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RADIATIVE MUON (PION) PAIR PRODUCTION IN HIGH-ENERGY ELECTRON–POSITRON ANNIHILATION (THE CASE OF SMALL INVARIANT PAIR MASS)

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The process of muon (pion) pair production with small invariant mass in electron–positron high-energy annihilation, accompanied by emission of a hard photon at large angles, is considered. We find that the Drell–Yan picture for the differential cross section is valid in the charge-even experimental set-up. Radiative corrections both for the electron block and for the final state block are taken into account.

Рассмотрен процесс образования мюонной (пионной) пары в процессе электрон-позитронной аннигиляции с образованием реального жесткого неколлинеарного фотона. Показано, что дифференциальное сечение описывается в рамках процесса Дрелла–Яна в зарядово-четной постановке эксперимента. Радиационные поправки к электронному блоку и конечному состоянию приведены в аналитическом виде.

INTRODUCTION

The radiative return method, when the hard initial state radiation is used to reduce the invariant mass of a hadronic system produced in the high-energy electron–positron annihilation, provides an important tool to study various hadronic cross sections in a wide range of invariant masses without actually changing the center-of-mass energy of the collider [1–6]. The very high luminosity of the modern meson factories makes the method competitive with the more conventional energy scan approach [7, 8]. Preliminary experimental studies both at KLOE [9] and at BABAR [10] confirm the excellent potential of the radiative return method. The case of resonance production by this mechanism was first considered in Ref. [11]. Recently a considerable effort was made to elucidate the theoretical understanding of the radiative return process, especially for the case of low-energy pion pair production [12–18].

The case when the invariant mass of hadron system $\sqrt{s_1}$ is small compared to the center-of-mass total energy $\sqrt{s} = 2\varepsilon$ is of special interest. Such a situation is realized, for example, in the BABAR radiative return studies, where such interesting physical quantities as form factors of the pion and the nucleon can be investigated. The processes of radiative annihilation into muon and pion pairs, considered here, play a crucial role in such studies, both for

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the normalization purposes and as one of the principal hadron production processes at low energies. Description of their differential cross sections with a rather high level of accuracy (better than 0.5% in the muon case) at the energies of the BABAR experiment is the goal of the present paper.

We specify the kinematics of the radiative muon (pion) pair creation process

$$e_-(p_-) + e_+(p_+) \to \mu_-(q_-) + \mu_+(q_+) + \gamma(k_1) \tag{1}$$

as follows:

$$\begin{gathered} p_\pm^2 = m^2, \quad q_\pm^2 = M^2, \quad k_1^2 = 0, \\ \chi_\pm = 2k_1 p_\pm, \quad \chi'_\pm = 2k_1 q_\pm, \quad s = (p_- + p_+)^2, \quad s_1 = (q_- + q_+)^2, \\ t = -2p_- q_-, \quad t_1 = -2p_+ q_+, \quad u = -2p_- q_+, \quad u_1 = -2p_+ q_-, \end{gathered} \tag{2}$$

where m and M are the electron and muon (pion) masses, respectively. Throughout the paper we will suppose

$$s \sim -t \sim -t_1 \sim -u \sim -u_1 \sim \chi_\pm \sim \chi'_\pm \gg 4M^2 \gg m^2, \tag{3}$$

and $s \gg s_1 > M^2$.

We will systematically omit the terms of the order of M^2/s and m^2/s_1 compared with the leading ones. In $\mathcal{O}(\alpha)$ radiative corrections, we will drop also terms suppressed by the factor s_1/s. A kinematical diagram of the process under consideration is drawn in Fig. 1.

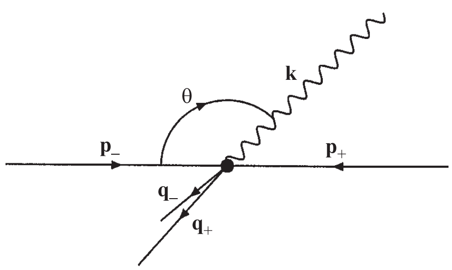

Fig. 1. Kinematical diagram for a radiative event with a small invariant mass s_1

In this paper we will consider only the charge-even part of the differential cross section, which can be measured in an experimental set-up blind to the charges of the created particles. So we omit the contribution from box-type FD, when virtual photon connects both muon and electron line. A detailed study of the charge-odd part of the radiative annihilation cross section in general kinematics will be presented elsewhere. Also we neglect the contributions from non-collinear final state radiation due to suppression factor s_1/s.

