Письма в ЭЧАЯ. 2011. Т. 8, № 4(167). С. 626–634

МЕТОДИКА ФИЗИЧЕСКОГО ЭКСПЕРИМЕНТА

AN ATTEMPT TO BUILD A STATISTICAL MODEL FOR A PIPS DETECTOR OPERATED IN A REAL-TIME MODE

Yu. S. Tsyganov[1]

Joint Institute for Nuclear Research, Dubna

Basic approaches by K.-H. Schmidt and V. B. Zlokazov to estimate the probability of registered multichain event to be explained by random coincidences are considered. A specific feature of the long-term experiments aimed at the synthesis of superheavy nuclei with the Dubna Gas-Filled Recoil Separator is usage of a real-time mode for radical suppression of background products. In fact, this assumes that the first correlation group, namely, recoil–alpha correlation, stops target irradiation for a short time and the forthcoming signals are detected in more favorable background conditions. Due to the application of this detection mode the first recoil–alpha chain can be considered as a «starter» for the detection of the forthcoming alpha-particle signals with high background suppression. This fact is taken into account in the given PIPS detector statistical model.

Рассматриваются базовые подходы К.-Х. Шмидта и В. Б. Злоказова для оценки вероятности того, что регистрируемое многозвенное событие может быть объяснено совокупностью случайных факторов. Специфической чертой длительных экспериментов по синтезу сверхтяжелых элементов на дубненском газонаполненном сепараторе ядер отдачи является использование детектирования в реальном масштабе времени для радикального подавления фоновых загрузок. Фактически это предполагает, что первая регистрируемая корреляционная группа сигналов, а именно ядро отдачи–альфа-распад, останавливает процесс облучения мишени и последующие сигналы детектируются в более предпочтительных условиях. Благодаря применению этого способа детектирования первое звено «ядро отдачи–альфа» является «стартом» для детектирования последующих сигналов альфа-распада с высоким подавлением фона. Этот факт принимается во внимание при построении статистической модели детектора PIPS.

PACS: 41.20.Cv; 07.57Kp

INTRODUCTION

^{48}Ca-induced fusion reactions on heavy actinide targets have been investigated since 1999 at FLNR, Dubna [1]. Results on the discovery of many isotopes and new superheavy elements (SHE) $Z = 113{-}118$ convincingly were reported since 2004 and presented in a review [2]. Recently, the discovery of ^{283}112 was confirmed [3, 4]. Production cross sections observed at FLNR reached a level of 5 pb for elements $Z = 114$ and $Z = 116$. The 1 pb level was reported for the formation of $Z = 112$ and $Z = 118$. Also the odd elements $Z = 115$

[1]E-mail: tyura@sungns.jinr.ru

and $Z = 113$ were produced in the range 1–3 pb. The mentioned orders of measured cross sections assume that not only experimental apparatus, especially the detection systems, operate well during the long-term experiments, but some statistical models for results interpretations are of appropriate quality. All these results were obtained at the Dubna Gas-Filled Recoil Separator (DGFRS), the mostly advanced facility of FLNR [5].

1. STATISTICS MODELS FOR RARE DETECTED SEQUENCES

In nuclear physics, especially in the experiments aimed at the discovery of SHE (or/and isotopes), the technique of delayed coincidences is widely used for detecting time–energy–position correlations among signals of different groups.

There are two aspects in establishing the significance for the existence of true correlation:

• Consideration of the possibility that the random background of uncorrelated events could simulate a correlation is required.

• Estimation of the compatibility of the parameters of the observed events with known properties of some numbers in the considered event chain should be under consideration too.

It was K.-H. Schmidt who first recognized the mentioned problems and epitomized a compact theoretical approach for numerical consideration [6].

Note that additionally to this approach, another theoretical model was formulated in [7, 8].

These theories are known as LDSC (Linked Decay Signal Combinations) and BSC (Background Signal Combinations) models, respectively (Fig. 1, schematically).

Having mentioned these approaches as classical, one should take into account the existence of a different approach to the problem basing on some Monte Carlo calculations [9].

