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SEMICLASSICAL APPROXIMATION OF THE DIRAC EQUATION WITH SUPERSYMMETRY

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The general scheme of the successive construction of semiclassical approximation for the classical Dirac equation in a background Yang–Mills field, where the usual Dirac operator is replaced by that with supersymmetry, is suggested. The first two terms of the semiclassical expansion in Planck's constant are derived in an explicit form. It is shown that supersymmetry of the initial Dirac operator leads to appearance of new additional terms in the classical equation of motion for spin of a particle and ipso facto requires appropriate modification for the Lagrangian of the spinning particle. The result obtained is used for the construction of one-to-one mapping between two Lagrangians of a classical color-charged spinning particle, one of which possesses local supersymmetry, and another one does not. It is demonstrated that for recovery of the one-to-oneness the additional terms obtained above in the semiclassical approximation of the Dirac operator with supersymmetry should be added to the Lagrangian without supersymmetry.

Предложена общая схема последовательного построения квазиклассического приближения для классического уравнения Дирака во внешнем поле Янга–Миллса, в котором обычный оператор Дирака заменен оператором Дирака с суперсимметрией. Явным образом вычислены два первых члена квазиклассического разложения по постоянной Планка. Показано, что наличие суперсимметрии в исходном операторе Дирака приводит к появлению новых дополнительных членов в классическом уравнении на движение спина частицы и тем самым требует соответствующей модификации лагранжиана спиновой частицы. Полученный результат используется для построения взаимнооднозначного отображения между двумя лагранжианами классической цветной спиновой частицы, один из которых обладает локальной суперсимметрией, а другой — нет. Показано, что для восстановления взаимной однозначности необходимо в лагранжиан без суперсимметрии добавить найденные выше дополнительные члены в квазиклассической аппроксимации оператора Дирака с суперсимметрией.

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1. STATEMENT OF PROBLEM

In our papers [1, 2] it was suggested the following model Lagrangian describing the interaction of a classical relativistic spinning color-charged particle with external non-Abelian

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gauge $A_\mu^a(x)$ and fermion $\Psi_\alpha^i(x)$ fields:

$$
\begin{gathered}
L = L_0 + L_m + L_\theta + L_\Psi, \quad L_0 = -\frac{1}{2e}\left(\frac{dx_\mu}{d\tau}\frac{dx^\mu}{d\tau}\right) + \frac{1}{2i}\left(\frac{d\bar{\psi}}{d\tau}\,\psi - \bar{\psi}\,\frac{d\psi}{d\tau}\right), \\
L_m = -\frac{e}{2}\,m^2, \quad L_\theta = i(\theta^{\dagger i}D^{ij}\theta^j) - e\,\frac{g}{4}\,Q^a F_{\mu\nu}^a(\bar{\psi}\sigma^{\mu\nu}\psi), \\
L_\Psi = -\frac{e}{\sqrt{2}}\,g\Big\{\theta^{\dagger i}(\bar{\psi}_\alpha\Psi_\alpha^i) + (\bar{\Psi}_\alpha^i\psi_\alpha)\theta^i\Big\} + \\
+ \frac{e}{\sqrt{2}}\,g\left(\frac{C_F}{2T_F}\right)Q^a\Big\{\theta^{\dagger j}(t^a)^{ji}(\bar{\psi}_\alpha\Psi_\alpha^i) + (\bar{\Psi}_\alpha^i\psi_\alpha)(t^a)^{ij}\theta^j\Big\},
\end{gathered}
\tag{1}
$$

where e is the one-dimensional vierbein field (we put throughout $c = 1$ for the speed of light) and $D^{ij} = \delta^{ij}\partial/\partial\tau + ig\dot{x}^\mu A_\mu^a(t^a)^{ij}$ is the covariant derivative along the direction of motion. The spin degree of freedom of the particle is represented here by a commuting c — number Dirac spinor $\psi = (\psi_\alpha)$, $\alpha = 1, \ldots, 4$. The equation of motion for this spinor is

$$
i\,\frac{d\psi(\tau)}{d\tau} = -\frac{g}{4m}\,\sigma^{\mu\nu}Q^a F_{\mu\nu}^a(x)\psi(\tau) + \big(\text{terms with fermion field } \Psi_\alpha^i(x)\big). \tag{2}
$$

