

## EFFECTS OF FINAL-STATE INTERACTIONS IN $B^+ \rightarrow D_s^+ \bar{K}^0$ DECAY

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We investigate the effects of final-state-interactions (FSI) contributions in the nonleptonic two-body  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay; however, the hadronic decay of  $B^+ \rightarrow D_s^+ \bar{K}^0$  is analyzed by using «QCD factorization» (QCDF) method and final-state interaction (FSI). First, the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay is calculated via QCDF method and only the annihilation graphs exist in that method. Hence, the FSI must be seriously considered to solve the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay and the  $D^0 \pi^+(D^{0*} \rho^+)$ ,  $D^+ \pi^0(D^{+*} \rho^0)$  and  $D^+ \eta_c(D^{+*} J/\psi)$  via the exchange of  $K^{+(*)}$ ,  $K^{0(*)}$  and  $D_s^{+(*)}$  mesons are chosen for the intermediate states. To estimate the intermediate states amplitudes, the QCDF method is again used. These amplitudes are used in the absorptive part of the diagrams. The experimental branching ratio of  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay is less than  $8 \cdot 10^{-4}$  and the predicted branching ratio is  $0.23 \cdot 10^{-9}$  in the absence of FSI effects and it becomes  $6.74 \cdot 10^{-4}$  when FSI contributions are taken into account.

В работе исследуется влияние эффектов взаимодействия в конечном состоянии (ВКС) в не-лептонном двухчастичном распаде  $B^+ \rightarrow D_s^+ \bar{K}^0$ , однако адронный распад  $B^+ \rightarrow D_s^+ \bar{K}^0$  анализируется с помощью метода КХД-факторизации (КХДФ) и с учетом ВКС. Сначала распад  $B^+ \rightarrow D_s^+ \bar{K}^0$  вычисляется методом КХДФ, когда принимаются в расчет только аннигиляционные диаграммы. ВКС необходимо учитывать для правильного описания распада  $B^+ \rightarrow D_s^+ \bar{K}^0$ , а также  $D^0 \pi^+(D^{0*} \rho^+)$ ,  $D^+ \pi^0(D^{+*} \rho^0)$  и  $D^+ \eta_c(D^{+*} J/\psi)$ , где в промежуточных состояниях рассматривается обмен  $K^{+(*)}$ -,  $K^{0(*)}$ - и  $D_s^{+(*)}$ -мезонами. Чтобы оценить величину амплитуд промежуточных состояний, был также использован метод КХДФ. Полученные амплитуды используются при вычислении поглощающих частей диаграмм. Известно, что экспериментальное значение вероятности распада  $B^+ \rightarrow D_s^+ \bar{K}^0$  меньше  $8 \cdot 10^{-4}$ , в то время как предсказываемое расчетами составляет  $0,23 \cdot 10^{-9}$  без учета эффектов ВКС и  $6,74 \cdot 10^{-4}$  — после учета ВКС.

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### INTRODUCTION

In the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay the  $B^+$  meson has  $u$  quark and  $\bar{b}$  antiquark and the  $D_s^+$ ,  $\bar{K}^0$  mesons are produced from pair quarks of  $c\bar{s}$  and  $s\bar{d}$  respectively, so no form factor can be produced between the  $B^+$  meson and final states  $D_s^+$ ,  $\bar{K}^0$  mesons, hence this process is pure annihilation type of the decay. For this reason the mentioned decay has a tiny branching ratio in the QCDF approach. However, the hadronic decay of  $B^+ \rightarrow D_s^+ \bar{K}^0$  is analyzed by using

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«QCD factorization» (QCDF) method and final-state interaction (FSI). The next-to-leading-order low-energy effective Hamiltonian is used for the weak-interaction matrix elements and the FSI. The importance of the FSI in hadronic processes has been identified for a long time. Recently, its applications in  $D$  and  $B$  decays have attracted extensive interest and attention of theorists.

Since the hadronic matrix elements are fully controlled by nonperturbative QCD, the most important problem is how to evaluate them properly. Factorization method enables one to separate the nonperturbative QCD effects from the perturbative parts and to calculate the latter in terms of the field theory order by order. Several factorization approaches have been proposed to analyze  $B$ -meson decays, such as the naive factorization approach, the QCD factorization approach, the perturbative QCD approach, and Soft-Collinear-Effective Theory; none provided an estimate of the FSI at the hadronic level. These approaches successfully explain many phenomena; however, there are still some problems which are not easy to describe within this framework.

These may be some hints for the need of FSI in  $B$  decays. FSI effects are nonperturbative in nature [1]. FSI is one of the ways to solve the nonperturbative QCD for the long-distance case. In many decay modes, the FSI may play a crucial role [2]. In this approach the CKM's most favored two-body intermediate states are the only ones that have been taken into consideration [3].

The FSI can be considered as a soft rescattering style for certain intermediate two-body hadronic channel  $B^+ \rightarrow D^0 \pi^+$  decay [4]. Therefore, the FSI are estimated via the one-particle-exchange processes at the hadronic loop level (HLL) as explained in Sec. 3.

As we know, the branching ratio of  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay has already been estimated by using the perturbative QCD approach and predicted  $2.01 \cdot 10^{-8}$  [5]. We calculated the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay according to the QCDF method. This process only occurs via annihilation between  $b$  and  $\bar{u}$ . We selected the leading-order Wilson coefficients at the scale  $m_b$  [6, 7] and obtained the  $\text{BR}(B^+ \rightarrow D_s^+ \bar{K}^0) = 0.23 \cdot 10^{-9}$ . The experimental result of this decay is  $\text{BR}(B^+ \rightarrow D_s^+ \bar{K}^0) < 8 \cdot 10^{-4}$ . In the FSI, rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams [8]. In the FSI effects, the intermediate states are  $D^0 \pi^+(D^{0*} \rho^+)$ ,  $D^+ \pi^0(D^{+*} \rho^0)$  and  $D^+ \eta_c(D^{+*} J/\psi)$ . We calculated the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay according to the HLL method [9]. In this case, the branching ratio of  $B^+ \rightarrow D_s^+ \bar{K}^0$  is  $6.74 \cdot 10^{-4}$ .

This paper is organized as follows. We present the calculation of QCDF for the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay in Sec. 1. In Sec. 2, we calculate the amplitudes of the intermediate states. Then, we present the calculation of HLL for the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay in Sec. 3. In Sec. 4, we give the numerical results, and in the last section, we have a short conclusion.

