# EFFECTS OF FINAL STATE INTERACTIONS IN PURE ANNIHILATION DECAY OF $B^{+} \rightarrow D^{+} K^{0^{*}}$ 

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The decay of the $B^{+}$meson to the $D^{+}$and $K^{0^{*}}$ mesons is a pure annihilation decay. For this reason, in the framework of the quantum chromodynamics factorization (QCDF) approach, this decay has a small amplitude and a small branching ratio. In this research, we find that before the $D^{+}$and $K^{0^{*}}$ mesons are produced in the final states, pair mesons, such as $D_{s}^{+*} \pi^{0}$ and $D_{s}^{+} \rho^{0}$, are produced in the intermediate states. The intermediate state mesons via the exchange of $\bar{K}^{0}\left(\bar{K}^{0^{*}}\right)$ and $D^{+}\left(D^{+*}\right)$ go to the $D^{+}$and $K^{0^{*}}$ final state mesons. However, we calculate the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay in two different frameworks. The first framework is the QCDF method and the second one is the final state interaction (FSI). The experimental branching ratio of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay is less than $3 \cdot 10^{-6}$, and our results obtained by the QCDF method and FSI are $(0.35 \pm 0.04) \cdot 10^{-6}$ and $(2.94 \pm 0.10) \cdot 10^{-6}$, respectively.

Распад $B^{+}$-мезона на $D^{+}$- и $K^{0^{*}}$-мезоны является чисто аннигиляционным. По этой причине в приближении квантовой хромодинамической факторизации (КХДФ) амплитуда этого распада мала, так же как мало отношение выхода частиц. В представленной работе показано, что, до того как в конечном состоянии рождаются мезоны $D^{+}$и $K^{0^{*}}$, в промежуточном состоянии появляются такие мезоны, как $D_{s}^{+{ }^{*}} \pi^{0}$ и $D_{s}^{+} \rho^{0}$. Эти промежуточные мезоны переходят в конечные мезоны $D^{+}$и $K^{0^{*}}$ в процессе обмена $\bar{K}^{0}\left(\bar{K}^{0 *}\right)$ и $D^{+}\left(D^{+*}\right)$. Распад $B^{+} \rightarrow D^{+} K^{0^{*}}$ вычисляется в двух различных приближениях. Первое приближение - это метод КХДФ, а второе - взаимодействие в конечном состоянии (ВКС). Экспериментальное отношение выхода частиц в распаде $B^{+} \rightarrow D^{+} K^{0^{*}}$ имеет величину меньше $3 \cdot 10^{-6}$, а вычисленные КХДФ и ВКС методами значения — $(0,35 \pm 0,04) \cdot 10^{-6}$ и $(2,94 \pm 0,10) \cdot 10^{-6}$ соответственно.

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## INTRODUCTION

The study of the two-body nonleptonic weak decay of $B^{+} \rightarrow D^{+} K^{0^{*}}$ may be useful in the search for new physics beyond the Standard Model, as $B^{+}$can decay to $D^{+} K^{0^{*}}$ through both current and penguin annihilation processes. Several useful methods have been created to calculate the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay, such as the perturbative QCD approach [1], QCDF by Beneke and Neubert [2], and the use of the FSI effects by Lu [3]. In perturbative QCD, Xiao et al. have considered complete twist-3 contributions, and in QCDF, state-of-the-art analysis was performed according to QCDF by four parameter scenarios. From

