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FRACTIONAL NONTOPOLOGICAL QUANTIZATION OF THE MAGNETIC FLUXES IN THE $U(1)$ GAUGED PLANAR SKYRME MODEL

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We discuss a $U(1)$ gauged version of the $(2+1)$-dimensional planar Skyrme model supplemented by the Maxwell term. We show that there exist a variety of static multisoliton configurations coupled to the noninteger fluxes of magnetic field, which revert to the usual planar Skyrmions in the limit of the gauge coupling constant vanishing. The structure of the multisoliton solutions of the model strongly depends on the particular choice of the potential term which may break the rotational invariance. We investigate the dependency of the shapes, masses, and magnetic fluxes of the gauged planar Skyrmions with broken symmetry on the gauge coupling constant. We further find that in the strong coupling limit the magnetic fluxes, associated with the parton components of the solitons, became quantized in fractional units of topological charge.

Рассматривается $U(1)$ калибровочная версия $(2+1)$-мерной обобщенной планарной модели Скирма–Максвелла. Показано, что существует набор различных статических мультисолитонных конфигураций, связанных с потоком магнитного поля, переходящий в обычные планарные солитонные конфигурации в пределе нулевой калибровочной константы связи. Показано, что структура построенных мультисолитонных решений сильно зависит от выбора потенциала модели, нарушающего ее вращательную инвариантность. Рассматривается зависимость профилей решений с нарушенной вращательной инвариантностью, их массы и магнитного потока от величины калибровочной константы связи. Показано, что в пределе сильной связи магнитный поток, ассоциированный с компонентами солитонных конфигураций, становится квантованным в дробных единицах топологического заряда.

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INTRODUCTION

The Skyrme model is a nonlinear $O(4)$ sigma model in $d = 3+1$ dimensions with topologically stable soliton solutions [1]. It can be derived from the expansion of the low-energy effective Lagrangian in the large N_c limit [2], then the topological charge of the multisoliton configuration is set into correspondence to the physical baryon number. Under certain assumption, the semiclassical quantization of rotations and isorotations of the Skyrmions allows us to get a good approximation to the isospinning atomic nuclei and describe the corresponding excitations [3]. Recently, the Skyrme model was successfully applied to the description of the rotational excitation bands of light nuclei [4].

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A peculiar feature of the Skyrme model is that the corresponding soliton solutions do not saturate the topological lower bound. In order to attain it and get a linear relation between the masses of the Skyrmions and their topological charges, one has, for example, to reduce the model to the BPS Skyrme model [5]. Furthermore, the contribution of the Coulomb electromagnetic energy is necessary to get a good agreement between the binding energies of heavy nuclei and the predictions of the reduced BPS Skyrme model [6]. Therefore, it is physically natural to extend the model by gauging it to describe various electromagnetic processes of nucleons.

The $U(1)$ gauged Skyrme model was originally proposed in [7], later the axially-symmetric gauged Skyrmions were considered in [8, 9]. It was noticed that the gauging of a $U(1)$ subgroup and the inclusion of a Maxwell term in the Lagrangian may stabilize the solitons even if the Skyrme term is dropped [10], furthermore, the BPS energy bound becomes saturated.

The planar reduction of the nonlinear sigma model is known as baby Skyrme model [11, 12], which resembles the basic properties of the genuine Skyrme model in many aspects. The corresponding Lagrangian includes the usual sigma model term, the Skyrme term, which is quartic in derivatives of the field, and the potential term which does not contain the derivatives. This low-dimensional model has a number of applications, e.g., in condensed matter physics, where Skyrmion configurations were observed experimentally [13], in the description of the topological quantum Hall effect [14, 15], or in brane cosmology, where the solitons of the model induce warped compactification of the 2-dimensional extra space [16]. Also it was found that there is the restricted baby Skyrme model in $2 + 1$ dimensions which has BPS soliton solutions saturating the Bogomolny bound [17].

