ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

NONLOCALITY EFFECTS ON SPIN-ONE PAIRING PATTERNS IN TWO-FLAVOR COLOR SUPERCONDUCTING QUARK MATTER AND COMPACT STAR APPLICATIONS

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We study the influence of nonlocality in the interaction on two spin-one pairing patterns of two-flavor quark matter: the anisotropic blue color pairing besides the usual two-color superconducting matter (2SCb), in which red and green colors are paired, and the color-spin locking phase (CSL). The effect of nonlocality on the gaps is rather large and the pairings exhibit a strong dependence on the form factor of the interaction, especially in the low density region. The application of these small spin-one condensates for compact stars is analyzed: the early onset of quark matter in the nonlocal models may help to stabilize hybrid star configurations. While the anisotropic blue quark pairing does not survive a big asymmetry in flavor space as imposed by the charge neutrality condition, the CSL phase as a flavor independent pairing can be realized as neutral matter in compact star cores. However, smooth form factors and the mismatch between the flavor chemical potential in neutral matter make the effective gaps of the order of magnitude $\simeq 10$ keV, and a more systematic analysis is needed to decide whether such small gaps could be consistent with the cooling phenomenology.

Изучается влияние нелокальности на взаимодействие пар частиц со спином 1 в двухароматовой кварковой материи: анизотропное спаривание синего цвета в дополнение к обычной двухцветной проводящей материи (2SCb), в которой спарены красный и зеленый цвета, и фаза запирания цветового спина (CSL). Влияние нелокальности на разрывы гораздо более сильное, и в спаривании проявляется сильная зависимость от формфактора взаимодействия, особенно в области низкой плотности. Проводится анализ применения этих малых односпиновых конденсатов для компактных звезд: использование кварковой материи в нелокальных моделях может помочь стабилизировать конфигурации гибридных звезд. До тех пор, пока анизотропное спаривание синих кварков не сохраняет большую симметрию в пространстве цветных состояний, как следует из условия цветовой нейтральности, CSL-фаза, как не зависящее от цвета спаривание, может реализовываться как нейтральная материя в ядрах компактных звезд. Однако гладкие формфакторы и несоответствие с химическим потенциалом в нейтральной материи делают эффективные разрывы величинами порядка $\simeq 10$ кэВ, и нужен более систематический анализ, чтобы решить, могут ли такие малые разрывы быть согласованы с феноменологией охлаждения.

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INTRODUCTION

The investigation of color superconducting phases in cold dense quark matter has attracted much interest in the last years since those phases could be relevant for the physics of compact star cores [1]. Nevertheless, it became clear that large spin-0 condensates, like the usual two-flavor superconductor (2SC), although having large pairing gaps (~ 100 MeV), and therefore a direct influence on the equation of state (EoS), may be disfavored by the charge neutrality condition if not unusually strong diquark coupling constants are considered [2–4].

Alternatively, spin-1 condensates are being investigated [5–7]. Due to their smallness, their influence on the EoS is negligible but they could strongly affect the transport properties in quark matter, and therefore have important consequences on observable phenomena like compact star cooling, see [8]. A recent investigation of neutrino emission and cooling rates of spin-1 color superconductors constructed for conserved total angular momentum allows for color-spin locking, planar, polar, and *A* phases [9]. However, none of these phases fulfills the requirements of cooling phenomenology that no ungapped quark mode should occur on which the direct Urca process could operate and lead to very fast cooling in disagreement with modern observational data [12].

In the present work, we consider spin-1 pairing patterns different from the above-mentioned, like the anisotropic third color (say, blue) quark pairing besides the usual 2SC phase (2SCb) [10] and the *s*-wave color-spin locking phase (CSL) [11], which have been introduced within the NJL model whereby small gaps in the region of some fractions of MeV have been obtained. Such small gaps could help to suppress efficiently the direct Urca process in quark matter, and thus to control the otherwise too rapid cooling of hybrid stars [12, 13].

One important feature is that the form of the regularization, i.e., via a sharp cutoff or a form factor function, is expected to have a strong impact on the resulting gaps due to the sensitive momentum dependence of the integrand in the gap equation [10]. Especially the behavior of quark matter in the density region of the suspected deconfinement transition plays a crucial role for determining the stability of compact star configurations. Models with a late onset of quark matter could eventually lead to unstable hybrid star configurations.

For example, in [11] it has been shown within the local NJL model how a possible pairing pattern for compact star matter which fulfills the constraints from compact star cooling phenomenology could be described. These require that all quark species should be paired and the smallest pairing gap should be of the order of 10 keV to 1 MeV with a decreasing density dependence. A caveat of the NJL model quark matter is, however, that a stable hybrid star can be realized only marginally, see [14].

