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QUANTUM FIELD THEORY WITH THREE-DIMENSIONAL VECTOR TIME

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The relativistic invariant plane wave decompositions of electromagnetic and spinor fields developing along arbitrary time trajectories in six-dimensional space-time are found. The quanta of the fields quantized in Hilbert space with indefinite metric have always positively defined energies. Though, in contrast to the customary one-time theory, the mass hyperboloid $p_\mu p^\mu = m^2$ is connected, the separation of particles and antiparticles can be done uniquely due to the condition of time irreversibility. Particles and antiparticles are always separated by an energy gap $\Delta E \geqslant 2m$. A Hamiltonian and the corresponding scattering matrix describing the interactions of particles with different time trajectories are deduced.

Найдены релятивистски-инвариантные разложения на плоские волны электромагнитного и спинорного полей, эволюционирующих вдоль произвольной временной траектории в шестимерном пространстве-времени. Кванты полей, проквантованных в гильбертовом пространстве с индефинитной метрикой, имеют положительно определенные энергии. Хотя в отличие от обычной одновременной теории полости массового гиперболоида $p_\mu p^\mu = m^2$ перекрываются, разделение квантов поля на частицы и античастицы может быть выполнено однозначно благодаря условию необратимости времени. Частица и античастица всегда разделены энергетической щелью $\Delta E \geqslant 2m$. Построены гамильтониан и соответствующая матрица рассеяния, описывающие взаимодействия частиц с произвольными временными траекториями.

INTRODUCTION

The exciting our imagination hypothesis for a possible multidimensionality of world time and an existence of hidden time axis has already been considered from various viewpoints (see [1–10] where one can find detailed bibliography). The investigations of theoretical and experimental consequences of this hypothesis did not discover any contradictions[2]. At present there are no evidences that warrant or, on the contrary, refuse the existence in our world of additional time axis. One may think that the answer will be found by means of the new huge gravitational detectors which can be able to observe the predicted multitime components of gravitational waves created in cosmic cataclysms [9] or the discovery of new heavy spinor particles predicted by the multitime Dirac equation [13].

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[2] The predicted in paper [12] too large advance of the Mercury perihelion (2.3 times larger than the observed one) is due to a used lame potential and occurs within the limits of the experimental errors under more accurate consideration [10].

One may also hope to observe the hidden times in virtual quantum processes occurring in small space-time intervals. To investigate such a possibility we need the respective quantum mechanical generalization of the multitime relations developed for macroscopic space-time scales. The first steps in this direction have been done in papers [8, 13, 14] where the multitime Dirac equation has been studied. At the next step one must develop a procedure of field quantization in six-dimensional space-time

$$\hat{\mathbf{x}} = (\mathbf{x}, \hat{\mathbf{t}}) = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{t}_1, \mathbf{t}_2, \mathbf{t}_3)$$

for arbitrary time vectors $\hat{t}$ of emitted and absorbed quanta[1]. That will allow one to construct the scattering matrix describing interactions of particles moving along different time directions.

Our goal is to check up how much the hypothesis of time multidimensionality is not contradictory in the region of microscopic phenomena. For this purpose we consider the relativistic invariant quantization of electromagnetic and spinor fields and the construction of the respective S-matrix.

1. RELATIVISTIC INVARIANT PLANE WAVE DECOMPOSITION IN SIX-DIMENSIONAL SPACE-TIME

The electromagnetic potential at a point $\hat{\mathbf{x}} = (\mathbf{x}, \hat{\eta}t)$, where $\hat{\eta}^2 = 1$, can be presented as a six-dimensional Fourier integral

$$\hat{\mathbf{A}}(\hat{\mathbf{x}}) = \int d^6p\, \delta(\hat{\mathbf{p}}^2)\hat{\mathbf{A}}(\hat{\mathbf{x}}, \hat{\mathbf{p}}) = \int d^3\tau\, \delta(\hat{\tau}^2 - 1)\hat{\mathbf{A}}_\tau(\hat{\mathbf{x}}), \tag{1}$$

where the bundle of waves along the time trajectory $\hat{\tau}$ is

$$\hat{\mathbf{A}}_\tau(\hat{\mathbf{x}}) = \int d^3p\, p^{-2}\hat{\mathbf{A}}(\hat{\mathbf{x}}, \hat{\mathbf{p}}) = \int d^3p \left[\hat{\mathbf{a}}(\hat{\mathbf{p}})\, \mathrm{e}^{i\hat{\mathbf{p}}\hat{\mathbf{x}}} + \hat{\mathbf{a}}^*(\hat{\mathbf{p}})\, \mathrm{e}^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}}\right]. \tag{2}$$

The amplitude is $\hat{\mathbf{a}}^*(\hat{\mathbf{p}}) = \hat{\mathbf{a}}(-\hat{\mathbf{p}})$, as a consequence of the relation $\hat{\mathbf{A}}(\hat{\mathbf{x}}) = \hat{\mathbf{A}}^*(\hat{\mathbf{x}})$, and the energy vector of every wave is directed along its time trajectory, $\hat{\mathbf{p}} = (\mathbf{p}, p\hat{\tau}),\ p = |\mathbf{p}|$. The scalar product is $\hat{\mathbf{p}}\hat{\mathbf{x}} = \mathbf{p}\mathbf{x} - pt\hat{\tau}\hat{\eta}$. The δ-function in Eq. (1) is due to the momentum-energy conservation law and shows that the integration is performed only over a sphere with unit radius.

