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A POSSIBILITY OF A PION–PION SCATTERING EXPERIMENT

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A possibility of a pion–pion collision experiment is discussed. This experiment permits one to perform very precise measurements of the s-wave scattering lengths at isospin state of $I = 0$ and $I = 2$ and a pion energy of 200 MeV – 1 GeV. Quark–antiquark condensate, effective Lagrangian theory and some features of the QCD lattice approach can be tested with very high precision.

Рассматривается возможность реализации экспериментов по пион-пионному взаимодействию. Такого рода эксперименты позволят осуществить точные измерения области пион-пионного взаимодействия для s-волн при изоспине $I = 0$ и $I = 2$ в диапазоне энергий пионов 200 МэВ – 1 ГэВ. Эксперименты дают возможность провести исследования кварк-антикваркового конденсата, проверить с высокой точностью теорию лагранжианов и рассмотреть некоторые особенности квантово-хромодинамических приближений.

INTRODUCTION

Pion interactions did play an important role all along the high-energy physics history, since the formulation of the Yukawa theory and the pion discovery. Nowadays, in the framework of QCD, pions are considered as (pseudo) Goldstone bosons of the spontaneous breaking chiral symmetry and will exhibit a mass M_π so that

$$M_\pi^2 = 2B_0 M_q^2 + 4AM_q^2 + \ldots, \quad (1)$$

where M_q is the quark mass (in the simplest assumption of only two equals up and down quarks); B_0 is the quark–antiquark condensate $B_0 = -\langle 0|qq|\rangle 0/f_\pi$; $A(s,t,u) = (S - M_\pi^2)/f_\pi^2$ is the amplitude; $s + t + u = 4M_\pi^2$. In general the QCD Lagrangian can be written as

$$L_{\rm QCD} = L_0 + L_I, \quad (2)$$

where L_0 describes the free quarks and gluons, L_I their interactions. Any physical observable $\langle O\rangle$ can be represented as a series expansion in the coupling constants α_s:

$$\langle O\rangle = \sum_n O_n \alpha_s^n, \quad (3)$$

where O_n are finite amplitudes. Expression (3) does not converge, reflecting the instability of the Fock-space vacuum; i.e., the quarks and gluons, due to the confinement, are not a good zeroth order approximation, a nonperturbative approach is needed to calculate the vacuum expectation values, which means composite operators are involved (condensates). From the phenomenological point of view the condensates can be related to the s-wave interaction lengths of the isospin state $I = 0$ and $I = 2$ in the $\pi^+\pi^-$ interactions. Such an analysis (often model-dependent) has been made recently by many authors [1–5].

From the experimental point of view the information is available from the $\pi\pi$ final state interaction in $\pi N \to \pi\pi N$ scattering or in K_{14} decays or, more recently, from the $\pi^+\pi^-$ pionium lifetime.

Several experiments are bringing new results: KLOE at DAFNE [6], E865 (BNL) [7] and NA48 [8] measuring the K_{14} decay, DIRAC (PS 212 at CERN) [9] has already presented results on the $\pi^+\pi^-$ atom lifetime, and an experiment at IUCF [10] is measuring the pionium lifetime in the reaction $p + d \to \mathrm{He}\pi\tilde{\pi}$.

Lastly we would like to emphasize that other interesting processes can be investigated by the proposed apparatus: four-quark hybrid states and $\pi\pi \to KK$.

1. PION–PION SCATTERING EXPERIMENT

We, first, shortly describe the main ideas of a pion–pion scattering experiment for two pion collection options.

