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SUPERRADIANCE REGIME OF LASER COOLING IN EXTENDED SOLIDS

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The kinetics of the extended crystal doped by rare-earth ions in regime of anti-Stokes laser cooling has been considered taking into account the collective radiation effects. The system of Markovian equations for impurities and pseudolocal phonons has been obtained. As would be expected, the collective radiation effects cause an acceleration in relaxation depletion of the phohon mode and, therefore, an increase in crystal cooling efficiency.

Рассмотрена кинетика протяженного кристалла с примесями редких земель в режиме антистоксового лазерного охлаждения с учетом коллективных излучательных эффектов. Получена система марковских кинетических уравнений для примесей и псевдолокальных фононов. Показано, что коллективные эффекты ускоряют процесс опустошения фононной моды и приводят, следовательно, к увеличению эффективности охлаждения кристалла.

INTRODUCTION

Laser cooling in solids is one of the most important problems in laser physics [1, 2]. The effect of laser cooling may be achieved using the anti-Stokes regime of fluorescence in crystal. This regime means that the transparent crystal material emits photons which have a higher mean energy than those it absorbs. The energy difference arises from thermal excitations in the sample. Effectively, heat is converted into light, which leaves the material. Successful fluorescent cooling with a high radiative quantum efficiency is required in order that nonradiative heating does not overwhelm the fluorescent cooling. A. Kastler (see in [2]) suggested that rare-earth ions in transparent solids could be effective fluorescent coolers because of their large quantum efficiencies. In 1995, R. Epstein et al. [3, 4] used rare-earth ions of (Yb^{3+}) in heavy-metal fluoride glass to perform a reliable experiment on laser cooling. The theory of the anti-Stokes regime of laser cooling in solids has been developed by many authors (see references in [2, 4]). V. Samartsev with collaborators proposed the model of pseudolocal phonons to obtain the qualitative and quantitative description of laser cooling in crystals doped with rare-earth ions in the regime of anti-Stokes fluorescence (see [2, 5] and references therein). Some years ago, the superradiance regime of laser cooling of impurity crystals has been suggested by V. Samartsev and S. Petrushkin [6] to increase the efficiency of laser cooling. Note that during the last two decades the superradiance of impurity solids

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has been the subject of intensive experimental investigations [7]. In 1999 superradiance was detected in experiments performed with rare-earth ions [7]. The authors of [6] have focused attention on radiative processes in the framework of the model of pseudolocal phonon modes and excluded the dynamics of phonon transitions from consideration. The analysis of phonon dynamics in the superradiance regime of laser cooling of impurity solids has been performed in our paper [8]. In the model that has been carried out in that paper the wavelength of the electromagnetic radiation λ is taken to be much larger than the characteristic dimension L of the crystal with impurity ions (so-called «point» Dicke model). The quantitative evaluation of the temperature lowering of the sample in the superradiance regime in the framework of such a nonrealistic model is unsuitable. In this paper we have generalized our consideration to extended sample ($\lambda \ll L$).

1. THE MODEL HAMILTONIAN

Let us consider an ensemble of N two-level ions in extended pencil-shaped molecular crystal with resonance frequency ω_0 interacting with quantum electromagnetic field on direct transitions 1–2 (see figure) and two coherent pumping fields — the continuous-wave laser radiation with frequency ω_1 obeying the condition $\omega_1 < \omega_0$ and the pulsed short laser radiation with frequency ω_0. The impurity ions interact also with the pseudolocal phonon mode with frequency Ω. The pseudolocal phonons are due to anisotropic ions vibrational librations with respect to their equilibrium positions in crystal. These librations modulate the constant of ion–photon interaction which gives rise to the indirect transitions, when a phonon is absorbed or emitted simultaneously with photon. Continious-wave pumping laser radiation with frequency $\omega_1 = \omega_0 - \Omega$ excites two-level ions from state 1 to state 2 with simultaneous absorbtion of a phonon with frequency Ω. Such transitions and indirect anti-Stokes transitions 2–1 are accompanied by decrease of phonons number in the pseudolocal mode. On the contrary, the indirect Stokes transitions 2–1 result in the emission of phonons. The direct spontaneous relaxation 2–1 does not vary the phonons quantity. The cooling takes place as long as the number of absorbed phonons exceeds the number of emitted phonons in a unit of time because the decrease of phonons number leads to lowering of effective temperature of the pseudolocal phonon mode. Such a situation is realized in rear-earth crystals. The temperature of the whole sample through the energy exchange between the phonon modes is lowered too. The pulsed laser radiation is necessary for realization of the superradiance regime of cooling.