[^0]these scenarios, considering large arbitrary numbers (the same work has been conducted in scenario 4 in [2]), the branching ratio (BR) becomes smaller than the experimental ones. In the FSI effects, the $D_{s}^{+*} \pi^{0}$ and $D_{s}^{+} \rho^{0}$ mesons have been considered as intermediate states. We mainly used the same framework in the QCDF approach and selected the leading order Wilson coefficients at the scale $m_{b}[2,4]$ and estimated the amplitude of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay while assuming the annihilation contributions. In this case, we obtained $(0.35 \pm 0.04) \cdot 10^{-6}$ for the branching ratio. Motivated by the above-mentioned study, we contributed the FSI corrections to the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay mode. Since, in $B$ decays, resonant FSI is expected to be suppressed owing to the absence of resonances at energies close to the mass of the $B$ meson, we considered only $t$ channels and estimated it via the one-particle exchange processes at the hadronic loop level (HLL) as explained in Sec.3. The FSI can give sizable corrections. Rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this decay, intermediate states are $D_{s}^{+{ }^{*}} \pi^{0}$ and $D_{s}^{+} \rho^{0}$. Then, we calculate the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay according to the HLL method. By the FSI method we obtain the branching ratio of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay, $(2.94 \pm 0.10) \cdot 10^{-6}$, and the experimental result of this decay is less than $3 \cdot 10^{-6}$ [5]. We present the calculation of QCDF for the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay in Sec. 1. In Sec. 2, we calculate the amplitudes of the intermediate states. Then, we present the calculation of HLL for the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay in Sec. 3. In Sec.4, we give the numerical results, and in Sec. 5, we have conclusion.

## 1. QCD FACTORIZATION OF THE $B^{+} \rightarrow D^{+} K^{0^{*}}$ DECAY

In this section, we calculate the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay by using the QCDF approach. The $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay just has annihilation diagrams shown in Fig. 1. According to QCDF, we write out the amplitude of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay as follows:

$$
\begin{equation*}
A\left(B^{+} \rightarrow D^{+} K^{0^{*}}\right)=\frac{i G_{F}}{\sqrt{2}} f_{B} f_{D} f_{K^{*}}\left\{b_{1} V_{c b} V_{u s}^{*}+b_{2} V_{u b} V_{c s}^{*}\right\} \tag{1}
\end{equation*}
$$

$f_{B}, f_{D}$ and $f_{K^{*}}$ are the decay constants. $V_{p b} V_{p s}^{*}(p=u, c)$ are the CKM matrix elements and $b_{1,2}$ correspond to the current-current annihilation. These nonsinglet annihilation coefficients are given as

$$
\begin{equation*}
b_{1,2}=\frac{C_{F}}{N_{c}^{2}} C_{1,2} A_{1,2}^{i} \tag{2}
\end{equation*}
$$



Fig. 1. Feynman annihilation diagrams for the $B^{+} \rightarrow D^{+} K^{0 *}$ decay
$C_{1,2}$ are the Wilson coefficients; $N_{c}$ is the color number and

$$
\begin{gather*}
A_{1}^{i} \approx-A_{2}^{i}=6 \pi \alpha_{s}\left[3\left(X_{A}-4+\frac{\pi^{2}}{3}\right)+r_{X}^{D^{+}} r_{X}^{K^{0^{*}}}\left(X_{A}^{2}-2 X_{A}\right)\right] \\
C_{F}=\frac{N_{c}^{2}-1}{2 N_{c}} \tag{3}
\end{gather*}
$$

There are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects. We parameterize these effects in terms of the divergent integrals $X_{A}$ (weak annihilation)

$$
\begin{equation*}
X_{A}=\left(1+\rho \mathrm{e}^{i \phi}\right) \ln \frac{m_{B}}{\Lambda_{h}}, \quad \rho \leqslant 1, \quad \Lambda_{h}=0.5 \mathrm{GeV} \tag{4}
\end{equation*}
$$

The ratios $r_{X}^{D^{+}}$and $r_{X}^{K^{0^{*}}}$ are defined as

$$
\begin{equation*}
r_{X}^{D^{+}}=\frac{2 m_{D}^{2}}{\left(m_{b}-m_{c}\right)\left(m_{d}+m_{c}\right)}, \quad r_{X}^{K^{0 *}}=\frac{2 m_{K^{*}}}{m_{b}} \frac{f_{K^{*}}^{\perp}}{f_{K^{*}}} \tag{5}
\end{equation*}
$$

## 2. WEAK AMPLITUDES OF INTERMEDIATE STATES

To consider the FSI effects in the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay, we must extract the accessible intermediate states and calculate the weak amplitude of them. According to Fig. 2, $D_{s}^{+*} \pi^{0}$ and $D_{s}^{+} \rho^{0}$ are produced for intermediate states via exchange meson of $\bar{K}^{0}\left(\bar{K}^{0^{*}}\right)$ and $D^{+}\left(D^{+*}\right)$.