The advantage of nonlocal models is that they can describe the regularization of the quark interaction via form factor functions, and therefore represent it in a smoother way, especially for low densities [15]. For the case of the 2SC phase it has already been shown that the effect of nonlocality in the low density region is rather large and the pairing exhibits a strong dependence on the form factor of the interaction [2, 16]. Moreover, the early onset of quark matter for the dynamical chiral quark model, in contrast to the NJL model, might help to stabilize hybrid star configurations.

As it has been shown in [17] within a nonlocal generalization of the NJL model [15], stable hybrid stars with large quark matter cores can be obtained. In order to describe the properties of these stars consistently, including their cooling phenomenology, the description

of diquark pairing gaps as well as emissivities and transport properties for the above-motivated CSL phase should be also given within a nonlocal quark model.

As the first step in this direction we will provide in the present paper the spin-1 pairing gaps for a nonlocal, instantaneous chiral quark model under neutron star constraints for later application in the cooling phenomenology. We give here the first discussion of the influence of nonlocality of the interaction in momentum space when compared to the NJL model case and discuss the possible role of spin-1 condensates for compact star applications.

The paper is organized as follows: in Sec. 1 we briefly review how the NJL model is modified when nonlocality is introduced using a three-dimensional nonlocal chiral quark model; in Secs. 2 and 3 we present the nonlocal version of the anisotropic blue color paring (2SCb) besides the usual two-flavor color superconducting (2SC) phase and of the color-spin locking phase (CSL), respectively. In Sec. 4 we present preliminary results for neutral matter in compact stars for the Gaussian form factor and discuss whether requirements for hybrid star cooling phenomenology could be met. Finally, we draw the Conclusions.

1. NONLOCAL CHIRAL QUARK MODEL

We investigate a nonlocal chiral model for two-flavor quark matter in which the quark interaction is represented in a separable way by introducing form factor functions $g(p)$ in the bilinears of the current–current interaction terms in the Lagrangian [2, 15–17]. It is assumed that this four-fermion interaction is instantaneous, and therefore the form factors depend only on the modulus of the three momentum $p = |\mathbf{p}|$.

In the mean field approximation the thermodynamical potential can be evaluated and is given by

$$\Omega(T,\mu) = -T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \mathrm{Tr} \ln \left(\frac{1}{T} S^{-1}(i\omega_n, \mathbf{p}) \right) + V, \tag{1}$$

where the sum is over fermionic Matsubara frequencies $\omega_n = (2n+1)\pi T$ and V is the quadratic contribution of the condensates considered. The specific form of V in dependence on the order parameters ϕ for chiral symmetry breaking and Δ for color superconductivity in the corresponding diquark pairing channels will be given below in Sec. 2.

In our nonlocal extension the inverse of the fermion propagator in Nambu–Gorkov space is modified in comparison to the NJL-model case by momentum-dependent form factor functions $g(p)$ as follows:

$$S^{-1}(p) = \begin{pmatrix} \not{p} + \hat{\mu}\gamma^0 - \hat{M}(p) & g(p)\hat{\Delta} \\ -g(p)\hat{\Delta}^\dagger & \not{p} - \hat{\mu}\gamma^0 - \hat{M}(p) \end{pmatrix}, \tag{2}$$

where $\hat{\mu}$ is the chemical potential matrix and the elements of $\hat{M}(p) = \mathrm{diag}\{M_f(p)\}$ are the dynamical masses of the quarks given by

$$M_f(p) = m_f + g(p)\phi_f. \tag{3}$$

The matrix $\hat{\Delta}$ represents the order parameters for diquark pairing which will be made explicit in Secs. 2 and 3, respectively.

In (2) and (3) we have introduced the same form factors $g(p)$ to represent the nonlocality of the interaction in the meson ($q\bar{q}$) and diquark (qq) channels. In our calculations we use the Gaussian (G), Lorentzian (L) and cutoff (NJL) form factors defined as

$$g_G(p) = \exp{(-p^2/\Lambda_G^2)}, \tag{4}$$

$$g_L(p) = [1 + (p^2/\Lambda_L^2)^2]^{-1}, \tag{5}$$

$$g_{\mathrm{NJL}}(p) = \theta(1 - p/\Lambda_{\mathrm{NJL}}). \tag{6}$$

The parameters for the above form factor models used in this work are presented in the table. They have been fixed by the pion mass, the pion decay constant and the constituent quark mass $M = M(0)$ at $T = \mu = 0$. In order to estimate the effect of the nonlocality on the results relative to the NJL model, we used g_G, g_L and g_{NJL} with parameters fixed such that $M = 380$ MeV (see table). For details of the parameterization, see [22].

