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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

SELF-SIMILARITY OF HIGH-p_T CUMULATIVE HADRON PRODUCTION IN $p + A$ COLLISIONS AT HIGH ENERGIES AT U70

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New data on inclusive transverse momentum spectra of charged hadrons produced in $p+A$ collisions at U70 are analyzed in the framework of the z-scaling approach. Self-similarity of the hadron production in the high-p_T cumualtive region is verified. Scaling function $\psi(z)$ for C, Al, Cu, W nuclei is constructed. It is expressed via the invariant cross section and the average multiplicity density of charged particles. Results of analysis are compared with the data obtained by J. Cronin, R. Sulyaev and D. Jaffe groups. Self-similarity of the hadron production in $p + A$ collisions over a wide kinematic range is verified. A microscopic scenario of $p + A$ interactions in terms of momentum fractions x_1, x_2 is discussed. Indication of self-similarity of the high-p_T cumulative hadron production in $p + A$ collisions over a wide kinematic range has been found. Based on the universality of the shape of the scaling function, the inclusive cross sections of h^- and h^+ hadrons produced in $p + A$ collisions on C, Al, Cu, W targets in the high-p_T deep-cumulative region are predicted.

Спектры заряженных адронов, измеренные в столкновениях $p + A$ на У-70 и охватывающие кумулятивную область при больших поперечных импульсах, анализируются в рамках теории z-скейлинга. Проверяется самоподобие рождения кумулятивных адронов. Построена скейлинговая функция $\psi(z)$ для ядер C, Al, Cu, W. Она выражается через инвариантное сечение и среднюю плотность множественности заряженных адронов. Результаты анализа сравниваются с данными, полученными группами Дж. Кронина, Р. Суляева и Д. Джаффе, при больших поперечных импульсах в некумулятивной области. Получено указание на самоподобие рождения кумулятивных адронов при больших p_T во взаимодействиях $p + A$ в широкой кинематической области. Обсуждается микроскопический сценарий взаимодействия $p + A$ в рамках долей импульсов x_1, x_2. В предположении универсальной формы функции $\psi(z)$ сделаны предсказания спектров заряженных адронов h^- и h^+ в столкновениях $p + A$ на ядрах C, Al, Cu, W в глубококумулятивной области.

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INTRODUCTION

Search for clear signatures of the phase transtion of the nuclear matter in collisions of hadrons and nuclei is the main goal of the heavy-ion experimental programs at the Relativistic Heavy Ion Collider at BNL [1, 2], Super Proton Synchrotron [3], and Large Hadron Collider

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at CERN [4–8]. The hypothesis of self-similarity of the hadron production is an important concept for data analysis in searching for new physics. The hypothesis is related with the scaling laws such as the Bjorken scaling — in the deep inelatic scattering, Feynman scaling — in the inclusive hadron production, P-KNO scaling — in the multiparticle production, and others. Among others there are quark counting rules describing the power asymptotics of the electromagnetic form factors of hadrons and cross sections of exclusive processes [9–19]. The phase transitions in the nuclear matter produced in the heavy-ion collisions at the high energy density and temperature are new phenomena related to collective interactions of quarks and gluons near the phase boundaries and the critical point.

It is well known that the general concepts in the critical phenomena are related with the notions of «scaling» and «universality» [20]. Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of the scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point. It is assumed that transition of the nuclear matter from the hadron to quark and gluon degrees of freedom near the critical point should reveal large fluctuations, correlations and discontinuty of some experimental quantities characterizing the system.

The high-density nuclear matter can be produced in cumulative processes. Production of any inclusive particle with a momentum far beyond the nucleon–nucleon kinematic region is accompanied by cumulation of a nucleus. The effect does not contradict the momentum conservation law. The cumulative processes have been extensively studied mainly at JINR, ITEP, and IHEP (see [13–15, 21] and references therein). After commisioning of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and the Large Hadron Collider (LHC) at CERN, the cumulative processes can be investigated in a new kinematic region [22]. High sensitivity of elementary constituent interactions to properties of the compressed nuclear matter is expected to be in this region [23, 24].