Similar analysis of the gauged baby Skyrmions [18] reveals very interesting features of the corresponding solitons which carry a nonquantized nontopological magnetic flux. Further, if the Chern–Simons term is additionally included in the Lagrangian, the planar Skyrmions become electrically charged [19]. Recently, the properties of the soliton configurations in the gauged BPS baby Skyrme model were investigated [20]. An interesting observation is that in the strong coupling limit the magnetic flux becomes quantized, though there are no topological reasons for that [18, 27].

Another peculiarity of the planar Skyrme model is related with the particular choice of the potential term. It gives a mass to the excitations of the scalar field, therefore, in the context of the usual Skyrme model it is referred to as “pion mass term” [21]. Although in $(3 + 1)$-dimensional Skyrme model this term is optional, its presence might strongly affect the structure of the solution configurations [22].

On the other hand, the form of the potential term in the baby Skyrme model is largely arbitrary, there are different choices related with various ways of symmetry breaking [23–26]. In particular, a suitable choice for the potential term allows us to separate the individual constituents of the planar Skyrmions, each of them being associated with a fractional part of the topological charge of the configuration [25, 26].

In this paper we discuss the topologically stable static soliton solutions of the full coupled gauged baby Skyrme–Maxwell system with symmetry breaking potential, which carry arbitrary magnetic flux. We study numerically the dependence of masses of these constituent solitons and the corresponding magnetic fluxes on the gauge coupling constant, both in perturbative and in the strong coupling limits. In the latter case we observe an effective fractional quantization of the magnetic fluxes associated with each of the partons.

1. THE MODEL

A gauged version of the $O(3)$ σ-model with the Skyrme term in $2+1$ dimensions [12]

$$L = \partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \frac{1}{4}(\partial_\mu \boldsymbol{\phi} \times \partial_\nu \boldsymbol{\phi})^2 - U[\boldsymbol{\phi}], \tag{1}$$

where $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ denotes a triplet of scalar fields which satisfy the constraint $|\boldsymbol{\phi}|^2 = 1$.

Topological restriction on the field ϕ^a is that it approaches its vacuum value at spacial boundary, i.e., $\phi^a_\infty = (0,0,1)$. This allows a one-point compactification of the domain space $\mathbb{R}^2$ to S^2, and the field of the finite energy solutions of the model is a map $\boldsymbol{\phi} : \mathbb{R}^2 \to \mathbb{S}^2$ which belongs to an equivalence class characterized by the topological charge $B = \pi_2(S^2) = \mathbb{Z}$. Explicitly,

$$B = \frac{1}{4\pi} \int \boldsymbol{\phi} \cdot \partial_1 \boldsymbol{\phi} \times \partial_2 \boldsymbol{\phi} \, d^2x. \tag{2}$$

Note that the first two terms in the functional (1) are invariant under the global $O(3)$ transformations, this symmetry becomes broken via the potential term. The standard choice of the potential of the baby Skyrme model is [12]

$$U[\boldsymbol{\phi}] = \mu^2[1 - \phi_3]. \tag{3}$$

Thus, the symmetry is broken to $SO(2)$ and there is a unique vacuum $\boldsymbol{\phi}_\infty = (0,0,1)$. The corresponding solitons of degree $B = 1, 2$ are axially symmetric [12], however, the rotational symmetry of the configurations of higher degree becomes broken [28].

The traditional approach to study the solitons of the model (1) is related with separation of the radial and angular variables [12, 28]; thus, the consideration becomes restricted to the case of rotationally invariant configurations and the corresponding Euler–Lagrange equations are reduced to a single ordinary differential equation on radial function $f(\rho)$. However, a more detailed analysis reveals that the higher charge $B \geqslant 3$ baby Skyrmions may not possess rotational symmetry [12, 29]; starting from some critical value of the mass parameter μ, the global minimum of the energy functional corresponds to the configurations with discrete symmetries.

