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HADRON AS COHERENT STATE ON THE HOROSPHERE OF THE LOBACHEVSKY MOMENTUM SPACE

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A model of hadron (proton) as a coherent state of transverse excitations on the horosphere in momentum space identified with partons is presented. The features of multiparticle production resulting from the existence of theoretical and experimental constants characterizing the processes with high multiplicity at the LHC are investigated.

Построена модель адрона (протона) как когерентного состояния поперечных возбуждений на орисфере в импульсном пространстве, отождествляемых с партонами. Исследованы особенности множественного рождения частиц, следующие из фактов существования теоретических и экспериментальных констант, характеризующих процессы с высокой множественностью на LHC.

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The investigation of the multiparticle production processes at high energies is designed to provide important information about the properties of the fundamental interactions. In this regard, the new results presented by the ATLAS and CMS collaborations require theoretical understanding both in terms of existing models and theories and search for new approaches.

Typical sizes that characterize the processes of pion production, i.e., mainly processes due to the strong interaction in the collision of two hadrons at centre-of-mass energy $\sqrt{S}$ have the following values:

$$\begin{gathered} r_{0S} = \frac{h}{m_\pi c} = 1.46 \text{ fm}, \quad r_0 \propto 2.33 \text{ fm}, \quad r \propto \frac{hc}{\sqrt{S}}, \\ \sqrt{S} = 7 \text{ TeV}, \quad r_{\text{eff}} \leqslant r_{0S} \ln P(S), \end{gathered} \tag{1}$$

where r_{0S} is the radius of the strong (nuclear) interaction, i.e., the Compton wavelength of a pion; r_0 is the experimental value of the correlation radius of the charged pions produced in the proton–proton collision (see the Figure), which can be regarded as the distance at

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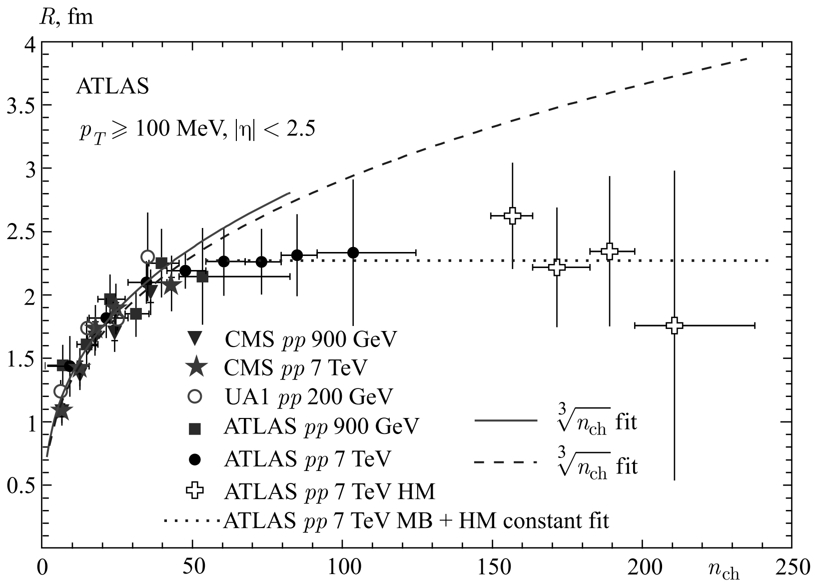

The dependence of the correlation radius of the pion pairs on the multiplicity of charged particles. It is evident that from the average multiplicity of charged particles 60 there is saturation — correlation radius does not change with increasing average multiplicity [1]

which the strong interactions are weak enough for the secondary hadrons formation; r is the de Broglie wavelength corresponding to the energy of the colliding particles; the last inequality is a limit on the possible increase in the effective radius of interaction in the strong interactions of hadrons, which follows from the general principles of quantum field theory, where $P(S)$ is a polynomial of degree less than 2 [2].

The main goal of this work is to develop a model of a hadron as a coherent state of its excitations interpreted as partons and to establish restrictions on the average multiplicity of produced particles resulting from the model based on the values (1).

Let us note that there are quite a number of physical models that describe more or less various aspects of multiparticle production processes [3]. The hydrodynamic model of multiparticle production, proposed by L. D. Landau and S. Z. Belenky in [4], indicates one characteristic dimension r_{0S}. Indeed, the hydrodynamic description of a system of particles is the approximation followed from the kinetic equations and it essentially depends on the characteristic linear dimension L of the existing problem in the study.

In this paper, we suppose that the radius of nuclear forces (Compton length of the pion) $L = 1.46 \cdot 10^{-15}$ m is the characteristic size of the investigated system.

Infinitesimal volumes used for the formulation of integral relations in hydrodynamics, thus, have to be much smaller than L^3 and much larger than the mean free path of the particles. It follows from (1) that the energy of the LHC from 7 TeV and above satisfies this condition.