## 1. QCD FACTORIZATION OF $B^+ \rightarrow D_s^+ \bar{K}^0$ DECAY

In the QCD factorization approach, the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay has annihilation contribution that Feynman diagram in Fig. 1 clearly shows. According to the diagram of Fig. 1, we obtained the annihilation amplitude as

$$A_{\text{QCD}}(B^+ \rightarrow D_s^+ \bar{K}^0) = i \frac{G_F}{\sqrt{2}} f_B f_K f_{D_s} b_2 V_{ub} V_{cd}^*, \quad (1)$$

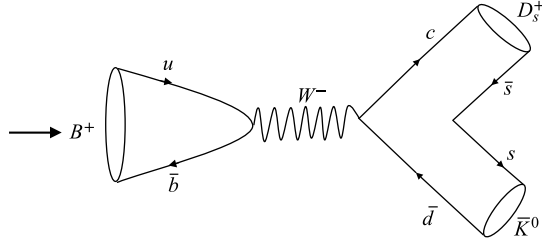


Fig. 1. Feynman annihilation diagram for  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay, the left arrow shows that the Feynman diagram is being read from left side

where

$$b_2 = \frac{C_F}{N_c^2} C_2 A_1^i, \quad A_1^i = 6\pi\alpha_s \left[ 3 \left( X_A - 4 + \frac{\pi^2}{3} \right) + r_\chi^K r_\chi^{D_s} (X_A^2 - 2X_A) \right], \quad (2)$$

$$X_A = (1 + \rho e^{i\phi}) \ln \frac{m_B}{\Lambda_{\text{QCD}}}, \quad r_\chi^K = \frac{2m_K^2}{(m_b + m_u)(m_u + m_s)}, \quad r_\chi^{D_s} = \frac{2m_{D_s}^2}{(m_b - m_c)(m_s + m_c)}.$$

## 2. AMPLITUDES OF INTERMEDIATE STATES

In this section, before analyzing FSI in the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay, we introduce the factorization approach of intermediate state in detail. In the FSI effects  $D^0 \pi^+ (D^{0*} \rho^+)$ ,  $D^+ \pi^0 (D^{+*} \rho^0)$  and  $D^+ \eta_c (D^{+*} J/\psi)$  are chosen for the intermediate states. The effective weak Hamiltonian for  $B$  decays consists of a sum of local operators  $Q_i$  multiplied by QCD coefficients  $c_i$  and products of elements of the quark mixing matrix [10]. The factorization approach of heavy-meson decays can be expressed in terms of different topologies of various decays mechanism such as tree, penguin and annihilation.

**2.1.  $B^+ \rightarrow D^0 \pi^+ (D^{0*} \rho^+)$  Decays.** When two intermediate mesons exchange  $u$  quark and two final-state mesons exchange  $s$  quark, the  $D^{0(*)}$  and  $\pi^+ (\rho^+)$  mesons are produced for intermediate state via exchange mesons of  $K^{+(*)}$ . Feynman diagrams for the  $B^+ \rightarrow D^0 \pi^+ (D^{0*} \rho^+)$  decays are shown in Fig. 2, and the amplitudes read

$$A(B^+ \rightarrow D^0 \pi^+) = i \frac{G_F}{\sqrt{2}} f_\pi \{ F^{B \rightarrow D} (m_\pi^2) (m_B^2 - m_D^2) a_1 V_{cb} V_{ud}^* + f_B f_D b_2 V_{ub} V_{cd}^* \},$$

$$A(B^+ \rightarrow D^{0*} \rho^+) = i \frac{G_F}{\sqrt{2}} \left\{ f_\rho m_\rho \left\{ (\epsilon_1 \cdot \epsilon_2) (m_B + m_{D^*}) A_1^{B \rightarrow D^*} (m_\rho^2) - \right. \right.$$

$$\left. \left. - (\epsilon_1 \cdot p_B) (\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow D^*} (m_\rho^2)}{m_B + m_{D^*}} \right\} a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* \right\}. \quad (3)$$

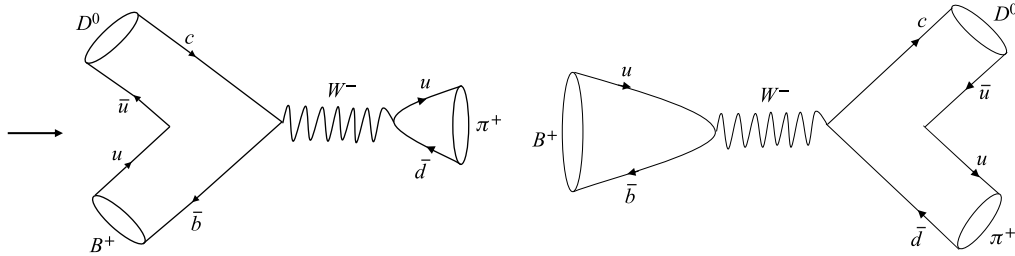


Fig. 2. Feynman diagrams for the  $B^+ \rightarrow D^0 \pi^+$  decay

**2.2.  $B^+ \rightarrow D^+ \pi^0 (D^{+*} \rho^0)$  Decays.** If  $d$  quark is exchanged between two intermediate mesons and  $s$  quark is exchanged between two final-state mesons, the  $D^{+(*)}$  and  $\pi^+ (\rho^+)$  mesons are produced for intermediate state via  $K^{0(*)}$  as the exchange mesons. Feynman diagrams for  $B^+ \rightarrow D^+ \pi^0 (D^{+*} \rho^0)$  decays are shown in Fig.3, and the amplitudes read

$$A(B^+ \rightarrow D^+ \pi^0) = i \frac{G_F}{\sqrt{2}} f_D \{ F^{B \rightarrow \pi} (m_D^2) (m_B^2 - m_\pi^2) a_1 V_{ub} V_{cd}^* + f_B f_\pi b_2 V_{ub} V_{cd}^* \},$$

$$A(B^+ \rightarrow D^{+*} \rho^0) = i \frac{G_F}{\sqrt{2}} f_{D^*} m_{D^*} \left\{ (\epsilon_1 \cdot \epsilon_2) (m_B + m_\rho) A_1^{B \rightarrow \rho} (m_{D^*}^2) - \right.$$

$$\left. - (\epsilon_1 \cdot p_B) (\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow \rho} (m_{D^*}^2)}{m_B + m_\rho} \right\} a_1 V_{ub} V_{cd}^* + i \frac{G_F}{\sqrt{2}} f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^*. \quad (4)$$