By virtue of the fact that the background fermion field $\Psi_\alpha^i(x)$ (which within the classical description is considered as the Grassmann-odd one) has, by definition, spinor index, a description of the spin degree of freedom of the particle in terms of the spinor ψ_α is very natural and simplest in technical respect. There is some vagueness with respect to Grassmann evenness of this spinor. In our papers [1, 3] in application to analysis of dynamics of a spinning color particle moving in a hot quark–gluon plasma, the spinor ψ_α was thought as the Grassmann-even one (although it is not improbable that the using simultaneously of spinors of the different Grassmann evenness may be required for a complete classical description of the spin dynamics in external fields of different statistics, i.e., it requires introducing a superspinor, see Summary).

An alternative approach most generally employed for the description of a spin for a massive particle is connected with introduction into consideration of the real pseudovector and pseudoscalar dynamical variables ξ_μ, $\mu = 1, \ldots, 4$, and ξ_5 that are elements of the Grassmann algebra [4–7]. For these variables an appropriate Lagrangian of the first-order time derivative was defined as follows:

$$
\begin{gathered}
L = L_0 + L_m + L_\theta, \qquad L_0 = -\frac{1}{2e}\,\dot{x}_\mu\dot{x}^\mu - \frac{i}{2}\,\xi_\mu\dot{\xi}^\mu + \frac{i}{2e}\,\chi\dot{x}_\mu\xi^\mu, \\
L_m = -\frac{e}{2}\,m^2 + \frac{i}{2}\,\xi_5\,\dot{\xi}_5 + \frac{i}{2}\,m\chi\xi_5, \qquad L_\theta = i\theta^{\dagger i}D^{ij}\theta^j + \frac{i}{2}\,eg\,Q^a F_{\mu\nu}^a\,\xi^\mu\xi^\nu,
\end{gathered}
\tag{3}
$$

where χ is the one-dimensional gravitino field. This field (as well as e) is not dynamical one. It is well known [5–7] that the Lagrangian possesses local supersymmetry. The description of the spin degree of freedom in terms of the odd pseudovector and pseudoscalar quantities is to some extent a more fundamental one in comparison with the description in terms of the even spinor ψ_α. For this reason the interesting question arises as to whether it is possible to define relation (mapping) between these variables, and, finally, to construct a mapping between Lagrangian (1) (without the interaction term L_Ψ) and Lagrangian (3). The construction of such a mapping in an explicit form is very important. The reason is that counterpart of the interaction term L_Ψ in Lagrangian (3) is unknown. Thus, having understood a connection between the Lagrangians without an external fermion field, one can define an explicit form of the interaction terms with the background Ψ-fields in terms of the Grassmann pseudovector and pseudoscalar variables ξ_μ and ξ_5 merely by means of an appropriate replacement of the ψ_α spinor by the mapping $\psi_\alpha = \psi_\alpha(\xi_\mu, \xi_5)$ in (1). In works [8, 9] it was shown that such a map can in principle be obtained if preliminarily to exclude the auxiliary variable χ in Lagrangian (3) with the help of the equation of motion

$$2\dot{\xi}_5 - m\chi = 0.$$

In this case, instead of L_0 and L_m in (3) we will have

$$L_0 = -\frac{1}{2e}\,\dot{x}_\mu\dot{x}^\mu - \frac{i}{2}\,\xi_\mu\dot{\xi}^\mu - \frac{i}{me}\,\dot{x}_\mu\xi^\mu\dot{\xi}_5, \quad L_m = -\frac{e}{2}\,m^2 - \frac{i}{2}\,\xi_5\,\dot{\xi}_5.$$

