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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

A PROBABILISTIC QUANTUM COMMUNICATION PROTOCOL USING MIXED ENTANGLED CHANNEL

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Qubits are realized as polarization states of photons or as superpositions of the spin states of electrons. In this paper, we propose a scheme to probabilistically teleport an unknown arbitrary two-qubit state using a nonmaximally entangled GHZ-like state and a nonmaximally entangled Bell state simultaneously as quantum channels. We also discuss the success probability of our scheme. We perform POVM in the protocol which is operationally advantageous. In our scheme, we show that the nonmaximal quantum resources perform better than maximal ones.

Кубиты реализованы как поляризационные состояния фотонов или суперпозиции спиновых состояний электронов. Предлагается схема вероятностного телепортирования неизвестного произвольного двухкубитового состояния с помощью немаксимально запутанного GHZ-подобного состояния и немаксимального состояния Бэлла одновременно в качестве квантовых каналов. Также обсуждается вероятность успеха предложенной схемы. Сделана положительная операторнозначная мера в протоколе, которая является операционно выгодной. В рамках данной схемы показано, что немаксимальные квантовые источники проявляют себя лучше максимальных.

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INTRODUCTION

Quantum teleportation is a method of transmission of a quantum state using entangled resources with classical communication and local operations. Originally the quantum teleportation was presented by Bennett et al. [1] for arbitrary single-qubit quantum states by using a maximally entangled Bell state. After that several theoretical and experimental aspects of different teleportation protocols with various types of quantum resources appeared in the literature, some of which are noted in [2–15]. We describe some of the works below. In 1994, Vaidman [2] proposed a teleportation protocol of system quantum states with continuous variables. In 2004, Zheng [5] presented a scheme for approximate conditional teleportation of an unknown atomic state without the Bell-state measurement. In 2005, Rigolin [6] proposed a quantum teleportation of an arbitrary two-qubit state and its relation to multipartite entanglement. In 2005, Cardoso et al. [7] proposed a teleportation protocol of entangled states without Bell-state measurement. In 2010, Tsai and Hwang [11] presented a teleportation of a pure EPR state using GHZ-like state. In 2014, Nandi and Mazumdar [12] presented a

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teleportation protocol of a more general two-qubit state using the GHZ-like state as quantum channel. In 2014, Zhu [13] presented a quantum teleportation of an arbitrary two-qubit state via maximally entangled GHZ-like states. In 2011, Nie et al. [14] attempted the teleportation of a three-qubit state which is an arbitrary GHZ-type state using four-qubit cluster states as quantum channel. In 2016, Li et al. [15] described a perfect teleportation protocol for a more general three-qubit state using four-qubit cluster states as quantum channel.

The protocol given by Bennett et al. [1] is a perfect teleportation protocol; that is, the transmission of qubit state is performed with fidelity one. Many of the subsequent teleportation protocols are not perfect — the state obtained after transmission has less fidelity than the one with the original state. Probabilistic teleportation is a method by which a state is teleported perfectly with some probability. There is a chance of failure with nonzero probability in which case the quantum information is totally lost. There are two different schemes of probabilistic teleportation, in one of which the sender performs an unambiguous quantum state discrimination [16], while in the other the receiver performs for extracting the quantum state [17]. In both schemes, the probability for total information loss is nonzero. Several works on probabilistic teleportation using different types of entanglement resources are obtainable in [17–21].

In 2000, Wan-Li et al. [17] presented a probabilistic teleportation of a single-qubit arbitrary state through a partially entangled quantum channel. In the same year, Shi et al. [18] proposed a probabilistic teleportation of two-particle entangled state by pure entangled three-particle state. In 2002, Agrawal and Pati [19] proposed a probabilistic teleportation protocol for an unknown two-qubit state using nonmaximally entangled state as a shared resource. In 2003, Yan and Wang [20] proposed a probabilistic and controlled teleportation scheme of the unknown one-particle quantum states and unknown two-particle quantum states. In 2013, Yu and Wu [21] proposed a probabilistic teleportation of an unknown three-qubit entangled state via a quantum channel of five-qubit nonmaximally entangled cluster state.