The Hamiltonian of the system under consideration can be written as

$$H = H_M + H_F + H_{\mathrm{MF}}, \tag{1}$$

where

$$H_M = \sum_{f=1}^{N} \hbar\omega_0 R_f^z + \sum_q \hbar\Omega_q b_q^+ b_q$$

is the Hamiltonian of free two-level impurity ions and free phonon field;

$$H_F = \sum_k \hbar\omega_k \, b_q^+ b_q$$

is the Hamiltonian of free quantum electromagnetic field and

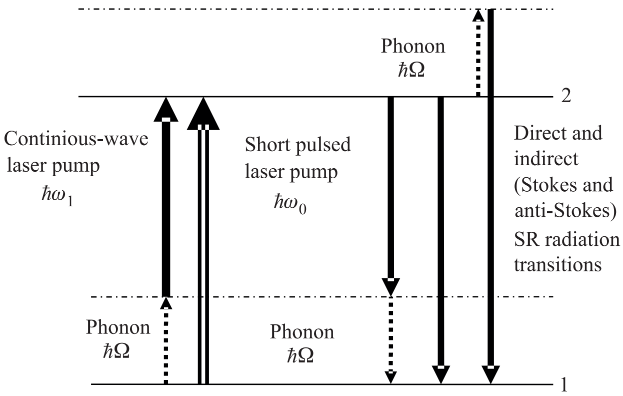
$$H_{\mathrm{MF}} = H_{\mathrm{MF}}^{(1)} + H_{\mathrm{MF}}^{(2)},$$

$$H_{\mathrm{MF}}^{(1)} = \sum_{k,f} g_k (\mathrm{e}^{\imath \mathbf{k}\mathbf{r}_f} a_k R_f^{+} + \mathrm{e}^{-\imath \mathbf{k}\mathbf{r}_f} a_k^{+} R_f^{-})$$

is the Hamiltonian of direct ion–photon interaction and

$$H_{\mathrm{MF}}^{(2)} = \sum_{k,f} h_{kq} \left\{ \mathrm{e}^{\imath(\mathbf{k}-\mathbf{q})\mathbf{r}_f} a_k R_f^{+} (b_{-q} + b_q^{+}) + \mathrm{e}^{-\imath(\mathbf{k}-\mathbf{q})\mathbf{r}_f} a_k^{+} R_f^{-} (b_q + b_{-q}^{+}) \right\}$$

is the Hamiltonian of indirect (Stokes and anti-Stokes) ion–photon interaction.



Scheme of energy levels and transitions in two-level impurity ion in the superradiance regime of laser cooling. 1 and 2 are the ground and excited states of ion, $\hbar\omega_1$ is the energy of continious-wave laser pump quantum, $\hbar\omega_0$ is the energy of pulsed laser pump quantum, $\hbar\Omega$ is the energy of the pseudolocal phonon

Here the index f numbers the ions in the crystal; $\mathbf{r}_f$ is the radius vector of the fth emitter; ω_0 is the resonance two-level emitter frequency; R_f^z is the inverse population operator of the fth emitter; $R_f^{\pm}$ are the operators describing the transitions in the fth emitter; $a_k^{+}(a_k)$ is the operator of creation (annihilation) of a photon with frequency ω_k, wave vector $\mathbf{k}$ and polarization $\mathbf{e}_\lambda$; $b_q^{+}(b_q)$ is the operator of creation (annihilation) of a phonon with frequency Ω_q, wave vector $\mathbf{q}$ and polarization $\mathbf{e}_\mu$; g_k and h_{kq} are coupling constants of direct and indirect interaction. Writing the Hamiltonian, we drop terms describing the interaction of the ions with pumping fields. To focus our attention on the the role of superradiance in cooling, we take into account the pumping processes in specifying the initial conditions for kinetic equations.

