434
75	Re	185	110	1478.341	7.99103	1477.0087	7.98383	1481.67469	8.00905
75	Re	187	112	1491.877	7.97795	1490.2904	7.96947	1497.36935	8.00732
76	Os	184	108	1469.921	7.9887	1469.0591	7.98402	1470.24809	7.99048
76	Os	186	110	1484.807	7.98283	1483.5856	7.97627	1486.09023	7.98973
76	Os	187	111	1491.097	7.97378	1489.7663	7.96666	1486.38408	7.94858
76	Os	188	112	1499.087	7.97387	1497.4093	7.96494	1501.9197	7.98893
76	Os	189	113	1505.007	7.963	1503.2513	7.95371	1502.04693	7.94734
76	Os	190	114	1512.799	7.9621	1510.5523	7.95028	1517.73611	7.98808
76	Os	192	116	1526.116	7.94852	1523.036	7.93248	1533.53906	7.98718
77	Ir	191	114	1518.088	7.9481	1516.0377	7.93737	1522.14121	7.96933
77	Ir	193	116	1532.058	7.93812	1529.0561	7.92257	1538.08144	7.96933
78	Pt	190	112	1509.851	7.94658	1508.3843	7.93886	1510.20996	7.94847

The end of Table

Z	Nucleus	A	N	B (EXP), MeV	B/A (EXP), MeV	B (LDM), MeV	B/A (LDM), MeV	B (INM), MeV	B/A (INM), MeV
78	Pt	192	114	1524.964	7.94252	1522.6015	7.93022	1526.27473	7.94935
78	Pt	194	116	1539.577	7.93596	1536.145	7.91827	1542.34012	7.95021
78	Pt	195	117	1545.682	7.92657	1541.8699	7.90703	1542.38144	7.90965
78	Pt	196	118	1553.604	7.92655	1549.0352	7.90324	1558.40616	7.95105
78	Pt	198	120	1567.007	7.91418	1561.2921	7.88531	1574.47287	7.95188
79	Au	197	118	1559.386	7.91566	1554.5095	7.89091	1562.51729	7.93156
80	Hg	196	116	1551.218	7.91438	1547.1158	7.89345	1550.06869	7.90851
80	Hg	198	118	1566.489	7.91156	1561.0472	7.88408	1566.36429	7.91093
80	Hg	199	119	1573.153	7.90529	1566.9765	7.87425	1566.48393	7.87178
80	Hg	200	120	1581.181	7.9059	1574.3316	7.87166	1582.67354	7.91337
80	Hg	201	121	1587.411	7.89757	1579.9487	7.86044	1582.63325	7.8738
80	Hg	202	122	1595.165	7.89686	1586.9882	7.85638	1598.99688	7.91583
80	Hg	204	124	1608.652	7.88555	1599.0356	7.83841	1615.33474	7.91831
81	Tl	203	122	1600.87	7.88606	1592.4387	7.84453	1602.80964	7.89561
81	Tl	205	124	1615.072	7.8784	1604.989	7.82921	1619.26432	7.89885
82	Pb	204	122	1607.507	7.87994	1598.9383	7.83793	1606.36585	7.87434
82	Pb	206	124	1622.325	7.87536	1611.9829	7.82516	1622.92633	7.87828
82	Pb	207	125	1629.063	7.86987	1617.5005	7.81401	1622.8084	7.83965
82	Pb	208	126	1636.431	7.86746	1624.4233	7.80973	1639.51406	7.88228
83	Bi	209	126	1640.23	7.84799	1629.8385	7.79827	1643.0243	7.86136
90	Th	232	142	1766.687	7.61503	1769.1834	7.62579	1764.19153	7.60427
92	U	234	142	1778.567	7.60071	1780.5253	7.60908	1769.14956	7.56047
92	U	235	143	1783.864	7.59091	1785.8988	7.59957	1786.7108	7.60302
92	U	238	146	1801.69	7.57013	1804.1704	7.58055	1803.59006	7.57811

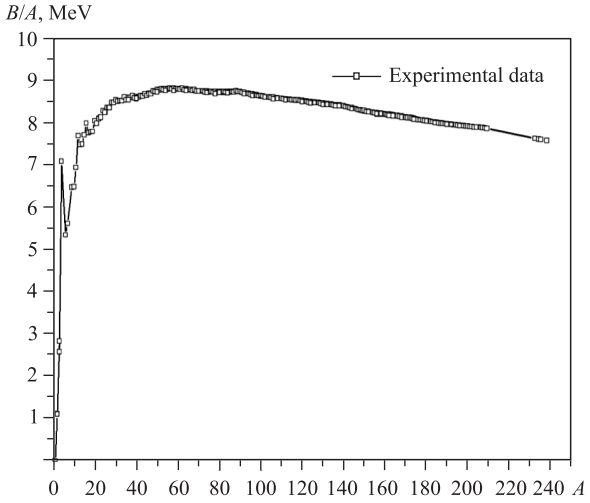

Fig. 1. Experimental data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

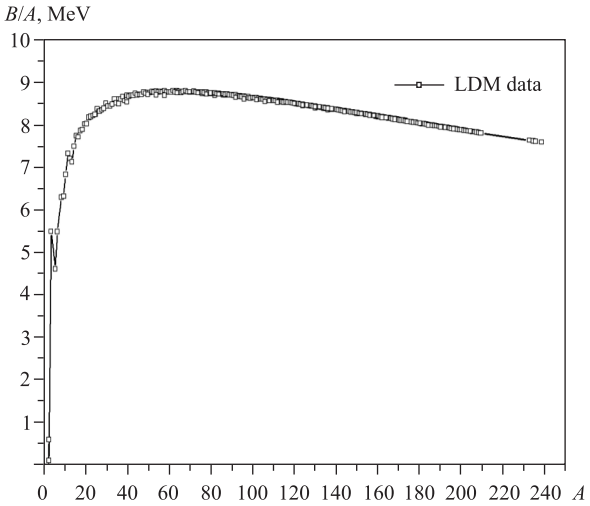

Fig. 2. Liquid drop model (LDM) data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

