Письма в ЭЧАЯ. 2011. Т. 8, № 7(170). С. 1276–1281

LIGHT QUARK FRAGMENTATIONS INTO THE PIONS

A. K. Edemskaya [a, 1], *D. V. Naumov* [b, 2], *O. B. Samoylov* [b, 3]

[a] Irkutsk State University, Irkutsk, Russia

[b] Joint Institute for Nuclear Research, Dubna

We discuss a process of hadronization of light quarks into charged pions in e^+e^- annihilations and in deep inelastic scattering of charged leptons and neutrino off nucleons. We write the corresponding semi-inclusive cross sections of pions production in terms of quark fragmentation functions and fracture functions. We suggest a new method of measurements of fragmentation and fracture functions based on analysis of semi-inclusive data.

Мы обсуждаем процесс адронизации легких кварков в заряженные пионы в e^+e^--столкновениях и глубоконеупругих реакциях взаимодействия заряженных лептонов и нейтрино с нуклонами. Соответствующие полуинклюзивные сечения рождения пионов записаны в терминах функций фрагментации и фрактурных функций. Предлагается новый способ измерения функций фрагментации кварков и фрактурных функций из анализа данных по полуинклюзивным сечениям.

PACS: 13.15.+g; 13.60.-r; 13.66.Bc; 13.60.Le; 13.87.Ce; 14.20.Dh

INTRODUCTION

Quark fragmentations in hard scattering processes are determined in terms of *fragmentation functions* [1]. Fragmentation functions are dimensionless functions that describe the final-state single-particle energy distributions in hard scattering processes like e^+e^- annihilation. This reaction provides a clean theoretical identification of the fragmentation function, which can be written for a hadron of type h as

$$F^h(z,s) = \frac{1}{\sigma_{\rm tot}} \frac{d\sigma}{dz}(e^+e^- \to hX), \tag{1}$$

where $z = 2E_h/\sqrt{s} \leqslant 1$ is the part of the c.m. energy $\sqrt{s}$ carried by hadron (in practice, the approximation $z = z_p = 2p_h/\sqrt{s}$ is often used). Its integral with respect to x gives the average multiplicity of those hadrons:

$$\langle n_h(s) \rangle = \int\limits_0^1 dx\, F^h(z,s). \tag{2}$$

[1] E-mail: aedemsk@gmail.com
[2] E-mail: naumov@nusun.jinr.ru
[3] E-mail: samoylov@nusun.jinr.ru

The fragmentation function can be rewritten also in terms of quark fragmentation functions $D_q^h \equiv D_q^h(z)$ into final hadron h. The quark fragmentation function $D_q^h(z)$ is interpreted as a probability for a quark q to produce a hadron h carrying a fraction of energy z with the following normalization:

$$\sum_q \int_0^1 dz\, z D_q^h(z) = 1.$$

In what follows we restrict our considerations of final hadrons to charged pions only. In e^+e^- annihilations via γ, Z exchange their fragmentation function reads [2]

$$F_{ee}^{\pi^\pm}(z,s) = \frac{1}{\sigma_0^{ee}} \sum_q C_q \left[D_q^{\pi^\pm} + D_{\bar{q}}^{\pi^\pm} \right] = \\ = \frac{1}{\sigma_0^{ee}} \left[(C_d + C_u)(D_u^{\pi^+} + D_d^{\pi^+}) + 2C_s D_s^{\pi^+} + 2C_c D_c^{\pi^+} \right], \quad (3)$$

where $\sigma_0^{ee} = 2\sum_q C_q$, $C_q = [e_q^2 + 2e_q v_e v_q \rho_1(s) + (v_e^2 + a_e^2)(v_q^2 + a_q^2)\rho_2(s)]$ — leading order coefficient for each quark with charge e_q in unit of positron charge, $v_i = T_{3i} - 2e_i \sin^2\theta_W$ and $a_i = T_{3i}$ are the vector and axial electroweak couplings, and $\rho_1(s)$, $\rho_2(s)$ are Z-resonance part to cross section. In the second line of (3) we neglected contributions from the third family of quarks and explicitly used the isotopic invariance for D_q^h functions. One could recognize from (3) that $F_{ee}^{\pi^+} = F_{ee}^{\pi^-}$ due to the isotopic invariance which suggests