The map linear in ψ and $\bar{\psi}$ has the following form:

$$(\bar{\theta}\theta)\,\psi = \kappa\,\xi_\mu(\gamma^\mu\gamma_5\theta) + \alpha\xi_5(\gamma_5\theta), \qquad (\bar{\theta}\theta)\,\bar{\psi} = -\kappa^*(\bar{\theta}\gamma_5\gamma^\mu)\xi_\mu - \alpha^*(\bar{\theta}\gamma_5)\xi_5. \tag{4}$$

Here, κ and α are unknown coefficient functions, $\theta = (\theta_\alpha)$ is an auxiliary Grassmann-odd Dirac spinor and the symbol * is a complex conjugation sign. Inverse mapping has the following form:

$$\xi_\mu = \frac{1}{2}\left\{\beta(\bar{\theta}\gamma_\mu\gamma_5\psi) - \beta^*(\bar{\psi}\gamma_5\gamma_\mu\theta)\right\}, \qquad \xi_5 = \frac{1}{2}\left\{\tilde{\beta}(\bar{\theta}\gamma_5\psi) - \tilde{\beta}^*(\bar{\psi}\gamma_5\theta)\right\}, \tag{5}$$

where β and $\tilde{\beta}$ are some new unknown coefficient functions. The explicit form of the coefficient functions was considered in [8, 9].

Our initial Lagrangian (1) written down in terms of the commutative variable ψ_α, is devoid of any supersymmetry. Therefore, it can be only mapped into the other nonsupersymmetric Lagrangian. The terms containing the fermion counterpart χ to the vierbein field e, namely

$$\frac{i}{2e}\,\chi\dot{x}_\mu\xi^\mu, \qquad \frac{im}{2}\,\chi\xi_5, \tag{6}$$

cannot appear under any map. These terms are important for the local supersymmetry of Lagrangian (3) and its counterparts a priori must be contained in the initial Lagrangian (1). In this notice we would like to show how the terms of this kind may really appear in (1).

2. SEMICLASSICAL APPROXIMATION

The main idea in determining such terms is to use an extended Hamiltonian or super-Hamiltonian in the construction of the «spinning» equation (2). Hamiltonians of this type have been considered in a few papers for different reasons. Thus, in the papers by Borisov, Kulish [10] and Fradkin, Gitman [11] they were used in the construction of the Green's function of a Dirac particle in background non-Abelian gauge field. Within the framework of operator formalism this superHamiltonian has the form

$$-2m\hat{H}_{\rm SUSY} = \Big(\hat{D}_\mu\hat{D}^\mu + \frac{1}{2}g\hat{\sigma}_{\mu\nu}F^{a\mu\nu}\hat{T}^a - m^2\Big) + i\chi\big(\hat{\gamma}_\mu\hat{D}^\mu + m\big). \tag{7}$$

All quantities with hats above represent operators acting in appropriate spaces of representations of the spinor, color and coordinate algebras; χ is an odd variable. Analog of introducing such a superHamiltonian in the massless limit can be also found in the work by Friedan and Windey [12] in the construction of the superheat kernel. The latter has been used in calculating the chiral anomaly. In the monograph by Thaller [13] within the supersymmetric quantum mechanics a notion of *the Dirac operator with supersymmetry* has been defined in the most general abstract form. The expression (7) is just its special case.

Before studying the general case of the Dirac operator with supersymmetry it is necessary to recall briefly the fundamental points of deriving the equation of motion for the commuting spinor ψ_α, Eq. (2). In quantum electrodynamics this equation arises when we analyze the connection of the relativistic quantum mechanics with the relativistic classical mechanics first performed by W. Pauli [14] within the so-called first-order formalism for fermions. In the book by Akhiezer and Beresteskii [15] this analysis has been performed on the basis of the second-order formalism [16]. Here, we will follow the second line.