In this paper, we present a probabilistic teleportation scheme for an arbitrary two-qubit quantum state by using simultaneously two types of entangled resources, namely, the nonmaximally entangled GHZ-like state and a nonmaximally entangled Bell state. The operations we perform are POVM measurement, local operations, and classical communication.

We mention two specialities of the present protocol. First, POVM is used in place of the usual quantum measurements with respect to the orthogonal bases. Second, the nonmaximally entangled resources perform better in our scheme than the maximally entangled ones. Demonstration of the latter is the main motivation of the paper.

1. THE MAIN RESULTS

Alice wants to transmit an unknown two-qubit entangled state to a distant receiver Bob. An arbitrary two-qubit pure quantum state is described as

$$|\psi\rangle_{12} = (a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle)_{12}, \tag{1}$$

where the parameter $\sum\limits_{i=0,j=0}^{1} |a_{ij}|^2 = 1$. Alice wants to teleport the state $|\psi\rangle_{12}$ to Bob.

Suppose two quantum channels shared between Alice and Bob: one nonmaximally entangled GHZ-like state and one nonmaximally entangled Bell state given by

$$|\psi\rangle_{345} = (a|000\rangle + b|011\rangle + c|101\rangle + d|110\rangle)_{345} \quad \text{and}$$
$$|\psi\rangle_{67} = (e|00\rangle + f|11\rangle)_{67},$$

where a, b, c, d, e, and f's are all real and nonzero, $a^2 + b^2 + c^2 + d^2 = 1$, and $e^2 + f^2 = 1$; the particles 3, 4, and 6 are in the possession of the sender Alice, the particles 5 and 7 belong to the receiver Bob. It may be noted that maximally entangled GHZ-like state $(1/2)(|000\rangle + |011\rangle + |101\rangle + |110\rangle)$ has been used in communication protocols in [13].

Therefore, the state of the whole system composed of an unknown two-qubit state and two quantum channels is given by

$$|\Psi\rangle_{1234567} = |\psi\rangle_{12} \otimes |\psi\rangle_{345} \otimes |\psi\rangle_{67}. \tag{2}$$

Consider GHZ-like category mutually orthogonal states given by

$$|\xi^{\pm}\rangle = \frac{1}{2}\{(|100\rangle + |111\rangle) \pm (|001\rangle + |010\rangle)\} \quad \text{and}$$
$$|\eta^{\pm}\rangle = \frac{1}{2}\{(|000\rangle + |011\rangle) \pm (|101\rangle + |110\rangle)\},$$

and the Bell basis given by

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad |\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).$$

We choose the POVM as follows:

$$\begin{aligned}
E_1 &= |\xi^+\rangle_{134}|\Phi^+\rangle_{26134}\langle\xi^+|_{26}\langle\Phi^+|, & E_2 &= |\xi^-\rangle_{134}|\Phi^+\rangle_{26134}\langle\xi^-|_{26}\langle\Phi^+|,\\
E_3 &= |\xi^+\rangle_{134}|\Phi^-\rangle_{26134}\langle\xi^+|_{26}\langle\Phi^-|, & E_4 &= |\xi^-\rangle_{134}|\Phi^-\rangle_{26134}\langle\xi^-|_{26}\langle\Phi^-|,\\
E_5 &= |\eta^+\rangle_{134}|\Phi^+\rangle_{26134}\langle\eta^+|_{26}\langle\Phi^+|, & E_6 &= |\eta^-\rangle_{134}|\Phi^+\rangle_{26134}\langle\eta^-|_{26}\langle\Phi^+|,\\
E_7 &= |\eta^+\rangle_{134}|\Phi^-\rangle_{26134}\langle\eta^+|_{26}\langle\Phi^-|, & E_8 &= |\eta^-\rangle_{134}|\Phi^-\rangle_{26134}\langle\eta^-|_{26}\langle\Phi^-|,\\
E_9 &= |\eta^+\rangle_{134}|\Psi^+\rangle_{26134}\langle\eta^+|_{26}\langle\Psi^+|, & E_{10} &= |\eta^-\rangle_{134}|\Psi^+\rangle_{26134}\langle\eta^-|_{26}\langle\Psi^+|,\\
E_{11} &= |\eta^+\rangle_{134}|\Psi^-\rangle_{26134}\langle\eta^+|_{26}\langle\Psi^-|, & E_{12} &= |\eta^-\rangle_{134}|\Psi^-\rangle_{26134}\langle\eta^-|_{26}\langle\Psi^-|,\\
E_{13} &= |\xi^+\rangle_{134}|\Psi^+\rangle_{26134}\langle\xi^+|_{26}\langle\Psi^+|, & E_{14} &= |\xi^-\rangle_{134}|\Psi^+\rangle_{26134}\langle\xi^-|_{26}\langle\Psi^+|,\\
E_{15} &= |\xi^+\rangle_{134}|\Psi^-\rangle_{26134}\langle\xi^+|_{26}\langle\Psi^-|, & E_{16} &= |\xi^-\rangle_{134}|\Psi^-\rangle_{26134}\langle\xi^-|_{26}\langle\Psi^-|,\\
E_{17} &= I - \sum_{i=0}^{16} E_i. & &
\end{aligned}$$