Parameter sets for the nonlocal chiral quark model (g_G, g_L) and for the NJL model (g_{NJL}); for all $M(0) = 380$ MeV is fixed

Form factor	Notation	Λ, MeV	$G\Lambda^2$	m, MeV
Gaussian	g_G	786.7	4.12	2.50
Lorentzian	g_L	637.2	2.76	2.59
NJL	g_{NJL}	596.1	2.36	5.54

The stationary points of the thermodynamical potential (1) are found from the condition of a vanishing variation

$$\delta\Omega = 0 \tag{7}$$

with respect to variations of the order parameters. Equation (7) defines a set of gap equations. Among the solutions of these equations the thermodynamically stable state corresponds to the set of order parameter values for which Ω has an absolute minimum.

2. ANISOTROPIC BLUE QUARK PAIRING FOR NONLOCAL MODEL

First, we consider the 2SCb phase in which two of three colors (e.g., red r and green g) pair in the standard spin-0 isospin singlet condensate (2SC phase) and the residual third color (consequently, it is blue b) pairs in a spin-1 condensate (symmetric in Dirac space, symmetric in color, antisymmetric in flavor) [10]. The matrix $\hat{\Delta}$ in the inverse quark propagator (2) for the 2SCb phase is then given by

$$\hat{\Delta}^{\mathrm{2SCb}} = \Delta(\gamma_5\tau_2\lambda_2)(\delta_{c,r} + \delta_{c,g}) + \Delta'\left(\sigma^{03}\ \tau_2\ \hat{P}_3^{(c)}\right)\delta_{c,b}, \tag{8}$$

where τ_2 is an antisymmetric Pauli matrix in the flavor space and λ_2 is an antisymmetric Gell-Mann matrix in the color space; $\sigma^{03} = \dfrac{i}{2}[\gamma^0, \gamma^3]$ and $\hat{P}_3^{(3)} = \dfrac{1}{3} - \dfrac{\lambda_8}{\sqrt{3}}$ is the projector

on the third color. If $\Delta' \neq 0$, this can be understood as a nonzero third component of a vector in color space which breaks the $\mathcal{O}(3)$ rotational symmetry spontaneously. The blue quark pairing is therefore anisotropic. We consider first symmetric matter, and thus we take the quark chemical potentials as $\mu_u = \mu_d = \mu$.

The thermodynamical potential is given by

$$
\Omega^{\mathrm{2SCb}}(T, \mu) = \frac{\phi^2}{4G_1} + \frac{|\Delta|^2}{4G_2} + \frac{|\Delta'|^2}{16G_3} -
$$
$$
- 4 \sum_{i=1}^{3} \int \frac{d^3p}{(2\pi)^3} \left[\frac{E_i^- + E_i^+}{2} + T \ln \left(1 + \mathrm{e}^{-E_i^-/T}\right) + T \ln \left(1 + \mathrm{e}^{-E_i^+/T}\right) \right], \quad (9)
$$

where the coupling constants G_1, G_2, G_3 follow the relation given by the instanton induced interaction [18]

$$
G_1 : G_2 : G_3 = 1 : 3/4 : 3/16. \tag{10}
$$

The dispersion law for the paired quarks (r, g) is given by

$$
E_{1,2}^{\mp}(\mathbf{p}) = E^{\mp}(\mathbf{p}) = \sqrt{(\epsilon \mp \mu)^2 + g^2(p)|\Delta|^2}, \tag{11}
$$

where $\epsilon = \sqrt{\mathbf{p}^2 + M^2}$ is the free particle dispersion relation and $M = M_u = M_d$.

For the anisotropic pairing of the blue quarks the dispersion relation can be written as

$$
E_3^{\mp}(\mathbf{p}) = \sqrt{(\epsilon_{\mathrm{eff}} \mp \mu_{\mathrm{eff}})^2 + g^2(p)|\Delta'_{\mathrm{eff}}|^2}, \tag{12}
$$

where the effective variables depend on the angle θ, with $\cos \theta = p_3/|\mathbf{p}|$, and are defined as

$$
\epsilon_{\mathrm{eff}}^2 = \mathbf{p}^2 + M_{\mathrm{eff}}^2, \tag{13}
$$
$$
M_{\mathrm{eff}} = M \frac{\mu}{\mu_{\mathrm{eff}}}, \tag{14}
$$
$$
\mu_{\mathrm{eff}}^2 = \mu^2 + g^2(p)|\Delta'|^2 \sin^2 \theta, \tag{15}
$$
$$
|\Delta'_{\mathrm{eff}}|^2 = |\Delta'|^2 \left(\cos^2 \theta + \frac{M^2}{\mu_{\mathrm{eff}}^2} \sin^2 \theta \right). \tag{16}
$$