Fig. 2. The $B^{+} \rightarrow D_{s}^{+*} \pi^{0}$ decay diagrams

We write out the amplitude of the $B^{+} \rightarrow D_{s}^{+*} \pi^{0}$ decay by using the QCDF approach as follows:

$$
\begin{equation*}
A\left(B^{+} \rightarrow D_{s}^{+*} \pi^{0}\right)=-i \sqrt{2} G_{F} f_{\pi}\left(\varepsilon_{D_{s}^{*}} \cdot p_{B}\right) A_{0}^{B \rightarrow D_{s}^{*}}\left(a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right) \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
A_{0}^{B \rightarrow D_{s}^{*}}=f_{0}\left(m_{B}+m_{D_{s}^{*}}\right)+f_{2}\left(m_{B}-m_{D_{s}^{*}}\right)+f_{3} \frac{m_{D}^{2}}{m_{B}+m_{D_{s}^{*}}} \tag{7}
\end{equation*}
$$

and

$$
\begin{align*}
& a_{i}=c_{i}^{\mathrm{eff}}+\frac{1}{N_{c}} c_{i+1}^{\mathrm{eff}} \quad(i=\mathrm{odd}) \\
& a_{i}=c_{i}^{\mathrm{eff}}+\frac{1}{N_{c}} c_{i-1}^{\mathrm{eff}} \quad(i=\mathrm{even}) \tag{8}
\end{align*}
$$

And there are also similar diagrams, such as the $B^{+} \rightarrow D_{s}^{+} \rho^{0}$ decay. So, the amplitude reads

$$
\begin{equation*}
A\left(B^{+} \rightarrow D_{s}^{+} \rho^{0}\right)=-i \sqrt{2} G_{F} f_{D_{s}}\left(\varepsilon_{\rho} \cdot p_{B}\right) A_{0}^{B \rightarrow \rho}\left(a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right) \tag{9}
\end{equation*}
$$

## 3. FINAL STATE INTERACTION OF THE $B^{+} \rightarrow D^{+} K^{0^{*}}$ DECAY

It is extremely difficult to calculate the FSI effects, but at the hadronic level formulated as rescattering processes with the $s$-channel resonances and one-particle exchange in the $t$ channel, the $s$-channel resonant FSI effects in the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay are expected to be vanished because of the lack of resonances. Therefore, one can model the FSI effects as rescattering processes of two-body intermediate states with one-particle exchange in the $t$ channel and compute the absorptive part via the optical theorem [6]. So, according to the hadronic loop level (HLL) diagrams, shown in Fig. 2, the absorptive part of the amplitude is calculated with the following formula:

$$
\begin{align*}
& \operatorname{Abs} M\left(B\left(p_{B}\right) \rightarrow M\left(p_{1}\right) M\left(p_{2}\right) \rightarrow M\left(p_{3}\right) M\left(p_{4}\right)\right)= \\
= & \frac{1}{2} \int \frac{d^{3} \mathbf{p}_{1}}{2 E_{1}(2 \pi)^{3}} \frac{d^{3} \mathbf{p}_{2}}{2 E_{2}(2 \pi)^{3}}(2 \pi)^{4} \delta^{4}\left(p_{B}-p_{1}-p_{2}\right) M\left(B \rightarrow M_{1} M_{2}\right) G\left(M_{1} M_{2} \rightarrow M_{3} M_{4}\right) \tag{10}
\end{align*}
$$

where $M\left(B \rightarrow M_{1} M_{2}\right)$ is the amplitude of the $B \rightarrow M_{1} M_{2}$ decay that is calculated via the QCDF method, and $G\left(M_{1} M_{2} \rightarrow M_{3} M_{4}\right)$ involves hadronic vertices factor defined as

$$
\begin{align*}
\left\langle D\left(p_{2}\right) \bar{K}^{0^{*}}\left(p_{3}, \varepsilon_{3}\right)\right| i \mathfrak{L}\left|D_{s}\left(p_{1}\right)\right\rangle & =-i g_{D_{s} K^{*} D} \varepsilon \cdot\left(p_{1}+p_{2}\right), \\
\left\langle D^{*}\left(\varepsilon_{2}, p_{2}\right) K^{0}(q)\right| i \mathfrak{L}\left|D_{s}^{*}\left(\varepsilon_{1}, p_{2}\right)\right\rangle & =-i g_{D_{s}^{*} D^{*} K} \varepsilon_{\mu \nu \alpha \beta} \varepsilon_{1}^{\mu} \varepsilon_{2}^{* \nu} q^{\alpha} p_{1}^{\beta} \tag{11}
\end{align*}
$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [7]:

$$
\begin{equation*}
\operatorname{Dis} M\left(m_{B}^{2}\right)=\frac{1}{\pi} \int_{s}^{\infty} \frac{\operatorname{Abs} M\left(s^{\prime}\right)}{s^{\prime}-m_{B}^{2}} d s^{\prime} \tag{12}
\end{equation*}
$$

where $s^{\prime}$ is the square of the momentum carried by the exchanged particle and $s$ is the threshold of intermediate states, in this case, $s \sim m_{B}^{2}$. Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integration.
3.1. Final State Interaction in the $B^{+} \rightarrow D_{s}^{+*} \pi^{0} \rightarrow D^{+} K^{0^{*}}$ Decay. The quark model diagram for $B^{+} \rightarrow D_{s}^{+*} \pi^{0} \rightarrow D^{+} K^{0^{*}}$ via the exchange of $\bar{K}^{0}$ is shown in Fig. 3, and the hadronic level diagrams are shown in Fig. 4.

The amplitude of the mode $B^{+} \rightarrow D_{s}^{+*}\left(\varepsilon_{1}, p_{1}\right) \pi^{0}\left(p_{2}\right) \rightarrow D^{+}\left(p_{3}\right) K^{0^{*}}\left(\varepsilon_{4}, p_{4}\right)$ via the exchange of $\bar{K}^{0}$ is given by

$$
\begin{align*}
& \operatorname{Abs}(4 a)=\frac{-i G_{F}}{2 \sqrt{2}} \int_{-1}^{1} \frac{d^{3} \mathbf{P}_{1}}{2 E_{1}(2 \pi)^{3}} \frac{d^{3} \mathbf{P}_{2}}{2 E_{2}(2 \pi)^{3}}(2 \pi)^{4} \delta^{4}\left(p_{B}-p_{1}-p_{2}\right) \times \\
& \times\left(-i g_{\pi K K^{*}}\right) \varepsilon_{3} \cdot\left(p_{1}+q\right)\left(-i g_{D K D_{s}^{*}}\right) \varepsilon_{2} \cdot(-q)\left\{2 m_{D_{s}^{*}}\left(\varepsilon_{2} \cdot p_{1}\right) f_{\pi} A_{0}^{B D_{s}^{*}}\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \times \\
& \quad \times \frac{F^{2}\left(q^{2}, m_{K}^{2}\right)}{T_{1}}=\frac{-i G_{F}}{8 \sqrt{2} \pi m_{B}} g_{\pi K K^{*}} g_{D K D_{s}^{*}} \int_{-1}^{1}\left|P_{1}\right| d(\cos \theta)\left\{2 H_{1} m_{D_{s}^{*}} f_{\pi} A_{0}^{B D_{s}^{*}} \times\right. \\
& \left.\times\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{K}^{2}\right)}{T_{1}}, \tag{13}
\end{align*}
$$

where

$$
\begin{gather*}
H_{1}=\left(\varepsilon_{2} \cdot p_{1}\right)\left(\varepsilon_{2} \cdot p_{4}\right)\left(\varepsilon_{3} \cdot p_{1}\right)=\left(-p_{1} \cdot p_{4}+\frac{\left(p_{1} \cdot p_{2}\right)\left(p_{2} \cdot p_{4}\right)}{m_{\rho}^{2}}\right)\left(\frac{E_{1}\left|p_{3}\right|-E_{3}\left|p_{1}\right| \cos \theta}{m_{B}\left|p_{3}\right|}\right) \\
T_{1}=\left(p_{1}-p_{3}\right)^{2}-m_{K}^{2}=p_{1}^{2}+p_{3}^{2}-2 p_{1}^{0} p_{3}^{0}+2 \mathbf{p}_{1} \cdot \mathbf{p}_{3}-m_{K}^{2},  \tag{14}\\
q^{2}=m_{1}^{2}+m_{3}^{2}-2 E_{1} E_{3}+2\left|\mathbf{p}_{1} \| \mathbf{p}_{3}\right| \cos \theta=m_{D_{s}^{*}}^{2}+m_{D}^{2}-2 p_{1}^{0} p_{3}^{0}+2\left|\mathbf{p}_{1}\right|\left|\mathbf{p}_{3}\right| \cos \theta
\end{gather*}
$$