Our paper is organized as follows. The next section is devoted to the Born-level cross section. Radiative corrections to the final and initial states are considered in Sec. 2. This is followed by concluding remarks. Some useful formulae are given in the appendix.

1. THE BORN-LEVEL CROSS SECTION

Within the Born approximation, the matrix element of the initial state emission process has the form

$$M_B = \frac{(4\pi\alpha)^{3/2}}{s_1} \bar{v}(p_+) \left[\gamma_\rho \frac{\hat{p}_- - \hat{k}_1 + m}{-2p_- k_1} \hat{e} + \hat{e} \frac{-\hat{p}_+ + \hat{k}_1 + m}{-2p_+ k_1} \gamma_\rho \right] u(p_-) J^\rho \tag{4}$$

with

$$J^{\rho} = \bar{u}(q_-)\gamma^{\rho}u(q_+) \tag{5}$$

for the muon pair production, and

$$J_{\pi}^{\rho} = (q_- - q_+)^{\rho} F_{\pi}^{\rm str}(s_1) \tag{6}$$

for the case of charged pions, $F_{\pi}^{\rm str}(s_1)$ being the pion strong interaction form factor, in which we include all the effects of strong interactions in two-pion formation.

The corresponding contribution to the cross section is

$$\begin{gathered}
\frac{d\sigma_B^j}{d\Gamma} = \frac{\alpha^3}{8\pi^2 s s_1^2} R^j, \qquad R^j = B^{\rho\sigma} i_{\rho\sigma}^{(0j)}, \qquad i_{\rho\sigma}^{(0j)} = \sum_{\rm pol} J_{\rho}^j (J_{\sigma}{}^j)^*, \quad j = \mu, \pi, \\
B_{\rho\sigma} = B_g g_{\rho\sigma} + B_{11}(p_- p_-)_{\rho\sigma} + B_{22}(p_+ p_+)_{\rho\sigma}, \\
B_g = -\frac{(s_1+\chi_+)^2 + (s_1+\chi_-)^2}{\chi_+\chi_-}, \quad B_{11} = -\frac{4s_1}{\chi_+\chi_-}, \quad B_{22} = -\frac{4s_1}{\chi_+\chi_-},
\end{gathered} \tag{7}$$

where we have used the short hand notations $(qq)_{\rho\sigma} = q_\rho q_\sigma$, $(pq)_{\rho\sigma} = p_\rho q_\sigma + q_\rho p_\sigma$. For the muon final state

$$i_{\rho\sigma}^{(0\mu)} = 4\left[(q_+ q_-)_{\rho\sigma} - g_{\rho\sigma}\frac{s_1}{2}\right]. \tag{8}$$

For the case of pions,

$$i_{\rho\sigma}^{(0\pi)} = |F_{\pi}^{\rm str}(s_1)|^2 (q_- - q_+)_{\rho}(q_- - q_+)_{\sigma}. \tag{9}$$

Note that the Born-level cross section for the $e^+e^- \to \mu^+\mu^-\gamma$ process was calculated in [19, 20] (see also [21]).

The phase space volume of the final particles is

$$d\Gamma = \frac{d^3q_-}{\varepsilon_-}\frac{d^3q_+}{\varepsilon_+}\frac{d^3k_1}{\omega_1}\delta^4(p_+ + p_- - q_+ - q_- - k_1). \tag{10}$$