The dispersion relation $E_3^{\pm}$ is, therefore, an anisotropic function of $\mathbf{p}$, and therefore the calculation of (9) should be performed as an integral over the modulus of $|\mathbf{p}|$ and over the angle θ. The dispersion relation $E_3^{\pm}$ has a minimum if $\theta = \pi/2$ and vanishes if $M = 0$ or $\Delta' = 0$.

As it has been pointed out in [10], the gap equations for Δ and Δ' are only indirectly coupled by their dependence on M. Therefore, since the equation for Δ' nearly decouples if M is small, we can illustrate the anisotropic contributions to the thermodynamical potential Ω fixing the variables (μ, T) and the order parameters (ϕ, Δ) and varying the angle θ. For this purpose, we consider

$$
d\Omega^{\mathrm{2SCb}} = d(\cos \theta) \Omega^{\mathrm{2SCb}} |_{\cos \theta}, \tag{17}
$$

and in Fig. 1 we plot $\Omega^{\mathrm{2SCb}} |_{\cos \theta}$ as a function of the gap Δ'. As θ increases from 0 to $\pi/2$ the position of the minimum of $\Omega^{\mathrm{2SCb}} |_{\cos \theta}$ moves to the lower values of the gaps Δ'. The value of Δ' that minimizes the thermodynamical potential is found once the integration over the angle θ is performed.
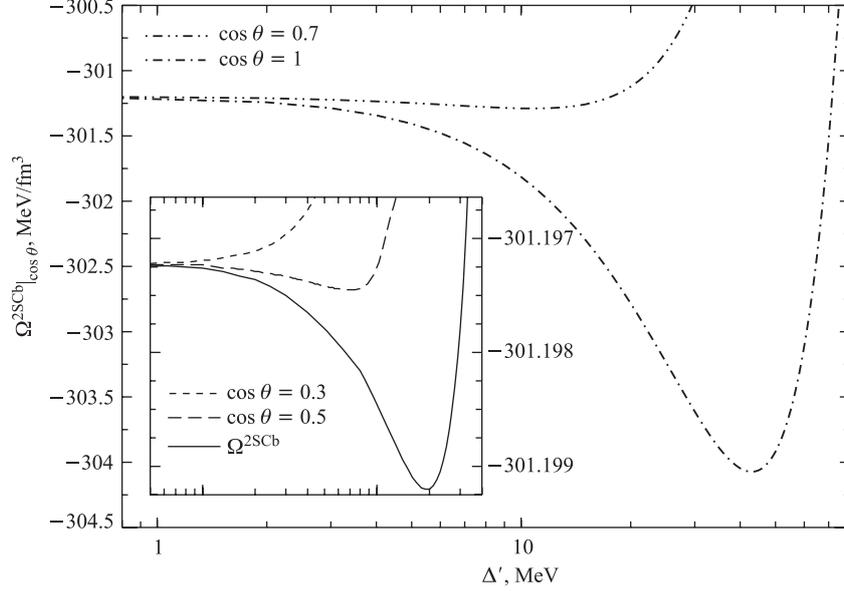
Fig. 1. Anisotropic contributions $\Omega^{2\text{SCb}}|_{\cos\theta}$ to the thermodynamical potential $\Omega^{2\text{SCb}}$ as a function of the spin-1 gap Δ' for fixed ($\mu = 500$ MeV, $T = 0$) and ($\phi = 20$ MeV, $\Delta = 106$ MeV). As θ increases the position of the minimum of $\Omega^{2\text{SCb}}|_{\cos\theta}$ moves to the lower values of the gaps Δ'. The low gap region is zoomed in the inset figure on the bottom left. For $\Omega^{2\text{SCb}}$ (solid line) we obtain that the minimum is placed at $\Delta' \simeq 2$ MeV/fm^3. The NJL parameterization is considered

2.1. Gap Equation Solutions. We search for the stationary points of $\Omega^{2\text{SCb}}$ (7) with respect to the order parameters solving the gap equations

$$\frac{\delta\Omega^{2\text{SCb}}}{\delta\phi} = \frac{\delta\Omega^{2\text{SCb}}}{\delta\Delta} = \frac{\delta\Omega^{2\text{SCb}}}{\delta\Delta'} = 0, \qquad (18)$$

using the dispersion relations (11) and (12). The results that are shown in this work are obtained for $T = 0$.