The possible outcomes for Bob after Alice performs the POVM measurement on her pairs of three qubits (1, 3, 4) and two qubits (2, 6) are given in Table 1.

For the case of E_{17}, the protocol fails.

Table 1

Alice's measurement	After the measurement outcome state of Bob
E_1	$\frac{1}{2\sqrt{2}}\{a_{10}(a+d)e\|00\rangle + a_{11}(a+d)f\|01\rangle + a_{00}(b+c)e\|10\rangle + a_{01}(b+c)f\|11\rangle\}_{57}$
E_2	$\frac{1}{2\sqrt{2}}\{a_{10}(a+d)e\|00\rangle + a_{11}(a+d)f\|01\rangle - a_{00}(b+c)e\|10\rangle - a_{01}(b+c)f\|11\rangle\}_{57}$
E_3	$\frac{1}{2\sqrt{2}}\{a_{10}(a+d)e\|00\rangle - a_{11}(a+d)f\|01\rangle + a_{00}(b+c)e\|10\rangle - a_{01}(b+c)f\|11\rangle\}_{57}$
E_4	$\frac{1}{2\sqrt{2}}\{a_{10}(a+d)e\|00\rangle - a_{11}(a+d)f\|01\rangle - a_{00}(b+c)e\|10\rangle + a_{01}(b+c)f\|11\rangle\}_{57}$
E_5	$\frac{1}{2\sqrt{2}}\{a_{00}(a+d)e\|00\rangle + a_{01}(a+d)f\|01\rangle + a_{10}(b+c)e\|10\rangle + a_{11}(b+c)f\|11\rangle\}_{57}$
E_6	$\frac{1}{2\sqrt{2}}\{a_{00}(a+d)e\|00\rangle + a_{01}(a+d)f\|01\rangle - a_{10}(b+c)e\|10\rangle - a_{11}(b+c)f\|11\rangle\}_{57}$
E_7	$\frac{1}{2\sqrt{2}}\{a_{00}(a+d)e\|00\rangle - a_{01}(a+d)f\|01\rangle + a_{10}(b+c)e\|10\rangle - a_{11}(b+c)f\|11\rangle\}_{57}$
E_8	$\frac{1}{2\sqrt{2}}\{a_{00}(a+d)e\|00\rangle - a_{01}(a+d)f\|01\rangle - a_{10}(b+c)e\|10\rangle + a_{11}(b+c)f\|11\rangle\}_{57}$
E_9	$\frac{1}{2\sqrt{2}}\{a_{01}(a+d)e\|00\rangle + a_{00}(a+d)f\|01\rangle + a_{11}(b+c)e\|10\rangle + a_{10}(b+c)f\|11\rangle\}_{57}$
E_{10}	$\frac{1}{2\sqrt{2}}\{a_{01}(a+d)e\|00\rangle + a_{00}(a+d)f\|01\rangle - a_{11}(b+c)e\|10\rangle - a_{10}(b+c)f\|11\rangle\}_{57}$
E_{11}	$\frac{1}{2\sqrt{2}}\{-a_{01}(a+d)e\|00\rangle + a_{00}(a+d)f\|01\rangle - a_{11}(b+c)e\|10\rangle + a_{10}(b+c)f\|11\rangle\}_{57}$
E_{12}	$\frac{1}{2\sqrt{2}}\{-a_{01}(a+d)e\|00\rangle + a_{00}(a+d)f\|01\rangle + a_{11}(b+c)e\|10\rangle - a_{10}(b+c)f\|11\rangle\}_{57}$
E_{13}	$\frac{1}{2\sqrt{2}}\{a_{11}(a+d)e\|00\rangle + a_{10}(a+d)f\|01\rangle + a_{01}(b+c)e\|10\rangle + a_{00}(b+c)f\|11\rangle\}_{57}$
E_{14}	$\frac{1}{2\sqrt{2}}\{a_{11}(a+d)e\|00\rangle + a_{10}(a+d)f\|01\rangle - a_{01}(b+c)e\|10\rangle - a_{00}(b+c)f\|11\rangle\}_{57}$
E_{15}	$\frac{1}{2\sqrt{2}}\{-a_{11}(a+d)e\|00\rangle + a_{10}(a+d)f\|01\rangle - a_{01}(b+c)e\|10\rangle + a_{00}(b+c)f\|11\rangle\}_{57}$
E_{16}	$\frac{1}{2\sqrt{2}}\{-a_{11}(a+d)e\|00\rangle + a_{10}(a+d)f\|01\rangle + a_{01}(b+c)e\|10\rangle - a_{00}(b+c)f\|11\rangle\}_{57}$