The concept of z-scaling [25, 26] is based on principles of self-similarity, locality, scale relativity and fractality reflecting the general features of constituent interactions in small scales. A-dependence of z-scaling in inclusive hadron production in $p + A$ collisions at a high transverse momentum and $\theta_{\rm cms} \simeq 90^\circ$ was studied in [27]. The independence of the function $\psi(z)$ from the center-of-mass energy $\sqrt{s}$ and the angle of the produced particle for different nuclei from D up to Pb was shown. The symmetry transformation, $z \to \alpha z$, $\psi \to \alpha^{-1}\psi$, was used to determine A-dependence of transformation parameter α. Self-similarity of low-p_T cumulative pion production in $p+A$ collisions at energy $\sqrt{s} \simeq 27.4$ GeV and angle $\theta_{\rm lab} = 70-160^\circ$ at FNAL [28] has been analyzed in [29]. The results of analysis have been compared with the high-p_T data sets obtained by J. Cronin, R. Sulyaev and D. Jaffe groups. It has been found that the shape of $\psi(z)$ is the same in the overlapping region.

In the present work, the high-p_T data [30] of cumulative [30] charged hadron production in $p + A$ collsions obtained at the U70 accelerator complex are analyzed. The self-similarity of particles produced in this kinematic region is verified. The paper is organized as follows. The z-scaling as the method of data analysis for hadron–nucleus collisions is briefly described in Sec. 1. Inclusive spectra of the cumulative charged hadrons produced in $p + A$ collisions obtained at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35^\circ$ are discussed in Sec. 2. Results of data analysis in the framework of z-scaling approach are given in Sec. 3. Comparison of the obtained results with the data sets measured by J. Cronin, R. Sulyaev and D. Jaffe groups is presented. In Sec. 4 the universality of the shape of scaling function is discussed. Predictions of charged hadron spectra in $p+A$ collisions in the deep-cumulative range at high p_T are given in Sec. 5.

In Sec. 6 a microscopic scenario of the elementary subprocess in terms of momentum fractions x_1 and x_2 is discussed. Discussion of the obtained results is presented in Sec. 7. Conclusions are given in the final section.

1. z-SCALING

In this section, we would like to remind the basic ideas of z-scaling dealing with the investigation of the inclusive process. We follow the approach developed in [27]. The main idea of z-scaling is based on the assumption that the gross feature of the inclusive particle distribution of the process $P_1 + P_2 \to p + X$ at high energies can be described in terms of the corresponding kinematic characteristics of the constituent subprocess written in the symbolic form:

$$(x_1 M_1) + (x_2 M_2) \to m_1 + (x_1 M_1 + x_2 M_2 + m_2), \tag{1}$$

satisfying the following condition [14]:

$$(x_1 P_1 + x_2 P_2 - p)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2. \tag{2}$$

The equation is the expression of locality of the hadron interaction at a constituent level. The x_1 and x_2 are the fractions of incoming momenta P_1 and P_2 of the colliding objects with masses M_1 and M_2. They determine the minimum energy necessary to produce the secondary particle with mass m_1 and four-momentum p. The parameter m_2 is introduced to satisfy the internal conservation laws (for charge, baryon, isospin, strangeness numbers, and so on).

Equation (2) reflects the minimum recoil mass hypothesis in the elementary subprocess. To connect kinematic and structural characteristics of the interaction, quantity Ω is introduced. It is chosen in the following form:

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2}. \tag{3}$$

Here, δ_1 and δ_2 are the fractal dimensions of the colliding objects. The fractions x_1 and x_2 are determined to maximize the value of $\Omega(x_1, x_2)$, simultaneously fulfilling condition (2):

$$\left.\frac{d\Omega(x_1, x_2)}{dx_1}\right|_{x_2 = x_2(x_1)} = 0. \tag{4}$$

The fractions x_1 and x_2 are equal to unity along the phase space limit and cover the full phase space accessible at any energy.

Self-similarity is a scale-invariant property related with dropping of certain dimensional quantities out of the physical picture of the interactions. It means that dimensionless quantities to describe the physical processes are used. The scaling function $\psi(z)$ depends in a self-similar manner on a single dimensionless variable z. The function is expressed via the measurable quantities and written in the following form:

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{\text{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{5}$$

Here, $Ed^3\sigma/dp^3$ is the invariant cross section; $dN/d\eta$ is the multiplicity density as a function of the center-of-mass collision energy squared s and pseudorapidity η; σ_{in} is the inelastic cross section; J is the corresponding Jacobian. The factor J is the known function of the kinematic variables, the momenta and masses of the colliding and produced particles.

The function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z)\, dz = 1. \tag{6}$$

The relation allows us to interpret the function as a probability density to produce a particle with the corresponding value of the variable z. We note that the existence of the function $\psi(z)$ depending on the single dimensionless variable z and revealing scaling properties (independence of $\psi(z)$ from the collision energy $\sqrt{s}$, an angle of the produced particle) is not evident in advance. The validity of the scaling is confirmed a posteriori.