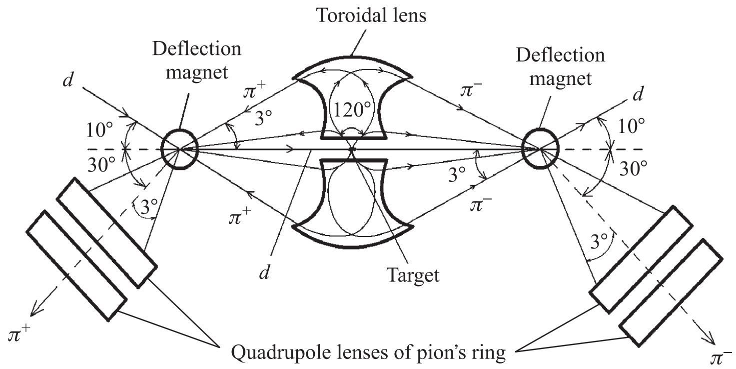

Fig. 1. Sketch of the pion production section with toroidal lens

In the first case a high-intensity ion (or proton) beam is directed to a carbon target positioned in the center of a toroidal lens (Fig. 1). The toroidal lens [11] permits one to collect practically isotropic pions. The produced π^+ and π^- particles entering the toroidal field, having a definite momentum, are deflected and exit the toroid practically parallel to the primary ion beam but in opposite directions. The two pion beams π^+ and π^- moved in opposite directions can then be focused and transported to an interaction region where a suitable detector should be placed (Fig. 2). Assuming a 5-T magnetic field, the toroid diameter can be limited to roughly 30 cm, for total center-of-mass energy of 1 GeV^2. Since as little material as possible will be desirable, to avoid pion loss, only thin-wall superconducting magnet will fulfill the requirements. A sketch of the focusing and deflecting pion beams is drawn in Fig. 2, together with an indication of a possible detector place: the pion ring diameter can be of the order of 10 m.

In the second pion collection mode the ion beam passes through the two pion production metallic targets with a pulse azimuth magnetic field of 1 MG (Fig. 3). The magnetized target was discussed in [12] for proton–antiproton collider. The high gradient target magnetic field is used to focus the pions produced along the target on its output. It reduces the pion phase space and increases its collection efficiency. The first target is applied for the positive pion production and efficient collection in the forward direction. The negative pions produced in this target are lost mainly. The length of the first target is equal to a half ion absorption length; the length of the second one corresponds to full absorption length. The second target with the opposite direction of the magnetic field is placed at a distance of a few meters from the first one (Fig. 3). A deflection magnet is installed between the two production targets to separate the ion beam passed through the first target and positive pion beam. The ion beam bombards the second target after interaction with the first one and a deflection in the dipole magnet. The second target is used for production and collection of the negative pions also moved in the forward direction. The positive pions from the second target are lost. The positive pions from the first target and the negative ones from the second target are matched with the pion ring. Both pion beams moved in opposite directions are collided in the detector area (Fig. 2).