The probabilities of obtaining the measurement results E_1 to E_{16} are given below:
for $E_i,\ i = 1,2,3,4,\ P_1 = \frac{1}{8}\{|a_{10}|^2(a+d)^2e^2 + |a_{11}|^2(a+d)^2f^2 + |a_{00}|^2(b+c)^2e^2 + |a_{01}|^2(b+c)^2f^2\}$;
for $E_i,\ i = 5,6,7,8,\ P_2 = \frac{1}{8}\{|a_{00}|^2(a+d)^2e^2 + |a_{01}|^2(a+d)^2f^2 + |a_{10}|^2(b+c)^2e^2 + |a_{11}|^2(b+c)^2f^2\}$;
for $E_i,\ i = 9,10,11,12,\ P_3 = \frac{1}{8}\{|a_{01}|^2(a+d)^2e^2 + |a_{00}|^2(a+d)^2f^2 + |a_{11}|^2(b+c)^2e^2 + |a_{10}|^2(b+c)^2f^2\}$;

for E_i, $i = 13, 14, 15, 16$, $P_4 = \frac{1}{8}\{|a_{11}|^2(a+d)^2e^2 + |a_{10}|^2(a+d)^2f^2 + |a_{01}|^2(b+c)^2e^2 + |a_{00}|^2(b+c)^2f^2\}$;
for E_{17}, $P_5 = 1 - P_1 - P_2 - P_3 - P_4$.

Now Alice informs Bob of her measurement result, and Bob gives a corresponding general evolution. To carry out the general evolution, Bob introduces an auxiliary qubit in an initial state $|0\rangle_{\text{aux}}$ and makes another unitary transformation U_{aux} under the basis $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}_{58\text{aux}}$, the form of collective unitary transformation U_{aux} which is a 8×8 matrix given by $U_{\text{aux}} = \begin{pmatrix} A & B \\ B & -A \end{pmatrix}$, where

$$A = \begin{pmatrix} \frac{(b+c)f}{(a+d)e} & 0 & 0 & 0 \\ 0 & \frac{(b+c)f}{(a+d)f} & 0 & 0 \\ 0 & 0 & \frac{(b+c)f}{(b+c)e} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$B = \begin{pmatrix} \sqrt{1 - \left\{\frac{(b+c)f}{(a+d)e}\right\}^2} & 0 & 0 & 0 \\ 0 & \sqrt{1 - \left\{\frac{(b+c)f}{(a+d)f}\right\}^2} & 0 & 0 \\ 0 & 0 & \sqrt{1 - \left\{\frac{(b+c)f}{(b+c)e}\right\}^2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

After applying the unitary transformation U_{aux}, Bob performs a measurement on the auxiliary particle. If the measurement result is $|0\rangle_{\text{aux}}$, then by applying the appropriate unitary operations U_1 to U_{16} given below, Bob gets the desired state corresponding to the 16 states available to Bob. If the measurement result is $|1\rangle_{\text{aux}}$, the teleportation protocol fails. The details are given in Table 2.