Self-similarity of an object revealing itself over a wide scale range is the general property of fractality. It means that the measure corresponding to the object diverges in terms of the resolution. In our case this measure is the variable z which has the following form:

$$z = z_0 \Omega^{-1}. \tag{7}$$

Here, $z_0 = \sqrt{\hat{s}_\perp}/[m(dN_{\rm ch}/d\eta)]$ is the finite part of z. It is expressed via the ratio of the transverse energy $\sqrt{\hat{s}_\perp}$ released in the binary collision of constituents and the average multiplicity density of charged particles $dN_{\rm ch}/d\eta$ at $\eta = 0$ and the nucleon mass m. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents a relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The δ_1 and δ_2 are fractal dimensions of the colliding objects. The momentum fractions x_1 and x_2 are determined to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of the measure z with respect to all possible subprocesses (1) under condition (2).

Note that ψ and z are the scale-dependent quantities. They both depend on the dimensional variables $\sqrt{s}$ and p_T. We assume that the hadron and nucleus interactions at high energies and transverse momenta are interactions of fractals. In this region the internal structure of hadrons, interactions of their constituents and mechanism of hadronization reveal self-similarity.

2. HIGH-p_T CUMULATIVE HADRON PRODUCTION IN $p + A$ COLLISIONS AT U70

Cumulative particles are the particles produced in the kinematic region forbidden for free nucleon–nucleon interactions [13–15] (see also [21]). Such particles can be produced only in the processes with participation of nuclei. Cumulative processes have been traditionally studied at small p_T. It corresponds to particle production in the backward semisphere (the large particle produced angle in the laboratory system frame). The other possibility is the production of cumulative particles at high p_T [16]. The interest in study of cumulative processes is motivated by searching for signatures of the phase transition in the high compressed nuclear matter.

The new data on inclusive momentum spectra for h^+ and h^- hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and angle $\theta_{\rm lab} = 35°$ were obtained at U70 [30]. The measurements were performed over the momentum range of $0.5 < p_T < 3.8$ GeV/c using C,

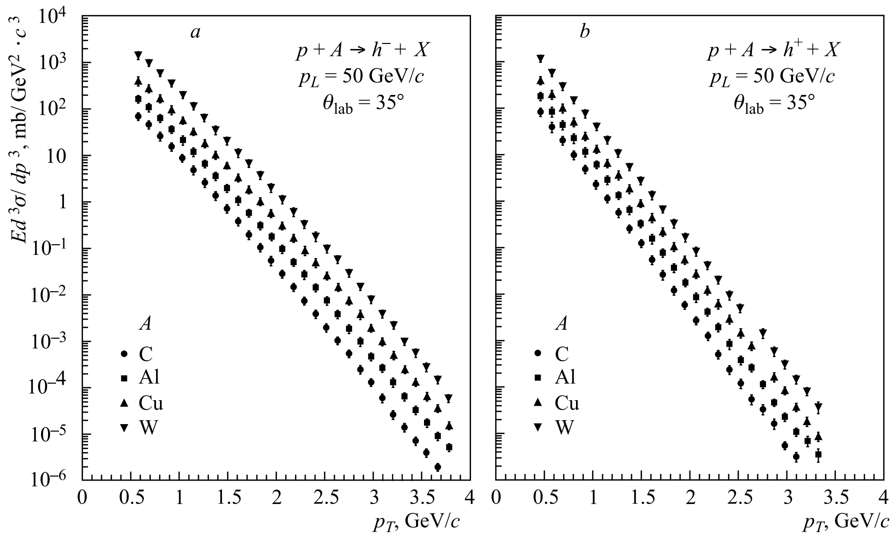

Fig. 1. Spectra of h^- (a) and h^+ (b) hadrons produced in $p+A$ collisions at incident proton momentum $p_L = 50$ GeV/c and angle $\theta_{\rm lab} = 35°$ as a function of transverse momentum p_T. The experimental data are taken from [30]

Al, Cu and W nuclear targets. The cumulative region corresponds to the momentum range $p_T > 2.5$ GeV/c.

Figure 1 shows the inclusive cross sections for h^- (a) and h^+ (b) hadrons produced in $p+A$ collisions at the momentum $p_{\rm lab} = 50$ GeV/c and angle $\theta_{\rm lab}$ of 35°. As seen from Fig. 1, the cross sections reveal the strong dependence on the transverse momentum p_T for all C, Al, Cu and W nuclear targets. The difference of cross sections at $p_T = 0.5$ and 3.5 GeV/c reaches more than 7 orders of the magnitude. The data have demonstrated more exponential rather than the power law behavior. Absolute hadron yields for h^- and h^+ hadrons have been found to be different.