For the case of small invariant mass of the created pair $s_1 \ll s$, it can be rewritten as (see Fig. 1)

$$d\Gamma = \pi^2 dx_- dc ds_1, \tag{11}$$

(note that s_1 is small due to $c \to 1$, but the energy of muon pair is large: $sx_{\pm}^2 \gg 4M^2$) and approximately

$$\begin{gathered}
x_{\pm} = \frac{\varepsilon_{\pm}}{\varepsilon}, \quad x_+ + x_- = 1, \quad \omega_1 = \frac{1}{2}\sqrt{s}, \quad \chi_{\pm} = \frac{s(1 \mp c)}{2}, \quad \chi'_{\pm} = sx_{\pm}, \\
t = \frac{-sx_-(1-c)}{2}, \quad t_1 = \frac{-sx_+(1+c)}{2}, \quad u = \frac{-sx_+(1-c)}{2}, \quad u_1 = \frac{-sx_-(1+c)}{2}, \\
c = \cos(\widehat{\mathbf{p}_-\mathbf{q}_-}) = \cos\theta.
\end{gathered}$$

We will assume that the emission angle of the hard photon lies outside the narrow cones around the beam axis: $\theta_0 < \theta_1 < \pi - \theta_0$, with $\theta_0 \ll 1$, $\theta_0 \varepsilon \gg M$.

When the initial state radiation dominates, the Born cross section take a rather simple forms

$$\begin{gathered} d\sigma_B^{(\mu)}(p_-, p_+; k_1, q_-, q_+) = \frac{\alpha^3(1+c^2)}{ss_1(1-c^2)}[2\sigma + 1 - 2x_-x_+]dx_-dcds_1, \\ d\sigma_B^{(\pi)}(p_-, p_+; k_1, q_-, q_+) = \frac{\alpha^3(1+c^2)}{ss_1(1-c^2)}\,|F_\pi^{\rm str}(s_1)|^2\,[-\sigma + x_-x_+]dx_-dcds_1, \\ \frac{1}{2}(1-\beta) < x_- < \frac{1}{2}(1+\beta), \quad \beta = \sqrt{1 - \frac{4M^2}{s_1}}, \quad \sigma = \frac{M^2}{s_1}. \end{gathered} \tag{12}$$

Here β is the velocity of the pair component in the center-of-mass reference frame of the pair.

2. RADIATIVE CORRECTIONS

Radiative corrections (RC) can be separated into three gauge-invariant parts. They can be taken into account by the formal replacement (see (7)):

$$\frac{R^j}{s_1^2} \longrightarrow \frac{K^{\rho\sigma} J^j_{\rho\sigma}}{s_1^2|1 - \Pi(s_1)|^2}, \tag{13}$$

where $\Pi(s_1)$ describes the vacuum polarization of the virtual photon (see Appendix); $K^{\rho\sigma}$ is the initial-state emission Compton tensor with RC taken into account; $J^j_{\rho\sigma}$ is the final state current tensor with $\mathcal{O}(\alpha)$ RC.

First we consider the explicit formulae for RC due to virtual, soft, and hard collinear final state emission. As regards RC to the initial state for the charge-blind experimental set-up considered here, we will use the explicit expression for the Compton tensor with heavy photon $K^{\rho\sigma}$ calculated in the paper [22] for the scattering channel and apply the crossing transformation (see also [15]). Possible contribution due to emission of an additional real photon from the initial state will be taken into account too. In conclusion, we will give the explicit formulae for the cross section, consider separately the kinematics of the collinear emission, and estimate the contribution of higher orders of perturbation theory (PT).

2.1. Corrections to the Final State. The third part is related to the lowest order RC to the muon (pion) current [27]

$$J_{\rho\sigma} = i^{(v)}_{\rho\sigma} + i^{(s)}_{\rho\sigma} + i^{(h)}_{\rho\sigma}. \tag{14}$$

The virtual photon contribution $i^{(v)}_{\rho\sigma}$ takes into account the Dirac and Pauli form factors of the muon current

$$J_\rho^{(v\mu)} = \bar{u}(q_-)\left[\gamma_\rho F_1(s_1) + \frac{\hat{q}\gamma_\rho - \gamma_\rho\hat{q}}{4M}F_2(s_1)\right]v(q_+), \quad q = q_+ + q_-, \quad s_1 = q^2. \tag{15}$$