$$\begin{aligned} &D_u^{\pi^+} = D_{\bar{d}}^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-}, \qquad D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_u^{\pi^-} = D_{\bar{d}}^{\pi^-}, \\ &D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^-}, \qquad D_c^{\pi^+} = D_{\bar{c}}^{\pi^+} = D_c^{\pi^-} = D_{\bar{c}}^{\pi^-}. \end{aligned} \quad (4)$$

The fragmentation functions $F_{\ell p(n)}^{\pi^\pm}$ can be measured in both charged leptons and neutrino $F_{\nu p(n)}^{\pi^\pm}$ and antineutrino $F_{\bar{\nu} p(n)}^{\pi^\pm}$ deep inelastic scattering (DIS) off proton (neutron). Such measurements are complementary to those at e^+e^- annihilations and provide new information. As one could notice, e^+e^- processes are sensitive to $D_q^h + D_{\bar{q}}^h$ combinations only while DIS processes could give access to individual quark fragmentation functions. In addition, the latter provide a scan over various Q^2. However, an interpretation of fragmentation functions in terms of D_q^h is not as straightforward as one usually assumes due to contributions from target remnants [3]. As an outcome, the functions D_q^h measured in e^+e^- processes can be different from those measured in DIS due to absorbtion of effects of target remnants fragmentation. The fragmentation of target remnant lepton–nucleon DIS can be taken into account with the help of more involved object — *fracture function* [4]. The fracture function $M^h(z, x_{\rm Bj}, Q)$ does depend not only on z but also on Bjorken $x_{\rm Bj}$ and on $Q = \sqrt{Q^2}$.

1. CHARGED LEPTONS AND (ANTI)NEUTRINO SEMI-INCLUSIVE DIS

The generalized expression for the fragmentation functions $F_{\ell p(n)}^{\pi^\pm}(z, x_{\rm Bj}, Q)$ taking into account the fracture function reads as follows:

$$F_{\ell N}^{\pi^\pm} = \frac{1}{\sigma_0^{\ell N}} \sum_q e_q^2 \left[x \left(f_q^N D_q^{\pi^\pm} + f_{\bar{q}}^N D_{\bar{q}}^{\pi^\pm} \right) + (1-x) \left(M_{q,N}^{\pi^\pm} + M_{\bar{q},N}^{\pi^\pm} \right) \right], \quad (5)$$

where $N = p, n$ are nucleons; e_q^2 — squared coupling constants of the corresponding process for the given quark (electric charge in units of positron in case of γ exchange or a combination of γ and Z-boson couplings in case of γ, Z interference), $f_q^{p,n} \equiv f_q^{p,n}(x, Q)$ are their parton distribution functions, $M_{q,N}^{\pi^\pm} \equiv M_{q,N}^{\pi^\pm}(z, x_{\rm Bj}, Q)$ and $\sigma_0^{\ell N} \equiv \sum_q e_q^2 x \left[f_q^N + f_{\bar q}^N\right]$.

In total, one has 16 fragmentation functions: $D_u^{\pi^\pm}, D_d^{\pi^\pm}, D_s^{\pi^\pm}, D_c^{\pi^\pm}, D_{\bar u}^{\pi^\pm}, D_{\bar d}^{\pi^\pm}, D_{\bar s}^{\pi^\pm}, D_{\bar c}^{\pi^\pm}$ and 24 fracture functions: $M_{d,N}^{\pi^\pm}, M_{s,N}^{\pi^\pm}, M_{u,N}^{\pi^\pm}, M_{\bar d,N}^{\pi^\pm}, M_{\bar s,N}^{\pi^\pm}, M_{\bar u,N}^{\pi^\pm}$ describing production of $\pi^\pm$ in lepton nucleon DIS. We neglect contributions from scattering off c quark in nucleon and thus $M_{c,N}^{\pi^\pm}, M_{\bar c,N}^{\pi^\pm}$ functions are neglected. However, not all of D and M functions are independent. Indeed, the isotopic invariance suggests apart of conventional $f_u^p = f_d^n$, $f_d^p = f_u^n$ 12 equalities for fragmentation functions (4) and 15 equalities for fracture functions:

$$M_{d,p}^{\pi^\pm} = M_{u,n}^{\pi^\mp},\ M_{u,p}^{\pi^\pm} = M_{d,n}^{\pi^\mp},\ M_{\bar d,p}^{\pi^\pm} = M_{\bar u,n}^{\pi^\mp},\ M_{\bar u,p}^{\pi^\pm} = M_{\bar d,n}^{\pi^\mp},$$
$$M_{s,p}^{\pi^+} = M_{\bar s,p}^{\pi^+} = M_{s,p}^{\pi^-} = M_{\bar s,p}^{\pi^-} = M_{s,n}^{\pi^+} = M_{\bar s,n}^{\pi^+} = M_{s,n}^{\pi^-} = M_{\bar s,n}^{\pi^-}. \qquad (6)$$

Therefore, in general the fragmentation function $F_{\ell N}^{\pi^\pm}(z, x_{\rm Bj}, Q)$ is described by 4 quark fragmentation functions ($D_d^{\pi^+}$, $D_u^{\pi^+}$, $D_s^{\pi^+}$, $D_c^{\pi^+}$) and 9 fracture functions ($M_{d,p}^{\pi^+}$, $M_{d,p}^{\pi^-}$, $M_{u,p}^{\pi^+}$, $M_{u,p}^{\pi^-}$,$M_{\bar d,p}^{\pi^+}$, $M_{\bar d,p}^{\pi^-}$, $M_{\bar u,p}^{\pi^+}$, $M_{\bar u,p}^{\pi^-}$, $M_{s,p}^{\pi^+}$). In total, one has 13 unknown functions to fit four data sets in $\ell N \to \pi^\pm X$ semi-inclusive DIS. Let us write down explicitly four fragmentation functions $F_{\ell N}^{\pi^\pm}(z, x_{\rm Bj}, Q)$:

$$F_{\ell p}^{\pi^+} = \frac{1}{\sigma_0^{\ell p}}\left[x\left((e_d^2 d + e_u^2 \bar u)D_d^{\pi^+} + (e_d^2 \bar d + e_u^2 u)D_u^{\pi^+} + e_s^2(s+\bar s)D_s^{\pi^+}\right) + \right.$$
$$\left. + (1-x)\left(M_{d,p}^{\pi^+} + M_{\bar d,p}^{\pi^+} + M_{u,p}^{\pi^+} + M_{\bar u,p}^{\pi^+} + 2M_{s,p}^{\pi^+}\right)\right], \qquad (7)$$

$$F_{\ell n}^{\pi^+} = \frac{1}{\sigma_0^{\ell n}}\left[x\left(e_d^2(u + \bar u)D_d^{\pi^+} + (e_d^2 \bar u + e_u^2 d)D_u^{\pi^+} + e_s^2(s+\bar s)D_s^{\pi^+}\right) + \right.$$
$$\left. + (1-x)\left(M_{u,p}^{\pi^-} + M_{\bar u,p}^{\pi^-} + M_{d,p}^{\pi^-} + M_{\bar d,p}^{\pi^-} + 2M_{s,p}^{\pi^+}\right)\right], \qquad (8)$$

$$F_{\ell p}^{\pi^-} = \frac{1}{\sigma_0^{\ell p}}\left[x\left((e_d^2 d + e_u^2 \bar u)D_u^{\pi^+} + (e_d^2 \bar d + e_u^2 u)D_d^{\pi^+} + e_s^2(s+\bar s)D_s^{\pi^+}\right) + \right.$$
$$\left. + (1-x)\left(M_{d,p}^{\pi^-} + M_{\bar d,p}^{\pi^-} + M_{u,p}^{\pi^-} + M_{\bar u,p}^{\pi^-} + 2M_{s,p}^{\pi^-}\right)\right], \qquad (9)$$