Fig. 3. Quark level diagram for the $B^{+} \rightarrow D_{s}^{+*} \pi^{0} \rightarrow D^{+} K^{0^{*}}$ decay


Fig. 4. HLL diagrams for long-distance $t$-channel contribution to the $B^{+} \rightarrow D_{s}^{+*} \pi^{0} \rightarrow D^{+} K^{0^{*}}$ decay
$\theta$ is the angle between $\mathbf{p}_{1}$ and $\mathbf{p}_{3} ; q$ is the momentum of the exchange $\bar{K}^{0}$ meson, and $F\left(q^{2}, m_{K}^{2}\right)$ is the form factor defined to take care of the off-shell of the exchange particles introduced as [6,7]:

$$
\begin{equation*}
F\left(q^{2}, m_{K}^{2}\right)=\left(\frac{\Lambda^{2}-m_{K}^{2}}{\Lambda^{2}-q^{2}}\right)^{n} \tag{15}
\end{equation*}
$$

The form factor (i.e., $n=1$ ) normalized to unity at $q^{2}=m_{K}^{2} \cdot m_{K}$ and $q$ are the physical parameters of the exchange particle, and $\Lambda$ is the phenomenological parameter.

It is obvious that for $q^{2} \rightarrow 0, F\left(q^{2}, m_{K}^{2}\right)$ becomes a number. If $\Lambda \gg m_{K}$, then $F\left(q^{2}, m_{K}^{2}\right)$ turns to be unity, whereas, as $q^{2} \rightarrow \infty$, the form factor approaches to zero and the distance becomes small and the hadron interaction is no longer valid. Since $\Lambda$ should not be far from $m_{K}$ and $q$, we choose

$$
\begin{equation*}
\Lambda=m_{K}+\eta \Lambda_{\mathrm{QCD}} \tag{16}
\end{equation*}
$$

where $\eta$ is the phenomenological parameter whose value in the form factor is expected to be of the order of unity and can be determined from the measured rates, and

$$
\begin{align*}
& \operatorname{Abs}(4 b)=\frac{-i G_{F}}{2 \sqrt{2}} \int_{-1}^{1} \frac{d^{3} \mathbf{P}_{1}}{2 E_{1}(2 \pi)^{3}} \frac{d^{3} \mathbf{P}_{2}}{2 E_{2}(2 \pi)^{3}}(2 \pi)^{4} \delta^{4}\left(p_{B}-p_{1}-p_{2}\right) \times \\
& \times\left(-i \sqrt{2} g_{\pi K^{*} K^{*}}\right) \varepsilon_{\mu \nu \alpha \beta} \varepsilon_{3}^{\mu} \varepsilon_{K^{*}}^{\nu} p_{1}^{\alpha} p_{3}^{\beta}\left(-i \sqrt{2} g_{D_{s}^{*} K^{*} D}\right) \varepsilon_{\rho \sigma \lambda \eta} \varepsilon_{2}^{\rho} \varepsilon_{K^{*}}^{\sigma} p_{2}^{\lambda} p_{4}^{\eta}\left\{2 m_{D_{s}^{*}}\left(\varepsilon_{2} \cdot p_{1}\right) f_{\pi} A_{0}^{B D_{s}^{*}} \times\right. \\
& \left.\times\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{K^{*}}^{2}\right)}{T_{2}}=\frac{i G_{F}}{8 \sqrt{2} \pi m_{B}} g_{D K^{*} D_{s}^{*}} g_{\rho K^{*} K} \int_{-1}^{1}\left|P_{1}\right| d(\cos \theta) \times \\
& \quad \times\left\{2 H_{3} m_{D_{s}^{*}}\left(\varepsilon_{2} \cdot p_{1}\right) f_{\pi} A_{0}^{B D_{s}^{*}}\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{K^{*}}^{2}\right)}{T_{2}} \tag{17}
\end{align*}
$$