The resulting symmetry of the multiskyrmion configuration depends on the particular choice of the potential term. In the model with double vacuum potential (or "easy-axis" potential) [11, 30]

$$U[\boldsymbol{\phi}] = \mu^2(1 - \phi_3^2), \tag{4}$$

the multisoliton solutions are rotationally invariant over the entire range of values of the mass parameter μ. The most general case of the one-parametric potential

$$U[\boldsymbol{\phi}] = \mu^2(1 - \phi_3)^s, \tag{5}$$

with $0 < s \leqslant 4$, was considered in [24]. Since the "old" potential (3) corresponds to the attractive force acting between the solitons, while the "holomorphic" potential $U[\boldsymbol{\phi}] = \mu^2(1-\phi_3)^4$ is repulsive [31, 32], the parameter s in the potential (5) is responsible for the balance of the repulsive and attractive interaction between the Skyrmions. Clearly, one can consider the linear combination of the "old" and "holomorphic" potentials [26]

$$U[\boldsymbol{\phi}] = \mu^2 \left[\lambda(1 - \phi_3) + (1-\lambda)(1-\phi_3)^4\right], \quad \lambda \in [0,1], \tag{6}$$

which corresponds to a short-range repulsion and a long-range attraction between the solitons. The resulting configuration is no longer rotationally invariant (cf. Fig. 1).

Note that all these potentials (3), (4), (5), and (6) are rotationally invariant. Another possibility is related with the choice of symmetry breaking potentials [23, 25, 33]. In this case the individual charge one skyrmion is not axially symmetric, furthermore, it is composed of a few partons. Indeed, one can directly introduce a potential term which would violate the $O(2)$ invariance of the baby Skyrme model [23, 25, 33], for example, Ward considered the potential

$$U[\boldsymbol{\phi}] = \mu^2(1 - \phi_3^2)(1 - \phi_1^2), \tag{7}$$

which breaks the symmetry to the dihedral group D_2; thus, the single charge one soliton is composed of two constituents, each being associated with topological charge 1/2 (cf. Fig. 1).

In Fig. 1, we present the plots of the energy density distributions of the $B = 2$ baby Skyrmion in the model with potentials (3), (4), and (7), respectively. Further generalization of this approach [25] yields the solution of the baby Skyrme model of degree $B = m \in \mathbb{Z}$, whose energy density distribution represents a "necklace", a ring with $2m$ half-Skyrmions which is symmetric with respect to the dihedral group D_{2m}.

Now we introduce a $U(1)$ gauge field A_μ defining the covariant derivative of the scalar field as (cf. [10, 18, 34])

$$D_\mu \phi^a = \partial_\mu \phi^a + g A_\mu \varepsilon_{abc} \phi^b \phi^c_\infty, \tag{8}$$

where g is the gauge coupling constant. Note that the field configuration has finite energy if $D_\mu \phi^a \to 0$ as $r \to \infty$.