In Fig. 2 we show the chiral gap ϕ, the 2SC diquark gap Δ and the spin-1 pairing gap of the blue quarks Δ' as functions of the quark chemical potential μ. The gaps Δ' are strongly density-dependent rising functions and typically of the order of magnitude of keV, e.g., for a fixed μ at least two orders of magnitude are smaller than the corresponding 2SC gaps. These small gaps are very sensitive to the form of the regularization and to the parameterization used. Obviously, the onset and the slope of the superconducting phases also strongly depend on the parameters used.

The effect of the nonlocality on the results is shown in Fig. 2, *b*: the smoothness of the Gaussian form factor reduces the Δ' gaps dramatically. For this case, the blue quark pairing gaps are about two orders of magnitude lower than the corresponding NJL model (both with fixed $M = 380$ MeV).
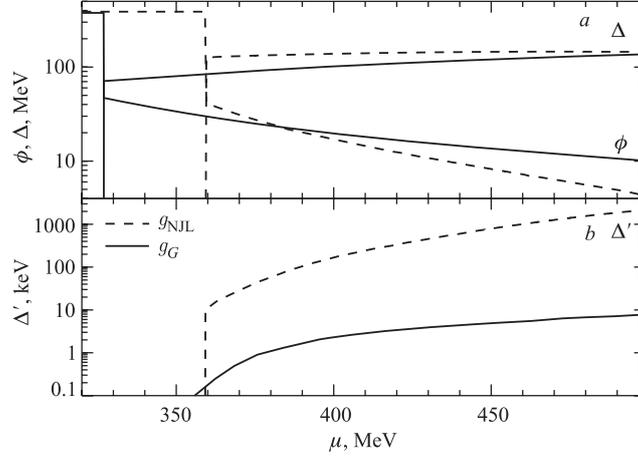
Fig. 2. Chiral gap ϕ, 2SC diquark gap Δ (*a*) and spin-1 pairing gap of the blue quarks Δ' (*b*) as a function of the quark chemical potential μ for the NJL model and for the Gaussian form factor

The dependence of the results on the parameters used is also rather strong and nonlinear as it is shown in Fig. 3 using, as example, the Gaussian form factor. When the coupling constant G_3 is doubled (dash-dotted line) the resulting gaps increase between two and three orders of magnitude, depending on the chemical potential.
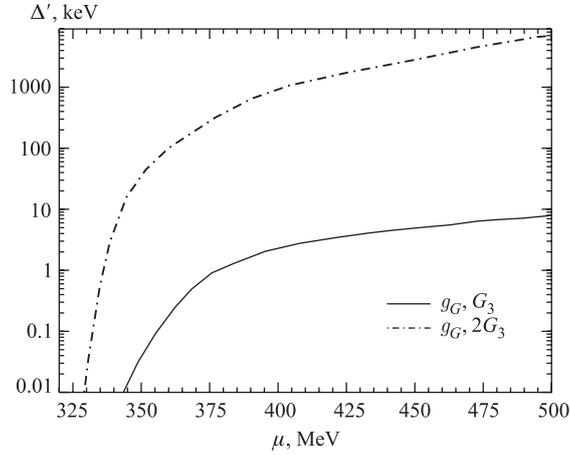
Fig. 3. Effect of the coupling constant G_3 on the spin-1 blue quarks energy gaps Δ' for the Gaussian form factor. The dash-dotted line corresponds to the case when the coupling constant G_3 is doubled

Since these small gaps are practically negligible in comparison to usual 2SC gaps, they would have no influence on the equation of state. On the other hand, it is well known that even small pairing energies could play an important role in the calculation of transport

properties of quark matter for temperatures below the order of magnitude of the gap para-meter. Nevertheless, it is unlikely that the blue quark pairing could survive the compact star constraints: the charge neutrality requires that the Fermi seas of the up and down quarks should differ by about 50–100 MeV and this is much larger than the gaps that we obtain for these condensates ($\sim$ keV) in the symmetric case.

In the following section we present the nonlocality effects on a flavor symmetric spin-1 pairing channel being a good candidate to survive the large mismatch between up and down quark Fermi seas in charge neutral quark matter.