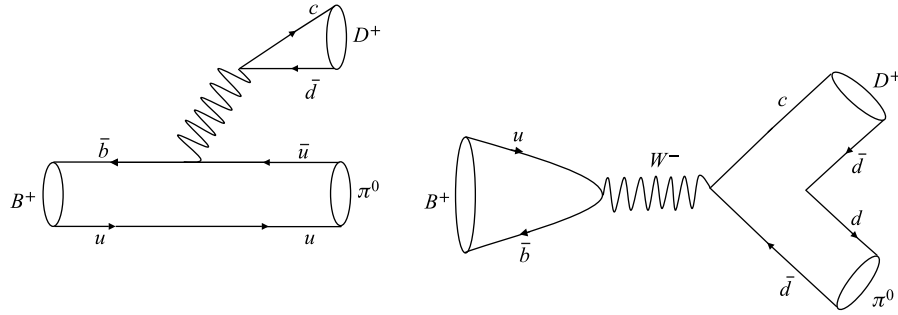


Fig. 3. Feynman diagrams for the  $B^+ \rightarrow D^+ \pi^0$  decay

**2.3.  $B^+ \rightarrow D^+ \eta_c (D^{+*} J/\psi)$  Decays.** If  $c$  quark is exchanged between two intermediate mesons and  $s$  quark is exchanged between two final-state mesons, the  $D^{+(*)}$  and  $\eta_c (J/\psi)$  mesons are produced for intermediate state via  $D_s^{+(*)}$  as the exchange mesons. Feynman diagram for  $B^+ \rightarrow D^+ \eta_c (D^{+*} J/\psi)$  decays is shown in Fig.4, and the amplitudes read

$$A(B^+ \rightarrow D^+ \eta_c) = i \frac{G_F}{\sqrt{2}} f_B f_D f_\eta b_2 V_{ub} V_{cd}^*,$$

$$A(B^+ \rightarrow D^{+*} J/\psi) = i \frac{G_F}{\sqrt{2}} f_B f_{D^*} f_{J/\psi} b_2 V_{ub} V_{cd}^*. \quad (5)$$

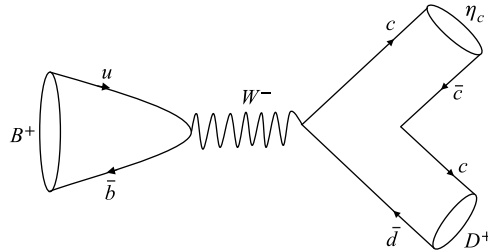


Fig. 4. Feynman annihilation diagram for the  $B^+ \rightarrow D^+ \eta_c$  decay

### 3. FINAL-STATE INTERACTION OF $B^+ \rightarrow D_s^+ \bar{K}^0$ DECAY

When the FSI for decay is calculated, two-body intermediate states such as  $D^0 \pi^+$  ( $D^{0*} \rho^+$ ),  $D^+ \pi^0$  ( $D^{+*} \rho^0$ ) and  $D^+ \eta_c$  ( $D^{+*} J/\psi$ ) are produced. The absorptive part of the HLL diagrams can be calculated with the following formula:

$$\begin{aligned} \text{Abs } A(B(p_B) \rightarrow M(p_1)M(p_2) \rightarrow M(p_3)M(p_4)) &= \frac{1}{2} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} \times \\ &\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) A(B \rightarrow M_1 M_2) G(M_1 M_2 \rightarrow M_3 M_4), \quad (6) \end{aligned}$$

for which both intermediate mesons ( $M_1, M_2$ ) are pseudoscalar. And

$$\begin{aligned} \text{Abs } A(B(p_B) \rightarrow M(p_1)M(p_2) \rightarrow M(p_3)M(p_4)) &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} \times \\ &\times (2\pi)^4 \delta^4(p_B - p_1 - p_2) f_V m_V V_{\text{CKM}} \left[ (\epsilon_1^* \cdot \epsilon_2^*) (m_B + m_1) A_1^{BM_1} (m_2^2) - \right. \\ &\quad \left. - (\epsilon_1^* \cdot p_B) (\epsilon_2^* \cdot p_B) \frac{2A_2^{BM_1} (m_2^2)}{m_B + m_1} \right] G(M_1 M_2 \rightarrow M_3 M_4), \quad (7) \end{aligned}$$

in which both mesons are vector. Also  $G(M_1 M_2 \rightarrow M_3 M_4)$  involves hadronic vertices factor, which are defined as [1, 11, 12]

$$\begin{aligned} \langle D_s(p_3) K^*(\epsilon_2, p_2) | i\mathcal{L} | D(p_1) \rangle &= -ig_{DD_s K^*} \epsilon_2 \cdot (p_1 + p_3), \\ \langle \pi(p_3) K^*(\epsilon_2, p_2) | i\mathcal{L} | K(p_1) \rangle &= -ig_{KK^* \pi} \epsilon_2 \cdot (p_1 + p_3), \\ \langle D_s(p_3) K(p_2) | i\mathcal{L} | D^*(\epsilon_1, p_1) \rangle &= -ig_{D_s D^* K} \epsilon_1 \cdot p_2, \\ \langle K(p_3) \rho(\epsilon_2, p_2) | i\mathcal{L} | K(p_1) \rangle &= -ig_{KK\rho} \epsilon_2 \cdot (p_1 + p_3), \\ \langle D_s(p_3) K^*(\epsilon_2, p_2) | i\mathcal{L} | D^*(\epsilon_1, p_1) \rangle &= -i\sqrt{2} g_{D_s D^* K^*} \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_2^{*\nu} p_1^\alpha p_3^\beta, \\ \langle K^*(\epsilon_3, p_3) \rho(\epsilon_2, p_2) | i\mathcal{L} | K(p_1) \rangle &= -ig_{KK^* \rho} \epsilon_{\mu\nu\alpha\beta} \epsilon_2^\mu \epsilon_3^{*\nu} p_1^\alpha p_2^\beta, \\ \langle D_s^*(\epsilon_3, p_3) \eta_c(p_2) | i\mathcal{L} | D_s(p_1) \rangle &= -ig_{D_s D_s^* \eta_c} \epsilon_3 \cdot p_2, \\ \langle D_s(p_3) \psi(\epsilon_2, p_2) | i\mathcal{L} | D_s(p_1) \rangle &= -ig_{D_s D_s \psi} \epsilon_2 \cdot (p_1 + p_3), \\ \langle D_s^*(\epsilon_3, p_3) \psi(\epsilon_2, p_2) | i\mathcal{L} | D_s(p_1) \rangle &= -i\sqrt{2} g_{D_s D_s^* \psi} \epsilon_{\mu\nu\alpha\beta} \epsilon_2^\mu \epsilon_3^{*\nu} p_1^\alpha p_2^\beta, \\ \langle D_s^*(\epsilon_3, p_3) K(p_2) | i\mathcal{L} | D^*(\epsilon_1, p_1) \rangle &= -ig_{D_s^* D^* K} \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_3^{*\nu} p_1^\alpha p_2^\beta. \end{aligned} \quad (8)$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [1, 13]

$$\text{Dis } M(m_B^2) = \frac{1}{\pi} \int_s^\infty \frac{\text{Abs } M(s')}{s' - m_B^2} ds', \quad (9)$$

where  $s'$  is the square of the momentum carried by the exchanged particle and  $s$  is the threshold of intermediate states, in this case  $s \sim m_B^2$ . Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integrations.