We have

$$B^{\rho\sigma} i_{\rho\sigma}^{(v\mu)} = B_g \sum_{\text{pol}} |J_\rho^{(v\mu)}|^2 + B_{11}\left[\sum_{\text{pol}} |p_- J^{(v\mu)}|^2 + \sum_{\text{pol}} |p_+ J^{(v\mu)}|^2\right]. \tag{16}$$

Here Σ means a sum over the muon spin states and

$$\begin{aligned}\sum_{\text{pol}} |J_\rho^{(v\mu)}|^2 &= \frac{\alpha}{\pi}\left[-8(s_1+2M^2) f_1^{(\mu)} - 12 s_1 f_2^{(\mu)}\right],\\ \sum_{\text{pol}} |J^{(v\mu)} p_\pm|^2 &= \frac{\alpha}{\pi} s^2 (1\pm c)^2 \left(x_+ x_- f_1^{(\mu)} + \frac{1}{4} f_2^{(\mu)}\right).\end{aligned} \tag{17}$$

For the pion final state we have

$$\begin{aligned} B^{\rho\sigma} i_{\rho\sigma}^{(v\pi)} &= 2\frac{\alpha}{\pi} B^{\rho\sigma} i_{\rho\sigma}^{(0\pi)} f_\pi^{\text{QED}},\\ B^{\rho\sigma} i_{\rho\sigma}^{(0\pi)} &= 2\frac{\alpha}{\pi} |F_\pi^{\text{str}}(s_1)|^2 \times \\ &\times \left[(4M^2 - s_1) B_g + \frac{1}{8} s^2 B_{11} (x_+ - x_-)^2 (1+c^2)\right] f_\pi^{\text{QED}}. \end{aligned} \tag{18}$$

The explicit expression for the $f_{1,2}^{\mu}$, f_π^{QED} form factors of pion and muon are given in the Appendix.

The soft photon correction to the final state currents reads

$$\begin{aligned} & i_{\rho\sigma}^{(s\pi)} = \frac{\alpha}{\pi}\Delta_{1'2'} i_{\rho\sigma}^{(0\pi)}, \qquad i_{\rho\sigma}^{(s\mu)} = \frac{\alpha}{\pi}\Delta_{1'2'} i_{\rho\sigma}^{(0\mu)},\\ & \Delta_{1'2'} = -\frac{1}{4\pi}\int \frac{d^3k}{\omega}\left(\frac{q_+}{q_+k} - \frac{q_-}{q_-k}\right)^2\Bigg|_{\omega\leqslant\Delta\varepsilon} = \left(\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta} - 1\right)\ln\frac{(\Delta\varepsilon)^2 M^2}{\varepsilon^2 x_+ x_- \lambda^2} + \\ & + \frac{1+\beta^2}{2\beta}\left[-g - \frac{1}{2}\ln^2\frac{1+\beta}{1-\beta} - \ln\frac{1+\beta}{1-\beta}\ln\frac{1-\beta^2}{4} - \frac{\pi^2}{6} - 2\mathrm{Li}_2\left(\frac{\beta-1}{\beta+1}\right)\right],\\ & g = 2\beta\int_0^1 \frac{dt}{1-\beta^2 t^2}\ln\left(1 + \frac{1-t^2}{4}\frac{(x_+ - x_-)^2}{x_+ x_-}\right) = \ln\left(\frac{1+\beta}{1-\beta}\right)\ln(1+z - z/\beta^2) + \\ & + \mathrm{Li}_2\left(\frac{1-\beta}{1+\beta/r}\right) + \mathrm{Li}_2\left(\frac{1-\beta}{1-\beta/r}\right) - \mathrm{Li}_2\left(\frac{1+\beta}{1-\beta/r}\right) - \mathrm{Li}_2\left(\frac{1+\beta}{1+\beta/r}\right),\\ & \beta = \sqrt{1 - \frac{4M^2}{s_1}}, \qquad z = \frac{1}{4}\left(\sqrt{\frac{x_+}{x_-}} - \sqrt{\frac{x_-}{x_+}}\right)^2, \qquad r = |x_+ - x_-|. \end{aligned} \tag{19}$$

This formula provides the generalization of known expression (see (25), (26) in [23]) for the case of small invariant mass $4M^2 \sim \sqrt{s_1} \ll \varepsilon_\pm$.