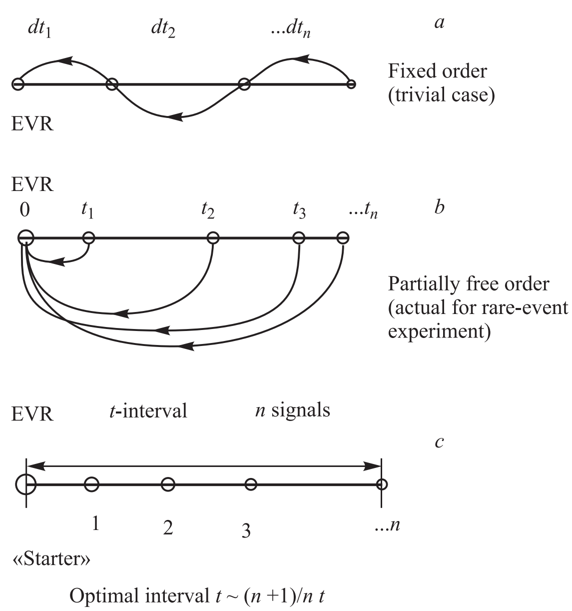

Fig. 1. a, b) Schematics for two LDSC scenarios; c) BSC method

The goal of the present paper is to modify the approaches reported in [6, 7] relative to the experimental method of «active correlations» which was extensively used to suppress background signals in heavy-ion-induced nuclear reactions [10–14]. An approach described in [9] is outside the scope of the present paper.

2. CORRELATION ANALYSIS: SCHMIDT EQUATION FOR CORRELATION CHAIN WITH PARTIALLY FREE ORDER

The a priori knowledge of the order of the events in a possible true event chain may be limited. In [6] (Subsec. 3.2) one case which is characterized by the condition that possible decay sequences are known to start with the events E_1 of group 1 is considered. The events of the other event groups (E_2 to E_K) may appear in any order, but at least one event E_i must appear within the time limit $\Delta t_{1,i}$. The equation for numbers of random events n_b was obtained in the form

$$n_b = \lambda_1 T \prod_{i=1}^{K-1} \left\{ \int\limits_0^{\Delta t_{1,i+1}} \left(\frac{dp_{1,i+1}}{dt} \right) dt \right\}, \tag{1}$$

where T is an effective time of the experiment (see [6, 9]); λ_i is the rate of events of i-type; K is the number of chains in the multievent; $dp_{1,i}/dt$ is the probability density that an event E_1 is followed by an event E_i after the time distance t. For the case of the condition $(\lambda_i + \lambda_1)\Delta t_{1,i} \ll 1$ being true the above equation can be simplified as

$$n_b \approx T \prod_{i=1}^{K} \lambda_i \prod_{i=2}^{K} \Delta t_{1,i}. \tag{2}$$

3. METHOD OF «ACTIVE CORRELATIONS» FOR BEAM ASSOCIATED BACKGROUNDS SUPPRESSION

Usually, to reach high total SHE experiment efficiency, one uses extremely high ($n \cdot 10^{12}$ to 10^{13} pps, $n > 1$) heavy-ion beam intensities. It means that not only irradiated target, sometimes (frequently) made on highly radioactive actinide material, should not be destroyed during long-term experiment, but the in-flight recoil separator and its detection system should provide backgrounds suppression in order to extract one or two events from the whole data flow. Typically, the DGFRS provides suppression of the beam-like and target-like backgrounds by factors of $\sim 10^{15}{-}10^{17}$ and $10^4{-}5 \cdot 10^4$, respectively. Nevertheless, under real circumstances, total counting rate above approximately one MeV threshold is about tens to one–three hundreds of events per second. Therefore, during, for example, one month of irradiation about $30 \times 10^5 \times 100 = 3\mathrm{E}+08$ multiparameter events are written to the hard disk during a typical SHE experiment.

To avoid a scenario that result of the SHE experiment (one to three decay chains per month) can be represented as a set of random signals, the real-time search technique to suppress the probability for detected event to be a random one has been designed and successfully applied.

Note that in the reactions with ^{48}Ca as a projectile, the efficiency of SHE recoiling products detection by both silicon and TOF detector is close to 100%. Namely, recoil-first (second) correlated alpha-decay signal was used as a triggering signal to switch off the cyclotron beam for a definite (seconds to minutes) and, therefore, detection of forthcoming alpha decays were in fact «background-free» (see Fig. 2). The basic idea to apply such a detection mode is to transform the main data flow to the discrete form [10–15].