3. INCLUSIVE $p + A$ SPECTRA IN z PRESENTATION

Below we follow the procedure of the data analysis used in [27]. The function ψ is calculated for every nucleus using Eq. (5) with the normalization factor $\sigma_{\rm in}^{pA}/\sigma_{\rm in}^{pp}$ instead of $\sigma_{\rm in}$. The factor $\sigma_{\rm in}^{pA}$ is the total inelastic cross section for $p+A$ interactions. The multiplicity density of charged particles for different nuclei is parametrized by the following formula:

$$\rho(s, A) \simeq 0.67 A^{0.18} \cdot s^{0.105}, \quad A \geqslant 2. \tag{8}$$

The scaling functions for different nuclei obtained in this way have revealed the energy and angular independence [27].

Figure 2 shows z presentation of the data [30] for h^- (a) and h^+ (b) hadrons. The data [31, 32] and [33] for the deuteron target are given for comparison. One can see that the curves found for C, Al, Cu, W nuclei are in agreement with the data z presentation for D nucleus at $p_L = 70, 400$ GeV/c and $\theta_{\rm cms} \simeq 90°$.

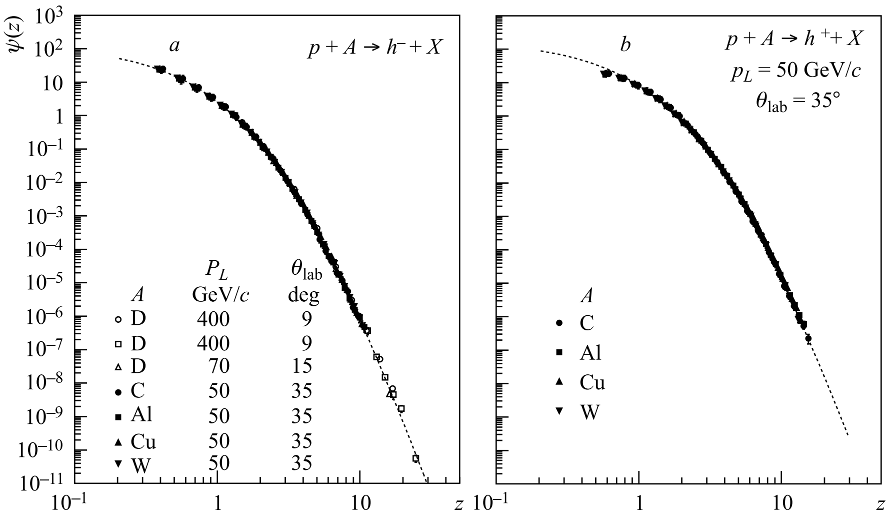

Fig. 2. Scaling function $\psi(z)$ versus variable z for h^- (a) and h^+ (b) hadrons produced in $p + A$ collisions at momentum $p_L = 50$ GeV/c and angle $\theta_{\rm lab} = 35°$. The dotted lines are the results of fitting. The points are z presentation of the experimental data taken from [30–33]

The symmetry transformation

$$z \to \alpha(A) \cdot z, \quad \psi \to \alpha^{-1}(A) \cdot \psi \tag{9}$$

of the function $\psi(z)$ and argument z should be used to compare the functions ψ for different nuclei. A-dependence of the parameter α was found in [27]. It is described by the expression $\alpha(A) = 0.9A^{0.15}$. We assume that the shape of the scaling curve should be the same as for the data points corresponding to the high-p_T region and $\theta_{\rm cms} \simeq 90°$. This hypothesis means the validity of self-similarity of the hadron production in the cumulative high-p_T region.

There are no experimental data on the angular dependence of $\rho(s, \eta, A)$ (η is a pseudorapidity, $\eta = -\ln(\tan(\theta_{\rm cms}/2)))$ for particles produced in a backward hemisphere to obtain a normalization factor for the scaling function. Therefore, we have verified a possibility to restore the shape of the $\psi(z)$ found from the analysis of the high-p_T data [31–33] using the cumulative high-p_T data [30]. A similar procedure for analysis of the low-p_T cumulative data [28] has been used in [29]. In this case, the function $\rho(s, \eta, A)$ has been parametrized in the following form: $\rho(s, \eta, A) = \rho(s, A) \cdot \chi(\theta_{\rm lab}, A)$. The angular dependence is described by $\chi(\theta_{\rm lab}, A)$. We have found that for h^- hadrons the values of χ at $\theta_{\rm lab} = 35°$ are equal to 1, 1.33, 1.67, 3.33 for C, Be, Cu, W nuclei, respectively. For h^+ hadrons the corresponding values have been found to be 1, 1.33, 2, 5. The multiplicity density $\rho(s, \eta, A)$ increases with A at the fixed energy and angle.