$$F_{\ell n}^{\pi^-} = \frac{1}{\sigma_0^{\ell n}}\left[x\left(e_d^2(u + \bar u)D_u^{\pi^+} + (e_d^2 \bar u + e_u^2 d)D_d^{\pi^+} + e_s^2(s+\bar s)D_s^{\pi^+}\right) + \right.$$
$$\left. + (1-x)\left(M_{u,p}^{\pi^+} + M_{\bar u,p}^{\pi^+} + M_{d,p}^{\pi^+} + M_{\bar d,p}^{\pi^+} + 2M_{s,p}^{\pi^+}\right)\right]. \qquad (10)$$

One could also generalize expressions for νN and $\bar{\nu} N$ semi-inclusive DIS taking into account the fracture functions:

$$\sigma_0^{\nu N} F_{\nu N}^{\pi^\pm} = \sum_{\substack{q=d,s \\ q'=u,c}} \left(|V_{q'q}|^2 \ x f_q^N D_{q'}^{\pi^\pm} + (1-x) M_{q,N}^{\pi^\pm}\right) + \\ + (1-y)^2 \sum_{\substack{\bar{q}=\bar{u} \\ \bar{q}'=\bar{d},\bar{s}}} \left(|V_{qq'}|^2 x f_{\bar{q}}^N D_{\bar{q}'}^{\pi^\pm} + (1-x) M_{\bar{q},N}^{\pi^\pm}\right), \quad (11)$$

$$\sigma_0^{\bar{\nu} N} F_{\bar{\nu} N}^{\pi^\pm} = \sum_{\substack{\bar{q}=\bar{d},\bar{s} \\ \bar{q}'=\bar{u},\bar{c}}} \left(|V_{q'q}|^2 \ x f_{\bar{q}}^N D_{\bar{q}'}^{\pi^\pm} + (1-x) M_{\bar{q},N}^{\pi^\pm}\right) + \\ + (1-y)^2 \sum_{\substack{q=u \\ q'=d,s}} \left(|V_{qq'}|^2 x f_q^N D_{q'}^{\pi^\pm} + (1-x) M_{q,N}^{\pi^\pm}\right), \quad (12)$$

where

$$\sigma_0^{\nu N} = \sum_{\substack{q=d,s \\ q'=u,c}} |V_{q'q}|^2 x f_q^N + (1-y)^2 \sum_{\substack{\bar{q}=\bar{u} \\ \bar{q}'=\bar{d},\bar{s}}} |V_{qq'}|^2 x f_{\bar{q}}^N,$$

$$\sigma_0^{\bar{\nu} N} = \sum_{\substack{\bar{q}=\bar{d},\bar{s} \\ \bar{q}'=\bar{u},\bar{c}}} |V_{q'q}|^2 x f_{\bar{q}}^N + (1-y)^2 \sum_{\substack{q=u \\ q'=d,s}} |V_{qq'}|^2 x f_q^N.$$

Equations (11), (12) can be rewritten taking into account isotopic relations (4), (6):

$$\sigma_0^{\nu p} F_{\nu p}^{\pi^+} = D_u^{\pi^+} x \left(d|V_{ud}|^2 + s|V_{us}|^2 + (1-y)^2 \bar{u}|V_{us}|^2\right) + D_s^{\pi^+} x (1-y)^2 \bar{u}|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(d|V_{cd}|^2 + s|V_{cs}|^2\right) + (1-x)\left(M_{d,p}^{\pi^+} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{u},p}^{\pi^+}, \quad (13)$$

$$\sigma_0^{\nu n} F_{\nu n}^{\pi^+} = D_u^{\pi^+} x \left(u|V_{ud}|^2 + s|V_{us}|^2 + (1-y)^2 \bar{d}|V_{us}|^2\right) + D_s^{\pi^+} x (1-y)^2 \bar{u}|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(u|V_{cd}|^2 + s|V_{cs}|^2\right) + (1-x)\left(M_{u,p}^{\pi^-} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{d},p}^{\pi^-}, \quad (14)$$