**3.1. Final-State Interaction in the  $B^+ \rightarrow D^0 \pi^+ (D^{0*} \rho^+) \rightarrow D_s^+ \bar{K}^0$  Decay.** The quark model and the hadronic level diagrams for the  $B^+ \rightarrow D^0 \pi^+ (D^{0*} \rho^+) \rightarrow D_s^+ \bar{K}^0$  decay are shown the Figs. 5 and 6.

In this framework we choose the  $t$ -channel one-particle-exchange processes. The absorptive part of the diagram (a) shown in Fig. 6, the amplitude of the mode  $B^+ \rightarrow D^0(p_1) \pi^+(p_2) \rightarrow D_s^+(p_3) \bar{K}^0(p_4)$  with the exchange of the  $K^{+*}$ , is given by

$$\begin{aligned} \text{Abs}(6a) &= \frac{1}{2} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-ig_{DD_s K^*}) \epsilon_q \cdot (p_1 + p_3) \times \\ &\quad \times (-ig_{KK^* \pi}) \epsilon_q \cdot (p_2 + p_4) A(B^+ \rightarrow D^0 \pi^+) \frac{F^2(q^2, m_i^2)}{T} = \\ &= - \int_{-1}^1 \frac{|\mathbf{p}_1| d(\cos \theta)}{16\pi m_B} g_{DD_s K^*} g_{KK^* \pi} A(B^+ \rightarrow D^0 \pi^+) \frac{F^2(q^2, m_i^2)}{T} H, \quad (10) \end{aligned}$$

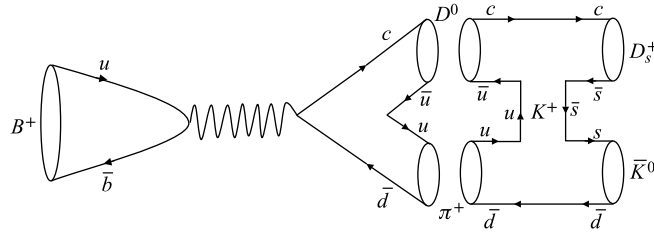


Fig. 5. Quark level diagram for  $B^+ \rightarrow D^0 \pi^+ \rightarrow D_s^+ \bar{K}^0$  decay

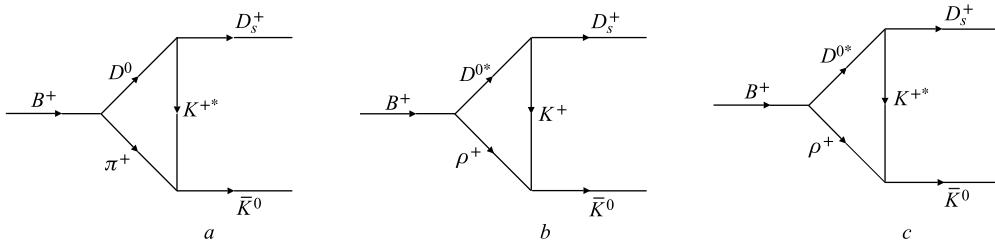


Fig. 6. HLL diagrams for long-distance  $t$ -channel contribution to  $B^+ \rightarrow D^0 \pi^+ (D^{0*} \rho^+) \rightarrow D_s^+ \bar{K}^0$

where  $\theta$  is the angle between  $\mathbf{p}_1$  and  $\mathbf{p}_3$ ,  $q$  and  $m_i$  are the momentum and mass of the exchange  $K^{+*}$  meson, respectively, and

$$\begin{aligned} H &= \epsilon_q \cdot (p_1 + p_3) \epsilon_q \cdot (p_2 + p_4) = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{K^*}^2} \right) (p_1 + p_3)^\mu (p_2 + p_4)^\nu = \\ &= -p_1 \cdot p_2 - p_1 \cdot p_4 - p_2 \cdot p_3 - p_3 \cdot p_4 + \frac{(m_1^2 - m_3^2)(m_4^2 - m_2^2)}{m_{K^*}^2}, \\ T &= q^2 - m_i^2 = m_D^2 + m_{D_s}^2 - m_{K^*}^2 - 2E_D E_{D_s} + 2|\mathbf{p}_D||\mathbf{p}_{D_s}| \cos \theta, \end{aligned} \quad (11)$$

and  $F(q^2, m_i^2)$  is the form factor defined to take care of the off-shell character of the exchange particles, defined as [14]

$$F(q^2, m_i^2) = \left( \frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2} \right)^n. \quad (12)$$

The form factor (i.e.,  $n = 1$ ) is normalized to unity at  $q^2 = m_i^2$ . The  $m_i$  and  $q$  are the physical parameters of the exchange particle and  $\Lambda$  is a phenomenological parameter. It is obvious that for  $q^2 \rightarrow 0$ ,  $F(q^2, m_i^2)$  becomes a number. If  $\Lambda \gg m_i$ , then  $F(q^2, m_i^2)$  turns to be unity, whereas as  $q^2 \rightarrow \infty$  the form factor approaches zero and the distance becomes small and the hadron interaction is no longer valid. Since  $\Lambda$  should not be far from the  $m_i$  and  $q$ , we choose

$$\Lambda = m_i + \eta \Lambda_{\text{QCD}}. \quad (13)$$

Likewise, for diagrams 6, *b* and 6, *c*, the amplitudes of the  $B^+ \rightarrow D^{0*}(\epsilon_1, p_1) \rho^+(\epsilon_2, p_2) \rightarrow D_s^+(p_3) \bar{K}^0(p_4)$  (where  $K^+$  and  $K^{+*}$  are exchanged at  $t$ -channel) are given by

$$\begin{aligned} \text{Abs}(6b) &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-i g_{D^* D_s K}) (\epsilon_1 \cdot q) \times \\ &\quad \times (-i g_{K K \rho}) \epsilon_2 \cdot (p_4 + q) \left\{ f_\rho m_\rho \left[ (\epsilon_1 \cdot \epsilon_2) (m_B + m_{D^*}) A_1^{B \rightarrow D^*}(m_\rho^2) - \right. \right. \\ &\quad \left. \left. - (\epsilon_1 \cdot p_B) (\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow D^*}(m_\rho^2)}{m_B + m_{D^*}} \right] a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* \right\} \frac{F^2(q^2, m_K^2)}{T_1} = \\ &= -i \frac{G_F}{8\sqrt{2}\pi m_B} g_{D^* D_s K} g_{K K \rho} \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \left\{ f_\rho m_\rho \left[ (m_B + m_{D^*}) A_1^{B \rightarrow D^*}(m_\rho^2) H_1 - \right. \right. \\ &\quad \left. \left. - \frac{2A_2^{B \rightarrow D^*}(m_\rho^2)}{m_B + m_{D^*}} H_2 \right] a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* H_3 \right\} \frac{F^2(q^2, m_K^2)}{T_1}, \end{aligned} \quad (14)$$