3. COLOR-SPIN LOCKING (CSL) PHASE FOR A NONLOCAL CHIRAL MODEL

In the s-wave CSL phase introduced in Ref. [11], which differs from the CSL phase in [7], each condensate is a component of the antisymmetric antitriplet in the color space and is locked with a vector component in the spin space. In the present paper, we study a nonlocal generalization of the CSL pairing pattern of Ref. [11] and consider the matrix $\hat{\Delta}$ in (2) for the CSL channel as

$$\hat{\Delta}^{\mathrm{CSL}} = \Delta_f(\gamma_3\lambda_2 + \gamma_1\lambda_7 + \gamma_2\lambda_5). \tag{19}$$

The thermodynamical potential can be decomposed into single-flavor components

$$\Omega^{\mathrm{CSL}}(T, \{\mu_f\}) = \sum_{f \in \{u,d\}} \Omega_f^{\mathrm{CSL}}(T, \mu_f), \tag{20}$$

where the contribution of each flavor is

$$\Omega_f^{\mathrm{CSL}}(T, \mu_f) = \frac{\phi_f^2}{8G_1} + 3\frac{|\Delta_f|^2}{8H_v} - \sum_{k=1}^{6} \int \frac{d^3p}{(2\pi)^3}(E_{f,k} + 2T\ln(1 + \mathrm{e}^{-E_{f,k}/T})). \tag{21}$$

The ratio of the two coupling constants

$$G_1 : H_v = 1 : \frac{3}{8} \tag{22}$$

is obtained via Fierz transformations of the color-currents for a one-gluon exchange interaction. To derive the dispersion relations $E_{f,k}$ we follow [11] and we extend the expressions for the nonlocal model introducing the form factors to modify the quark interaction. We obtain that $E_{f;1,2}$ could be brought in the standard form

$$E_{f;1,2}^2 = (\varepsilon_{f,\mathrm{eff}} \mp \mu_{f,\mathrm{eff}})^2 + |\Delta_{f,\mathrm{eff}}|^2 g^2(p) \tag{23}$$

if the effective variables are now defined as

$$\varepsilon_{f,\mathrm{eff}}^2 = \mathbf{p}^2 + M_{f,\mathrm{eff}}^2, \tag{24}$$

$$M_{f,\mathrm{eff}} = \frac{\mu_f}{\mu_{f,\mathrm{eff}}}M_f(p), \tag{25}$$

$$\mu_{f,\mathrm{eff}}^2 = \mu_f^2 + |\Delta_f|^2 g^2(p), \tag{26}$$

$$\Delta_{f,\mathrm{eff}} = \frac{M_f(p)}{\mu_{f,\mathrm{eff}}}|\Delta_f|. \tag{27}$$

For $E_{f;k}$, $k = 3, \ldots, 6$, the particle and the antiparticle branches split

$$E_{f;3,5}^2 = (\varepsilon_f - \mu_f)^2 + a_{f;3,5}|\Delta_f|^2 g^2(p), \tag{28}$$

$$E_{f;4,6}^2 = (\varepsilon_f + \mu_f)^2 + a_{f;4,6}|\Delta_f|^2 g^2(p), \tag{29}$$

where the momentum-dependent coefficients $a_{f;k}$, $k = 3, \ldots, 6$ are given by

$$a_{f;3,5} = \frac{1}{2}\left[5 - \frac{\mathbf{p}^2}{\varepsilon_f \mu_f} \pm \sqrt{\left(1 - \frac{\mathbf{p}^2}{\varepsilon_f \mu_f}\right)^2 + 8\frac{M_f^2(p)}{\varepsilon_f^2}}\right], \tag{30}$$

$$a_{f;4,6} = \frac{1}{2}\left[5 + \frac{\mathbf{p}^2}{\varepsilon_f \mu_f} \pm \sqrt{\left(1 + \frac{\mathbf{p}^2}{\varepsilon_f \mu_f}\right)^2 + 8\frac{M_f^2(p)}{\varepsilon_f^2}}\right] \tag{31}$$

and

$$\varepsilon_f^2 = \mathbf{p}^2 + M_f^2(p). \tag{32}$$

We solve the gap equations

$$\frac{\delta\Omega_f^{\mathrm{CSL}}}{\delta\phi_f} = \frac{\delta\Omega_f^{\mathrm{CSL}}}{\delta\Delta_f} = 0 \tag{33}$$

and present the results of the global minimum of Ω_f^{CSL} in the next subsection.

3.1. Gap Equation Solutions for Each Flavor. The mass gaps and the CSL gaps are shown in Fig. 4 for different form factors of the quark interaction.