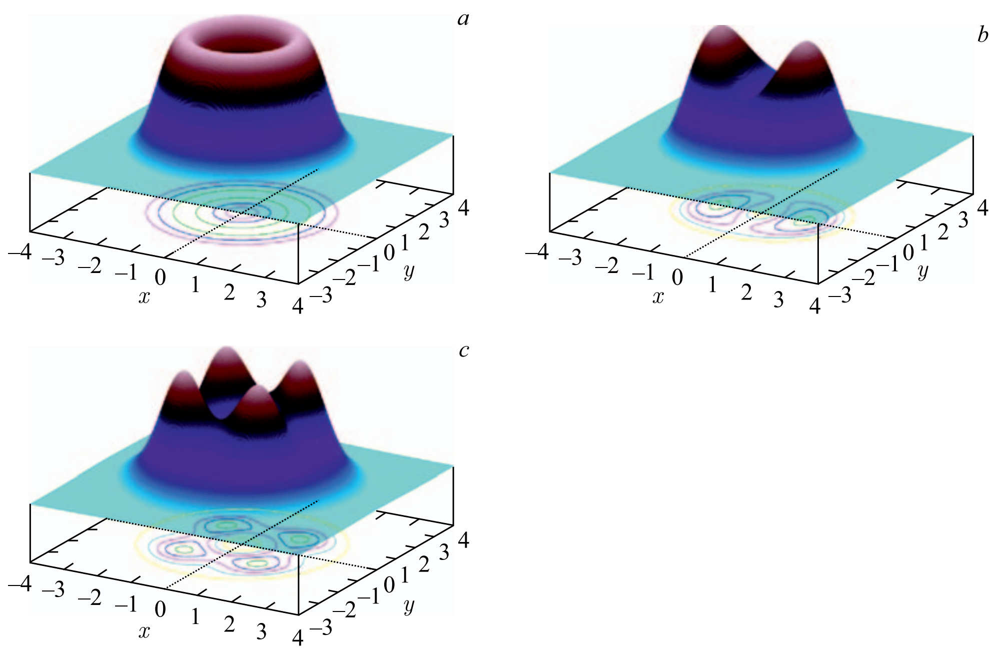

Fig. 1. Energy density plots of the $B = 2$ baby Skyrmions in the model with potentials (4), (6), and (7), respectively, at $\mu^2 = 0.1$ (from *a* to *c*)

The total Lagrangian of the gauged baby Skyrme–Maxwell model with symmetry breaking potential (7) then can be written as

$$\mathcal{L} = \frac{1}{32\pi^2\sqrt{2}}\biggl(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^a D^\mu\phi^a - \\ - \frac{1}{2}(\varepsilon_{abc}\phi^a D_\mu\phi^b D_\nu\phi^c)^2 - \mu^2(1-\phi_3^2)(1-\phi_1^2)\biggr), \quad (9)$$

where we introduced the usual Maxwell term, and the field strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Note that the symmetry breaking potential (7) is not invariant with respect to the gauge transformations defined via (8). However, the model preserves a symmetry under combined gauge-rotational symmetry.

The Euler–Lagrange equations can be written as

$$\partial^\mu F_{\mu\nu} = j_\nu = \varepsilon_{abc}\phi^a D_\nu\phi^b\phi^c_\infty + \varepsilon_{abc}D^\mu\phi^a\phi^b D_\nu\phi^c(D_\mu\phi^k\phi^k_\infty), \quad (10)$$

where j_ν is the electromagnetic current.

In $2+1$ dimensions we can consider purely magnetic field generated by the axially symmetric Maxwell potential

$$A_0 = A_r = 0, \quad A_\theta = a(r,\theta), \quad (11)$$

where $a(r,\theta)$ is an arbitrary function, and the gauge fixing condition is used to exclude the radial component of the vector-potential. Note we do not use the rotationally-invariant parameterization given by the ansatz

$$\phi_1 = \sin f(r)\cos(B\theta), \quad \phi_2 = \sin f(r)\sin(B\theta), \quad \phi_3 = \cos f(r). \quad (12)$$

However, we applied it to generate initial configurations in the given topological sector. Indeed, if input profile function is defined as $f(r) = 4\arctan \mathrm{e}^{-r}$, this corresponds to the configuration of degree B with usual boundary conditions on the profile function $f(r)$, i.e., $f(0)=\pi, f(\infty)=0$. Then the triplet of the scalar fields was considered as a set of dynamical variables in the full unrestricted system of the field equations.