where

$$\begin{aligned}
H_1 &= (\epsilon_1 \cdot \epsilon_2)(\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) = \\
&= p_3 \cdot p_4 - \frac{(p_2 \cdot p_4)(p_2 \cdot p_3)}{m_2^2} - \frac{(p_1 \cdot p_3)(p_1 \cdot p_4)}{m_1^2} + \frac{(p_1 \cdot p_3)(p_1 \cdot p_2)(p_2 \cdot p_4)}{m_1^2 m_2^2}, \\
H_2 &= (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B)(\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) = \\
&= \left[ -p_3 \cdot p_B + \frac{(p_1 \cdot p_B)(p_1 \cdot p_3)}{m_1^2} \right] \left[ -p_4 \cdot p_B + \frac{(p_2 \cdot p_B)(p_2 \cdot p_4)}{m_2^2} \right], \quad (15) \\
H_3 &= (\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) = \left( \frac{E_3 |\mathbf{p}_1| - E_1 |\mathbf{p}_3| \cos \theta}{m_B |\mathbf{p}_1|} \right) \left( \frac{E_4 |\mathbf{p}_2| - E_2 |\mathbf{p}_4| \cos \theta}{m_B |\mathbf{p}_4|} \right), \\
T_1 &= m_{D^*}^2 + m_\rho^2 - m_K^2 - 2E_{D^*} E_\rho + 2|\mathbf{p}_{D^*}| |\mathbf{p}_\rho| \cos \theta.
\end{aligned}$$

$$\begin{aligned}
\text{Abs (6c)} &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3 \mathbf{p}_1}{2E_1 (2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2 (2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \times \\
&\quad \times (-i\sqrt{2}g_{D^* D_s K^*}) \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{K^*}^\nu p_1^\alpha p_3^\beta (-ig_{KK^*\rho}) \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{K^*}^\sigma p_2^\lambda p_4^\eta \times \\
&\quad \times \left\{ f_\rho m_\rho \left[ (\epsilon_1 \cdot \epsilon_2)(m_B + m_{D^*}) A_1^{B \rightarrow D^*}(m_\rho^2) - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow D^*}(m_\rho^2)}{m_B + m_{D^*}} \right] \times \right. \\
&\quad \left. \times a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_2} = \\
&= -i \frac{G_F}{16\pi m_B} g_{D^* D_s K^*} g_{KK^*\rho} \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \left\{ f_\rho m_\rho \left[ (m_B + m_{D^*}) A_1^{B \rightarrow D^*}(m_\rho^2) H_4 - \right. \right. \\
&\quad \left. \left. - \frac{2A_2^{B \rightarrow D^*}(m_\rho^2)}{m_B + m_{D^*}} H_5 \right] a_1 V_{cb} V_{ud}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* H_6 \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_2}, \quad (16)
\end{aligned}$$

$$H_4 = \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{K^*}^\nu p_1^\alpha p_3^\beta \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{K^*}^\sigma p_2^\lambda p_4^\eta (\epsilon_1 \cdot \epsilon_2) = 2(p_1 \cdot p_4)(p_2 \cdot p_3) - 2(p_1 \cdot p_2)(p_3 \cdot p_4),$$

$$\begin{aligned}
H_5 &= \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{K^*}^\nu p_1^\alpha p_3^\beta \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{K^*}^\sigma p_2^\lambda p_4^\eta (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) = \\
&= m_B^2 [(p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_2)(p_3 \cdot p_4)] + \\
&+ (p_1 \cdot p_B)(p_2 \cdot p_B)(p_3 \cdot p_4) - (p_2 \cdot p_B)(p_3 \cdot p_B)(p_1 \cdot p_4) - \\
&- (p_1 \cdot p_B)(p_4 \cdot p_B)(p_2 \cdot p_3) + (p_3 \cdot p_B)(p_4 \cdot p_B)(p_1 \cdot p_2), \quad (17)
\end{aligned}$$

$$H_6 = \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{K^*}^\nu p_1^\alpha p_3^\beta \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{K^*}^\sigma p_2^\lambda p_4^\eta,$$

$$T_2 = m_{D^*}^2 + m_\rho^2 - m_{K^*}^2 - 2E_{D^*} E_\rho + 2|\mathbf{p}_{D^*}| |\mathbf{p}_\rho| \cos \theta.$$



As the bridge between the dispersive part of FSI amplitude and the absorptive part, the dispersion relation is

$$\text{Dis } 6(m_B^2) = \frac{1}{\pi} \int_s^\infty \frac{\text{Abs}_{6a}(s') + \text{Abs}_{6b}(s') + \text{Abs}_{6c}(s')}{s' - m_B^2} ds'. \quad (18)$$

**3.2. Final-State Interaction in the  $B^+ \rightarrow D^+\pi^0(D^{+*}\rho^0) \rightarrow D_s^+\bar{K}^0$  Decay.** The quark model diagram for the  $B^+ \rightarrow D^+\pi^0(D^{+*}\rho^0) \rightarrow D_s^+\bar{K}^0$  decay is shown in Fig. 7 and the hadronic level diagrams are shown in Fig. 8. The amplitude of the mode  $B^+ \rightarrow D^+(p_1)\pi^0(p_2) \rightarrow D_s^+(p_3)\bar{K}^0(p_4)$  (where  $K^{0*}$  is exchanged at  $t$ -channel) is given by

$$\begin{aligned} \text{Abs}(8a) &= \frac{1}{2} \int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-ig_{DD_sK^*}) \epsilon_q \cdot (p_1 + p_3) \times \\ &\times (-ig_{KK^*\pi}) \epsilon_q \cdot (p_2 + p_4) A(B^+ \rightarrow D^+\pi^0) \frac{F^2(q^2, m_{K^*}^2)}{T} = \\ &= - \int_{-1}^1 \frac{|\mathbf{p}_1| d(\cos \theta)}{16\pi m_B} g_{DD_sK^*} g_{KK^*\pi} A(B^+ \rightarrow D^+\pi^0) \frac{F^2(q^2, m_{K^*}^2)}{T} H, \quad (19) \end{aligned}$$