In the second-order formalism the initial QCD Dirac equation for the wave function Ψ is replaced by its quadratic form

$$-2m\hat{H}\Phi = \Big(D_\mu D^\mu + \frac{1}{2}g\sigma_{\mu\nu}F^{\mu\nu} - m^2\Big)\Phi = 0, \tag{8}$$

where a new spinor Φ is connected with the initial one by the relation

$$\Psi = \frac{1}{m}\big(\gamma_\mu D^\mu + m\big)\Phi.$$

In what follows we restore Planck's constant $\hbar$ in all formulae. Since we are interested in the interaction of the spin degree of freedom of a particle with an external gauge field most, then for the sake of simplicity we will consider Eq. (8) for the case of the interaction with an Abelian background field (with the replacement of the strong coupling g by electric charge q). The presence of the color degree of freedom can result in qualitatively new features, one of them is appearing a mixed spin–color degree of freedom [17]. In this respect our initial model Lagrangian (1) is the simplified one and it corresponds to perfect factorization of the spin and color degrees of freedom of the particle. The non-Abelian case also requires appreciable complication of the usual WKB-method in the analysis of Eq. (8) that is beyond the scope of our work (see, for example, [18, 19]).

A solution of Eq. (8) in the semiclassical limit is defined as a series in powers of $\hbar$:

$$\Phi = \mathrm{e}^{iS/\hbar}(f_0 + \hbar f_1 + \hbar^2 f_2 + \ldots), \tag{9}$$

where S, f_0, f_1, … are some functions of coordinates and time. Substituting this series into (8) and collecting the terms of the same power of $\hbar$, we obtain the following equations correct to the first order in $\hbar$:

$$\hbar^0 : \left(\frac{\partial S}{\partial x_\mu} + qA_\mu\right)^2 - m^2 = 0, \tag{10}$$

$$\hbar^1 : \left[\frac{1}{i}\,\frac{\partial}{\partial x_\mu}\left(\frac{\partial S}{\partial x^\mu} + qA_\mu\right)\right] f_0 + \frac{2}{i}\left(\frac{\partial S}{\partial x^\mu} + qA_\mu\right)\frac{\partial f_0}{\partial x_\mu} + \frac{q}{2}\,\sigma_{\mu\nu}F^{\mu\nu}f_0 = 0. \tag{11}$$

Furthermore, we introduce into consideration a flux fermion density

$$s_\mu \equiv \bar{\Psi}_0 \gamma_\mu \Psi_0, \tag{12}$$

where as Ψ_0 we take the following expression:

$$\Psi_0 = \frac{1}{m}\left(\gamma_\mu D^\mu - m\right) f_0\, \mathrm{e}^{iS/\hbar} \simeq \frac{1}{m}\, \mathrm{e}^{iS/\hbar}\left[\pi_\mu \gamma^\mu - m\right] f_0,$$
$$\pi_\mu \equiv \frac{\partial S(x, \boldsymbol{\alpha})}{\partial x^\mu} + qA_\mu(x).$$

Here, $\boldsymbol{\alpha}$ designates three arbitrary constants defining a solution for the action S, Eq. (10). In terms of the spinor f_0 the flux density (12) has the form

$$s_\mu = \frac{2}{m^2}\, \pi_\mu \left[\bar{f}_0(\gamma_\nu \pi^\nu - m) f_0\right]$$

and by virtue of Eqs. (10) and (11) it satisfies the equation of continuity

$$\frac{\partial s_\mu}{\partial x_\mu} = 0.$$

Equation (2) (without the terms with the external fermion field) arises from an analysis of the equation for the spinor f_0 (11). The latter in terms of the function π_μ can be written in a more compact form

$$\frac{\partial \pi_\mu}{\partial x_\mu}\, f_0 + 2\pi_\mu\, \frac{\partial f_0}{\partial x_\mu} + \frac{iq}{2}\, \sigma_{\mu\nu} F^{\mu\nu} f_0 = 0. \tag{13}$$