In the one-time limit, Eq. (1) is an identity

$$A_k(\hat{\mathbf{x}}) = (1/2)\left[A_{\tau,k}(\mathbf{x}, t) + A_{-\tau,k}(\mathbf{x}, t)\right] = A_k(\mathbf{x}, t), \tag{3}$$

$k = 1, \ldots, 4,\ \tau = 1$.

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[1]In what follows the three-dimensional vectors in x- and t-subspaces will be denoted, respectively, by bold symbols and by a hat, six-dimensional vectors will be marked by bold symbols with a hat. (In manuscripts it is convenient to use the notations $\bar{x}$, $\hat{x}$ and $\hat{\bar{x}}$.) We suppose also that co- and contravariant vectors are distinguished by the sign of their time components, e.g., $(\hat{\mathbf{x}})_\mu = (\mathbf{x}, -c\hat{t})^T_\mu$, $(\hat{\mathbf{x}})^\mu = (\mathbf{x}, c\hat{t})^{T\mu}$. So, the scalar product $\hat{\mathbf{a}}\hat{\mathbf{b}} = g^{\mu\nu}a_\mu b_\nu = \mathbf{ab} - \hat{a}\hat{b}$ where the metric tensor $g^{\mu\nu} \equiv g_{\mu\nu}$ has the diagonal elements $(1, 1, 1, -1, -1, -1)$. The «six-dimensional nabla» $\hat{\boldsymbol{\nabla}} = (\nabla, -\hat{\nabla})$ and $\nabla_\mu \equiv \partial/\partial x_\mu$. As a rule, we shall also suppose that the Latin and Greek indices take values $k = 1, \ldots, 3, \mu = 1, \ldots, 6$ and the constants $\hbar = c = 1$.

Similarly, the relativistic invariant plane wave decomposition of a spinor field is

$$\psi(\hat{\mathbf{x}}) = \int d^6p\,\delta(\hat{\mathbf{p}}^2)\psi(\hat{\mathbf{x}},\hat{\mathbf{p}}) = \int d^3\tau\,\delta(\hat{\tau}^2-1)\psi_\tau(\hat{\mathbf{x}}), \tag{4}$$

$$\psi_\tau(\hat{\mathbf{x}}) = \sum_{s=1}^{4}\int d^3p\,\left[a_s(\hat{\mathbf{p}})U_s(\hat{\mathbf{p}})\,\mathrm{e}^{i\hat{\mathbf{p}}\hat{\mathbf{x}}} + b_s^+(\hat{\mathbf{p}})V_s(-\hat{\mathbf{p}})\,\mathrm{e}^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}}\right] \tag{5}$$

with $\hat{\mathbf{p}} = (\mathbf{p}, \hat{\tau}E_p)$, and $E_p = (\mathbf{p}^2+m^2)^{1/2}$, $\hat{\mathbf{p}}\hat{\mathbf{x}} = \mathbf{px} - E_p t\hat{\tau}\hat{\eta}$. The four eight-component spinors U_s and V_s are the linearly independent solutions of the multitime Dirac equations

$$(\hat{\mathbf{p}}\hat{\boldsymbol{\gamma}}+m)U_s(\hat{\mathbf{p}}) = 0, \quad (\hat{\mathbf{p}}\hat{\boldsymbol{\gamma}}+m)V_s(\hat{\mathbf{p}}) = 0 \tag{6}$$

with the (8×8)-matrices γ_μ satisfying the commutation relation $[\gamma_\mu,\gamma_\nu]_+ = -2g_{\mu\nu}$. The spinors have the normalization

$$\bar{U}_r(\hat{\mathbf{p}})U_s(\hat{\mathbf{p}}) = -\bar{V}_r(\hat{\mathbf{p}})V_s(\hat{\mathbf{p}}) = (-1)^r(m/E_p)\delta_{rs}. \tag{7}$$

Like Eq. (1), the decomposition (4) describes a bundle of plane waves with different time trajectories $\hat{\tau}$.

The six-dimensional potential $\hat{\mathbf{A}}(\hat{\mathbf{x}})$ and the multitime solutions $\psi(\hat{\mathbf{x}})$ of Dirac equation are described more comprehensively in [4, 15] and [8, 13].

2. QUANTIZATION OF ELECTROMAGNETIC FIELD

A six-dimensional momentum-energy vector of electromagnetic field at a three-dimensional time point $\hat{t} = t\hat{\eta}$, where $\hat{\eta}$ is again a unit time vector, is given by

$$\begin{aligned}P_\mu = \int d^3x\,T^{\mu 3+k}\eta_k = \int d^3x\left(\frac{\partial A_\alpha}{\partial x_{3+k}}\frac{\partial A_\alpha}{\partial x_\mu} - \frac{1}{2}g_{\mu 3+k}\frac{\partial A_\alpha}{\partial x_\beta}\frac{\partial A_\alpha}{\partial x_\beta}\right)\eta^k = \\ = (2\pi)^3\int d^3\tau\,d^3\tau'\,d^3p\,d^3p'\,\delta(\hat{\tau}^2-1)\delta(\hat{\tau}'^2-1)p'_\mu p\hat{\tau}\hat{\eta}\times \\ \times\left[\hat{\mathbf{a}}(\hat{\mathbf{p}})\hat{\mathbf{a}}^+(\hat{\mathbf{p}}')\,\mathrm{e}^{-i(\hat{p}-\hat{p}')\hat{t}} + \hat{\mathbf{a}}^+(\hat{\mathbf{p}})\hat{\mathbf{a}}(\hat{\mathbf{p}}')\,\mathrm{e}^{i(\hat{p}-\hat{p}')\hat{t}}\right],\end{aligned} \tag{8}$$