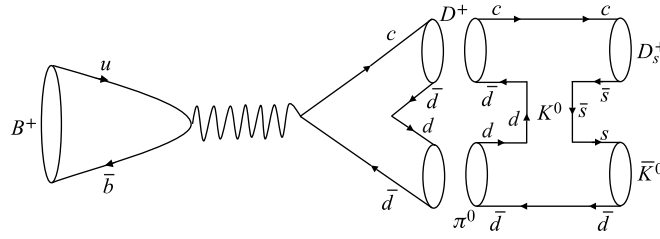


Fig. 7. Quark level diagram for the  $B^+ \rightarrow D^+\pi^0 \rightarrow D_s^+\bar{K}^0$  decay

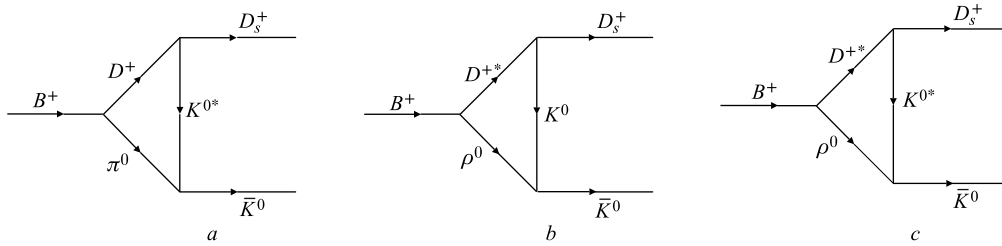


Fig. 8. HLL diagrams for long-distance  $t$ -channel contribution to  $B^+ \rightarrow D^+\pi^0(D^{+*}\rho^0) \rightarrow D_s^+\bar{K}^0$

The amplitudes of the mode  $B^+ \rightarrow D^{*+}(\epsilon_1, p_1)\rho^0(\epsilon_2, p_2) \rightarrow D_s^+(p_3)\bar{K}^0(p_4)$  (where  $K^0$  and  $K^{0*}$  are exchanged at  $t$ -channel) are given by

$$\begin{aligned}
 \text{Abs}(8b) &= i \frac{G_F}{4} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-ig_{D^*D_s K})(\epsilon_1 \cdot q) \times \\
 &\quad \times (-ig_{KK\rho}) \epsilon_2 \cdot (p_4 + q) \left\{ f_{D^*} m_{D^*} \left[ (\epsilon_1 \cdot \epsilon_2)(m_B + m_\rho) A_1^{B \rightarrow \rho}(m_{D^*}^2) - \right. \right. \\
 &\quad \left. \left. - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow \rho}(m_{D^*}^2)}{m_B + m_\rho} \right] a_1 V_{ub} V_{cd}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_1} = \\
 &= -i \frac{G_F}{32\pi m_B} g_{D^*D_s K} g_{KK\rho} \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \left\{ f_{D^*} m_{D^*} \left[ (m_B + m_\rho) A_1^{B \rightarrow \rho}(m_{D^*}^2) H_1 - \right. \right. \\
 &\quad \left. \left. - \frac{2A_2^{B \rightarrow \rho}(m_{D^*}^2)}{m_B + m_\rho} H_2 \right] a_1 V_{ub} V_{cd}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* H_3 \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_1}, \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 \text{Abs}(8c) &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \times \\
 &\quad \times (-i\sqrt{2}g_{D^*D_s K^*}) \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{K^*}^\nu p_1^\alpha p_3^\beta (-ig_{KK^*\rho}) \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{K^*}^\sigma p_2^\lambda p_4^\eta \times \\
 &\quad \times \left\{ f_{D^*} m_{D^*} \left[ (\epsilon_1 \cdot \epsilon_2)(m_B + m_\rho) A_1^{B \rightarrow \rho}(m_{D^*}^2) - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) \frac{2A_2^{B \rightarrow \rho}(m_{D^*}^2)}{m_B + m_\rho} \right] \times \right. \\
 &\quad \left. \times a_1 V_{ub} V_{cd}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_2} = \\
 &= -i \frac{G_F}{16\sqrt{2}\pi m_B} g_{D^*D_s K^*} g_{KK^*\rho} \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \left\{ f_{D^*} m_{D^*} \left[ (m_B + m_\rho) A_1^{B \rightarrow \rho}(m_{D^*}^2) H_4 - \right. \right. \\
 &\quad \left. \left. - \frac{2A_2^{B \rightarrow \rho}(m_{D^*}^2)}{m_B + m_\rho} H_5 \right] a_1 V_{ub} V_{cd}^* + f_B f_{D^*} f_\rho b_2 V_{ub} V_{cd}^* H_6 \right\} \frac{F^2(q^2, m_{K^*}^2)}{T_2}. \quad (21)
 \end{aligned}$$

The dispersion relation is

$$\text{Dis} 8(m_B^2) = \frac{1}{\pi} \int_s^\infty \frac{\text{Abs}_{8a}(s') + \text{Abs}_{8b}(s') + \text{Abs}_{8c}(s')}{s' - m_B^2} ds'. \quad (22)$$

**3.3. Final-State Interaction in the  $B^+ \rightarrow \eta_c D^+(J/\psi D^{+*}) \rightarrow D_s^+ \bar{K}^0$  Decay.** The quark model and the hadronic level diagrams for the  $B^+ \rightarrow \eta_c D^+(J/\psi D^{+*}) \rightarrow D_s^+ \bar{K}^0$  decay are shown in Figs. 9 and 10. The amplitude of the mode  $B^+ \rightarrow \eta_c(p_1) D^+(p_2) \rightarrow D_s^+(p_3) \bar{K}^0(p_4)$  with the exchange of the  $D_s^{+*}$  is given by

$$\begin{aligned} \text{Abs}(10a) &= \frac{1}{2} \int \frac{d^3 \mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3 \mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-ig_{D_s^* D_s \eta_c}) \epsilon_q \cdot p_1 \times \\ &\quad \times (-ig_{DD_s^* K}) \epsilon_q \cdot p_4 A(B^+ \rightarrow \eta_c D^+) \frac{F^2(q^2, m_{D_s^*}^2)}{T_3} = \\ &= - \int_{-1}^1 \frac{|\mathbf{p}_1| d(\cos \theta)}{16\pi m_B} g_{D_s^* D_s \eta_c} g_{DD_s^* K} A(B^+ \rightarrow \eta_c D^+) \frac{F^2(q^2, m_{D_s^*}^2)}{T_3} H_7, \end{aligned} \quad (23)$$

where

$$H_7 = \epsilon_q \cdot p_1 \epsilon_q \cdot p_4 = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D_s^*}^2} \right) p_1^\mu p_4^\nu = -p_1 \cdot p_4 + \frac{(m_1^2 - p_1 \cdot p_3)(m_4^2 - p_2 \cdot p_4)}{m_{D_s^*}^2}, \quad (24)$$

$$T_3 = q^2 - m_i^2 = m_{D_s}^2 + m_{\eta_c}^2 - m_{D_s^*}^2 - 2E_{D_s} E_{\eta_c} + 2|\mathbf{p}_{D_s}| |\mathbf{p}_{\eta_c}| \cos \theta.$$