The CSL gaps are strongly μ_f-dependent rising functions in the domain that is relevant to compact star applications. There is a systematic reduction of the CSL gaps as the form factors become smoother (from NJL to Gaussian) and the condensates in the nonlocal extension are at least one order of magnitude smaller than in the NJL case.
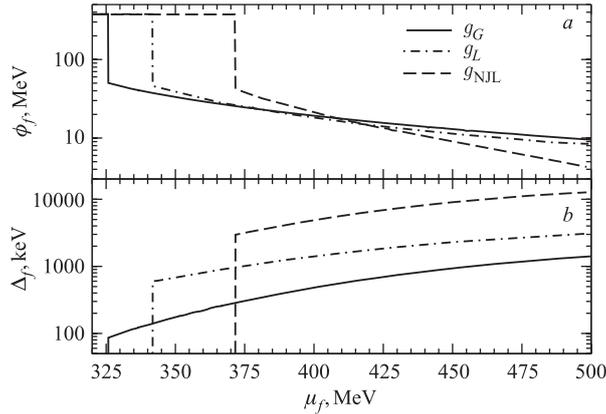

Fig. 4. The dependence of the chiral gap ϕ (*a*) and the CSL pairing gap Δ_f (*b*) on the chemical potential μ_f for different form factors of the nonlocal interaction and for the NJL model

Note that especially the low-density region is qualitatively determined by the form of the interaction. Since the nonlocality also affects the chiral gap, the breakdown of which is a prerequisite for the occurrence of color superconducting phases, we observe for the nonlocal models an earlier onset of the superconducting quark matter ($\mu_{f,\mathrm{crit}} \leqslant 350$ MeV) than in the NJL model cases. As it has been pointed out in [17], the position of the onset is crucial to stabilize hybrid star configurations. In general, NJL models present a later onset than nonlocal ones [16] and might disfavor the occurrence of stable hybrid star configurations with a quark matter core [19].
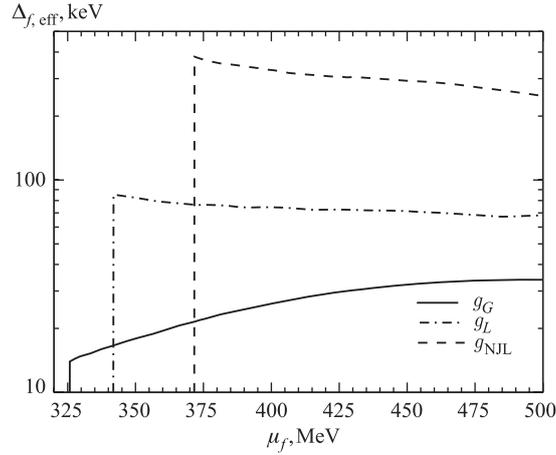
Fig. 5. The effective CSL pairing gap $\Delta_{f,\mathrm{eff}}$ for different form factors as a function of the chemical potential μ_f

In Fig. 5 we plot the effective CSL gaps setting the explicit dependence of the form factor $g(p) = 1$ in (27) in order to compare the order of magnitude of them with sharp cutoff models. We obtain that the Gaussian is an increasing function of the chemical potential from approximately 15 to 35 keV and the Lorentzian is nearly constant of the order of 80–90 keV. Both exhibit gaps that are much smaller than the corresponding NJL ones which are in the range $\Delta_{f,\mathrm{eff}} \simeq 300{-}200$ keV.

4. RESULTS FOR MATTER IN COMPACT STARS

Since the CSL pairing is symmetric in flavor, we can easily construct electrically neutral quark matter in β-equilibrium for compact star applications. We consider stellar matter in the quark core of compact stars consisting of $\{u, d\}$ quarks and leptons $\{e, \nu_e, \bar{\nu}_e, \mu, \nu_\mu, \bar{\nu}_\mu\}$. The particle densities n_j are conjugate to the corresponding chemical potentials μ_j according to

$$n_j = -\left.\frac{\partial \Omega}{\partial \mu_j}\right|_{\phi_0, \Delta_0; T}, \tag{34}$$

where the index j denotes the particle species. We consider matter in β-equilibrium with only electrons and since we assume that neutrinos leave the star without being trapped ($\mu_{\bar{\nu}_e} = -\mu_{\nu_e} = 0$, $\mu_{\bar{\nu}_\mu} = -\mu_{\nu_\mu} = 0$)

$$\mu_e = \mu_d - \mu_u. \tag{35}$$

We are interested in neutral matter. Therefore, we impose that the total electric charge should vanish

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e = 0. \tag{36}$$

The CSL condensates are color neutral such that no color chemical potentials are needed and no further constraints need to be obeyed.

We calculate the CSL gaps for each flavor as a function of the quark chemical potential $\mu = (\mu_u + 2\mu_d)/3$,

$$\begin{aligned}
\Delta_f &= \Delta_f(\mu_f(\mu)), \\
\Delta_{f,\mathrm{eff}} &= \Delta_{f,\mathrm{eff}}(\mu_f(\mu)),
\end{aligned} \tag{37}$$

where the functional relation $\mu_f(\mu)$ is taken from the β-equilibrated and neutral normal quark matter equation of state.