where $\hat{\mathbf{p}} = (\mathbf{p}, \hat{\tau}p)$; $\hat{\mathbf{p}}' = (\mathbf{p}, \hat{\tau}'p)$, and we took into account that

$$p'_\mu\hat{p}\hat{\eta} - (1/2)\hat{\mathbf{p}}\hat{\mathbf{p}}'\delta_{\mu 3+k}\eta^k = p'_\mu p\hat{\tau}\hat{\eta}.$$

Let us replace now $4p(\pi)^{3/2}\,\hat{\mathbf{a}} \to \hat{\mathbf{a}}$ and suppose that the new amplitudes satisfy the commutation relations

$$\left[a_\mu(\hat{\mathbf{p}}), a_\nu^+(\hat{\mathbf{p}}')\right] = g_{\mu\nu}\delta(\hat{\mathbf{p}}-\hat{\mathbf{p}}')\varepsilon(\hat{p}), \tag{9}$$

$$\left[a_\mu(\hat{\mathbf{p}}), a_\nu(\hat{\mathbf{p}}')\right] = \left[a_\mu^+(\hat{\mathbf{p}}), a_\nu^+(\hat{\mathbf{p}}')\right] = 0, \tag{10}$$

where the presence of $g_{\mu\nu}$ means that the operators of creation and absorption of the particles with the polarizations $\mu > 3$ switch the roles. (Like the customary one-time theory, such a quantization assumes the indefinite metric in Hilbert space [16, 17].) The necessity of the staircase function $\varepsilon(\hat{p}) = \varepsilon(p\tau_1)\varepsilon(p\tau_2)\varepsilon(p\tau_3)$, where $\varepsilon(p\tau_k) = 1$, if $p\tau_k \geqslant 0$ and $\varepsilon(p\tau_k) = 0$ for $p\tau_k < 0$, is stipulated by the time irreversibility $d\hat{t}(t)/dt \geqslant 0$ due to which the time trajectories pass from the seventh into the first quadrant of time subspace.

If we take now into account the commutation relation and note that the exponents with $\hat{\mathbf{p}} \neq \hat{\mathbf{p}}'$ in Eq. (8) give zeros[1], then Eq. (8) can be reduced to the form

$$P_\mu = \int d^3\tau\, \delta(\hat{\tau}^2 - 1) P_\mu(\hat{\tau}, \hat{\eta}) = \\ = 2\int d^3\tau\, \delta(\hat{\tau}^2 - 1)\hat{\tau}\hat{\eta} \int d^3p\, p_\mu \sum_{\nu=1}^{6} n_\nu(\hat{\mathbf{p}})\varepsilon(\hat{p}) + P_{0\mu}, \quad (11)$$

where $\hat{\mathbf{p}} = (\mathbf{p}), \mathbf{p}\hat{\tau}$. The quantities $n_k(\hat{\mathbf{p}}) = \mathbf{a}_k^+(\hat{\mathbf{p}})\mathbf{a}_k(\hat{\mathbf{p}})$ and $n_{k+3}(\hat{\mathbf{p}}) = -\hat{a}_{k+3}^+(\hat{\mathbf{p}})\hat{a}_{k+3}(\hat{\mathbf{p}})$, $k \leqslant 3$, are the numbers of photons with the momentum-energy $\hat{\mathbf{p}}\varepsilon(\hat{p})$ and the polarization ν moving along the time direction $\hat{\tau}$. Due to the staircase functions $\varepsilon(p_k)$ all components of particle energies $\hat{\mathbf{p}}$ have always positive values. The vacuum energy $\hat{P}_0$ is an infinite large but also a positive quantity.

One should also note that $n_3 = n_4 = 0$ since the subsidiary Lorentz condition excludes the fields with the polarizations $\nu = 3, 4$. In the one-time limit

$$\sum_{k=1}^{6} \int d^3\hat{\tau}\, \delta(\hat{\tau}^2 - 1) \;\to\; (1/2)\sum_{k=1}^{3} \int d\hat{\tau}$$

and Eq. (11) gets the same form as in the customary electrodynamics.

We can also see that $\hat{\mathbf{P}}$ does not depend on t; i.e., if we take into account all the contributions of the fields evolving along all crossing time trajectories at the point $\hat{t}$, the momentum-energy obeys, as in the one-time theory, the conservation law $dP_\mu/dt = 0$.