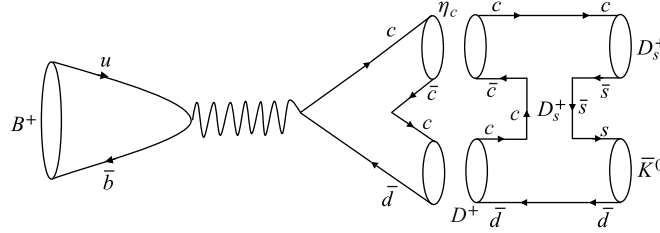


Fig. 9. Quark level diagram for the  $B^+ \rightarrow \eta_c D^+ \rightarrow D_s^+ \bar{K}^0$  decay

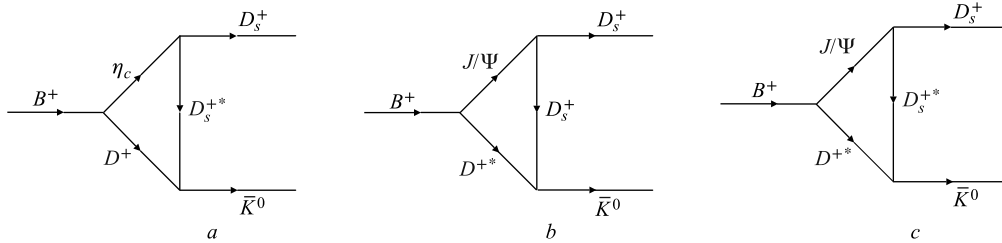


Fig. 10. HLL diagrams for long-distance  $t$ -channel contribution to  $B^+ \rightarrow \eta_c D^+(J/\psi D^{+*}) \rightarrow D_s^+ \bar{K}^0$

The amplitudes of the mode  $B^+ \rightarrow J/\psi(\epsilon_1, p_1)D^{+*}(\epsilon_2, p_2) \rightarrow D_s^+(p_3)\bar{K}^0(p_4)$  with the exchange of the  $D_s^+$  and  $D_s^{+*}$  mesons are given by

$$\begin{aligned} \text{Abs}(10b) &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) (-ig_{D_s D_s \psi}) \times \\ &\quad \times (-\epsilon_1) \cdot (p_1 - q) (-ig_{D^* D_s K}) \epsilon_2 \cdot p_4 f_B f_{D^*} f_{J/\psi} b_2 V_{ub} V_{cd}^* \frac{F^2(q^2, m_{D_s}^2)}{T_4} = \\ &= i \frac{G_F}{16\sqrt{2}\pi m_B} g_{D_s D_s \psi} g_{D^* D_s K} f_B f_{D^*} f_{J/\psi} b_2 V_{ub} V_{cd}^* \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \frac{F^2(q^2, m_{D_s}^2)}{T_4} H_8, \end{aligned} \quad (25)$$

where

$$\begin{aligned} H_8 &= (\epsilon_1 \cdot p_3)(\epsilon_2 \cdot p_4) = \left( \frac{E_3 |\mathbf{p}_1| - E_1 |\mathbf{p}_3| \cos \theta}{m_B |\mathbf{p}_1|} \right) \left( \frac{E_4 |\mathbf{p}_2| - E_2 |\mathbf{p}_4| \cos \theta}{m_B |\mathbf{p}_4|} \right), \\ T_4 &= q^2 - m_i^2 = m_{D_s}^2 + m_{J/\psi}^2 - m_{D_s}^2 - 2E_{D_s} E_{J/\psi} + 2|\mathbf{p}_{D_s}| |\mathbf{p}_{J/\psi}| \cos \theta. \end{aligned} \quad (26)$$

$$\begin{aligned} \text{Abs}(10c) &= i \frac{G_F}{2\sqrt{2}} \int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \times \\ &\quad \times (-i\sqrt{2}g_{D_s D_s^* \psi}) \epsilon_{\mu\nu\alpha\beta} (-\epsilon_1^\mu) \epsilon_{D_s^*}^\nu (-p_1^\alpha) (-p_3^\beta) (-ig_{D^* D_s K}) \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho (-\epsilon_{D_s^*}^\sigma) p_2^\lambda p_4^\eta \times \\ &\quad \times f_B f_{D^*} f_{J/\psi} b_2 V_{ub} V_{cd}^* \frac{F^2(q^2, m_{D_s^*}^2)}{T_5} = \\ &= -i \frac{G_F}{16\sqrt{2}\pi m_B} g_{D_s D_s^* \psi} g_{D^* D_s K} f_B f_{D^*} f_{J/\psi} b_2 V_{ub} V_{cd}^* \int_{-1}^1 |\mathbf{p}_1| d(\cos \theta) \frac{F^2(q^2, m_{D_s^*}^2)}{T_5} H_9, \end{aligned} \quad (27)$$

where

$$\begin{aligned} H_9 &= \epsilon_{\mu\nu\alpha\beta} \epsilon_1^\mu \epsilon_{D_s^*}^\nu p_1^\alpha p_3^\beta \epsilon_{\rho\sigma\lambda\eta} \epsilon_2^\rho \epsilon_{D_s^*}^\sigma p_2^\lambda p_4^\eta, \\ T_5 &= q^2 - m_i^2 = m_{D_s}^2 + m_{J/\psi}^2 - m_{D_s^*}^2 - 2E_{D_s} E_{J/\psi} + 2|\mathbf{p}_{D_s}| |\mathbf{p}_{J/\psi}| \cos \theta. \end{aligned} \quad (28)$$

The dispersion relation is

$$\text{Dis } 10(m_B^2) = \frac{1}{\pi} \int_s^\infty \frac{\text{Abs}_{10a}(s') + \text{Abs}_{10b}(s') + \text{Abs}_{10c}(s')}{s' - m_B^2} ds'. \quad (29)$$

The decay amplitude of  $B^+ \rightarrow D_s^+ \bar{K}^0$  via the HLL diagrams is

$$\begin{aligned} A(B^+ \rightarrow D_s^+ \bar{K}^0) &= \text{Abs}(6a) + \text{Abs}(6b) + \text{Abs}(6c) + \text{Dis } 6 + \text{Abs}(8a) + \\ &\quad + \text{Abs}(8b) + \text{Abs}(8c) + \text{Dis } 8 + \text{Abs}(10a) + \text{Abs}(10b) + \text{Abs}(10c) + \text{Dis } 10. \end{aligned} \quad (30)$$