Considering in the above-described process a definite order correlated pair recoil–alpha $E_1 \rightarrow E_2$ as a starter signal $\hat{E}_1 \equiv (E1 \cap E2)$ for forthcoming sequences of «α» decays and following the philosophy of [6], one can rewrite Eq. (1) for the given case in the form

$$n_b = \hat{\lambda}_1 T \prod_{i=2}^{K-1} \left\{ \int_0^{\Delta t_{2,i+1}} \left(\frac{dp_{2,i+1}}{dt} dt \right) \right\}. \quad (3)$$

Here the parameter $\hat{\lambda}_1$ denotes not any single signal rate per pixel, but a rate of correlations/pauses generated by the detection system during a long-term experiment. Therefore, if $N_{\rm stop}$ is a total number of pauses measured in an experiment, to a first approximation one can consider $N_{\rm stop} = \psi \hat{\lambda}_1 T$ and

$$n_b = \psi N_{\rm stop} \prod_{i=2}^{K-1} \left\{ \int_0^{\Delta t_{2,i+1}} \left(\frac{dp_{2,i+1}}{dt} dt \right) \right\}. \quad (4)$$

The simplified formula (2) is rewritten as:

$$n_b \approx \psi N_{\rm stop} \bar{\lambda}^{K-2} \prod_{i=3}^{K} \Delta t_{2,i}, \quad (5)$$

where $\bar{\lambda}$ is a mean counting rate value for alpha-decay signals measured in beam-off pauses by the focal plane detector.

When a more common case of detecting of α particles by side detector and with finishing spontaneous fission signal is taken into account, one should rewrite (5) in the form of (6):

$$n_b \approx \psi N_{\rm stop} \bar{\lambda}^{K-3-m} \bar{\lambda}_{\rm ESC}^{m} \lambda_{\rm FF} \prod_{i=3}^{K} \Delta t_{2,i}. \quad (6)$$

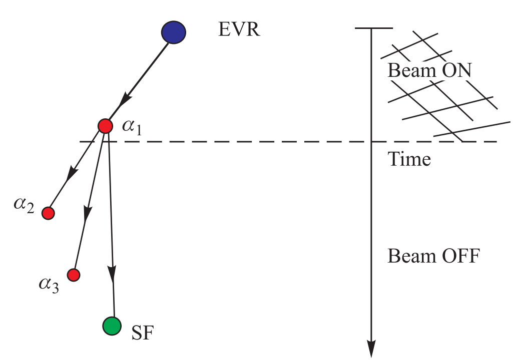

Fig. 2. Correlation graph for the method of «active correlations» in the form EVR-α

In this equation $\bar{\lambda}_{\rm ESC}$ is the mean rate per detector of escaping α particles; $\lambda_{\rm FF}$ is the rate of fission fragment signals imitators per pixel; m is the number of escaping α particles detected out off beam. Of course, in (5), (6) it is assumed, like in [6], that

$$(\lambda_2 + \lambda_i)\Delta t_i \ll 1.$$

The parameter ψ denotes an effective time part[1] $\psi \approx 2t_{\text{EVR-}\alpha}/t_{\rm PRS}$, where $t_{\text{EVR-}\alpha}$ is the measured recoil-alpha correlation time and $t_{\rm PRS}$ is the pre-setting time parameter ($t_{\rm PRS} > t_{\text{EVR-}\alpha}$, or even $t_{\rm PRS} \gg t_{\text{EVR-}\alpha}$) for beam stopping process. Factor two for an optimal interval is explained in [8]. Or, in the case of nonuniform distribution: $\psi = \dfrac{1}{N_{\rm stop}} \displaystyle\int\limits_0^{\frac{m+1}{m} t_{\text{EVR-}\alpha m}} \left(\frac{dN_{\rm stop}}{dt}\right) dt$, where m is the number of α-particle signals creating beam-stop process and $\hat{E}_1 \equiv (E_1 \cap E_2 \cap E_3 \ldots \cap E_{m+1})$.

Note that in [16] nearly the same conclusion was drawn by using BSC philosophy and on separate elementary events spaces representation for groups «beam ON» and «beam OFF». It is not excluded that the higher-order correlated event may be considered in a similar way as an event «starter».