The found values of χ allow us to restore the scaling behavior of the function. Note that $\rho(s, \eta, A)$ is the normalization factor for the inclusive cross section at the fixed angle $\theta_{\rm lab}$. The fact that the single factor can restore the shape of $\psi(z)$ over a wide range of z is not an evident result. Although the outcome of the procedure was a success, the measurements of the angular dependence of the multiplicity density of charged hadrons are necessary to verify this assumption directly.

As seen from Fig. 2, the shape of $\psi(z)$ is restored using the angular dependence of the multiplicity density. We would like to note that the data points [30] cover both the preasymptotic ($z < 4$) and asymptotic ($z > 4$) regions. The latter is characterized by the power behavior of $\psi(z)$. We see that the low- and high-p_T data overlap. They demonstrate a deviation from the law $\psi(z) \sim z^{-\beta}$ for $z < 4$. Note that the function $\psi(z)$ reveals a power behavior in the cumulative region and for z up to 10. We have observed no indication of the deviation of the shape of $\psi(z)$ from the shape found in the noncumulative region corresponding to high z. The values of the fractal dimensions for proton and nucleus for both regions are found to be the same: $\delta_N = \delta = 0.5$ and $\delta_A = A\delta$. It was expected (see [23, 24] and references therein) that these dimensions should change in the cumulative region. This effect was substantiated by the fact that the compressed matter produced in the cumulative region should be characterized by a different dependence than the additive law $\delta_A = A\delta$. Therefore, it is of interest to verify the power behavior of $\psi(z)$ in the cumulative region for $z > 10$. We assume that the discontinuity of the fractal dimension δ_A at high z is the signature of the phase transition.

4. THE SHAPE OF $\psi(z)$

One of the features of z-scaling is the universality of the shape of the function $\psi(z)$. It has been found [27, 29] that this property is valid for charged hadrons produced in $p + A$ collisions over a wide range of momenta and angles of the inclusive particle [28, 31–33].

The parametrization of the scaling function in the Tsallis form [34]

$$\psi(z) = C[1 + (q - 1)z/T]^{1/1-q} \tag{10}$$

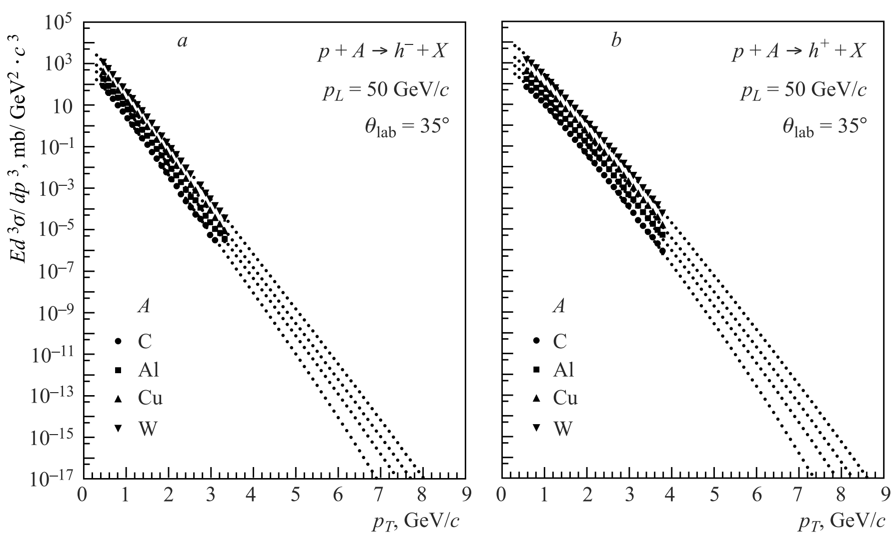

Fig. 3. Spectra of h^- (*a*) and h^+ (*b*) hadrons produced in $p+A$ collisions at momentum $p_L = 50$ GeV/*c* and angle $\theta_{\rm lab} = 35°$ in p_T presentation. The dotted lines are predictions based on z-scaling. The points are the experimental data taken from [30]