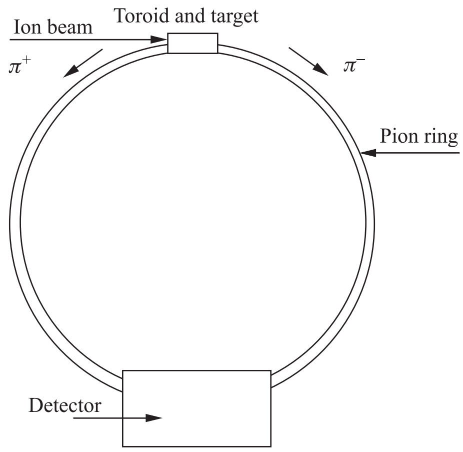

Fig. 2. Sketch of the pion–pion collision experiment

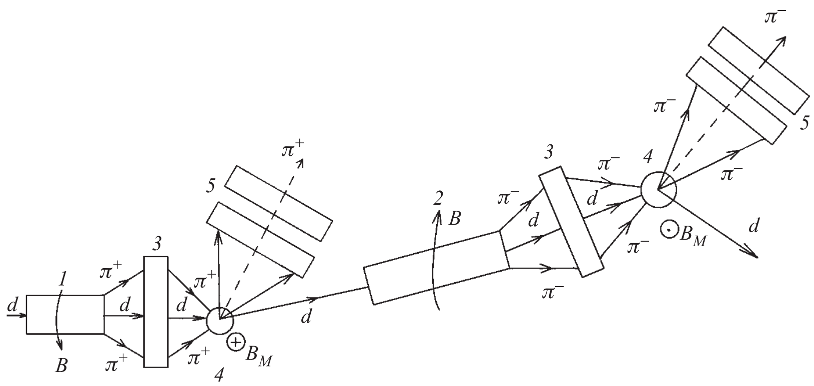

Fig. 3. Sketch of the pion production section with two magnetized targets. *1* is the first magnetized pion production target applied for positive pion collection; *2* is the second magnetized pion production target used for collection of the negative pions; *3* are the quadrupole lenses; *4* are the dipole deflection magnets; *5* are the quadrupole lenses of the pion ring

2. PION PRODUCTION AND COLLECTION IN A TARGET PLACED IN TOROIDAL LENS

The total negative pion production cross section corresponds to $\sigma_\pi = 70$ mb [13] at collisions of 1 GeV/u deuterons with a carbon target. The colliding particles which have the isotopic spin $I = 0$ produce pions π^+, π^- and π^0 with equal probability. The pion production efficiency (Fig. 2) is estimated as $\xi \approx \dot{N}_\pi/\dot{N}_i \approx n_{\rm nucl} d\sigma_\pi \approx 3.5 \cdot 10^{-2}$ at single ion collision with target, here $d = 5$ cm is the thickness of the carbon target, $n_{\rm nucl} \approx 10^{23}$ cm^{-3} is the nucleon target density, $\dot{N}_i$ is the ion intensity. The toroidal magnet (Fig. 1) permits one to collect practically all pions generated in the carbon target. The pion beam phase volume on production target is estimated as $F_\pi \approx S\Omega \approx \pi r^2(1 + 2d/r)4\pi \approx 200$ cm$^2 \cdot$ sr, where $r \approx 0.5$ cm is the ion beam radius on target, $\Omega \approx 4\pi$ is the pion angle on target. The emittance of the pion beam formed in the toroidal magnet is rather high compared with the pion ring acceptance $\varepsilon_{\rm accept} \approx 0.25$ cm·rad. The total acceptance of the pion ring corresponds to $F_{\rm ring} \approx (\pi\varepsilon_{\rm accept})^2 \approx 0.6$ cm^2·sr. The efficiency of pion collection is proportional to a ratio of the pion beam phase space to the pion ring acceptance $\eta = (F_{\rm ring}/F_\pi) \approx 3 \cdot 10^{-3}$. The longitudinal acceptance of pion ring is about $\Delta p/p \cong 10^{-1}$. The estimated pion production rate is

$$\dot{N}_{\pi^+} \approx \dot{N}_{\pi^-} \approx \dot{N}_i \xi \eta (\Delta p/p) \approx 10^{-5} \dot{N}_i. \quad (4)$$

A similar π^+-production efficiency was obtained at the JINR Phasotron at a pion energy of 200–400 MeV (Fig. 4) [14]. The maximum pion yield was realized at a pion energy of 200–300 MeV for a proton energy of 660 MeV (Fig. 3). For proton beams the π^- yield is less by one order of magnitude than that for π^+. The Phasotron pion channel angle acceptance is 15 msr, the longitudinal acceptance is 10%. The intensity of proton beam corresponds to