where

$$
\left.\begin{array}{rl}
H_{3}= & m_{3}^{2}\left(p_{1} \cdot p_{2}\right)-\left(p_{1} \cdot p_{3}\right)\left(p_{2} \cdot p_{3}\right)+\left(\frac{E_{2}\left|p_{3}\right|-E_{3}\left|p_{2}\right| \cos \theta}{m_{B}\left|p_{3}\right|}\right) \times \\
& \times\left[\left(p_{B} \cdot p_{1}\right)\left(p_{3} \cdot p_{4}\right)-\left(p_{B} \cdot p_{3}\right)\left(p_{1} \cdot p_{4}\right)\right]
\end{array}\right\} \begin{gathered}
T_{2}=\left(p_{1}-p_{3}\right)^{2}-m_{K^{*}}^{2}=p_{1}^{2}+p_{3}^{2}-2 p_{1}^{0} p_{3}^{0}+2 \mathbf{p}_{1} \cdot \mathbf{p}_{3}-m_{K^{*}}^{2}  \tag{18}\\
q^{2}=m_{1}^{2}+m_{3}^{2}-2 E_{1} E_{3}+2\left|\mathbf{p}_{1}\left\|\mathbf{p}_{3}\left|\cos \theta=m_{D_{s}^{*}}^{2}+m_{D}^{2}-2 p_{1}^{0} p_{3}^{0}+2\right| \mathbf{p}_{1}\right\| \mathbf{p}_{3}\right| \cos \theta
\end{gathered}
$$

The dispersion relation is

$$
\begin{equation*}
\operatorname{Dis} 4\left(m_{B}^{2}\right)=\frac{1}{\pi} \int_{s}^{\infty} \frac{\operatorname{Abs} 4 a\left(s^{\prime}\right)+\operatorname{Abs} 4 b\left(s^{\prime}\right)}{s^{\prime}-m_{B}^{2}} d s^{\prime} \tag{19}
\end{equation*}
$$

3.2. Final State Interaction in the $B^{+} \rightarrow D_{s}^{+} \rho^{0} \rightarrow D^{+} K^{0^{*}}$ Decay. The quark model diagram for the $B^{+} \rightarrow D_{s}^{+} \rho^{0} \rightarrow D^{+} K^{0^{*}}$ decay is shown in Fig. 5, and the hadronic level


Fig. 5. Quark level diagram for the $B^{+} \rightarrow D_{s}^{+} \rho^{0} \rightarrow D^{+} K^{0 *}$ decay


Fig. 6. HLL diagrams for long-distance $t$-channel contribution to the $B^{+} \rightarrow D_{s}^{+} \rho^{0} \rightarrow D^{+} K^{0 *}$ decay
diagrams are presented in Fig. 6. The amplitude of the mode $B^{+} \rightarrow \rho^{0}\left(\varepsilon_{1}, p_{1}\right) D_{s}^{+}\left(p_{2}\right) \rightarrow$ $D^{+}\left(p_{3}\right) K^{0^{*}}\left(\varepsilon_{4}, p_{4}\right)$ via the exchange of $D^{+}$is given by

$$
\begin{align*}
\operatorname{Abs}(6 a)= & \frac{-i G_{F}}{2 \sqrt{2}} \int_{-1}^{1} \frac{d^{3} \mathbf{P}_{1}}{2 E_{1}(2 \pi)^{3}} \frac{d^{3} \mathbf{P}_{2}}{2 E_{2}(2 \pi)^{3}}(2 \pi)^{4} \delta^{4}\left(p_{B}-p_{1}-p_{2}\right) \times \\
\times & \left(-i g_{D_{s} D K^{*}}\right) \varepsilon_{3} \cdot\left(p_{1}+q\right)\left(-i g_{\rho D D}\right) \varepsilon_{2} \cdot(-q)\left\{2 m_{\rho}\left(\varepsilon_{2} \cdot p_{1}\right) f_{D_{s}} A_{0}^{B \rho} \times\right. \\
& \left.\times\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{D}^{2}\right)}{T_{1}}=\frac{-i G_{F}}{8 \sqrt{2} \pi m_{B}} g_{D_{s} D K^{*}} g_{\rho D D} \times \\
& \times \int_{-1}^{1}\left|P_{1}\right| d(\cos \theta)\left\{2 H_{1} m_{\rho} f_{D_{s}} A_{0}^{B \rho}\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{D}^{2}\right)}{T_{1}}, \tag{20}
\end{align*}
$$