The complete set of the field equations, which follows from the variation of the action of the baby Skyrme–Maxwell model (9), can be solved when we impose the boundary conditions. As usually, they follow from the regularity on the symmetry axis and symmetry requirements, as well as the condition of finiteness of the energy and the topology. In particular, we have to take into account that the magnetic field is vanishing on the spacial asymptotic. Explicitly, in agreement with (12), we impose

$$\phi_1\big|_{r\to\infty} \to 0, \quad \phi_2\big|_{r\to\infty} \to 0, \quad \phi_3\big|_{r\to\infty} \to 1, \quad \partial_r a\big|_{r\to\infty} \to 0 \quad (13)$$

at infinity and

$$\phi_1\big|_{r\to0} \to 0, \quad \phi_2\big|_{r\to0} \to 0, \quad \phi_3\big|_{r\to0} \to -1, \quad a\big|_{r\to0} \to 0 \quad (14)$$

at the origin. The condition of regularity of the fields on the symmetry axis yields

$$\partial_\theta\phi_1\big|_{\theta\to0,\pi} \to 0, \quad \phi_2\big|_{\theta\to0,\pi} \to 0, \quad \partial_\theta\phi_3\big|_{\theta\to0,\pi} \to 1, \quad \partial_\theta a\big|_{\theta\to0,\pi} \to 0. \quad (15)$$

2. NUMERICAL RESULTS

The numerical calculations are mainly performed on an equidistant grid in polar coordinates r and θ, employing the compact radial coordinate $x = r/(1+r) \in [0:1]$ and $\theta \in [0, 2\pi]$. To find solutions of the Euler–Lagrange equations, which follow from the Lagrangian (9) and depend parametrically on coupling constant g, we implement a simple forward differencing scheme on a rectangular lattice with lattice spacing $\Delta x = 0.01$. Typical grids used have sizes 120×70. The resulting system is solved iteratively until convergence is achieved.

All numerical calculations have been performed by using the professional package CADSOL, which uses a Newton–Raphson finite difference method with an arbitrary grid and arbitrary consistency order (a detailed description of this package is given in [35]). This code solves a given system of nonlinear partial differential equations subject to a set of boundary conditions on a rectangular domain. Apart from some initial guess for the solution, CADSOL requires also the Jacobian matrices for the equations with respect to the unknown functions and their first and second derivatives, and the boundary conditions. This software package provides also error estimates for each function, which allows one to judge the quality of the computed solution. The relative errors of the solutions we found are of the order of 10^{-4} or smaller. We also introduce an additional Lagrangian multiplier to constrain the field to the surface of unit sphere.

Each of our simulations began at $g = 0$ at fixed value of μ, then we proceed by making small increments in g.

In Fig. 2, we have plotted the graphs of energy of gauged static baby Skyrmion defined by the functional (9) and magnetic energy as function of the gauge coupling. Here we used the normalized units of energy per unit charge.

As the gauge coupling increases from zero, the energy of the gauged planar Skyrmions decreases since the magnetic flux is formed. The effect it causes is to squeeze the configuration down, as shown in Fig. 3, where we exhibited the energy density plots and contour plots of the gauged $B = 1, 2, 3, 4$ baby Skyrmions in the model with potential (7) at $g = 0$ and $g = 1$, respectively.

Note that as the coupling remains smaller than one, the electromagnetic energy $E_{\rm em}$ is increasing, however, in the strong coupling limit its contribution begins to decrease as g con-

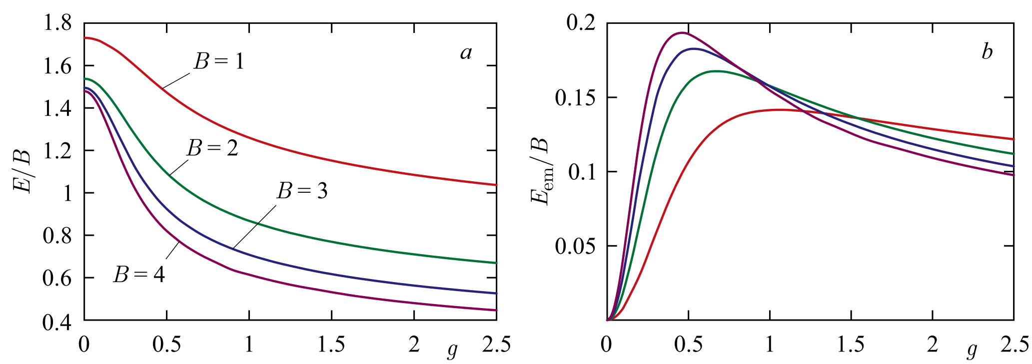

Fig. 2. The normalized energy E of the $B = 1, 2, 3, 4$ gauged baby Skyrmions (*a*) and the corresponding magnetic energy (*b*) as a function of the coupling constant g at $\mu = 1$