One should remember that the wave function in Hilbert space with indefinite metric norm $\bar{\Psi}\Psi$, where $\bar{\Psi} = \Psi^+\eta$ and η is the hermitian Pauli operator [16, 17], can have a negative value. In contrast to the one-time theory where due to the Lorentz condition we can restrict ourselves to the subspace with the positive norm, in the multitime case due to a creation of quanta with the polarizations $\nu = 5, 6$ we have to deal with the complete Hilbert space. As in the case of multitime Dirac spinors considered in [12], one must distinguish between the sign-indefinite norm $\bar{\Psi}\Psi$ and the positive probability $\Psi^+\Psi$. The physical interpretation of $\bar{\Psi}$ needs more detailed study in connection with calculations of concrete physical effects.

Using the commutation relations (9) and (10) defined above, one can obtain for a commutator of electromagnetic potentials with the fixed time trajectories τ and τ'

$$[A_{\tau\mu}(\hat{\mathbf{x}}), A_{\tau'\nu}(\hat{\mathbf{x}}')] =$$

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[1] For $\hat{q} = \hat{p} - \hat{p}' \neq 0$

$$\int_0^\pi \hat{\mathbf{a}}(\hat{\mathbf{p}})\hat{\mathbf{a}}^+(\hat{\mathbf{p}}')\, \mathrm{e}^{ipt\cos\vartheta} \sin\vartheta d\vartheta \sim \langle \hat{\mathbf{a}}\hat{\mathbf{a}}^+\rangle t^{-1} \sin pt \to 0,$$

where $\langle \hat{\mathbf{a}}\hat{\mathbf{a}}^+\rangle$ is the value of the product of the amplitudes at some intermediate value of ϑ and $t \to \infty$, since the time frame origin can be placed arbitrarily far in the past.

$$= (1/2)(2\pi)^{-3} g_{\mu\nu} \int d^3p\, \varepsilon(\hat{p}) \delta(\hat{\tau} - \hat{\tau}') p^{-1} \mathrm{e}^{i\mathbf{p}(\mathbf{x}-\mathbf{x}')} \left[\mathrm{e}^{-i\hat{p}(\hat{t}-\hat{t}')} - \mathrm{e}^{i\hat{p}(\hat{t}-\hat{t}')} \right] =$$
$$= ig_{\mu\nu} \varepsilon(\hat{\tau}) \delta(\hat{\tau} - \hat{\tau}') \mathcal{D}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}), \quad (12)$$

where we took into account that this expression is invariant under the replacement $\mathbf{p} \to -\mathbf{p}$, the time vector $\hat{t} = \hat{\eta} t$, $\hat{\eta}^2 = 1$, and the function

$$\mathcal{D}(\hat{\mathbf{x}}, \hat{\tau}) = -\mathcal{D}(\hat{\mathbf{x}}, -\hat{\tau}) = -i(2\pi)^{-3} \int d^4p\, \delta(\hat{\mathbf{p}}^2) \varepsilon(p)\, \mathrm{e}^{i\mathbf{p}\mathbf{x} - ipt\hat{\tau}\hat{\eta}} \quad (13)$$

is a generalization of the well known Pauli–Jordan function $\mathcal{D}(\mathbf{x})$.

The commutator of the potentials is

$$[A_\mu(\hat{\mathbf{x}}), A_\nu(\hat{\mathbf{x}}')] = 2ig_{\mu\nu} \int d^3\tau\, \delta(\hat{\tau}^2 - 1) \varepsilon(\hat{\tau}) \mathcal{D}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}). \quad (14)$$

In the general case, in contrast to the commutator (12) which vanishes outside the light cone with the axis along the vector $\hat{t} - \hat{t}'$, the commutation bracket (14) is not zero, because it includes the contributions of many intersecting light cones.

Analogously (repeating, for example, the reasoning presented in book [17]) one can calculate the vacuum average $\langle T(A_{\tau\mu}(\hat{\mathbf{x}}) A_{\tau'\nu}(\hat{\mathbf{x}}')) \rangle_0$ which is distinguished from the expressions of the one-time theory by the additional factor $\delta(\hat{\tau} - \hat{\tau}')\varepsilon(\hat{\tau})$, and the scalar time t must be replaced by the product $t\hat{\tau}\hat{\eta}$:

$$\langle T(A_{\tau\mu}(\hat{\mathbf{x}}) A_{\tau'\nu}(\hat{\mathbf{x}}')) \rangle_0 = \delta(\hat{\tau} - \hat{\tau}') \varepsilon(\hat{\tau}) \mathcal{D}^c_{\mu\nu}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau} - \hat{\tau}'), \quad (15)$$

$$\mathcal{D}^c_{\mu\nu}(\hat{\mathbf{x}}, \hat{\tau}) = -i(2\pi)^{-4} g_{\mu\nu} \int d^4p\, \mathrm{e}^{i\mathbf{p}\mathbf{x} - ipt\hat{\tau}\hat{\eta}} / (p^2 - \hat{p}^2). \quad (16)$$

The chronologization is performed with respect to the time t. Reducing to the one-time limit, we get again the well known relations.