and

$$
\begin{align*}
& \operatorname{Abs}(6 b)=\frac{-i G_{F}}{2 \sqrt{2}} \int_{-1}^{1} \frac{d^{3} \mathbf{P}_{1}}{2 E_{1}(2 \pi)^{3}} \frac{d^{3} \mathbf{P}_{2}}{2 E_{2}(2 \pi)^{3}}(2 \pi)^{4} \delta^{4}\left(p_{B}-p_{1}-p_{2}\right) \times \\
& \times\left(-i \sqrt{2} g_{D_{s} D^{*} K^{*}}\right) \varepsilon_{\mu \nu \alpha \beta} \varepsilon_{3}^{\mu} \varepsilon_{D^{*}}^{\nu} p_{1}^{\alpha} p_{3}^{\beta}\left(-i \sqrt{2} g_{\rho D^{*} D}\right) \varepsilon_{\rho \sigma \lambda \eta} \varepsilon_{2}^{\rho} \varepsilon_{D^{*}}^{\sigma} p_{2}^{\lambda} p_{4}^{\eta}\left\{2 m_{\rho}\left(\varepsilon_{2} \cdot p_{1}\right) f_{D_{s}} A_{0}^{B \rho} \times\right. \\
& \left.\times\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{D^{*}}^{2}\right)}{T_{2}}=\frac{i G_{F}}{8 \sqrt{2} \pi m_{B}} g_{D_{s} D^{*} K^{*}} g_{\rho D^{*} D} \int_{-1}^{1}\left|P_{1}\right| d(\cos \theta) \times \\
& \quad \times\left\{2 H_{3} m_{\rho}\left(\varepsilon_{2} \cdot p_{1}\right) f_{D_{s}} A_{0}^{B \rho}\left[a_{1} V_{u b} V_{c s}^{*}+a_{2} V_{c b} V_{u s}^{*}\right]\right\} \frac{F^{2}\left(q^{2}, m_{D^{*}}^{2}\right)}{T_{2}} . \tag{21}
\end{align*}
$$

Equations $H_{1}, H_{3}, T_{1}, T_{2}$ and $q^{2}$ can be written similar to those in the previous section. The dispersion relation is

$$
\begin{equation*}
\operatorname{Dis} 6\left(m_{B}^{2}\right)=\frac{1}{\pi} \int_{s}^{\infty} \frac{\operatorname{Abs} 6 a\left(s^{\prime}\right)+\operatorname{Abs} 6 b\left(s^{\prime}\right)}{s^{\prime}-m_{B}^{2}} d s^{\prime} \tag{22}
\end{equation*}
$$

The decay amplitude of $B^{+} \rightarrow D^{+} K^{0^{*}}$ via the HLL diagrams is

$$
\begin{align*}
\mathrm{A}\left(B^{+} \rightarrow D^{+} K^{0^{*}}\right)=\operatorname{Abs}(4 a)+\operatorname{Abs}(4 b)+\operatorname{Abs}(6 a) & +\operatorname{Abs}(6 b)+ \\
& +\operatorname{Dis} 4\left(m_{B}^{2}\right)+\operatorname{Dis} 6\left(m_{B}^{2}\right) \tag{23}
\end{align*}
$$

## 4. NUMERICAL RESULTS

Numerical values of effective coefficients $a_{i}$ for $\bar{b} \rightarrow \bar{d}$ transition at $N_{c}=3$ are given by [8]:

$$
\begin{array}{ll}
a_{1}=1.05, & a_{2}=0.053 \\
a_{3}=0.0048, & a_{4}=-0.046-0.012 i \\
a_{5}=-0.0045, & a_{6}=-0.059-0.012 i \\
a_{7}=0.00003-0.00018 i, & a_{8}=0.0004-0.00006 i \\
a_{9}=-0.009-0.00018 i, & a_{10}=-0.0014-0.00006 i \tag{24}
\end{array}
$$