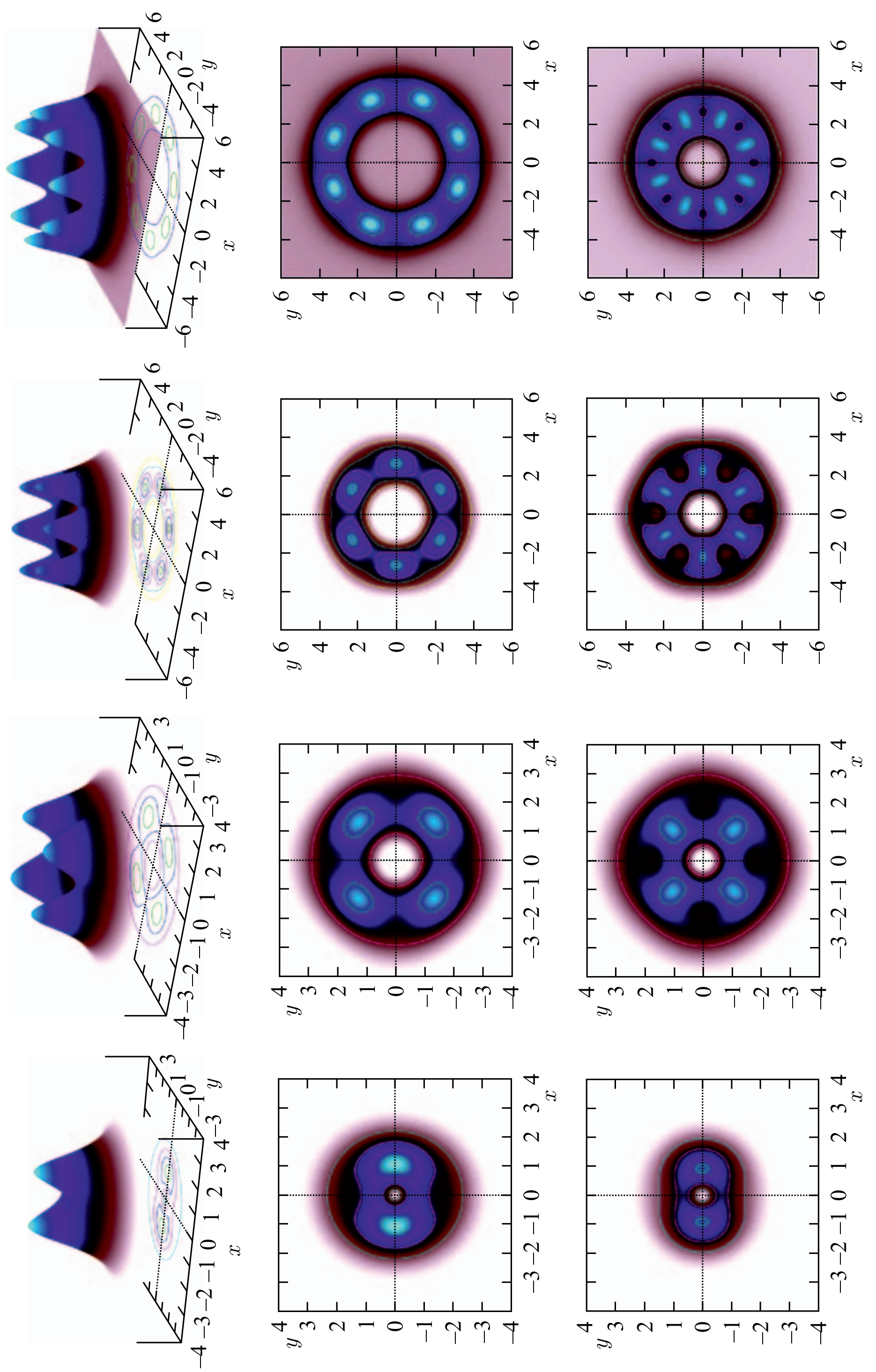

Fig. 3. Energy density plots for $B = 1, 2, 3, 4$ gauged planar Skyrmions in the model (9) at $g = 0$ (top row), contour energy density plots at $g = 0$ (middle row), and at $g = 1$ (bottom row)

tinues to grow, see Fig. 2, *b*. We can understand this effect if we note that the conventional rescaling of the potential $A_\mu \to gA_\mu$ leads to $F_{\mu\nu}^2 \to (1/g^2)F_{\mu\nu}^2$. Thus, the very large gauge coupling effectively removes the Maxwell term leaving the limiting configuration of gauged planar Skyrmions coupled to a circular magnetic vortex of constant flux. Apparently, in such a limit the strong coupling with a vortex yields an effective potential term which, unlike the usual symmetry breaking term in (9), affects the field components ϕ_1 and ϕ_2. An interplay between such an effective potential and the mass term may drastically affect the stability of the configuration.

Thus, the solitons carry magnetic flux $\Phi = \int d^2xB$, which is in general nonquantized. The flux of the gauged baby Skyrmions is associated with the position of the solitons, it is orthogonal to the $x-y$ plane [18]. In the usual model with rotationally invariant potential (4) there is a single magnetic flux through the center of the soliton, in the model, where the rotational invariance becomes violated, each unit charge constituent of the multisoliton configuration is coupled to a flux.