4. SUPPLEMENT 1: ON THE ROLE OF THE EVENT CONFIGURATION FACTOR[2]

Only in ideal case one can consider relation «*effect* $\neq$ *background*» as an absolutely true one. In real cases 1/0 this relation sometimes is under question, and probably, quaternary logic may take place (Fig. 3). So, in fact we consider, in additional to 1/0 states, the states 10, 01 that means: false effect and false background [17, 18]. In this case it is reasonable to add a necessary condition of event identification by some configuration probability factors, which can be measured (or calculated) on the base of a priori information about event. For example, for short EVR-α-SF like events detection one can definitely use information about the probability $P(x,n)$ to detect x full chains from $n \geqslant x$ events.

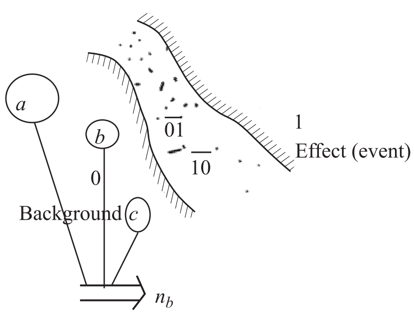

Fig. 3. A schematic representation of the event. a, b, c — any mathematical model for background description

It is reasonable to consider effective parameter for each individual event relative to $P(x,n)$ value, for instance, in the simple form $\tilde{n}_b \approx n_b/P(x,n)^3$. In this case, «harder» criteria for «event» existence should be considered as $\tilde{n}_b \ll 1$ (as a sufficient condition), whereas case of ~ 1 corresponds to a mixed and nondefinite area 0110 (see Fig. 3). For example, if one considers the event of $Z = 114$ element reported in [19–21], then the parameter

[1] For the case of flat distribution.

[2] In the form of hypothesis.

[3] Or, in more common form: $\tilde{n}_b = n_b/\prod\limits_{i=1}^{k} P_i$, if we have k a priori independent probability parameters of the event.

$\tilde{n}_b$ should be calculated as $\tilde{n}_b \approx \dfrac{0.08}{(0.66)^3 \cdot 0.4} = 0.7 \sim 1$[1]. Therefore, the case of a «true event» should be considered as a pessimistic scenario.

5. SUPPLEMENT 2: A MODIFIED BSC APPROACH

Let us consider an application of the method of «active correlations» when pointer to a potential ER–alpha correlation is used to stop the process of radioactive target irradiation and to minimize the beam associated background contribution. It is usual that $\tau < t_0$ or even $\tau \ll t_0$. Here τ is duration of the beam-off time interval if one considers ER signal as a «starter» and t_0 is a correlation time window [11, 16].

If one tries to estimate the statistical significance of the event, three different alpha-particle groups may be considered, namely:

1) signal is registered with the main focal plane PIPS detector;
2) signal is registered by only side detector;
3) signal is registered by both focal plane and side detector (composite nature signal);
4) ... there may be some others.

Additionally, one can consider a distribution per time and per strip of beam-off pauses number to be well known from the experiment.

Under these circumstances, considering a whole PIPS detector as an array of the definite size pixels [9] and according to the BSC approaches [7, 8, 11, 16], one can write the following relationships:

$$\Delta N_R^{i,j} = P_{i,j}(n) \int_0^{\tau_{\rm opt}} \frac{dN_{i,j}^{\rm meas}}{dt} dt \text{ and, therefore, for a whole detector}$$

$$N_R = \sum_{i,j}^{M,\text{strips}} \Delta N_R^{i,j} = \sum_{i,j}^{M,\text{strips}} P_{i,j}(n) \int_0^{\tau_{\rm opt}} \frac{dN_{i,j}^{\rm meas}}{dt} dt. \qquad (7)$$

Here $P_{i,j}(n)$ is the probability to detect n random signals in a pause at a definite position-pixel; strips is the number of position sensitive strips; M is the effective number of pixels per strip [9], N_R is random event expectation value.

In (1) an integration interval $\tau_{\rm opt}$ can be chosen according to for instance [8], namely $m + 1/m \cdot \tau$, where m is signal number after «starter» (ER). In our case it is exactly n number. In the framework of BSC model for $P(n)$ value one could write

$$P(n) = p_s \prod_{k=1}^{3} Q_{k_m}(t_{\rm opt}), \quad Q_{k_i} = \frac{(\lambda_{k_m} t)^{k_m}}{k_m!} \exp(-\lambda_{k_m} t), \qquad (8)$$

and $\sum k_m = n$.