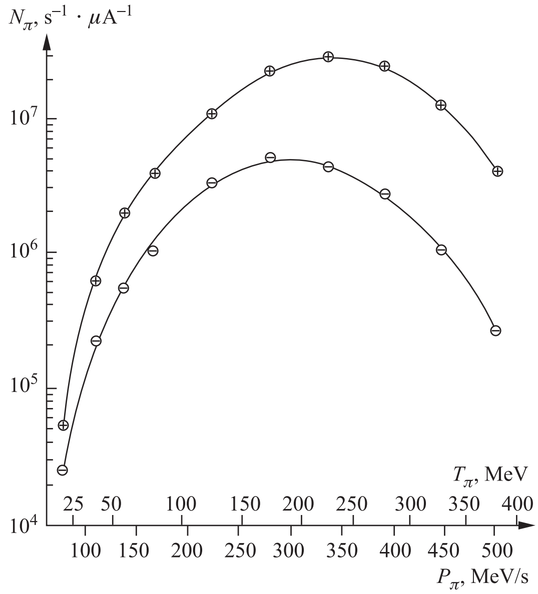

Fig. 4. Dependence of π^+ ($\oplus$) and π^- ($\ominus$) intensity on the JINR Phasotron energy

1 μA ($6.25 \cdot 10^{12}$ s^{-1}). The JINR Phasotron efficiencies of proton conversion into positive and negative pions are equal to $\dot{N}_{\pi^+} \approx 5 \cdot 10^{-6} \dot{N}_i$ and $\dot{N}_{\pi^-} \approx 8 \cdot 10^{-7} \dot{N}_i$ (Fig. 4).

An ion synchrotron with an electron cooler improves the pion collection 15–20 times compared with (4). Electron cooler installed in a synchrotron provides a low horizontal and vertical emittance of 1 $\pi \cdot$ mm $\cdot$ mrad for a cooled ion beam. As a result, the ion beam radius corresponds to $r \approx 0.3$ mm on a production pion target at a beta function of 10 cm in this area. The pion beam phase space is estimated as $F_\pi \approx S\Omega \approx \pi r^2 (1 + 2d/r) 4\pi \approx 0.6$ cm$^2 \cdot$ sr in the toroidal magnet at application of bombarded cooled ion beam and production target thickness of $d \approx 2.5$ mm. The pion production efficiency at a small target thickness of $d \approx 2.5$ mm is estimated as $\xi \approx \dot{N}_\pi / \dot{N}_i \approx n_{\rm nucl} d\sigma_\pi \approx 1.7 \cdot 10^{-3}$. The efficiency of the pion collection is $\eta = (F_{\rm ring}/F_\pi) \approx 1$ in this case. The pion production–collection rate for a synchrotron with electron cooler corresponds to

$$\dot{N}_{\pi^+} \approx \dot{N}_{\pi^-} \approx \dot{N}_i \xi \eta (\Delta p/p) \approx 1.7 \cdot 10^{-4} \dot{N}_i. \quad (5)$$

A thin internal carbon target of $d \approx 2.5$ mm permits one to increase the pion production efficiency for ions which many times pass through this target. The internal target is installed in a synchrotron bypass through which ions pass with a repetition time of 10 ms. The number of deuteron turns in a synchrotron is $k = 1(n_{\rm nucl} d\sigma_{\rm in}) \approx 80$, where $\sigma_{\rm in} = 450$ mb is the total deuteron cross section for inelastic collisions with carbon target at an ion energy of 1 GeV/u [15]. The upper limit of pion production efficiency corresponds to $\xi_{\max} \approx k\xi \approx \sigma_\pi/\sigma_{\rm in} \approx 0.15$ when ions pass through the thin target k times. The pion production–collection rate for a synchrotron with electron cooler and internal thin target is estimated as

$$\dot{N}_{\pi_{\rm multi}} \approx k \dot{N}_{\pi_{\rm single}} \approx \dot{N}_i \xi_{\max} \eta (\Delta p/p) \approx 1.5 \cdot 10^{-2} \dot{N}_i. \quad (6)$$

3. PION PRODUCTION AND COLLECTION IN TWO MAGNETIZED TARGETS

The pulse azimuth magnetic field of $H_{\rm targ} \approx 1$ MG is formed in two pion production metallic targets to reduce the pion phase space (Fig. 3). The current running through the target generates the field. The magnetic field has opposite direction in each target, it permits one to collect the positive pions produced in the forward direction with a large differential cross section in the first target and the negative ones moved also in the forward direction in the second target.