The vacuum average taking into account the creation and absorption of virtual quanta along all time trajectories is obtained by the three-dimensional integration with the condition $|\hat{p}| = p$:

$$\langle T(A_\mu(\hat{\mathbf{x}}) A_\nu(\hat{\mathbf{x}}')) \rangle_0 = \int d^3\hat{p} d^3\hat{p}' \delta(|\hat{p}| - p) \langle T(A_{\tau\mu}(\hat{\mathbf{x}}) A_{\tau'\nu}(\hat{\mathbf{x}}')) \rangle_0 =$$
$$= \delta(\hat{\tau} - \hat{\tau}') \varepsilon(\hat{\tau}) \mathcal{D}^c{}_{\mu\nu}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}). \quad (17)$$

3. QUANTUM RELATIONS FOR SPINOR FIELDS

The momentum-energy vector and the electric charge for spinor field at a time point $\hat{t} = t\hat{\eta}$ are

$$P_\mu = -i \int d^3x\, T^{\mu k} \eta_k = -(i/2) \int d^3x \left(\bar{\psi}(\hat{\gamma}\hat{\eta}) \nabla_\mu \psi - \nabla_\mu \bar{\psi}(\hat{\gamma}\hat{\eta}) \psi \right) =$$
$$= (2\pi)^3 \sum_{k=1}^{4} \int d^3\tau\, d^3\tau'\, d^3p\, d^3p' \delta(\hat{\tau}^2 - 1) \delta(\hat{\tau}'^2 - 1) (\hat{p}\hat{\eta})_\mu B_s^{(+)}(\hat{p}, \hat{p}', \hat{t} - \hat{t}'), \quad (18)$$

$$Q = \int d^3x\, Q^k \eta_k = q \int d^3x\, \bar{\psi}(\hat{\gamma}\hat{\eta})\psi =$$
$$= q(2\pi)^3 \sum_{k=1}^{4} \int d^3\tau\, d^3\tau'\, d^3p\, d^3p'\, \delta(\hat{\tau}^2 - 1)\delta(\hat{\tau}'^2 - 1)(\hat{p}\hat{\eta}) B_s^{(-)}(\hat{p}, \hat{p}', \hat{t} - \hat{t}'), \quad (19)$$

where

$$B_s^{(\pm)}(\hat{p}, \hat{p}', \hat{t} - \hat{t}') = (-1)^s \left[a_s^+(\hat{\mathbf{p}}) a_s(\hat{\mathbf{p}}')\, e^{i(-\hat{p}+\hat{p}')\hat{t}} \pm b_s(\hat{\mathbf{p}}) b_s^+(\hat{\mathbf{p}}')\, e^{i(\hat{p}-\hat{p}')\hat{t}} \right]; \quad (20)$$

q is the electron charge; $\bar{\psi} = \psi^+\Gamma$; the matrix $\Gamma = \Gamma^+ = i\gamma_4\gamma_5\gamma_6$ [8, 13]; the energy-momemtum vectors $\hat{\mathbf{p}} = (\mathbf{p}, \hat{\tau}E_p)$, $\hat{\mathbf{p}}' = (\mathbf{p}, \hat{\tau}'E_p)$, and summation includes all linearly independent solutions of Eq. (6). We have also taken into account that, due to Eqs. (6) and (7),

$$\bar{U}_s(\hat{\mathbf{p}})\gamma_\mu U_r(\hat{\mathbf{p}}) = -(2m)^{-1}\bar{U}_s(\hat{\mathbf{p}})(\gamma_\nu\gamma_\mu + \gamma_\mu\gamma_\nu)p^\nu U_r(\hat{\mathbf{p}}) =$$
$$= m^{-1}p_\mu \bar{U}_s(\hat{\mathbf{p}})U_r(\hat{\mathbf{p}}) = (-1)^s (m/E_p)p_\mu \delta_{rs}, \quad (21)$$

$$\bar{V}_s(-\hat{\mathbf{p}})\gamma_\mu V_r(-\hat{\mathbf{p}}) = m^{-1}p_\mu \bar{V}_s(-\hat{\mathbf{p}})V_r(-\hat{\mathbf{p}}) = -(-1)^s (m/E_p)p_\mu \delta_{rs}. \quad (22)$$

It is also evident that

$$U_s(\hat{\mathbf{p}})\hat{\gamma}\hat{\tau}V_r(-\hat{\mathbf{p}}) = \bar{U}_s(\hat{\mathbf{p}})\hat{\gamma}\hat{\tau}V_r(-\hat{\mathbf{p}}) = \bar{U}_s(\hat{\mathbf{p}})\hat{\gamma}\hat{\tau}\bar{V}_r(-\hat{\mathbf{p}}) = 0. \quad (23)$$

If we replace now the quantities $(2\pi)^{3/2}(2p)^{1/2}a_s \to a_s$, do a similar replacement for b_s and define the commutation rules

$$\left[a_s^+(\hat{\mathbf{p}}), a_r(\hat{\mathbf{p}}')\right]_+ = \left[b_s^+(\hat{\mathbf{p}}), b_r(\hat{\mathbf{p}}'')\right]_+ = (-1)^r \delta_{rs}\delta(\hat{\mathbf{p}} - \hat{\mathbf{p}}')\varepsilon(\hat{p}), \quad (24)$$

$$\left[a_s(\hat{\mathbf{p}}), a_r(\hat{\mathbf{p}}')\right]_+ = \left[b_s(\hat{\mathbf{p}}), b_r(\hat{\mathbf{p}}')\right]_+ = \left[a_s(\hat{\mathbf{p}}), b_r(\hat{\mathbf{p}}')\right]_+ = \left[a_s^+(\hat{\mathbf{p}}), b_r^+(\hat{\mathbf{p}}')\right]_+ = 0, \quad (25)$$