Let us consider the collision of two hadrons at high energy, for example, proton–proton collision in the Large Hadron Collider. We suppose that colliding protons have a 4-momenta

$$p_1 = (p_{01}, \mathbf{p}_1), p_2 = (p_{02}, \mathbf{p}_2),$$
$$p_1^2 = \mathbf{p}_1^2 - p_{01}^2 = \mathbf{p}_2^2 - p_{02}^2 = -m_p^2, \tag{2}$$

where m_p is the proton mass. We use a system of units where $c = h = 1$.

The collision is carried out at centre-of-mass energy $\sqrt{S}$ which is determined as

$$S = -(p_1^2 + p_2^2) = -P^2 = -P_x^2 - P_y^2 - P_z^2 + P_0^2 =$$
$$= -(p_{x1} + p_{x2})^2 - (p_{y1} + p_{y2})^2 - (p_{z1} + p_{z2})^2 + (p_{01} + p_{02})^2, \tag{3}$$

where

$$P = (\mathbf{P}, iP_0) = [p_{x1} + p_{x2}, p_{y1} + p_{y2}, p_{z1} + p_{z2}, +i(p_{01} + p_{02})]. \tag{4}$$

It should be noted that in the laboratory frame (the rest system of the second proton)

$$P = (\mathbf{P}, iP_0) = [p_x, p_y, p_z, i(p_0 + m_p)], \tag{5}$$

where $p = (p_x, p_y, p_z, i(p_0)) = (\mathbf{p}, ip_0)$ is four-momentum of the incident proton.

We introduce quasi-Cartesian coordinates in Lobachevsky space realized on the upper sheet of the hyperboloid (3) in the momentum space [5] as

$$P_z = \frac{\sqrt{S}}{2}\left[\exp\left(\frac{2q_z}{\sqrt{S}}\right) + \left(\frac{q_x^2 + q_y^2}{S} - 1\right)\exp\left(\frac{-q_z}{\sqrt{S}}\right)\right],$$
$$P_x = q_x \exp\left(\frac{-2q_z}{\sqrt{S}}\right), \quad P_y = q_y \exp\left(\frac{-2q_z}{\sqrt{S}}\right), \tag{6}$$
$$P_0 = \frac{\sqrt{S}}{2}\left[\exp\left(\frac{2q_z}{\sqrt{S}}\right) + \left(\frac{q_x^2 + q_y^2}{S} + 1\right)\exp\left(\frac{-q_z}{\sqrt{S}}\right)\right].$$

The formula inverse to formula (6) is

$$q_x = \frac{P_x\sqrt{S}}{P_0 - P_z}, \quad q_y = \frac{P_y\sqrt{S}}{P_0 - P_z}, \quad q_z = \sqrt{S}\ln\frac{\sqrt{S}}{P_0 - P_z}. \tag{7}$$

The metric element has the form

$$dS^2 = \exp\left(\frac{-2q_z}{\sqrt{S}}\right)(dq_x^2 + dq_y^2) + dq_z^2, \tag{8}$$

and the volume element is

$$dV_m = \sqrt{g}\, dq_x\, dq_y\, dq_z = \exp\left(-\frac{2q_z}{\sqrt{S}}\right) dq_x\, dq_y\, dq_z. \tag{9}$$

The introduced quasi-Cartesian coordinates (6) allow us to separate variables q_x, q_y, and q_z. That is impossible in four-dimensional space (3). Therefore, we can consider the physics in the plane of the variables q_x, q_y only.

In addition, considering that Euclidean plane geometry is realized on the horosphere of Lobachevsky space, the Fourier transformation F of the function $\phi_1(q_x, q_y)\phi_2(q_z)$ defined on that plane (horosphere) defines the function in the coordinate plane also with the Euclidean geometry. That is not correct for the variable q_z as is evident from (9),

$$\Psi_1(x, y)\Psi_2(z) \leftrightarrow F\phi_1(q_x, q_y)\phi_2(q_z). \tag{10}$$

We note that quasi-Cartesian coordinates (6) and (7) automatically ensure the scale invariance of the theory in the plane of q_x, q_y, i.e., invariance under the following transformations:

$$P'_x = \lambda P_x, \quad P'_y = \lambda P_y, \quad P'_z = \lambda P_z, \quad P'_0 = \lambda P_0, \tag{11}$$

which is valid for any $\sqrt{S}$.

The fundamental role of scale invariance in processes of multiparticle production has been pointed out by V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze in [6, 7].

Let us build the quantum mechanics of the system described by four-momentum (6).