The ion beam is focused on a pion converter by a special X-type lens in a spot diameter of $2r = 5$ mm. The positive pions produced along the whole target length in the angle of $\alpha \approx \pm r K^{1/2} \approx \pm 22^\circ$ are focused by the converter magnetic field gradient $G = dH_{\rm targ}/dr \approx 4 \cdot 10^6$ G/cm on the target output, here $K = eG/pc$ is equal to 2.5 cm^{-2} at a pion momentum $p = 0.5$ GeV/c. The phase space of these pions is $F_\pi \approx S\Omega \approx \pi r^2 \pi \alpha^2 \approx 0.1$ cm$^2 \cdot$ sr. It does not depend on the converter length for a magnetized target. This value is less than the pion ring phase space $F_{\rm ring}$. The collection efficiency of these positive pions is estimated as $\eta \approx (\pi\alpha^2/\sigma_\pi) d\sigma_\pi/d\Omega \approx 0.1$, where $d\sigma_\pi/d\Omega \approx 50$ mb/sr is the differential pion production cross section at a small angle α and $\sigma_\pi \approx 250$ mb is the total one at collisions of 1 GeV/u deuterons with a copper target [16]. A quadrupole lens focuses the positive pions produced in the target in the forward direction. Then they are deflected by a dipole magnet installed between the two pion production targets and are matched with the pion ring (Fig. 3).

The ion beam bombards the second target after an interaction with the first target and deflection in the dipole magnet. The negative pions produced in the second target at the angle $\pm\alpha$ relative to the forward direction are focused on the target output at opposite direction of the magnetic field. The positive and negative pion production and collection rate in each converter is estimated as

$$\dot{N}_{\pi^+} \approx \dot{N}_{\pi^-} \approx \dot{N}_i \frac{\xi_{\max}}{2} \eta(\Delta p/p) \approx 7 \cdot 10^{-4} \dot{N}_i, \qquad (7)$$

where $\xi \approx \xi_{\max}/2$ is the ion conversion efficiency into the positive pions in the first target and the negative pions in the second one.

4. LUMINOSITY

Below we consider a possibility of pion production on the basis of the JINR Nuclotron. The Nuclotron intensities of the ion beams correspond to 10^{11} particle per pulse (ppp) (d), 109 ppp (^{12}C), $2 \cdot 10^7$ ppp (^{24}Mg) and 10^6 ppp (^{18}Ar) [17] at slow extraction. The ion beam intensities can be increased up to 10^{13} ppp (d), 10^{12} ppp (^{12}C) and $2 \cdot 10^9$ ppp (^{18}Ar) at construction of the Nuclotron booster [17].

The pion lifetime is equal to $\tau_\pi \approx 26\gamma_\pi \approx 200$ ns at a pion energy of 1 GeV. The slow ion extraction permits one to collide only few pions in the ring (Fig. 2) $N_{\pi^+,\pi^-} \approx \dot{N}_{\pi^+,\pi^-}\tau_\pi \approx 2$ during pion lifetime at a pion production efficiency (4) and an ion intensity of $\dot{N}_i = 10^{12}$ p/s.

A fast extraction of bunched ion beam makes it possible to increase the number of colliding pions in the ring. When the pulse duration of extracted ion beam is comparable with a pion lifetime, the number of the colliding pions in the ring is equal to $N_\pi \approx 10^7$ ppp at a pion production efficiency (4) and an ion intensity of $N_i = 10^{12}$ ppp. The peak luminosity of the colliding pion beams is equal to $L_{\text{peak}} = N_{\pi^+}N_{\pi^-}f/S \approx 1.5 \cdot 10^{20}$ cm$^2 \cdot$ s^{-1}, where $f =$ 10 MHz is the pion revolution frequency, $a_\pi = \sqrt{\varepsilon_{\text{accept}}\beta_{\min}} \approx 1.5$ cm and $\beta_{\min} \approx 10$ cm are the pion beam radius and beta function in the collision point, $S = \pi\alpha_\pi^2 \approx 7$ cm^2 is the pion beam cross section. However, the average luminosity is small:

$$L = L_{\text{peak}}\tau_\pi/\tau_{\text{ext}} \approx 2 \cdot 10^{-7} L_{\text{peak}} \approx 3 \cdot 10^{13}\ \text{cm}^2 \cdot \text{s}^{-1} \qquad (8)$$

caused by a small ratio of pion lifetime to repetition time of $\tau_{\text{ext}} \approx 1$ s at a fast ion extraction. The luminosity is rather small at the routine Nuclotron parameters [17].