2. DYNAMICS OF PSEUDOLOCAL PHONON MODE IN SUPERRADIANCE REGIME

Using the Bogolubov method of elimination of boson (field) variables and standard decouplings for ion–phonon correlation functions [8], one can obtain for the considered pencil-shaped crystal the closed set of Markovian equations for ion–phonon subsystem:

$$\frac{dW}{dt} = -\left\{ \frac{1}{\tau} + \frac{1}{\tau_{(s)}}(1+n) + \frac{1}{\tau_{(as)}} n \right\} (N+W) - 2 \left\{ \frac{\mu}{\tau} + \frac{\mu_{(s)}}{\tau_{(s)}}(1+n) + \frac{\mu_{(as)}}{\tau_{(as)}} n \right\} S, \quad (7)$$

$$\frac{dn}{dt} = \frac{1}{2}\left\{\frac{1}{M\tau_{(s)}}(1+n) - \frac{1}{M\tau_{(as)}}n\right\}(N+W) + \left\{\frac{\mu_{(s)}}{M\tau_{(s)}}(1+n) - \frac{\mu_{(as)}}{M\tau_{(as)}}n\right\}S, \quad (8)$$

$$\frac{dS}{dt} = -\frac{1}{2}W\frac{dW}{dt}, \quad (9)$$

where

$$\tau^{-1} = 2\sum_k \pi g_k^2/\hbar^2\delta(\omega_k - \omega_0), \quad \tau_{(s)}^{-1} = 2M\sum_k \pi h_{kq_0}^2/\hbar^2\delta(\omega_k - \omega_0 + \Omega),$$

$$\tau_{(as)}^{-1} = 2M\sum_k \pi h_{kq_0}^2/\hbar^2\delta(\omega_k - \omega_0 - \Omega)$$

are inverse times of spontaneous emission on direct, Stokes and anti-Stokes transitions,

$$\mu = 2\tau\sum_k \pi g_k^2/\hbar^2\Gamma(\mathbf{k} - \mathbf{k}_0)\delta(\omega_k - \omega_0),$$

$$\mu_{(s)} = 2\tau_{(s)}M\sum_k \pi h_{kq_0}^2/\hbar^2\Gamma(\mathbf{k} - \mathbf{q}_0 - \mathbf{k}_0)\delta(\omega_k - \omega_0 + \Omega),$$

$$\mu_{(as)} = 2\tau_{(as)}M\sum_k \pi h_{kq_0}^2/\hbar^2\Gamma(\mathbf{k} + \mathbf{q}_0 - \mathbf{k}_0)\delta(\omega_k - \omega_0 - \Omega)$$

and

$$\Gamma(\boldsymbol{\kappa} - \mathbf{k}_0) = \left|\frac{1}{N}\sum_f e^{\imath(\boldsymbol{\kappa} - \mathbf{k})\mathbf{r}_f}\right|^2 \quad (10)$$

is the geometrical factor which has been calculated by Rehler and J. H. Eberly for different symmetrical pencil-shaped extended system (see in [9]), $\mathbf{k}_0$ is the wave vector of short pulsed pump. We also introduce, following [9], the collective population of the excited levels in two-level media $W = 2\sum_f \langle R_f^+\rangle$ and two-particle collective correlator S in the following manner:

$$\langle R_f^+ R_{f'}^-\rangle = \frac{1}{N^2}S\,e^{\imath\mathbf{k}_0(\mathbf{r}_f - \mathbf{r}_{f'})}.$$

In deriving (7)–(9) we take into account that the pseudolocal phonons characterize the enhanced density in narrow spectral range and, therefore, all the phonon numbers, except for the phonon with a frequency Ω, should be put equal to zero in this case. Then, $n = n(t)$ is the mean value of pseudolocal phonons with frequency Ω, q_0 is the modulus of the wave vector of pseudolocal phonon, and M is the number of phonon states involved in the cooling process. In deriving equation (9) we drop for simplicity the noncollective terms.