The relevant input parameters are used as follows:

$$
\begin{gather*}
m_{B}=(5279 \pm 0.3) \mathrm{MeV}, \quad m_{D}=(187 \pm 0.2) \mathrm{MeV}, \quad m_{D^{*}}=(2010.2 \pm 0.17) \mathrm{MeV} \\
m_{K}=(493.6 \pm 0.016) \mathrm{MeV}, \quad m_{K^{*}}=(891 \pm 0.26) \mathrm{MeV}, \quad m_{D_{s}}=(197 \pm 0.34) \mathrm{MeV} \\
m_{D_{s}^{*}}=(2010.2 \pm 0.17) \mathrm{MeV}, \quad m_{\pi}=139.5 \mathrm{MeV}, \quad m_{\rho}=(775.4 \pm 0.34) \mathrm{MeV}, \\
f_{B}=(176 \pm 42) \mathrm{MeV}, \quad f_{D}=(222.6 \pm 19.5) \mathrm{MeV}, \quad f_{D^{*}}=(230 \pm 20) \mathrm{MeV} \\
f_{K^{*}}=(217 \pm 5) \mathrm{MeV}, \quad f_{\pi}=(130.7 \pm 0.46) \mathrm{MeV}, \quad f_{\rho}=211 \mathrm{MeV},  \tag{25}\\
V_{u b}=0.0043 \pm 0.0003, \quad V_{u d}=0.974 \pm 0.0002, \quad V_{u s}=0.2257 \pm 0.002, \\
V_{c s}=0.9745 \pm 0.11, \quad V_{c d}=0.230 \pm 0.011, \quad V_{c b}=0.0416 \pm 0.0006[7,9] \\
A_{0}^{B \rho}=0.3, \quad \phi=-70^{\circ}(V P), \quad \phi=-20^{\circ}(P V), \quad \rho=0.5, \quad \Lambda_{\mathrm{QCD}}=0.225 \mathrm{GeV}, \\
G_{F}=1.166 \cdot 10^{-5}[8,10], \quad g_{\rho D D}=2.52, \quad g_{\rho D^{* D}}=2.82[11], \quad g_{D_{s}^{*} K D}=18.34, \\
g_{D_{s}^{*} K^{*} D}=2.99, \quad g_{D_{s} D K^{*}}=2.59, \quad g_{D_{s} D^{*} K^{*}}=2.79[6], \quad g_{\pi K K^{*}}=4.6[12]
\end{gather*}
$$

By using the input parameters and according to the QCDF method of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay, we get

$$
\begin{equation*}
\mathrm{BR}\left(B^{+} \rightarrow D^{+} K^{0^{*}}\right)=(0.35 \pm 0.04) \cdot 10^{-6} \tag{26}
\end{equation*}
$$

We note that our estimate of branching ratio of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay according to the QCDF method seems less than the experimental result. Before calculating the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay amplitude via FSI, we have to compute the intermediate state amplitude. We are able to calculate the branching ratio of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay with different values of $\eta$, which are shown in the Table.

The branching ratio of the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay with $\eta=2.1-2.6$ and experimental data (in units of $10^{-6}$ )

| $\eta$ | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | Exp. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BR | $0.61 \pm 0.03$ | $0.86 \pm 0.05$ | $1.18 \pm 0.06$ | $1.71 \pm 0.08$ | $2.37 \pm 0.09$ | $2.94 \pm 0.10$ | $<3$ |

## 5. SUMMARY

We have analyzed the $B^{+} \rightarrow D^{+} K^{0^{*}}$ decay in the QCD factorization approach and then we have added the final state interaction effects. For evaluating the FSI effects, we have only considered the absorptive part of the HLL, because both hadrons which are produced via the weak interaction are on their mass shells. The experimental result of this decay is less than $3 \cdot 10^{-6}$. According to QCDF and FSI, our results are $\mathrm{BR}\left(B^{+} \rightarrow D^{+} K^{0^{*}}\right)=(0.35 \pm 0.04)$. $10^{-6}$ and $(2.94 \pm 0.10) \cdot 10^{-6}$, respectively. The main phenomenological parameter in the FSI effects is $\eta$, which is determined from the measured ratios. Its value in form factor is expected to be of the order of unity. In this work, we have considered $\eta=2.1-2.6$ and the best result obtained by $\eta=2.6$.

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