#### 4. NUMERICAL RESULTS

The Wilson coefficients  $c_i$  have been calculated in different schemes. In this paper we will use consistently the naive dimensional regularization (NDR) scheme. The values of  $c_i$  at  $\mu = m_b$  with the next-to-leading order (NLO) QCD corrections are given by [7, 15]

$$\begin{aligned}
 c_1 &= 1.117, & c_2 &= -0.257, \\
 c_3 &= 0.017, & c_4 &= -0.044, \\
 c_5 &= 0.011, & c_6 &= -0.056, \\
 c_7 &= -1 \cdot 10^{-5}, & c_8 &= 5 \cdot 10^{-4}, \\
 c_9 &= -0.010, & c_{10} &= 0.002.
 \end{aligned} \tag{31}$$

The relevant input parameters used are:  $m_B = 5.279$ ,  $m_K = 0.49$ ,  $m_{K^*} = 0.89$ ,  $m_\rho = 0.78$ ,  $m_\pi = 0.139$ ,  $m_D = 1.87$ ,  $m_{D^*} = 2.01$ ,  $m_{D_s} = 1.97$ ,  $m_{D_s^*} = 2.11$ ,  $m_{J/\psi} = 3.1$ ,  $m_{\eta_c} = 3.0$ ,  $m_b = 4.20$ ,  $m_c = 1.27$ ,  $m_u = 0.00240$ ,  $m_s = 0.104$ ,  $f_B = 0.176$ ,  $f_K = 0.16$ ,  $f_\pi = 0.13$ ,  $f_D = 0.223$ ,  $f_{\eta_c} = 0.35$ ,  $f_{J/\psi} = 0.416$ ,  $f_\rho = 0.216$ ,  $f_{D^*} = 0.23$ ,  $f_{D_s} = 0.294$  [16] (values of masses and decay constants are in GeV);  $r_\chi^K = 1.09$ ,  $r_\chi^{D_s} = 1.93$ ,  $\phi = -55^\circ(PP)$ ,  $\phi = -20^\circ(PV)$ ,  $\phi = -70^\circ(VP)$ ,  $\rho = 0.5$ ,  $\Lambda_{\text{QCD}} = 0.225$  GeV,  $C_F = 4/3$ ,  $\alpha_s = 0.2244$ ,  $N = 3$ ,  $G_F = 1.166 \cdot 10^{-5}$ ,  $V_{ub} = 0.0043$ ,  $V_{ud} = 0.974$ ,  $V_{cb} = 0.042$ ,  $V_{cd} = 0.230$  [16];  $F^{B \rightarrow \pi} = 0.33$ ,  $A_1^{B \rightarrow \rho} = A_2^{B \rightarrow \rho} = 0.28$  [17];  $F^{B \rightarrow D} = 0.6$ ,  $A_1^{B \rightarrow D^*} = 0.8$ ,  $A_2^{B \rightarrow D^*} = 0.99$  [15];  $g_{DD_s K^*} = \sqrt{\frac{m_{D_s}}{m_D}} g_{DD\rho} = 3.83$ ,  $g_{D^* D_s K^*} = \sqrt{\frac{m_{D_s}}{m_D^*}} g_{DD\rho} = 2.79$ ,  $g_{DD_s^* K} = \sqrt{\frac{m_{D_s^*}}{m_D^*}} g_{D^* D\rho} = 3$ ,  $g_{D^* D_s^* K} = \sqrt{\frac{m_{D_s^*}}{m_D^*}} g_{D^* D^* \pi} = 6.6$ ,  $g_{D^* D_s K} = \sqrt{\frac{m_{D_s}}{m_D}} g_{D^* D\pi} = 12.83$  [1] ( $g_{D^* D\pi} = 12.5$  [3, 8]);  $g_{KK^* \pi} = 4.6$ ,  $g_{KK\rho} = 4.28$  [18];  $g_{KK^* \rho} = 6.48$  [19];  $g_{DD^* \eta_c} = m_{\eta_c}/f_{\eta_c} = 8.57$  [20];  $g_{DD\psi} = 7.71$ ,  $g_{DD^* \psi} = 8.64$  [14]. Using the parameters relevant for the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay, we get flavor averaged branching ratio for the QCDF method as

$$\text{BR}(B^+ \rightarrow D_s^+ \bar{K}^0) = 0.23 \cdot 10^{-9}. \tag{32}$$

After calculating the amplitudes of the intermediate states, we obtain:

$$\begin{aligned}
 A(B^+ \rightarrow D^0 \pi^+) &= 0.56 \cdot 10^{-6}, \\
 A(B^+ \rightarrow D^+ \pi^0) &= 0.74 \cdot 10^{-7}, \\
 A(B^+ \rightarrow D^+ \eta_c) &= 0.37 \cdot 10^{-7}.
 \end{aligned} \tag{33}$$

Now, according to FSI, we can obtain the branching ratios of the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay with different values of  $\eta$  which are shown in table.

**The branching ratio of the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay with  $\eta = 1-2.5$  and experimental data (in units of  $10^{-4}$ )**

$\eta$				Experiment [16]
1	1.5	2	2.5	
0.35	1.31	3.36	6.74	< 8

## CONCLUSIONS

In this work, we have calculated the contribution of the  $t$ -channel FSI, i.e., inelastic re-scattering processes to the branching ratio of the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay. For evaluating the FSI effects, we have only considered the absorptive and dispersive parts of the HLL, because both the hadrons which were produced via the weak interaction were on their mass shells.

We have calculated the branching ratio of the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay by using QCDF and FSI. The experimental result of this decay is  $\text{BR}(B^+ \rightarrow D_s^+ \bar{K}^0) < 8 \cdot 10^{-4}$  [16]. The branching ratio of this decay in [5] was  $2.01 \cdot 10^{-8}$ . According to QCDF and FSI, our results are  $\text{BR}(B^+ \rightarrow D_s^+ \bar{K}^0) = 0.23 \cdot 10^{-9}$  and  $6.74 \cdot 10^{-4}$ , respectively.

There exist some phenomenological parameters in our calculations on FSI such as  $\eta$  in (13) and many other sources of uncertainties, for example, the coupling constant  $g_{D^* D_s k}$  etc., the neglected subdominant contributions in the FSI, the estimate of pure QCDF contribution, etc. We have introduced the phenomenological parameter  $\eta$ ; that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. For a given exchanged particle, we have used  $\eta = 1-2.5$ . If  $\eta = 2.5$  is selected, the branching ratio of the  $B^+ \rightarrow D_s^+ \bar{K}^0$  decay approaches the upper bound of experimental result.

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