An interesting observation is that, as the gauge coupling becomes stronger, the magnetic flux of the degree n baby Skyrmions grows from 0 to $-2\pi n$, i.e., in the strong coupling regime the magnetic flux is quantized though there are no topological reasons for it [18].

Indeed, in Fig. 4 we display the results of our numerical calculations of the integrated magnetic field of the gauged planar Skyrmion through the $x-y$ plane. The density distribution of the magnetic field of the $B = 1, 2, 3, 4$ gauged planar Skyrmions in the model (9) at $g = 1$ is displayed in Fig. 5.

Evidently, this is the field of the circular vortices which encircles the partons.

As the gauge coupling increases, the radius of each vortex is getting smaller and the magnitude of the magnetic field increases significantly. Effectively, using the Maxwell equation $\nabla \times \mathbf{B} = \mathbf{j}$, one can set this magnetic field into correspondence with a circular electric current $\mathbf{j}$ (10). In the strong coupling limit the integrated total magnetic flux of the gauged planar Skyrmion through the $x-y$ plane becomes quantized in units of 2π (see Fig. 4). However, each parton is now associated with the fractional flux which in the model with D_2 symmetry breaking potential (7) is just half of integer. Evidently, since in the model with easy plane

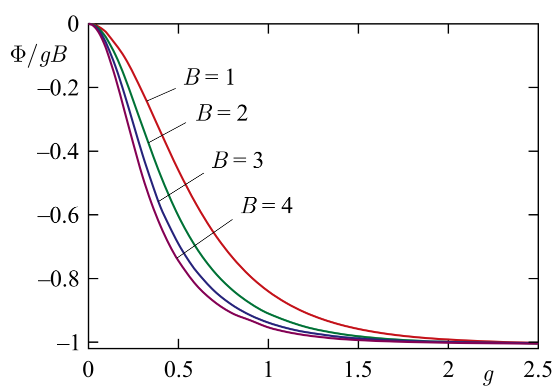

Fig. 4. The magnetic flux through the $x-y$ plane as a function of the coupling constant g for the solutions of degree $Q = 1, 2, 3, 4$ at $\mu = 1$