Let us concentrate our attention on analysis of phonon mode dynamics in the regime of superradiance. To consider such a regime of cooling, let us suppose that a system of two-level ions is pumped not only by the continuous-wave laser radiation with frequency ω_1 but the short pulsed π laser radiation with frequency ω_0 and fixed repetition rate. Such a short π pump impulse creates additional population inversion in two-level ions and gives rise to superradiance. Let the pulsed pump impulse duration be less than times of collective radiation and all relaxation times for pseudolocal-phonon mode and impurity subsystem. Then, at the instant the π pumped impulse is terminated ($t = 0$) we can write the initial conditions for equations (7)–(9) in the form: $\langle W(0)\rangle = N$, $n(0) = n_{\rm st}$, $S = 0$, where $n_{\rm st}$ is the stationary number of pseudolocal phonons under the action of the continuous-wave laser radiation [2]. Let us consider the evolution of our system on the time interval $0 \leqslant t \leqslant t_D$, where t_D is the delay time of superradiance. We can consider for system with sufficiently large number N the emitters evolution provided that $n(t) \approx n_{\rm st}$. Under this approximation the solutions of (7), (9) for collective population and collective two-particle correlator are

$$W(t) = -N\left\{\left(1 + \frac{1}{N\mu_{\rm st}}\right)\operatorname{th}\left(\frac{t - t_D}{2\tilde{\tau}_R}\right) - \frac{1}{N\mu_{\rm st}}\right\}, \quad S(t) = (N^2 - W(t)), \tag{11}$$

where $t_D = \tilde{\tau}_R \ln(N\mu_{\rm st})$ is the delay time of collective pulse, $\tilde{\tau}_R^{-1} = (1 + N\mu_{\rm st})\tau_1^{-1}$ is the inverse time of correlation self-induction which define the collective pulse width and

$$\frac{1}{\tau_1} = \frac{1}{\tau} + \frac{1}{\tau_{(s)}}(1 + n_{\rm st}) + \frac{1}{\tau_{(as)}}n_{\rm st},$$

$$\mu_{\rm st} = \frac{\tau_1}{\tau}\mu + \frac{\tau_1}{\tau_{(s)}}\mu_{(s)}(1 + n_{\rm st}) + \frac{\tau_1}{\tau_{(as)}}\mu_{(as)}n_{\rm st}.$$

Substituting formulae (11) in the right-hand side of equation (8), one can easily obtain for sample with ($N\mu_{\rm st} \gg 1$) the time behaviour of the mean phonon number at instants of time $t \sim t_D$ (in other words, at the end of superradiance process):

$$n(t) \approx \overline{n} + [n_{\rm st} - \overline{n}]\exp\left\{-\frac{\beta\tau_1 N}{2\mu_{\rm st}}\right\}, \tag{13}$$

where $\overline{n} = \alpha/\beta$, and $\alpha = \dfrac{\mu_{(s)}}{M\tau_{(s)}}$, $\beta = \dfrac{1}{M}\left(\dfrac{\mu_{(as)}}{\tau_{(as)}} - \dfrac{\mu_{(s)}}{\tau_{(s)}}\right)$. In the high-temperature approximation, $n_{\rm st} \gg \overline{n}$, one can see from (13) that the mean number of phonons decreases in the processes of superradiance. Such a decrease leads to lowering of effective temperature of the pseudolocal phonon mode and to additional cooling of the crystal.

CONCLUSION

Thus, considering the dynamics of the pseudolocal phonon mode, we have shown that efficiency of laser cooling in the superradiance regime is higher than the efficiency of laser cooling in the regime of usual fluorescence. In this paper we have restricted ourselves to qualitative consideration of laser cooling in extended sample. The main problem in quantitative evaluation of the temperature lowering in the superradiance regime consists in calculation of the geometrical factor (10) for real experimental sample of heavy-metal fluoride glass doped by (Yb^{3+}) rare-earth ions [1]. This will be a subject of our subsequent papers.

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