was used in [29]. This form is flexible enough to describe also the inclusive cross sections (see [35] and references therein). As seen from (10), the fitting parameters (C, T and q) are dimensionless. The asymptotics of ψ are as follows: $\psi(z) \to C$ if $z \to 0$, and $\psi(z) \to z^{1/1-q}$ if $z \to \infty$. We have used the high-p_T data for D nucleus [31–33] to determine the fitting parameters. The values of the parameter q for h^- and h^+ hadrons have been found to be equal to each other, $q = 1.083$. The parameter C is responsible for the absolute normalization of the cross section. The values of the parameters C and T for h^- and h^+ particles are equal to 125, 0.2189 and 177, 0.2850, respectively. The dotted lines shown in Figs. 3, a and 3, b correspond to our calculations of $\psi(z)$.

5. HIGH-p_T SPECTRA IN CUMULATIVE REGION

The universal shape of the scaling function allows us to predict inclusive spectra in the region which has not been available up to now. It is expected that the mechanisms of the hadron production in the cumulative and noncumulative regions differ from each other. As a result, the additive law, $\delta_A = A \cdot \delta$, for fractal dimensions of nuclei should be violated. The results obtained in Sec. 3 have shown that the shape of $\psi(z)$ is the same over a wide z region. It means that experimental study of the hadron production in the cumulative region for $p_T > 3.5$ GeV/c ($z > 10$) is highly desirable.

The main physical motivation to investigate the cumulative processes is related with the assumption that the cumulative region corresponds to the regime of particle production in which the nuclear matter is strongly compressed. We also assume that a nucleus size in the deep cumulative region (the region near the kinematic boundary of the reaction) could be of the order of a nucleon size. For this process the momentum of the inclusive particle should be fully balanced by the momentum of the recoil system consisting of very slow constituents. The system in this state should demonstrate the property of collectivity. Therefore, a transition regime from single constituent interactions to collective phenomena is expected.

Our predictions are based on self-similarity of constituent interactions in the noncumulative region at high p_T. Therefore, extrapolation of the cross sections far from the nucleon–nucleon kinematic boundary could allow us to verify simultaneously the power law, $\psi(z) \simeq z^{-\beta}$, and search for its violation. Verification of self-similarity of hadron production in the cumulative region and for high p_T could give us a new insight into collective phenomena of the nuclear matter.

Figure 3 shows the dependence of the inclusive cross section $Ed^3\sigma/dp^3$ on the transverse momentum p_T of h^- (a) and h^+ (b) hadrons produced in $p + A$ collisions at the incident proton momentum $p_L = 50$ GeV/c and angle $\theta_{\text{lab}} = 35°$. The U70 data [30] are shown by symbols. Our calculations are drawn by the dotted lines. The cross sections, as seen from Fig. 3, rapidly decrease with the momentum. They drop for the carbon nucleus by more than 19 orders of magnitude at $p_T = 0.5$ and 7.0 GeV/c, respectively. A kinematic boundary

Kinematic boundary for h^- hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and $\theta_{\text{lab}} = 35°$

A	Proton	C	Al	Cu	W
θ_{lab}	$p_T^{\max}$, GeV/c				
35°	2.6	15.6	20.7	24.4	26.7

for $p + A \to h^- + X$ process is shown in table. The value of the mass of the unidentified hadron was taken to be 0.31 GeV. We expect that experimental measurements of spectra up to $p_T = 7{-}8$ GeV/c at $\theta_{\rm lab} = 35°$ (the deep-cumulative region) would allow us to test the power law of $\psi(z)$ up to $z \simeq 10^2$.

Figures 4 and 5 show the dependence of the fractions x_1 (*a*) and x_2 (*b*) on the momentum p_T for h^- and h^+ hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$, respectively. The fraction x_1 corresponds to fragmentaion of the incident proton and x_2 to