The luminosity of colliding pion beams can be essentially improved compared with (8) at a fast ion beam extraction and an electron cooler realized in an ion synchrotron. The pion intensity in the ring is estimated as $N_\pi \approx 5 \cdot 10^9$ ppp at a deuteron beam intensity of $N_i = 3 \cdot 10^{13}$ ppp [17] and a pion production–collection efficiency (5). The peak luminosity corresponds to $L_{\text{peak}} \approx 4 \cdot 10^{25}$ cm$^2 \cdot$ s^{-1} at a fast ion extraction with pulse duration smaller than the pion lifetime. The average luminosity is equal to $L_{\text{single}} \approx 2 \cdot 10^{-7} L_{\text{peak}} \approx 8 \cdot 10^{18}$ cm$^2 \cdot$ s^{-1}.

Installation of a thin internal carbon target in synchrotron bypass through which ions pass $k \approx 80$ times permits one to increase by this factor the average luminosity

$$L_{\text{multi}} \approx kL_{\text{single}} \approx 6.5 \cdot 10^{29}\ \text{cm}^2 \cdot \text{s}^{-1}. \qquad (9)$$

The number of events at a pion–pion cross section of $\sigma \approx 100$ mb corresponds to

$$n = L_{\rm multi}\,\sigma \approx 6.4 \cdot 10^{-5}\ \text{s}^{-1} \approx 5\ \text{day}^{-1}. \qquad (10)$$

The intensity of extracted deuteron beam of $3 \cdot 10^{13}$ ppp at a repetition time of 1 s is close to a limit [18] related to the radiation resistance of the materials used in the toroidal magnet (Fig. 1). The radiation flux is estimated as $3.7\ \text{Gy} \cdot \text{s}^{-1}$ for the secondary charge particle flux of $6 \cdot 10^9\ \text{cm}^{-2} \cdot \text{s}^{-1}$ produced in the target. The absorption dose per one year corresponds to 10^8 Gy. It is close to the threshold absorption dose for the organic isolation materials used in the toroidal magnet.

The two magnetized metallic targets installed in the ion extraction channel provide the pion intensity $N_\pi \approx 2 \cdot 10^{10}$ ppp at a deuteron beam intensity of $N_i = 3 \cdot 10^{13}$ ppp [17] and a pion production–collection efficiency (7). The peak luminosity for magnetized target corresponds to $L_{\rm peak} \approx 5 \cdot 10^{26}\ \text{cm}^2 \cdot \text{s}^{-1}$ and the average luminosity is estimated as $L \approx 10^{20}\ \text{cm}^2 \cdot \text{s}^{-1}$. The luminosity of pion–pion scattering experiment based on the magnetized target is a few times less than the luminosity based on the thin internal target.

CONCLUSION

A proposal of pion–pion scattering experiment for two pion collection options is discussed. In one scheme the high-intensive ion beam bombarded the carbon production target installed in the center of the toroidal magnet and two pion beams π^+ and π^- are deflected in opposite directions and then both beams are collided in a pion ring. The luminosity of the pion colliding beams is $6 \cdot 10^{20}\ \text{cm}^2 \cdot \text{s}^{-1}$.

Two magnetized metallic targets can be used for production of positive and negative pion beams with rather small emittances. The luminosity of the colliding pion beams corresponds to $10^{20}\ \text{cm}^2 \cdot \text{s}^{-1}$ for a scheme based on the pulse-magnetized targets.

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