$$\sigma_0^{\nu p} F_{\nu p}^{\pi^-} = D_d^{\pi^+} x \left(d|V_{ud}|^2 + s|V_{us}|^2 + (1-y)^2 \bar{u}|V_{us}|^2\right) + D_s^{\pi^+} x (1-y)^2 \bar{u}|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(d|V_{cd}|^2 + s|V_{cs}|^2\right) + (1-x)\left(M_{d,p}^{\pi^-} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{u},p}^{\pi^-}, \quad (15)$$

$$\sigma_0^{\nu n} F_{\nu n}^{\pi^-} = D_d^{\pi^+} x \left(u|V_{ud}|^2 + s|V_{us}|^2 + (1-y)^2 \bar{d}|V_{us}|^2\right) + D_s^{\pi^+} x (1-y)^2 \bar{u}|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(u|V_{cd}|^2 + s|V_{cs}|^2\right) + (1-x)\left(M_{u,p}^{\pi^+} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{d},p}^{\pi^+}, \quad (16)$$

$$\sigma_0^{\bar{\nu} p} F_{\nu p}^{\pi^+} = D_d^{\pi^+} x \left(\bar{d}|V_{ud}|^2 + \bar{s}|V_{us}|^2 + (1-y)^2 u|V_{us}|^2\right) + D_s^{\pi^+} x (1-y)^2 u|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(\bar{d}|V_{cd}|^2 + \bar{s}|V_{cs}|^2\right) + (1-x)\left(M_{u,p}^{\pi^+} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{d},p}^{\pi^+}, \quad (17)$$

$$\sigma_0^{\bar{\nu}n} F_{\nu n}^{\pi^+} = D_d^{\pi^+} x \left(\bar{u}|V_{ud}|^2 + \bar{s}|V_{us}|^2 + (1-y)^2 d|V_{us}|^2\right) + D_s^{\pi^+} x(1-y)^2 d|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(\bar{u}|V_{cd}|^2 + \bar{s}|V_{cs}|^2\right) + (1-x)\left(M_{d,p}^{\pi^-} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{u},p}^{\pi^-}, \quad (18)$$

$$\sigma_0^{\bar{\nu}p} F_{\nu p}^{\pi^-} = D_u^{\pi^+} x \left(\bar{d}|V_{ud}|^2 + \bar{s}|V_{us}|^2 + (1-y)^2 u|V_{us}|^2\right) + D_s^{\pi^+} x(1-y)^2 u|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(\bar{d}|V_{cd}|^2 + \bar{s}|V_{cs}|^2\right) + (1-x)\left(M_{u,p}^{\pi^-} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{d},p}^{\pi^-}, \quad (19)$$

$$\sigma_0^{\bar{\nu}n} F_{\nu n}^{\pi^-} = D_u^{\pi^+} x \left(\bar{u}|V_{ud}|^2 + \bar{s}|V_{us}|^2 + (1-y)^2 d|V_{us}|^2\right) + D_s^{\pi^+} x(1-y)^2 d|V_{ud}|^2 + \\ + D_c^{\pi^+} x \left(\bar{u}|V_{cd}|^2 + \bar{s}|V_{cs}|^2\right) + (1-x)\left(M_{d,p}^{\pi^+} + M_{s,p}^{\pi^+}\right) + (1-x)(1-y)^2 M_{\bar{u},p}^{\pi^+}. \quad (20)$$

2. COMBINED ANALYSIS

As one can see, these additional 8 observables depend on the same 4 fragmentation and 9 fracture functions. Fit of experimental data gives an access to these functions. The fit is what one should do having a limited amount of experimental data sets. Such fits should assume functional forms of D and M functions with usually quite large number of free (generally correlated) parameters. This results in an additional uncertainty due to unknown form of the fragmentation and fracture functions.