then we get

$$P_\mu = \int d^3\tau \delta(\hat{\tau}^2 - 1) P_\mu(\hat{\tau}) =$$
$$= 2\sum_{k=1}^{4}(-1)^s \int d^3\tau\, \delta(\tau^2 - 1)(\hat{\tau}\hat{\eta}) \int d^3p\, p_\mu \varepsilon(\hat{p}) \left[n_s^+(\hat{\mathbf{p}}) + n_s^-(\hat{\mathbf{p}})\right] + P_{0\mu}, \quad (26)$$

$$Q = \int d^3\tau \delta(\hat{\tau}^2 - 1)\hat{\tau} =$$
$$= 2q\sum_{s=1}^{4}(-1)^s \int d^\tau\, \delta(\tau^2 - 1) \int (\hat{\tau}\hat{\eta}) d^3p\, \varepsilon(\hat{p}) \left[n_s^+(\hat{\mathbf{p}}) + n_s^-(\hat{\mathbf{p}})\right] + Q_0. \quad (27)$$

From these expressions one must conclude that the products $n^+ = a_s^+(\hat{\mathbf{p}})a_s(\hat{\mathbf{p}})$ and $n^- = -b_s^+(\hat{\mathbf{p}})b_s(\hat{\mathbf{p}})$ of the amplitudes with $s = 1, 3$, transforming in the one-time limit into the customary solutions of Dirac equation [13], have the meaning of the numbers of particles and antiparticles moving along a time trajectory $\hat{\tau} = \hat{p}/p$. As in the case of electromagnetic waves considered above, due to the step function $\varepsilon(\hat{p})$ all particle energy components $E_p t \tau_k$ are positively defined. The vacuum energy P_0 is also positive, and the summary vacuum charge $Q_0 = 0$.

In contrast to the one-time theory, in the multitime case the mass hyperboloid $\hat{\mathbf{p}}^2 = -m^2$ is connected; i. e., at energy $E_i > m$ there is no gap between positive and negative energies, and one can think that we cannot distinguish between the positive- and negative-frequency field states, and quite free transitions of particles and antiparticles into each other are possible. However, this is not the case since energy and time vectors are connected by the relation $\hat{E} = E\hat{\tau}$, and due to the condition of time irreversibility $d\hat{\tau}/dt \geqslant 0$ all energy components must be positive. The positive- and negative-frequency waves $\exp(+i\hat{\mathbf{x}}\hat{\mathbf{p}})$ and $\exp(-i\hat{\mathbf{x}}\hat{\mathbf{p}})$ are separated uniquely. From a geometrical viewpoint, all particle and antiparticle trajectories must be placed inside the first octant of t-subspace $t_i \geqslant 0$. Particle and antiparticle are always separated by a gap $\Delta E \geqslant 2m$ and the creation of a particle–antiparticle pair demands the energy $E \geqslant 2m$[1].

Using the integral expression (5) and the commutation rules (24) and (25), one can present the spinor field anticommutator as

$$\left[\psi_\tau(\mathbf{x}, \hat{\tau}t), \bar{\psi}_{\tau'}(\mathbf{x}', \hat{\tau}t')\right]_+ = (1/2)(2\pi)^{-3} \int d^3p\, \varepsilon(\hat{p})\delta(\hat{\tau} - \hat{\tau}')\times$$
$$\times \left[\mathrm{e}^{i\hat{\mathbf{p}}(\hat{\mathbf{x}}-\hat{\mathbf{x}}')} \sum_{r=1}^{4} (-1)^r U_r(\hat{\mathbf{p}})\bar{U}_r(\hat{\mathbf{p}}) + \mathrm{e}^{-i\hat{\mathbf{p}}(\hat{\mathbf{x}}-\hat{\mathbf{x}}')} \sum_{r=1}^{4} (-1)^r V_r(-\hat{\mathbf{p}})\bar{V}_s(-\hat{\mathbf{p}}) \right], \quad (28)$$

where $\hat{p} = \hat{\tau}E_p$; $\hat{p}(\hat{x} - \hat{x}') = E_p(t - t')$.

Calculations analogous to the ones performed in the one-time theory (see the Appendix) give

$$\sum_{s=1}^{4} (-1)^s U_s(\hat{\mathbf{p}})\bar{U}_s(\hat{\mathbf{p}}) = (m - \hat{\boldsymbol{\gamma}}\hat{\mathbf{p}})/2E_p, \quad (29)$$

$$\sum_{s=1}^{4} (-1)^s V_s(-\hat{\mathbf{p}})\bar{V}_s(-\hat{\mathbf{p}}) = -(m + \hat{\boldsymbol{\gamma}}\hat{\mathbf{p}})/2E_p. \quad (30)$$

So, the anticommutator

$$\left[\psi_\tau(\mathbf{x}, \hat{\tau}t), \bar{\psi}_\tau(\mathbf{x}', \hat{\tau}t')\right]_+ = \varepsilon(\hat{\tau})\delta(\hat{\tau} - \hat{\tau}')\mathcal{S}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}), \quad (31)$$

where

$$\mathcal{S}(\hat{\mathbf{x}}, \hat{\tau}) = -i(2\pi)^{-3}(m - \hat{\gamma}\hat{\mathbf{p}}) \int d^4p\, \delta(\hat{\mathbf{p}}^2 - m^2)\varepsilon(p)\, \mathrm{e}^{i\mathbf{p}\mathbf{x} - ip\hat{\tau}\hat{\eta}}. \quad (32)$$

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[1]It is known from experiments that inside the space-time intervals accessible now the time regulation takes place. However, it may happen that inside much smaller intervals such a regulation is violated and all time directions become possible [18]. Then the sign of energy becomes indefinite and particle and antiparticle are undistinguished. We do not consider such a case.