At this point we introduce a new variable

$$\eta \equiv \frac{2}{m^2}\left[\bar{f}_0(\gamma_\nu \pi^\nu - m) f_0\right],$$

such that $s_\mu = \pi_\mu \eta$. Owing to the continuity equation we have an important relation for the η function

$$\frac{\partial \pi_\mu}{\partial x_\mu}\, \eta = -\pi_\mu\, \frac{\partial \eta}{\partial x_\mu}. \tag{14}$$

At the final stage we substitute $f_0 = \sqrt{\eta}\varphi_0$ into Eq. (13). With allowance for (14) this equation takes the following form for a new spinor function φ_0:

$$\pi_\mu \frac{\partial \varphi_0}{\partial x_\mu} = -\frac{iq}{4}\, \sigma_{\mu\nu} F^{\mu\nu} \varphi_0.$$

In book [15] a solution of the equation derived just above was expressed in terms of the solution of the Schrödinger equation for the wave function $\psi_\alpha(\tau)$, Eq. (2). The latter describes the motion of a spin in a given gauge field $F_{\mu\nu}(x)$. This field is defined along the trajectory of the particle $x_\mu = x_\mu(\tau, \boldsymbol{\alpha}, \boldsymbol{\beta})$ which in turn is defined from a solution of the equation

$$m\, \frac{dx_\mu}{d\tau} = \pi_\mu(x, \boldsymbol{\alpha})$$

with the initial value given by a vector $\boldsymbol{\beta}$.

3. EXTENSION OF THE SEMICLASSICAL APPROXIMATION

Let us now consider the question of a modification of the expressions obtained above in the case when instead of the usual Hamilton operator in Eq. (8) one takes its supersymmetric extension

$$-2m\hat{H}_{\rm SUSY}\Phi \equiv \left\{\left(D_\mu D^\mu + \frac{q\hbar}{2}\,\sigma_{\mu\nu}F^{\mu\nu} - m^2\right) + i\chi\big(\gamma_\mu\gamma_5 D^\mu + m\gamma_5\big)\right\}\Phi = 0. \qquad (15)$$

Here, in the second expression following [11] in parentheses we have introduced the γ_5 matrix into the definition of the linear Dirac operator. This operator $\big(\gamma_\mu\gamma_5 D^\mu + m\gamma_5\big)$ should be believed as an odd function. We will seek a solution of Eq. (15) also in the form of a power series (9) only on condition that the function S is considered as a usual commuting function, whereas the spinor functions $f_0, f_1, \ldots$ should be considered as those containing both Grassmann even and odd parts. Equations (10) and (11) are modified as follows:

$$\begin{aligned}
\hbar^0 :&\quad \big(\pi^2 - m^2\big)f_0 + i\chi\big(\pi_\mu\gamma^\mu\gamma_5 + m\gamma_5\big)f_0 = 0,\\
\hbar^1 :&\quad \big(\pi^2 - m^2\big)f_1 + i\chi\big(\pi_\mu\gamma^\mu\gamma_5 + m\gamma_5\big)f_1 +\\
&\qquad + \left[\frac{1}{i}\,\frac{\partial\pi_\mu}{\partial x_\mu}\,f_0 + \frac{2}{i}\,\pi_\mu\,\frac{\partial f_0}{\partial x_\mu} + \frac{q}{2}\,\sigma_{\mu\nu}F^{\mu\nu}f_0\right] + \chi\gamma_\mu\gamma_5\,\frac{\partial f_0}{\partial x_\mu} = 0.
\end{aligned}$$

The next step is to present the spinors f_0 and f_1 as a sum of even and odd parts

$$\begin{cases} f_0 = f_0^{(0)} + \chi f_0^{(1)},\\ f_1 = f_1^{(0)} + \chi f_1^{(1)}. \end{cases} \qquad (16)$$