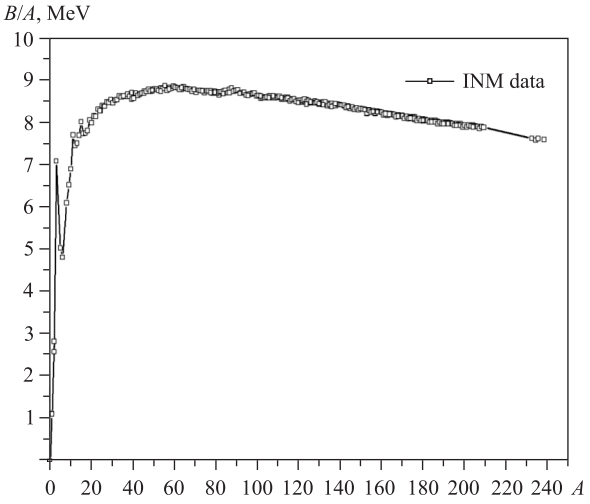

Fig. 3. Our integrated model data of nuclear binding energy per nucleon in terms of mass number for most of the known stable nuclei

2. DISCUSSION AND CONCLUSION

The adjusting coefficient in binding energy formula (6) may be attributed to our first assumption, namely, the nuclear binding energy is of the order of about one percent of the energy of the total rest mass of the constituent nucleons which implies that we have room for limited approximation which includes microscopic corrections, for example, one has to do with the defined nuclear region [12] in which the density remains constant. In other words, it

is assumed that in about 10 percent of outer nuclear region the density is no longer constant and falls rapidly and is ignored in the integrated model. The other has to do with the quark distribution inside the nuclides as stated in the quark plasma nuclear model [13–15] due to the fact that each nucleon is made of 3 quarks and existence of a new three-fold symmetry in this model. Attention should be paid to the fact that for $A < 5$ such as ${}^{4}_{2}\mathrm{He}$ and ${}^{3}_{2}\mathrm{H}$ the factor 3 in the second term of binding energy equation (6) changes to 1 and for other light nuclei our given formula needs minor correction due to the fact that for these light nuclides at least two nucleons should participate and the spherical distribution of the nucleons inside the nuclide changes, the problem that exists in other models too.

The semi-empirical mass formula (1), based upon only liquid drop model contains at least five terms to be calculated, whereas in our formula (6) only two terms are calculated. Careful consideration of the Table and Figs. 2 and 3 reveals the meaningful accuracy of our integrated model compared to the liquid drop model with respect to experimental data (Fig. 1). Special features of the experimental diagram such as having maximum value for Fe and its local extremums coincide with the calculated values from formula (6). The binding energy formula (6) is extracted from various existing models and that is why it is called integrated model. In this model the constituent nuclear particles are considered «free» in a dense plasma-type media. It is interesting that in such a plasma model of nuclei, based on a statistical view, all magic numbers and the new magic number, namely, 184, are also obtained with no spherical potential and spin–orbit coupling assumptions [13, 14]. Here attempts are made to conceptualize an integrated nuclear model capable of providing all nuclear characteristics such as binding energy per nucleon, magic number, excited states and magnetic moments. Such concepts may lead us to understand a realistic picture of nuclei.

We believe the results obtained from the integrated model are not only simple to understand but also more physical and relatively closer to the experimental data than other models. Other characteristics of nuclei are being studied in the framework of the integrated model in our group.

REFERENCES

1. *Von Weizsäcker C. F.* // Z. Physik. 1935. V. 96. P. 431.
2. *Bohr N., Wheeler J. A.* // Phys. Rev. 1939. V. 56. P. 426.
3. *Bethe H. A.* // Rev. Mod. Phys. 1936. V. 8. P. 82.
4. *Goeppert-Mayer M.* // Phys. Rev. 1949. V. 75. P. 1969.
5. *Haxel O., Jensen J. D. H., Suess H. E.* // Z. Phys. 1950. V. 128. P. 295.
6. *de-Shalit A., Talmi I.* Nuclear Shell Theory. N. Y.: Academic Press Inc., 1963.
7. *Garvey G. T., Kelson I.* // Phys. Rev. Lett. 1966. V. 16. P. 1967.
8. *Kelson I., Garvey G. T.* // Phys. Lett. 1966. V. 23. P. 689.
9. *Garvey G. T. et al.* // Rev. Mod. Phys. 1969. V. 41. P. s1.
10. *de-Shalit A., Feshbach H.* Theoretical Nuclear Physics. V. 1. N. Y.: John Wiley & Sons Inc., 1974.

11. *Cottingham W. N., Greenwood D. A.* An Introduction to Nuclear Physics. Cambridge: Cambridge Univ. Press, 2001.

12. *Meyerhof W. E.* Elements of Nuclear Physics. N. Y.: McGraw-Hill Series in Fundamentals of Phys., 1967.

13. *Ghahramany N. et al.* // Phys. Essays. 2008. V. 21, No. 3. P. 200.

14. *Ghahramany N., Ghanaatian M., Hooshmand M.* // Iranian Phys. J. 2007. V. 1, No. 2. P. 35.

15. *Osman F., Ghahramany N., Hora H.* // Laser Part. Beams. 2005. V. 23. P. 461.

Received on June 12, 2010.