Since the horosphere of three-dimensional Lobachevsky space includes the geometry of the two-dimensional Euclidean space, we can introduce conjugate coordinates in momentum space in the standard way [8]:

$$q_x, x = -ih\frac{\partial}{\partial q_x}, \quad q_y, y = -ih\frac{\partial}{\partial q_y}. \tag{12}$$

There is a Heisenberg–Weyl algebra

$$\begin{gathered} [x, q_x] = [y, q_y] = ihI, \\ [x, y] = [q_x, q_y] = 0, \\ [x, I] = [y, I] = [q_x, I] = [q_y, I] = 0, \end{gathered} \tag{13}$$

where I is the identity operator.

The expressions (12) and (13) allow us to construct quantum coherent states on the horosphere.

The extra dimensional constant characterizing the system is required to lead the coordinates and momenta (12) to the same dimension. It is needed for the construction of creation and annihilation operators. It is natural to take the size of hadron (proton) as such constant. This size provides, due to the uncertainty relation, nonzero components x, y, even for a hadron moving along the axis z, which in turn implies the existence of nonzero components q_x, q_y in accordance with (7).

Then the creation and annihilation operators can be written in the following manner:

$$\begin{aligned} a_x &= \frac{Rq_x + i\dfrac{x}{R}}{\sqrt{2}}, \quad a_x^+ = \frac{Rq_x - i\dfrac{x}{R}}{\sqrt{2}}, \\ a_y &= \frac{Rq_y + i\dfrac{y}{R}}{\sqrt{2}}, \quad a_y^+ = \frac{Rq_y - i\dfrac{y}{R}}{\sqrt{2}}. \end{aligned} \tag{14}$$

Heisenberg–Weyl algebra in terms of the creation and annihilation operators is

$$[a_k, a_l^+] = \delta_{kl}I, \quad [a_k^+, a_l^+] = [a_k, a_l] = [a_k, I] = [a_k^+, I] = 0, \tag{15}$$

where $k,\ l = 1,\ 2$ correspond to x or y and $h = 1$. Coherent states are known to be defined as a state of its own annihilation operators with complex eigenvalues

$$a_x|z_1\rangle = z_1|z_1\rangle, \quad a_y|z_2\rangle = z_2|z_2\rangle. \tag{16}$$

The coherent states (16) satisfy the following conditions:

$$\langle z_1|z_1\rangle = \mathrm{e}^{|z_1|^2}, \quad \langle z_2|z_2\rangle = \mathrm{e}^{|z_2|^2}. \tag{17}$$

The expression for the total space of coherent states of two-dimensional problem on the horosphere is the tensor product of states that are constructed using operators of the same mode. These coherent states are determined by the formula

$$|z_1, z_2\rangle = \mathrm{e}^{z_1 a_x^+}\mathrm{e}^{z_2 a_y^+}|0,0\rangle, \tag{18}$$

where the vacuum state $|0,0\rangle$ is determined by the condition

$$a_x|0,0\rangle = a_y|0,0\rangle = 0. \tag{19}$$

There is the completeness criterion for coherent states which for the states on the horosphere (18) has the form

$$\int |z_1,z_2\rangle\langle z_1,z_2|d\mu(z_1,z_2) = \int |z_1\rangle\langle z_1|d\mu(z_1)\int |z_2\rangle\langle z_2|d\mu(z_2) = I, \tag{20}$$

and the uncertainty relations are

$$\Delta x\Delta q_x = \frac{h}{2}, \quad \Delta y\Delta q_y = \frac{h}{2}. \tag{21}$$

Thus, if the uncertainty of x or y is of the order R, then the uncertainty of momentum will be $h/2R$.

As you know, in the laboratory frame the incident particle (hadron) is flattened in the direction of the movement due to the Lorentz contraction. In this case, transverse degrees of freedom (x and y) are important, since at high energies the kinetic energy of the hadron constituents (partons) is much larger than the energy of their interaction.

Therefore, hadron moving at a speed close to the speed of light can be seen as a set of almost free partons. Since components of hadron move in unison before collision, therefore this state of a hadron can be considered as a coherent state of its transverse excitations, i.e., partons.

The main hypothesis is that the incident particle is a coherent state of partons, i.e., transverse excitations of a hadron.

It should be noted that expressions mentioned above are covariant.