This function differs essentially from the corresponding one-time function due to the replacing of the Dirac 4×4 matrices by the 8×8 γ matrices of the multitime theory.

The anticommutator of spinor bundles is

$$\left[\psi(\mathbf{x}), \bar{\psi}(\mathbf{x}')\right]_+ = \int d^3\tau \, \delta(\hat{\tau}^2 - 1)\varepsilon(\hat{\tau})\mathcal{S}(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}). \tag{33}$$

For the vacuum average we have

$$\langle T(\psi_\tau(\hat{\mathbf{x}})\bar{\psi}_{\tau'}(\hat{\mathbf{x}}'))\rangle_0 = i\varepsilon(\hat{\tau})\delta(\hat{\tau} - \hat{\tau}')\mathcal{S}^c(\hat{\mathbf{x}} - \hat{\mathbf{x}}', \hat{\tau}), \tag{34}$$

$$\mathcal{S}^c(\hat{\mathbf{x}}, \hat{\tau}) = -i(2\pi)^{-4} \int d^4p \, (m - \hat{\gamma}\hat{\mathbf{p}}) \, \mathrm{e}^{i\mathbf{px} - ipt\hat{\tau}\hat{\eta}}/(\hat{\mathbf{p}}^2 + m^2), \tag{35}$$

where instead of the Dirac matrices the 8×8 matrices γ_μ are used again.

4. HAMILTONIAN AND MULTITIME S-MATRIX

In order to deduce a multitime generalization of Hamiltonian we, following [14], rewrite the Dirac equation

$$(i\hat{\boldsymbol{\gamma}}\hat{\boldsymbol{\nabla}} + q\hat{\boldsymbol{\gamma}}\hat{\mathbf{A}} - m)\Psi = 0$$

in the form

$$(i\Theta d/dt + \boldsymbol{\gamma}\boldsymbol{\nabla} + q\hat{\boldsymbol{\gamma}}\hat{\mathbf{A}} - m)\Psi = 0. \tag{36}$$

Here t is the proper time on the trajectoty $\hat{t} = t\hat{\tau}$.

One can write now

$$(id/dt + H)\Psi = 0, \tag{37}$$

where

$$H \equiv H_0 + H_{\text{int}} = \Theta\boldsymbol{\gamma}\boldsymbol{\nabla} + \Theta\hat{\boldsymbol{\gamma}}\hat{\mathbf{A}} - \Theta m$$

is the sought Hamiltonian. The fields $\hat{\mathbf{A}}(\hat{\mathbf{x}})$ and $\psi(\hat{\mathbf{x}})$ are bundles of waves with energy vectors which in the general case are directed not only along the considered trajectory $\hat{t}$.

In the interaction representation we get the formal solution for Eq. (37)

$$\Phi(\hat{t}(t)) = S\left(\hat{t}(t), \hat{t}(t_0)\right)\Phi(\hat{t}(t_0)), \tag{38}$$

with the scattering matrix

$$S\left(\hat{t}(t), \hat{t}(t_0)\right) = \int_{t_0}^{t} dt d^3x \, H_{\text{int}}(\hat{\mathbf{x}}) = \\ = T \, \exp\left(-i\int_{t_0}^{t} dt \, H_{\text{int}}(t)\right) = T \, \exp\left(-iq\int d^4x \, H_{\text{int}}(\hat{\mathbf{x}})\right) \tag{39}$$

and the chronologization of the proper time of the trajectory $\hat{t}$.

CONCLUSION

Of many variants of the multitime theory considered in the literature (see the bibliography in [8, 10]) the variant proposed by E. A. B. Cole [3] is the best. It does not encounter now theoretical or experimental contradictions in the region of macroscopic phenomena and allows for the relativistic invariant quantization. Due to the relation $\hat{E} = E\hat{\tau}$, connecting in the multitime theory time vectors and energy, the demand $\hat{E} \geqslant 0$ is equivalent to the time irreversibility condition $d\hat{t}/dt > 0$.

The expression for scattering matrix (39) together with the wave decompositions (1)–(5) allows one to calculate the influence of hidden time dimensions on the experimentally observed quantum processes. However, one should note that the Hamiltonian $H_{\rm int}$ is non-Hermitian, which reflects the possibility of changing the trajectory and energy. This peculiarity of the multitime S-matrix needs closer examination [14].

The integration over x-subspace and over the proper time t provides the fulfillment of the momentum-energy conservation law $\delta(\hat{\mathbf{p}}_\mathbf{1} + \hat{\mathbf{p}}_\mathbf{2} + \hat{\mathbf{p}}_\mathbf{3})$ for created and absorbed particles at every interaction.