The contribution of an additional hard photon emission (with momentum k_2) by the muon block, provided $\tilde{s}_1 = (q_+ + q_- + k_2)^2 \sim s_1 \ll s$, can be found by the expression

$$B^{\rho\sigma} i_{\rho\sigma}^{(h\mu)} = \frac{\alpha}{4\pi^2}\int \frac{d^3k_2}{\omega_2} B^{\rho\sigma} \sum J_\rho^{(\gamma)} (J_\sigma^{(\gamma)})^*\Big|_{\omega_2 \geqslant \Delta\varepsilon}, \tag{20}$$

with

$$\sum |J_\rho^{(\gamma)}|^2 = 4Q^2(s_1 + 2k_2q_- + 2k_2q_+ + 2M^2) - 8\frac{(k_2q_-)^2 + (k_2q_+)^2}{(k_2q_-)(k_2q_+)},$$
$$Q = \frac{q_-}{q_-k_2} - \frac{q_+}{q_+k_2},$$

and

$$\sum |J^{(\gamma)}p_\pm|^2 = -8Q^2(q_-p_\pm)(q_+p_\pm) + \\ + 8(p_\pm k_2)\left(Qq_+\frac{p_\pm q_-}{q_+k_2} - Qq_-\frac{p_\pm q_+}{q_-k_2}\right) + 8(p_\pm k_2)\left(\frac{p_\pm q_-}{q_+k_2} + \frac{p_\pm q_+}{q_-k_2}\right) + \\ + 8(p_\pm Q)(p_\pm q_+ - p_\pm q_-) - 8\frac{(k_2p_\pm)^2M^2}{(k_2q_+)(k_2q_-)}. \quad (21)$$

For the case of charged pion pair production, the radiative current tensor has the form

$$i_{\rho\sigma}^{(h\pi)} = -\frac{\alpha}{4\pi^2}\int |F_\pi^{\mathrm{str}}(\tilde{s}_1)|^2\frac{d^3k_2}{\omega_2}\left[\frac{M^2}{\chi_{2-}^2}(Q_1Q_1)_{\rho\sigma} + \frac{M^2}{\chi_{2+}^2}(Q_2Q_2)_{\rho\sigma} - \right. \\ \left.\left. - \frac{q_+q_-}{\chi_{2+}\chi_{2-}}(Q_1Q_2)_{\rho\sigma} + g_{\rho\sigma} - \frac{1}{\chi_{2-}}(Q_1q_-)_{\rho\sigma} + \frac{1}{\chi_{2+}}(Q_2q_+)_{\rho\sigma}\right]\right|_{\omega_2>\Delta\varepsilon}, \quad (22)$$
$$Q_1 = q_- - q_+ + k_2, \qquad Q_2 = q_- - q_+ - k_2, \qquad \chi_{2\pm} = 2k_2q_\pm.$$

One can check that the Bose symmetry and the gauge invariance condition is valid for the pionic current tensor. Namely, it is invariant with regard to the permutation of the pion momenta and turns to zero after conversion with 4-vector q.

The sum of soft and hard photon corrections to the final current does not depend on $\Delta\varepsilon/\varepsilon$.

2.2. Corrections to the Initial State. Let us now consider the Compton tensor with RC, which describe virtual corrections to the initial state. In our kinematical region it will be convenient to rewrite the tensor explicitly extracting large logarithms. We will distinguish two kinds of large logarithms:

$$l_s = \ln\frac{s}{m^2}, \quad l_1 = \ln\frac{s}{s_1}. \quad (23)$$

We rewrite the Compton tensor [22] in the form

$$K_{\rho\sigma} = \left(1 + \frac{\alpha}{2\pi}\rho\right)B_{\rho\sigma} + \frac{\alpha}{2\pi}\left[\tau_g g_{\rho\sigma} + \tau_{11}(p_-p_-)_{\rho\sigma} + \tau_{22}(p_+p_+)_{\rho\sigma} - \frac{1}{2}\tau_{12}(p_-p_+)_{\rho\sigma}\right], \quad (24)$$
$$\rho = -4\ln\frac{m}{\lambda}(l_s - 1) - l_s^2 + 3l_s - 3l_1 + \frac{4}{3}\pi^2 - \frac{9}{2},$$