Let us write expressions of coherent states in the occupation-number representation as

$$\begin{aligned} a_x^+|z_1\rangle &= a_x^+\sum\frac{z_1^{n_1}}{\sqrt{n_1!}}|n_1\rangle = \sum\frac{n_1 z_1^{n_1-1}}{\sqrt{n_1!}}|n_1\rangle, \\ a_y^+|z_2\rangle &= a_y^+\sum\frac{z_2^{n_2}}{\sqrt{n_2!}}|n_2\rangle = \sum\frac{n_2 z_2^{n_2-1}}{\sqrt{n_2!}}|n_2\rangle. \end{aligned} \tag{22}$$

The average number of quanta of excitation in each coherent state is defined by (see [9])

$$\bar{n}_1 = \mathrm{e}^{-|z_1|^2}\langle z_1|a_x^+ a_x|z_1\rangle = |z_1|^2, \\ \bar{n}_2 = \mathrm{e}^{-|z_2|^2}\langle z_2|a_y^+ a_y|z_2\rangle = |z_2|^2. \tag{23}$$

The total average number of excitations in both degrees of freedom is

$$\bar{n} = \bar{n}_1 + \bar{n}_2 = \mathrm{e}^{-z_1^2 - z_2^2}\langle z_1 z_2|a_x^+ a_x + a_y^+ a_y|z_2 z_1\rangle = |z_1|^2 + |z_2|^2, \tag{24}$$

and the distribution of the number of excitations for each of the degrees of freedom obeys a Poisson law

$$P(n) = \frac{\mathrm{e}^{-\bar{n}}\,\bar{n}^n}{n!}, \tag{25}$$

where $\bar{n} = n_1$ or $\bar{n} = n_2$.

Therefore, the number of excitations corresponding to coherent state of hadron (25) is a Poisson distribution and coincides with the multiplicity distribution in the multiperipheral model [3].

The coordinate representation of a coherent state is given by (see [10])

$$\langle x, y|z_1, z_2\rangle \propto \exp\left[\frac{i\sqrt{2}}{R}(\beta_1 x + \beta_2 y)\right]\exp\left\{\frac{-1}{2R^2}[(x - \sqrt{2}R\alpha_1)^2 + (y - \sqrt{2}R\alpha_2)^2]\right\}. \tag{26}$$

The density distribution of the coordinates is

$$|\langle x, y|z_1, z_2\rangle|^2 \propto \exp\left\{\frac{-1}{R^2}[(x - \sqrt{2}R\alpha_1)^2 + (y - \sqrt{2}R\alpha_2)^2]\right\}. \tag{27}$$

The corresponding momentum representation has the form

$$\langle q_x, q_y|z_1, z_2\rangle \propto \exp\left[iR\sqrt{2}(\alpha_1 q_x + \alpha_2 q_y)\right] \times \\ \times \exp\left\{\frac{-R^2}{2}\left[\left(q_x - \frac{\sqrt{2}}{R}\beta_1\right)^2 + \left(q_y - \frac{\sqrt{2}}{R}\beta_2\right)^2\right]\right\}, \tag{28}$$

and therefore the density distribution is

$$|\langle q_x, q_y|z_1, z_2\rangle|^2 \propto \exp\left\{-R^2\left[\left(q_x - \frac{\sqrt{2}}{R}\beta_1\right)^2 + \left(q_y - \frac{\sqrt{2}}{R}\beta_2\right)^2\right]\right\}, \tag{29}$$

where the following notation is used:

$$z_1 = \alpha_1 + i\beta_1, \quad \alpha_1 = |z_1|\cos\theta_1, \quad \beta_1 = |z_1|\sin\theta_1, \\ z_2 = \alpha_2 + i\beta_2, \quad \alpha_2 = |z_2|\cos\theta_2, \quad \beta_2 = |z_2|\sin\theta_2. \tag{30}$$

According to the Gaussian distribution in (21), $\sqrt{2}R\alpha_1$ and $\sqrt{2}R\alpha_2$ are the average coordinates of the particles. We note that in this case the coordinates characterize the size of hadron. Let us estimate minimal value of $\bar{n}$ assuming $n = n_1$ in (25). Using the size of the proton 0.84 fm (see [11]) and the relation

$$\sqrt{2}R\alpha_1 = \sqrt{2}R|z_1|\cos\theta_1 = \sqrt{2n}R\cos\theta_1 = r_0, \tag{31}$$

we get the minimum value $\bar{n} \approx 4$ for $\cos\,\theta_1 = 1$. We can obtain the number of excitations (partons) arbitrarily large varying $\cos\,\theta_1$.

At this stage, restrictions on the phase change can be offered only on the basis of heuristic arguments, for example, the symmetry. We consider $\theta_1 = \pi/4$ based on assumption of symmetry between the coordinate and momentum representations which follows from the explicit expressions (27) and (29). Then $\cos\,\theta_1 = \sin\,\theta_2 = 1/\sqrt{2}$ and we obtain $\bar{n} \approx 7.5$.

If using the expression $\sqrt{2n}R\cos\theta_1 = r_{0S}$ instead of (31) at the same θ_1, θ_2 we get $\bar{n} = 3$. It is obvious that $\theta_{1,2} \to \pi/2$ at high multiplicity n and correspondingly high energy.

Thus, we constructed a model of hadron as a coherent state of excitations on the horosphere of the Lobachevsky momentum space identified with partons and hadron structure functions which depend on number of partons.

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