[1] 0.66 is alpha-particle energy & nonzero position signal detection efficiency; 0.4 is both fission fragments detection efficiency.

Considering configuration parameter p_s as

$$p_s = \frac{\prod\limits_m k_m!}{n!}$$

and after substitution in (2.6), one can derive the relation

$$N_R = \sum_{i,j}^{M,\mathrm{strips}} \Delta N_R^{i,j} = \frac{\prod\limits_{m=1}^{3} k_i!}{n!} \sum_{i,j}^{M,\mathrm{strips}} \left(\prod \frac{(\lambda_{k_m}^{i,j} \tau_{\mathrm{opt}})^{k_m}}{k_m!} \exp\left(-\lambda_{k_m} \tau_{\mathrm{opt}}\right) \right) \int\limits_0^{\tau_{\mathrm{opt}}} \frac{dN_{i,j}^{\mathrm{meas}}}{dt}\, dt. \qquad (9)$$

In practice, position window for the beam stop pause is usually slightly greater than the one used for a statistical estimate taking into account a vertical position resolution. For this reason, one should include an additional factor of slightly less than one into the relationship (9):

$$\xi \approx N|_{\Delta Y \leqslant \Delta_{\mathrm{eff}}} / N_{\mathrm{tot}},$$

where N_{tot} is total pauses number, $N_{\Delta Y}$ is a value that corresponds to the effective vertical position size of the pixel. Therefore, finally one could write

$$N_R = \frac{N|_{\Delta Y \leqslant \Delta_{\mathrm{eff}}}}{N_{\mathrm{tot}}} \frac{\prod\limits_{m=1}^{3} k_i!}{n!} \sum_{i,j}^{M,\mathrm{strips}} \left(\prod_{k_m} \frac{(\lambda_{k_m}^{i,j} \tau_{\mathrm{opt}})^{k_m}}{k_m!} \exp\left(-\lambda_{k_m} \tau_{\mathrm{opt}}\right) \right) \int\limits_0^{\tau_{\mathrm{opt}}} \frac{dN_{i,j}^{\mathrm{meas}}}{dt}\, dt. \qquad (10)$$

6. SUPPLEMENT 3: ON POISSON STOCHASTIC PROCESS FORMALISM

Let us consider a multievent of t_0 duration time consisting of

1. ER as a «starter» signal and α_n as a «finisher» signal;
2. α_1 as a first alpha-decay signal following within t_0/N_1 time interval after a starter;
3. α_2 as a second alpha-decay signal following within t_0/N_2 time interval after an α_1 signal;

...

n. α_n as a «finisher» signal, following within t_0/N_n time after an α_{n-1} signal.

Additionally, $N_j \gg N_i$ for any $j < i$.

Of course, the evident relation $t_0 = \sum\limits_{i=1}^{n} \dfrac{t_0}{N_i}$ holds.

Additional assumption is $N_i \gg 1$ for any index i. Note that sometimes in practice it corresponds to the decay pattern for SHE.

It is well known that for Poisson stochastic process, three basic features [8] are considered as defining ones, namely:

a) stationarity;

b) independence of the event prehistory;

c) rareness of the events: $Q_{k>1}(\delta t) = o(\delta t)$, where δt is a small value and Q is a probability to observe k counts within time interval.

Let us consider time interval $t_k = t_0/N_k + \varepsilon$, where $\varepsilon \ll 1$ and $k < n$.

In that case, $t_k \gg t_0$. On the other hand, definitely $k\ (t_k) > 1$ and one may consider the c) condition as a quite questionable one.

In this connection let us try to build a simple algorithm to smooth partially this contradiction.

Let the detected event consist of n registered times $t_1, t_2, \ldots, t_n$ with ER signal as a starter.

First step: for signals ER and α_1 the effective time interval is considered as $2t_1$. If there is no signal within this interval, then the first correlation group is built and we start to build another one in a similar manner starting from α_2. If, nevertheless, α_2 signal is within the mentioned interval, we consider next interval $(3/2)t_2$ and so on: $(j+1)/jt_j$. Finally, we shall build independent groups and one may apply Poisson formalism (or more exactly BSC approach) within these intervals. Note that in the present case one might consider this approach as a compromise between LDSC and BSC approaches.