The results for the CSL gaps and the effective gaps as a function of the chemical potential μ are shown in Figs. 6 and 7, respectively. From the parameterizations we listed in the table, we choose the Gaussian set because we consider it the most promising for stable hybrid star configurations due to the early onset of chiral and superconducting phase transitions in quark matter.
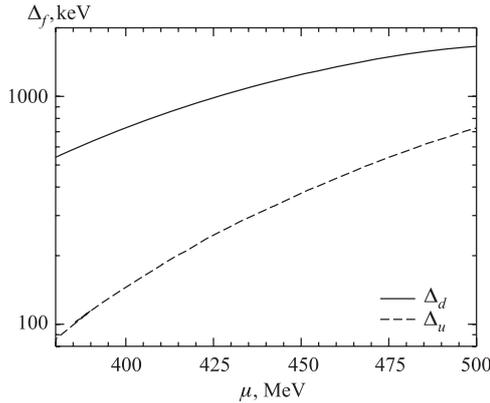

Fig. 6. CSL gaps for β-equilibrated and neutral quark matter as a function of the quark chemical potential μ for the Gaussian set
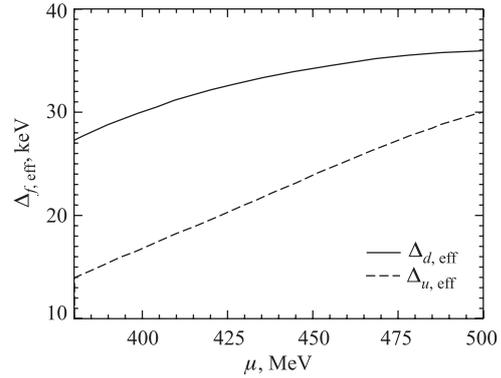
Fig. 7. Effective CSL gaps for β-equilibrated and neutral quark matter as a function of μ for the Gaussian set

From Figs. 6 and 7, we see that the two branches of the gap functions corresponding to the up and down quarks are put apart by the charge neutrality condition (thick lines). The smallest gap Δ_u runs from ≈ 100 keV near the onset to ≈ 500 keV at $\mu = 500$ MeV while for the d quarks Δ_d increases from ≈ 380 keV to 1.4 MeV in the same range.

On the other hand, the effective gaps $\Delta_{f,\mathrm{eff}}$ are of the order of magnitude of $\simeq 10$ keV, showing an approximate linear behaviour with μ. It remains to be investigated whether such small effective gaps could effectively suppress the direct Urca process in quark matter which is a requirement of compact star cooling phenomenology.

In this respect, we found here that two facts produce a strong reduction of the CSL energy gaps. First, when we include smooth form factors in the effective interactions, the

values of the gaps decrease dramatically relative to the NJL case (an order of magnitude from $\simeq 100$ keV for NJL to $\simeq 10$ keV for Gaussian). Second, when neutrality constraints are considered, Δ_u and the effective gaps $\Delta_{f,\text{eff}}$ are reduced further.

However, as we have shown, there is a strong dependence of our results on the parameterization, and a more systematic investigation on the smoothness of the form factor could be helpful to decide whether these phases could be suitable for compact star applications. Moreover, this study should be seen as a preparatory step for subsequent investigations where, for example, a covariant generalization of the formalism for the inclusion of nonlocality effects [20, 21] should be considered.

CONCLUSION

We have studied the effect of nonlocality on spin-1 condensates in two-flavor quark matter: the 2SC + spin-1 pairing of the blue quarks (2SCb) and the color-spin locking (CSL) phase. We found that the size of these small gaps is very sensitive to the form of the regularization. The nonlocality has a strong impact on the low density region and we obtain an earlier onset for the superconducting phases. This might be crucial to stabilize quark matter cores in hybrid stars.

On the other hand, due to the flavor asymmetry, we find that the 2SCb pairing phase can be ruled out for compact stars applications. The CSL phase is, in contrast, flavor independent, and therefore inert against the constraint of electric neutrality. For electrically neutral quark matter in β-equilibrium we obtain effective CSL gaps which are of the order of magnitude of 10 keV, which might help to suppress the direct Urca process, in accordance with recent results from compact star cooling phenomenology. Nevertheless, since our results are strongly-dependent on the parameters used and on the form of the regularization, more systematic studies are needed in order to decide whether the CSL phase could be applied in the description of compact stars.

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