In the decomposition of (16) we believe the functions $(f_0^{(0)}, f_1^{(0)})$ to be the even ones, and $(f_0^{(1)}, f_1^{(1)})$ to be the odd ones. The opposite case of partition into Grassmann evenness will be mentioned at the end of the paper. By using (16) the equation of the zeroth order in $\hbar$ is decomposed into two equations

$$\begin{aligned}
&\big(\pi^2 - m^2\big)f_0^{(0)} = 0,\\
&\big(\pi^2 - m^2\big)f_0^{(1)} + i\big(\pi_\mu\gamma^\mu\gamma_5 + m\gamma_5\big)f_0^{(0)} = 0,
\end{aligned}$$

the first of which defines the Hamilton–Jacobi equation for the action S, Eq. (10), and the second one is reduced to the matrix algebraic equation for the spinor $f_0^{(0)}$

$$\big(\pi_\mu\gamma^\mu\gamma_5 + m\gamma_5\big)f_0^{(0)} = 0.$$

Furthermore, the equation of the first order in $\hbar$ is also decomposed into two equations which with the use of (10) take the form

$$\begin{aligned}
&\frac{1}{i}\left(\frac{\partial\pi_\mu}{\partial x_\mu}\right)f_0^{(0)} + \frac{2}{i}\,\pi_\mu\frac{\partial f_0^{(0)}}{\partial x_\mu} + \frac{q}{2}\,\sigma_{\mu\nu}F^{\mu\nu}f_0^{(0)} = 0,\\
&\frac{1}{i}\left(\frac{\partial\pi_\mu}{\partial x_\mu}\right)f_0^{(1)} + \frac{2}{i}\,\pi_\mu\frac{\partial f_0^{(1)}}{\partial x_\mu} + \frac{q}{2}\,\sigma_{\mu\nu}F^{\mu\nu}f_0^{(1)} + \gamma_\mu\gamma_5\,\frac{\partial f_0^{(0)}}{\partial x_\mu} = \big(\pi_\mu\gamma^\mu\gamma_5 + m\gamma_5\big)f_1^{(0)}.
\end{aligned} \qquad (17)$$

Notice that the term on the right-hand side of Eq. (17) represents the contribution of a quantum correction in contrast to the other terms. The first equation for the even spinor $f_0^{(0)}$ is analyzed similar to Eq. (13) by the replacement

$$f_0^{(0)} = \sqrt{\eta}\,\varphi_0^{(0)}, \quad \eta \equiv \frac{2}{m^2}\big[\bar{f}_0^{(0)}(\gamma_\mu \pi^\mu - m) f_0^{(0)}\big]. \tag{18}$$

For the odd spinor $f_0^{(1)}$ we define a similar replacement introducing a new odd spinor $\theta_0^{(1)}$ by the rule

$$f_0^{(1)} = \sqrt{\eta}\,\theta_0^{(1)}, \tag{19}$$

with the same scalar function η as it was defined in (18). Taking into account the continuity equation in the form (14) and the replacement (19), we obtain instead of (17)

$$\pi_\mu \frac{\partial \theta_0^{(1)}}{\partial x_\mu} + \frac{iq}{4}\,\sigma_{\mu\nu} F^{\mu\nu} \theta_0^{(1)} + i\gamma_\mu\gamma_5\,\frac{1}{2\sqrt{\eta}}\frac{\partial \sqrt{\eta}\,\varphi_0^{(0)}}{\partial x_\mu} = \frac{1}{2}\left(\pi_\mu \gamma^\mu \gamma_5 + m\gamma_5\right)\varphi_1^{(0)}, \tag{20}$$

where on the right-hand side we also have set $f_1^{(0)} \equiv \sqrt{\eta}\,\varphi_1^{(0)}$. The equation obtained can be connected with the equation of motion for a spin in external gauge field in the form (2), but instead of the even spinor $\psi_\alpha(\tau)$, here we have the odd spinor $\theta_{0\alpha}^{(1)}(\tau)$. The latter can be identified with the auxiliary Grassmann spinor $\theta_\alpha(\tau)$ we have introduced into mapping (4). Further, the spinor $\varphi_1^{(0)}$ on the left-hand side of (20) is the even one and it can be related to our commuting spinor ψ_α by setting