Here the unitary operations U_i's are given by

$$U_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad U_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad U_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$U_4 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad U_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

Table 2

Alice's measurement	After applying $U_{\rm aux}$ and the measurement outcome $\|0_{\rm aux}\rangle$	Unitary operator to achieve the desired state
E_1	$\{a_{10}\|00\rangle + a_{11}\|01\rangle + a_{00}\|10\rangle + a_{01}\|11\rangle\}_{57}$	U_1
E_2	$\{a_{10}\|00\rangle + a_{11}\|01\rangle - a_{00}\|10\rangle - a_{01}\|11\rangle\}_{57}$	U_2
E_3	$\{a_{10}\|00\rangle - a_{11}\|01\rangle + a_{00}\|10\rangle - a_{01}\|11\rangle\}_{57}$	U_3
E_4	$\{a_{10}\|00\rangle - a_{11}\|01\rangle - a_{00}\|10\rangle + a_{01}\|11\rangle\}_{57}$	U_4
E_5	$\{a_{00}\|00\rangle + a_{01}\|01\rangle + a_{10}\|10\rangle + a_{11}\|11\rangle\}_{57}$	U_5
E_6	$\{a_{00}\|00\rangle + a_{01}\|01\rangle - a_{10}\|10\rangle - a_{11}\|11\rangle\}_{57}$	U_6
E_7	$\{a_{00}\|00\rangle - a_{01}\|01\rangle + a_{10}\|10\rangle - a_{11}\|11\rangle\}_{57}$	U_7
E_8	$\{a_{00}\|00\rangle - a_{01}\|01\rangle - a_{10}\|10\rangle + a_{11}\|11\rangle\}_{57}$	U_8
E_9	$\{a_{01}\|00\rangle + a_{00}\|01\rangle + a_{11}\|10\rangle + a_{10}\|11\rangle\}_{57}$	U_9
E_{10}	$\{a_{01}\|00\rangle + a_{00}\|01\rangle - a_{11}\|10\rangle - a_{10}\|11\rangle\}_{57}$	U_{10}
E_{11}	$\{-a_{01}\|00\rangle + a_{00}\|01\rangle - a_{11}\|10\rangle + a_{10}\|11\rangle\}_{57}$	U_{11}
E_{12}	$\{-a_{01}\|00\rangle + a_{00}\|01\rangle + a_{11}\|10\rangle - a_{10}\|11\rangle\}_{57}$	U_{12}
E_{13}	$\{a_{11}\|00\rangle + a_{10}\|01\rangle + a_{01}\|10\rangle + a_{00}\|11\rangle\}_{57}$	U_{13}
E_{14}	$\{a_{11}\|00\rangle + a_{10}\|01\rangle - a_{01}\|10\rangle - a_{00}\|11\rangle\}_{57}$	U_{14}
E_{15}	$\{-a_{11}\|00\rangle + a_{10}\|01\rangle - a_{01}\|10\rangle + a_{00}\|11\rangle\}_{57}$	U_{15}
E_{16}	$\{-a_{11}\|00\rangle + a_{10}\|01\rangle + a_{01}\|10\rangle - a_{00}\|11\rangle\}_{57}$	U_{16}

$$U_7 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad U_8 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U_9 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$U_{10} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad U_{11} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad U_{12} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

$$U_{13} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad U_{14} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad U_{15} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$U_{16} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

As an illustration, corresponding to E_1, the unitary transformation $U_{\rm aux}$ transforms the state

$$\frac{1}{2\sqrt{2}}\{a_{10}(a+d)e|00\rangle|0\rangle_{\rm aux} + a_{11}(a+d)f|01\rangle|0\rangle_{\rm aux} + a_{00}(b+c)e|10\rangle|0\rangle_{\rm aux} + a_{01}(b+c)f|11\rangle|0\rangle_{\rm aux}\}$$

to the state

$$|\Psi\rangle_{58\mathrm{aux}} = \frac{(b+c)f}{2\sqrt{2}}\{a_{10}|00\rangle + a_{11}|01\rangle + a_{00}|10\rangle + a_{01}|11\rangle\}_{57}|0\rangle_{\mathrm{aux}}+$$
$$+\frac{1}{2\sqrt{2}}\{a_{10}(a+d)\,e\sqrt{1-\left\{\frac{(b+c)f}{(a+d)e}\right\}^2}|00\rangle + a_{11}(a+d)f\sqrt{1-\left\{\frac{(b+c)f}{(a+d)f}\right\}^2}|01\rangle+$$
$$+a_{00}(b+c)\,e\sqrt{1-\left\{\frac{(b+c)f}{(b+c)e}\right\}^2}|10\rangle\}_{57}|1\rangle_{\mathrm{aux}}.$$