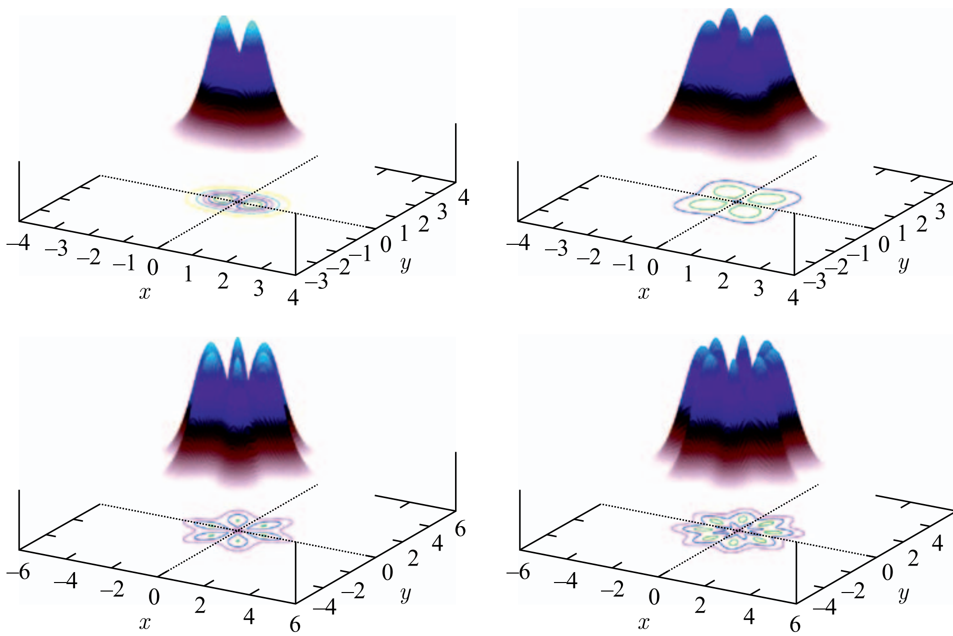

Fig. 5. Magnetic field distribution in the x–y plane for the solutions of degree $B = 1, 2, 3, 4$ at $g = 1$

potential [25] the symmetry is broken to D_N and each parton bears a fractional topological charge B/N, in the strong coupling limit the corresponding fluxes will be effectively quantized in units of fractional flux Φ/N.

CONCLUSIONS

The main purpose of this work was to present a new type of gauged solitons in the planar Skyrme–Maxwell theory. In the model with dihedral symmetry the individual solitons are composed out of the constituents with fractional topological charge. Interestingly, since the original $(3 + 1)$-dimensional Skyrme model is posed to describe hadrons, such constituents may be considered as an analogue of constituent "quarks" in the effective low-energy theory.

Similar to the corresponding solutions in the Skyrme model and Faddeev–Skyrme model [27], they are topologically stable and in the weak coupling regime they carry a nonquantized magnetic flux which is orthogonal to the x–y plane and penetrates the Skyrmion. In the strong coupling limit the magnetic flux, associated with the partons, becomes quantized in fractional units of topological charge.

We confirm that the mass of the static configuration decreases when the electromagnetic coupling constant is increased; thus, a baby Skyrmion can lower its mass by interacting with the electromagnetic field.

Finally, note that the planar Skyrmions appear as quasiparticles in various systems, in particular, they are natural objects at the description of the integer quantum Hall effect [15]. The vortices coupled to the planar Skyrmion constituents in the strong coupling limit may carry fractionally quantized magnetic flux. Vortices of this type may appear in multicomponent superconductors [36]; thus, as avenues for further research, it would be interesting to extend the solutions in this work to the effective condensed matter systems.

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