We observe however (and this is the main result of our work) that combining one observable $F_{ee}^{\pi^\pm}(z,s)$ from e^+e^- annihilations, four observables $F_{\ell N}^{\pi^\pm}$ and eight observables $F_{\nu N}^{\pi^\pm}, F_{\bar{\nu} N}^{\pi^\pm}$, one has in total 13 observables which are linear combinations of 13 unknown fragmentation and fracture functions. Thus, hosting 13 observables into a vector $\mathcal{F}$, one can relate it with a vector of 13 unknown fragmentation and fracture functions $\mathcal{D}$ through the matrix $\mathcal{E}$ with matrix elements which can easily be recovered from equations (3), (7)–(10) and (13)–(20):

$$\mathcal{F} = \mathcal{E}\mathcal{D}, \quad (21)$$

which can trivially be solved at every point z, x_{Bj}, Q as

$$\mathcal{D} = \mathcal{E}^{-1}\mathcal{F}. \quad (22)$$

Such a method is free from assumptions about a functional form of the fragmentation and fracture functions. Thus, in general it has a potential to measure these functions with smaller uncertainties.

CONCLUSIONS

We explicitly derived expressions for fragmentation functions of charged pions produced in both charged lepton and (anti)neutrino DIS off protons (neutrons) taking into account contributions from fracture functions. We suggest a combined analysis of 13 data sets of $\pi^\pm$ production from e^+e^- annihilations, charged lepton–nucleon DIS and (anti)neutrino–nucleon DIS to measure 13 unknown functions: 4 quark fragmentation functions ($D_d^{\pi^+}$, $D_u^{\pi^+}$, $D_s^{\pi^+}$, $D_c^{\pi^+}$) and 9 fracture functions: ($M_{d,p}^{\pi^+}$, $M_{d,p}^{\pi^-}$, $M_{u,p}^{\pi^+}$, $M_{u,p}^{\pi^-}$, $M_{\bar{d},p}^{\pi^+}$, $M_{\bar{d},p}^{\pi^-}$, $M_{\bar{u},p}^{\pi^+}$, $M_{\bar{u},p}^{\pi^-}$, $M_{s,p}^{\pi^+}$). The proposed combined analysis is free from explicit assumptions of functional dependence of these objects.

Acknowledgements. We thank Federico Ceccopieri for useful discussions of fracture functions. Also we gratefully acknowledge the following funding agencies: Russian Foundation for Basic Research (RFBR, grant 07-02-00215-a), Russian Federal Targeted Program «Research and Research-Human Resources for Innovating Russia» (contracts 02.740.11.5154, P1197, 16.740.11.0154), the Dynasty Foundation grant for students (2010), grant of the Russian President (grant NSh-3810.2010.2), Analytical Targeted Program «Development of the Scientific Potential for the High School» (projects RNP.2.2.1.1/1483, RNP.2.1.1/1539), Russia.

REFERENCES

1. *Nakamura K. et al. (Particle Data Group).* Review of Particle Physics // J. Phys. G. 2010. V. 37. P. 075021.
2. *Nason P., Webber B. R.* Scaling Violation in e^+e^- Fragmentation Functions: QCD Evolution, Hadronization and Heavy Quark Mass Effects // Nucl. Phys. B. 1994. V. 421. P. 473; Erratum. Nucl. Phys. B. 1996. V. 480. P. 755.
3. *Ellis J. R. et al.* Longitudinal Polarization of Lambda and Anti-Lambda Hyperons in Lepton Nucleon Deep-Inelastic Scattering // Eur. Phys. J. C. 2007. V. 52. P. 283;
   Ellis J. R., Kotzinian A., Naumov D. V. Intrinsic Polarized Strangeness and Lambda0 Polarization in Deep-Inelastic Production // Eur. Phys. J. C. 2002. V. 25. P. 603.
4. *Trentadue L., Veneziano G.* Fracture Functions: An Improved Description of Inclusive Hard Processes in QCD // Phys. Lett. B. 1994. V. 323. P. 201;
   Trentadue L. Fracture Functions // Nucl. Phys. Proc. Suppl. 1995. V. 39BC. P. 50.