Then from the measurement on the auxiliary particle it follows that if the measurement result is $|0\rangle_{\mathrm{aux}}$, then the corresponding state is given by $\{a_{10}|00\rangle + a_{11}|01\rangle + a_{00}|10\rangle + a_{01}|11\rangle\}_{57}$; and after applying the unitary transformation U_1, Bob gets the desired state which is given by $\{a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle\}_{57}$, that is, the teleportation is successful. The probability of getting $|0\rangle_{\mathrm{aux}}$ in the single-qubit measurement is $\frac{(b+c)^2f^2}{8}$. So the probability of success in case of the POVM yielding E_1 is $P_1\frac{(b+c)^2f^2}{8}$. Similarly we can calculate the probabilities corresponding to the measurements $E_2, E_3, \ldots, E_{16}$ being performed. The protocol fails for the measurement E_{17}. Then the total probability of success for the teleportation process is

$$P = 4P_1\frac{(b+c)^2f^2}{8} + 4P_2\frac{(b+c)^2f^2}{8} + 4P_3\frac{(b+c)^2f^2}{8} + 4P_4\frac{(b+c)^2f^2}{8} =$$
$$= \frac{(b+c)^2f^2}{2}\{P_1+P_2+P_3+P_4\} =$$
$$= \frac{(b+c)^2f^2}{2\times 8}\{(a+d)^2e^2 + (a+d)^2f^2 + (b+c)^2e^2 + (b+c)^2f^2\} =$$
$$= \frac{(b+c)^2f^2}{16}\{(a+d)^2 + (b+c)^2\} =$$
$$= \frac{(b+c)^2f^2}{16}\{a^2+d^2+b^2+c^2+2ad+2bc\} = \frac{(b+c)^2f^2}{16}\{1+2ad+2bc\}.$$

2. DISCUSSION AND CONCLUSIONS

The teleportation scheme was first advanced with a maximally entangled Bell state [1]. Gradually it has been established that nonmaximally entangled states have also their roles in problems of quantum information and communication. There are entanglement concentration protocols (ECPs) for generating maximal entanglements. But the use of nonmaximally entangled states in communications can be more encouraging rather than passing through an ECP and then opting for a communication protocol with maximally entangled channels. This is one of the goals of probabilistic schemes.

Normally the probabilistic teleportation protocols use nonmaximally entangled quantum channels in contrast to the perfect teleportations which are generally performed with the help of maximally entangled states. There are also exceptions to the latter fact. For instance, it is possible to teleport perfectly arbitrary single-qubit states using class of W states which are not maximally entangled [22]. In several cases the probabilistic teleportation tends to perfect teleportation, as we use more entangled resources, and reduces to that when the quantum channel is maximal. Thus, the success probability can be increased to any degree by proper adjustment of parameters characterizing the quantum channel thereby making it nearer to a maximally entangled channel.

Our protocol differs from the above category in that particular choice of the channels, as maximally entangled resources do not supply us with the maximum possible probability of success. This is the reason why the success probability cannot be arbitrarily increased by adjustment of parameters. In fact, the use of maximal entanglement which results with the particular choice of parameters as $a = b = c = d = 1/2$, and $e = f = 1/\sqrt{2}$, gives a success probability 0.0625, whereas the maximum probability of success in our scheme is 0.25. Thus, one implication of our result is that the maximal resource need not always give the maximum probability of success. Neither of our protocols is related in the limit to the perfect teleportation protocol. The nonmaximal resources perform better in our scheme.

Another speciality of our protocol is that we use POVM which is operationally more advantageous compared to measurements with respect to orthogonal bases which are generally performed in perfect as well as probabilistic teleportations.

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