$$\varphi_1^{(0)} \equiv m\psi.$$

The expression in parentheses on the right-hand side of (20) should be considered as the Grassmann odd one by virtue of oddness of the initial operator expression which correlates with it (see the text after formula (15)). The oddness of this expression can be displayed explicitly if we reintroduce the Grassmann scalar χ as a multiplier. Taking into account all the above-mentioned and also the relation $\dot{x}_\mu = \pi_\mu/m$, we obtain the final expression of equation for the odd spinor θ_α:

$$\frac{1}{i}\frac{d\theta}{d\tau} + \frac{q}{4m}\,\sigma_{\mu\nu} F^{\mu\nu}\theta + \ldots = \frac{m}{2i}\,\chi \dot{x}_\mu\big(\gamma^\mu\gamma_5\psi\big) + \frac{m}{2i}\,\chi\big(\gamma_5\psi\big). \tag{21}$$

Here, the dots denote the contribution of the last term on the left-hand side of Eq. (20). Its physical meaning is not clear. The terms on the right-hand side of (21) can be obtained by varying with respect to $\bar{\theta}$ from the following terms, which must be added to Lagrangian (1):

$$L = \ldots + \left\{\left(\frac{im}{2}\,\chi \dot{x}_\mu\big(\bar{\theta}\gamma^\mu\gamma_5\psi\big) + \frac{im}{2}\,\chi\big(\bar{\theta}\gamma_5\psi\big)\right) + (\text{conj. part})\right\}. \tag{22}$$

Finally, in turn, under the mapping of Lagrangian (1) into (3) the expressions in braces in (22) should be identified with the Grassmann pseudovector ξ_μ and pseudoscalar ξ_5 by the rule (the inverse mapping (5))

$$\xi_\mu \sim \big(\bar{\theta}\gamma_\mu\gamma_5\psi\big) + (\text{conj. part}),$$
$$\xi_5 \sim \big(\bar{\theta}\gamma_5\psi\big) + (\text{conj. part}),$$

and, thereby, we can obtain the missing terms (6) in our map. Although we have obtained here, the equation of motion for the odd spinor θ_α, Eq. (21), a similar equation can be obtained for the even spinor ψ_α by changing the Grassmann evenness of the spinors $(f_0^{(0)}, f_1^{(0)})$ and $(f_0^{(1)}, f_1^{(1)})$ in the decomposition of (16) to the opposite one.

4. SUMMARY

In this notice it was shown that to construct the map into a complete supersymmetric Lagrangian (3), the initial Lagrangian (1) also has to possess a supersymmetry. To accomplish these ends, we must add the terms (22) in an explicit form containing auxiliary anticommuting classical spinor θ_α to the initial expression (1). Furthermore, the obtained Eq. (21) for the odd spinor serves as a hint that the spinor should generally be considered as an *independent* dynamical variable subject to own dynamical equation similar to the equation for ψ_α. This odd spinor θ_α should be related to its superpartner: the even spinor ψ_α, and thus we have to consider a single *superspinor*

$$\Theta_\alpha = \theta_\alpha + \eta\psi_\alpha,$$

as was done, for instance, in paper [20]. Here, η is a real odd scalar.