APPENDIX

Analogously to the one-time theory (see [17]) let us consider the function

$$G = (\hat{\boldsymbol{\gamma}}\hat{\mathbf{p}} + m)^{-1} = (m - \hat{\boldsymbol{\gamma}}\hat{\mathbf{p}})/(\hat{\mathbf{p}}^2 + m^2) = (m - \hat{\boldsymbol{\gamma}}\hat{\mathbf{p}})/({E_p}^2 - {\lambda_p}^2), \tag{40}$$

satisfying the equation

$$(\hat{\boldsymbol{\gamma}}\hat{\mathbf{p}} + m)G = 1, \tag{41}$$

where $\lambda_p \neq E_p \equiv (\mathbf{p}^2 + m^2)^{1/2}$, since Eq. (41) is distinguished from the Dirac equation.

The function G can be presented as a series

$$G = \sum_{r=1}^{4} \left[U_r(\hat{\mathbf{p}})c_{1r}U_r + V_r(\hat{\mathbf{p}})c_{2r}\right], \tag{42}$$

where U_r and V_r are the solutions for the Dirac equations (6). Inserting this series into Eq. (41) and taking into account the Dirac equations

$$(\boldsymbol{\gamma}\mathbf{p} + m)U_r(\hat{\mathbf{p}}) = E_p\theta U_r(\hat{\mathbf{p}}), \quad (\boldsymbol{\gamma}\mathbf{p} + m)V_r(\hat{\mathbf{p}}) = E_p\theta V_r(\hat{\mathbf{p}}) \tag{43}$$

with the matrix $\theta = \theta^{-1} = \hat{\gamma}\hat{\tau}$ and matrix coefficients c_{ir}, we get

$$\sum_{r=1}^{4} \left[\theta(E_p - \lambda_p)U_r(\hat{\mathbf{p}})c_{1r} - \theta(E_p + \lambda_p)V_r(\hat{\mathbf{p}})c_{2r}\right] = 1. \tag{44}$$

Multiplying now this relation from the left firstly by $\bar{U}_s(\hat{\mathbf{p}})$ and then by $\bar{V}_s(\hat{\mathbf{p}})$, we find the coefficients c_{ir}:

$$c_{1r} = (-1)^r\bar{U}_r(\hat{\mathbf{p}})(E_p + \lambda_p)/({E_p}^2 - {\lambda_p}^2), \tag{45}$$

$$c_{2r} = -(-1)^r\bar{V}_r(\hat{\mathbf{p}})(E_p - \lambda_p)/({E_p}^2 - {\lambda_p}^2). \tag{46}$$

The series (42) can be written now as

$$G = ({E_p}^2 - {\lambda_p}^2)^{-1} \sum_{s=1}^{4} (-1)^s \left[U_s(\hat{\mathbf{p}}) \bar{U}_s(\hat{\mathbf{p}})(E_p + \lambda_p) - V_s(\hat{\mathbf{p}}) \bar{V}_s(\hat{\mathbf{p}})(E_p - \lambda_p) \right]. \quad (47)$$

Comparing this expression with Eq. (40) by $\lambda_p \to \pm E_p$, we get finally the relation (29) and then (30) by replacing $\hat{\mathbf{p}} \to -\hat{\mathbf{p}}$.

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REFERENCES

1. *Cole E. A. B.* // Nuovo Cim. A. 1972. V. 40. P. 171.
2. *Cole E. A. B.* // Phys. Lett. A. 1979. V. 75. P. 29.
3. *Cole E. A. B.* // J. Phys. A: Math. Gen. 1980. V. 13. P. 109.
4. *Cole E. A. B.* // Nuovo Cim. A. 1980. V. 60. P. 1.
5. *Pavšič M.* // J. Phys. A: Math. Gen. 1981. V. 14. P. 3217.
6. *Cole E. A. B., Buchanan S. A.* // J. Phys. A: Math. Gen. 1982. V. 15. P. L255.
7. *Cole E. A. B.* // Nuovo Cim. B. 1985. V. 85. P. 105.
8. *Boyling J. B., Cole E. A. B.* // Intern. J. Theor. Phys. 1993. V. 32. P. 801.
9. *Barashenkov V. S., Pestov A. B., Yuriev M. Z.* // Gen. Relat. & Grav. 1997. V. 29. P. 1345.
10. *Barashenkov V. S.* // Found. Phys. 1998. V. 28. P. 485.
11. *Barashenkov V. S., Yuriev M. Z.* Extensions of Quantum Physics / Ed. A. Horzgela, E. Kapuscik. Montreal, 2001. P. 53.
12. *Cole E. A. B.* // Nuovo Cim. B. 1980. V. 55. P. 269.
13. *Barashenkov V. S., Yuriev M. Z.* // Part. Nucl., Lett. 2002. No. 6[115]. P. 38.
14. *Patty C. E., Smalley L. I.* // Phys. Rev. D. 1985. V. 32. P. 891.
15. *Barashenkov V. S., Yur'iev M. Z.* // Nuovo Cim. B. 1997. V. 112. P. 117.
16. *Pauli W.* // Rev. Mod. Phys. 1943. V. 15. P. 175.
17. *Bogolubov N. N., Shirkov D. V.* Introduction to Quantum Field Theory. N.Y.: Waley Intersc. Publ., 1976.
18. *Blokhintsev D. I.* Space and Time in Microworld. M.: Nauka, 1970.

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