In this connection, a correction factor (with respect to BSC) should be as follows: $\psi = \dfrac{\prod\limits_{i=1}^{n} \lambda_\alpha \tau_i \,\mathrm{e}^{-\lambda_\alpha \tau_i}}{\dfrac{(\lambda_\alpha \tau)^n}{n!}\,\mathrm{e}^{-\lambda_\alpha \tau}} = \dfrac{n! \prod\limits_{i=1}^{n} \tau_i \,\mathrm{e}^{-\lambda_\alpha \tau_i}}{\tau^n \,\mathrm{e}^{-\lambda_\alpha \tau}}.$

Here $\tau_i = 2t_i$ and $\tau = \dfrac{n+1}{n} \sum\limits_{i=1}^{n} t_i$.

SUMMARY

The Schmidt equation with partially free order was used to build simple statistical model for the case of application of an active correlations technique. This model can be used to estimate statistical significance of rare decay events measured in experiments aimed at the synthesis of SHE. An approach based on the BSC model was considered too. Additionally, the present formulae may be used to establish the limitations for the method application in different heavy-ion-induced nuclear reactions. The author plans a prolongation of his effort aimed at building a phenomenological model which will be in fact a compromise model between BSC and LDSC ones. In the immediate future the author plans to consider an alternative to a mathematical statistics approach, namely that reported in [22].

This work was supported in part by the Russian Foundation for Basic Research, grant No. 09-02-12060.

REFERENCES

1. *Armbruster P.* // Eur. Phys. J. A. 2008. V. 37. P. 159–167.
2. *Oganessian Yu. Ts. et al.* // J. Phys. G: Nucl. Part. Phys. 2007. V. 34. P. R.165.
3. *Eichler R. et al.* // Nature. 2007. V. 83. P. 487.
4. *Hofmann S. et al.* // Eur. Phys. J. A. 2007. V. 32. P. 251.
5. *Subotic K. et al.* // Nucl. Instr. Meth. A. 2002. V. 481. P. 71–80.

6. *Shmidt K.-H. et al.* // Z. Phys. A. 1984. V. 316. P. 19–26.
7. *Zlokazov V. B.* // Eur. Phys. J. A. 2002. V. 14. P. 147.
8. *Zlokazov V. B.* // Yad. Fiz. 2003. V. 66b, No. 9. P. 1714–1718.
9. *Stoyer N. et al.* // Nucl. Instr. Meth. A. 2000. V. 455. P. 433.
10. *Tsyganov Yu. S.* // Part. Nucl., Lett. 2007. V. 4, No. 4(140). P. 608–613.
11. *Tsyganov Yu. S.* // Part. Nucl., Lett. 2009. V. 6, No. 1(150). P. 96–102.
12. *Tsyganov Yu. S. et al.* // Nucl. Instr. Meth. A. 2003. V. 513. P. 413–416.
13. *Tsyganov Yu. S. et al.* // Nucl. Instr. Meth. A. 2004. V. 525. P. 213–216.
14. *Tsyganov Yu. S.* // Nucl. Instr. Meth. A. 2007. V. 573. P. 161–164.
15. *Oganessian Yu. Ts. et al.* // Phys. Rev. C. 2007. V. 76. P. 011601(R).
16. *Tsyganov Yu. S.* JINR Commun. P13-2008-92. Dubna, 2008.
17. *Nadderd L. et al.* // Proc. of XXI Intern. Symp. on Nuclear Electronics & Computing, NEC'2007, Varna, Bulgaria, Sept. 10–17, 2007. Dubna, 2008. P. 362–366.
18. *Chernyshev S. L.* Quaternary Logic of Measurements. M., 2008. P. 17.
19. *Oganessian Yu. Ts. et al.* // Revista Mexicana de Fisika. 2000. V. 46, Suppl. 1. P. 35–41.
20. *Tsyganov Yu. S.* JINR Commun. E7-2008-144. Dubna, 2008.
21. *Tsyganov Yu. S.* JINR Commun. P13-2009-79. Dubna, 2009.
22. *Pytyev Yu. P.* Vozmozhnost'. Elementy teorii i primeneniya. M.: Editorial. URSS, 2000 (in Russian).

Received on July 22, 2010.