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REFERENCES

1. *Markov Yu. A., Markova M. A.* Nonlinear Dynamics of Soft Fermion Excitations in Hot QCD Plasma II: Soft-Quark-Hard Particle Scattering and Energy Losses // Nucl. Phys. A. 2007. V. 784. P. 443.
2. *Markov Yu. A., Markova M. A., Shishmarev A. A.* The Equations of Motion for a Classical Color Particle in Background Non-Abelian Bosonic and Fermionic Fields // J. Phys. G. 2010. V. 37. P. 105001(25).
3. *Markov Yu. A., Markova M. A., Vall A. N.* Nonlinear Dynamics of Soft Fermion Excitations in Hot QCD Plasma: Soft-Quark Bremsstrahlung and Energy Losses // Intern. J. Mod. Phys. A. 2010. V. 25. P. 685.
4. *Berezin F. A., Marinov M. S.* Classical Spin and Grassmann Algebra // JETP Lett. 1975. V. 21. P. 678;
   Berezin F. A., Marinov M. S. Particle Spin Dynamics as the Grassmann Variant of Classical Mechanics // Ann. Phys. (NY). 1977. V. 104. P. 336.
5. *Barducci A., Casalbuoni R., Lusanna L.* Supersymmetries and the Pseudoclassical Relativistic Electron // Nuovo Cim. A. 1976. V. 35. P. 377;
   Barducci A., Casalbuoni R., Lusanna L. Classical Scalar and Spinning Particles Interacting with External Yang–Mills Fields // Nucl. Phys. B. 1977. V. 124. P. 93.
6. *Brink L. et al.* Local Supersymmetry for Spinning Particles // Phys. Lett. B. 1976. V. 64. P. 435;
   Brink L., Di Vecchia P., Howe P. S. A Lagrangian Formulation of the Classical and Quantum Dynamics of Spinning Particles // Nucl. Phys. B. 1977. V. 118. P. 76.

7. *Balachandran A. P. et al.* Classical Description of a Particle Interacting with a Non-Abelian Gauge Field // Phys. Rev. D. 1977. V. 15. P. 2308.
8. *Markov Yu. A. et al.* Equations of Motion for a Classical Color Particle in Background Non-Abelian Fermionic and Bosonic Fields: Inclusion of Pseudoclassical Spin. hep-th/1112.2056.
9. *Markov Yu. A. et al.* Correspondence between Classical and Pseudoclassical Descriptions of Spin Degree of Freedom of Relativistic Particle in External Fields // Russ. Phys. J. 2012. V. 55.
10. *Borisov N. V., Kulish P. P.* Path Integral in Superspace for a Relativistic Spinor Particle in an External Gauge Field // Theor. Math. Phys. 1982. V. 51. P. 535.
11. *Fradkin E. S., Gitman D. M.* Path-Integral Representation for the Relativistic Particle Propagators and BFV Quantization // Phys. Rev. D. 1991. V. 44. P. 3230.
12. *Friedan D., Windey P.* Supersymmetric Derivation of the Atiyah–Singer Index and the Chiral Anomaly // Nucl. Phys. B. 1984. V. 235. P. 395.
13. *Thaller B.* The Dirac Equation. Berlin; Heidelberg: Springer-Verlag, 1992.
14. *Pauli W.* Diracs Wellengleichung des Elektrons und geometrische Optik // Helv. Phys. Acta. 1932. V. 5. P. 179.
15. *Akhiezer A. I., Beresteskii V. B.* Quantum Electrodynamics. M.: Nauka, 1969.
16. *Morgan A. G.* Second-Order Femions in Gauge Theories // Phys. Lett. B. 1995. V. 351. P. 249.
17. *Arodz H.* Colored, Spinning Classical Particle in an External Non-Abelian Gauge Field // Phys. Lett. B. 1982. V. 116. P. 251.
18. *Arodz H.* A Remark on the Classical Mechanics of Colored Particles // Ibid. P. 255;
Arodz H. Limitation of the Concept of the Classical Colored Particle // Acta Phys. Polonica B. 1983. V. 14. P. 13.
19. *Belov V. V., Kondrat'eva M. F.* «Classical» Equations of Motion in Quantum Mechanics with Gauge Fields // Theor. Math. Phys. 1992. V. 92. P. 722.
20. *Sorokin D. P. et al.* From the Superparticle Siegel Symmetry to the Spinning Particle Proper-Time Supersymmetry // Phys. Lett. B. 1989. V. 216. P. 302.

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