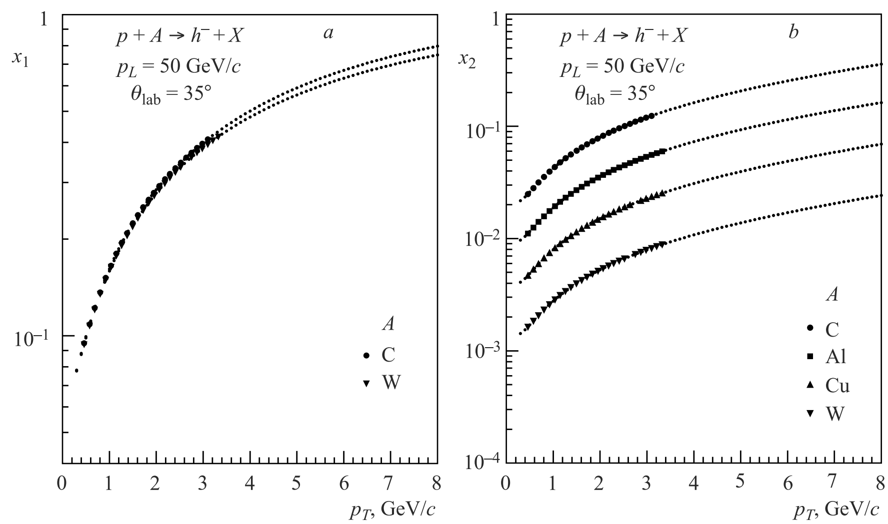

Fig. 4. Dependence of the fractions x_1 (*a*) and x_2 (*b*) on the momentum p_T of h^- hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$

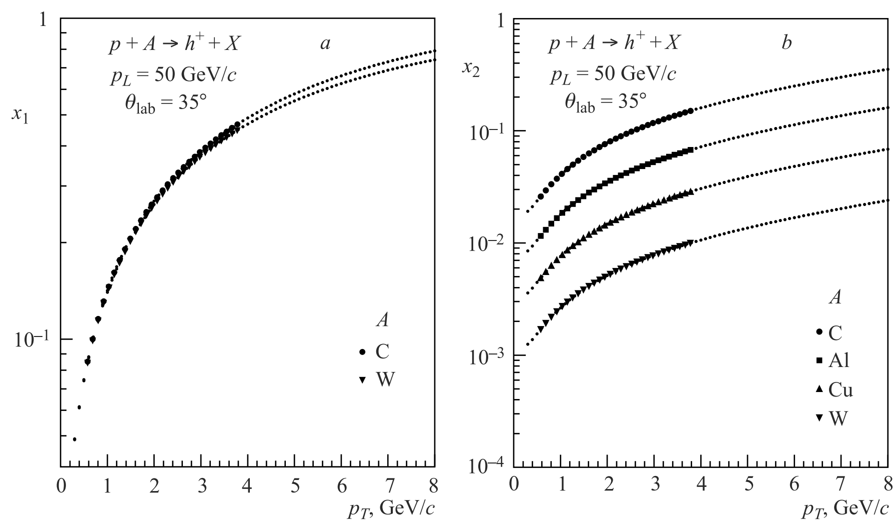

Fig. 5. Dependence of the fractions x_1 (*a*) and x_2 (*b*) on the momentum p_T of h^+ hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$

fragmentaion of the nucleus. They are both restricted: $0 < x_1,\ x_2 < 1$. The normalization conditions — the momentum fraction x_1 per nucleon and x_2 per nucleus — were used in the paper. We see that the fractions increase with the transverse momentum. The noncumulative region covers the range $x_2 < 1/A$ and the cumulative one — $x_2 > 1/A$ for fragmentation of a nucleus. The cumulative region in this kinematics ($p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$) corresponds to $p_T > 2.5$ GeV/c.

6. MOMENTUM FRACTIONS x_1, x_2 AND z–p_T PLOT

Figure 6 demonstrates the dependence of the scaling variable z on the momentum p_T of h^- (a) and h^+ (b) hadrons at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$ for different nuclei. The plot allows us to choose the most suitable kinematic region to verify the power law for $\psi(z)$. The cumulative region correponds to $z > 10$.

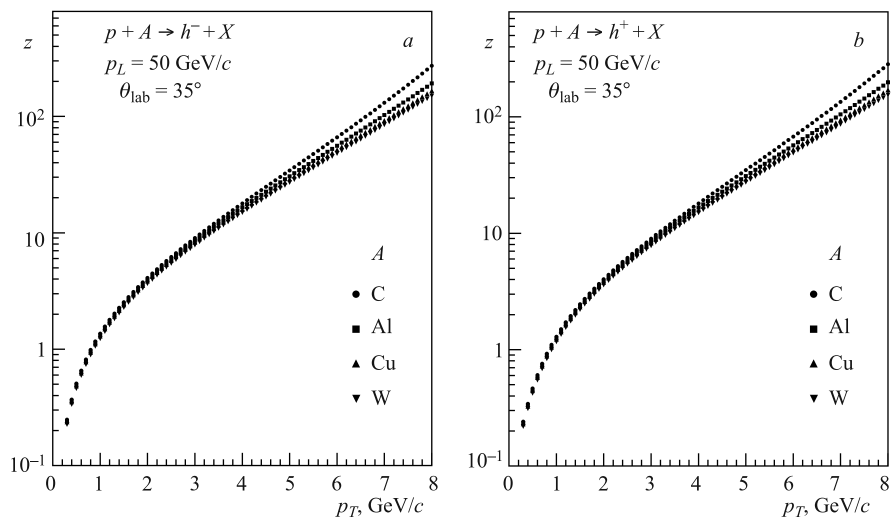

Fig. 6. The z–p_T plot for h^- (a) and h^+ (b) hadrons produced in $p + A$ collisions at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$. Symbols correspond to nuclei C, Al, Cu, W

7. DISCUSSION

Self-similarity of constituent interactions at high energies was studied in the z-scaling approach for different inclusive processes in $p + p$ and $\bar{p} + p$ collisions. The flavor independence of the scaling function over a wide range of z has been found [26]. This result cogently indicates fractal properties of the internal hadron structure and constituent interactions. A more sophisticated approach developed in [25] allowed us to analyze the hadron spectra in $A + A$ collisions as well [36–38]. The main goal of the study is to search for signatures of the critical point and phase transitions of the nuclear matter. A constituent energy loss as a function of energy and centrality of the collision and transverse momentum of the inclusive particle is assumed to be a good feature of the produced medium. The energy loss was found to increase with energy $\sqrt{s}$ and multiplicity density and decrease while the momentum p_T increases.

Complementary information on constituent interactions and properties of multiparticle system can be obtained in $p + A$ collisions. We expect that such information would allow us to clarify properties of transition from the hadron to nuclear medium at different scales. Modification of an elementary subprocess is assumed to be stronger in the region forbidden for particle production on free nucleons. This region is known as a cumulative one. It can be experimentally reached at a collider in the central rapidity range and in backward semisphere production in the fixed target experiments. Thus, the high-p_T and low-p_T regions can be complementarily studied in the z-scaling approach.

The experiment [30] performed at U70 is the first one which has measured the spectra of charged hadrons produced in $p+A$ collisions in the cumulative high-p_T region. The maximal value of transverse momentum p_T up to 3.5 GeV/c was reached. The decrease of the cross section is more than seven orders of magnitude for $p_T = 3.5$ GeV/c. The function $\psi(z)$ has demonstrated transition to the power behavior for $z > 4$. The fact that the shape of $\psi(z)$ found at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$ in the cumulative region coincides with the shape of $\psi(z)$ found at $p_L = 70, 400$ GeV/c and $\theta_{\rm cms} \simeq 90°$ in the noncumulative one is an unexpected result. These kinematic regions are quite different. This result means that self-similarity of the hadron production takes place in both regions. Therefore, it is of interest to verify the power behavior of $\psi(z)$ for the cumulative production in $p + A$ collisions at higher p_T, up to 7–8 GeV/c. We expect the change of the slope parameter β of the scaling function, $\psi(z) \sim z^{-\beta}$, in this region.

CONCLUSIONS

The experimental data on inclusive spectra of the charged hadrons produced in $p + A$ collisions at the incident proton momentum $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$ obtained at U70 were analyzed in the framework of z-scaling. Self-similarity of cumulative hadron production in the high-p_T region up to 3.5 GeV/c was verified. The scaling variable z and scaling function $\psi(z)$ for $p + A \to h + X$ process were constructed. They are expressed in terms of the measurable quantities such as the invariant cross section, average multiplicity density, momenta and masses of colliding and produced particles. The quantity z is a self-similarity parameter. It has a property of the fractal measure. The parameters δ_1, δ_2 are the fractal dimensions of colliding particles. They characterize the self-similarity of the internal structure of proton and nucleus.

In the pesent paper we have verified the hypothesis that z-scaling reflects the fundamental symmetries such as locality, self-similarity, and fractality of hadron interactions at a constituent level in cumulative processes at high p_T. Comparison of our results with the high-p_T data sets obtained by J. Cronin, R. Sulyaev and D. Jaffe groups in high-p_T noncumulative processess has shown that the shape of $\psi(z)$ is the same in the overlapping region. We have found the power behavior of $\psi(z)$ in the cumulative region. The microscopic scenario of $p + A$ interactions in terms of the momentum fractions x_1 and x_2 is supported by the obtained results.

The parametrization of the scaling function in the Tsalis form has been used to predict inclusive cross sections of h^- and h^+ hadrons produced in $p+A$ collisions at $p_L = 50$ GeV/c and $\theta_{\rm lab} = 35°$ on C, Al, Cu, W targets up to the transverse momentum p_T of 7–8 GeV/c.

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