with $\tau_i = a_i l_1 + b_i$ and

$$a_{11} = -\frac{2s_1}{\chi_+\chi_-}\left[\frac{2b^2}{\chi_+\chi_-} + \frac{4s}{a} + \frac{4(s^2+b\chi_-)}{a^2} - \frac{b^2(2c-\chi_-)}{c^2\chi_-} - \frac{2s+\chi_+}{\chi_+}\right], \tag{25}$$

$$\begin{aligned} b_{11} &= \frac{2}{\chi_+\chi_-}\left[-s_1\left(1+\frac{s^2}{\chi_+^2}\right)G_- - s_1\left(2+\frac{b^2}{\chi_-^2}\right)G_+ - \frac{s_1 b^2(2c-\chi_-)}{c^2\chi_-}\ln\frac{s}{\chi_+} - \right.\\ &- \left.\frac{s_1}{\chi_+}(2s+\chi_+)\ln\frac{s}{\chi_-} - \frac{4(s^2+b\chi_-)}{a} - 4s - 2s_1 - \chi_- - \frac{b^2}{c}\right], \end{aligned} \tag{26}$$

$$\begin{aligned} a_{12} &= -\frac{2s_1}{\chi_+\chi_-}\left[-\frac{4ss_1}{\chi_+\chi_-} + \frac{8(\chi_+\chi_- - s^2)}{a^2} - \frac{4s}{a} + 4ss_1\left(\frac{1}{c\chi_-}+\frac{1}{b\chi_+}\right) + \right.\\ &+ \left.(2ss_1 + 4\chi_+\chi_-)\left(\frac{1}{c^2}+\frac{1}{b^2}\right)\right], \end{aligned} \tag{27}$$

$$\begin{aligned} b_{12} &= \frac{2}{\chi_+\chi_-}\left[\frac{2s_1}{\chi_+^2}(sc-\chi_-\chi_+)G_- + \frac{2s_1}{\chi_-^2}(sb-\chi_-\chi_+)G_+ + \right.\\ &+ s_1\left(\frac{2ss_1+4\chi_-\chi_+}{c^2} + \frac{4ss_1}{c\chi_-}\right)\ln\frac{s}{\chi_+} + s_1\left(\frac{2ss_1+4\chi_-\chi_+}{b^2}+\frac{4ss_1}{b\chi_+}\right)\ln\frac{s}{\chi_-} + \\ &+ \left.\frac{8(s^2-\chi_+\chi_-)}{a} - 2s\left(\frac{\chi_+}{c}+\frac{\chi_-}{b}\right) + 2s_1 + 10s\right], \end{aligned} \tag{28}$$

$$a_g = -2s\left(\frac{s_1}{\chi_+\chi_-} - \frac{2}{a}\right) + \frac{c}{\chi_+}\left(\frac{3s}{b}-1\right) + \frac{b}{\chi_-}\left(\frac{3s}{c}-1\right), \tag{29}$$

$$\begin{aligned} b_g &= -\frac{1}{\chi_+}\left(\frac{ss_1}{\chi_+} + \frac{2sb+\chi_+^2}{\chi_-}\right)G_- - \frac{1}{\chi_-}\left(\frac{ss_1}{\chi_-}+\frac{2sc+\chi_-^2}{\chi_+}\right)G_+ - \\ &- \frac{c}{\chi_+}\left(\frac{3s}{b}-1\right)\ln\frac{s}{\chi_-} - \frac{b}{\chi_-}\left(\frac{3s}{c}-1\right)\ln\frac{s}{\chi_+} + \frac{2s^2-\chi_+^2-\chi_-^2}{2\chi_+\chi_-}, \end{aligned} \tag{30}$$

$$G_- = -\ln^2\frac{\chi_-}{s} + \frac{\pi^2}{3} - 2\mathrm{Li}_2\left(1-\frac{s_1}{s}\right) + 2\mathrm{Li}_2\left(-\frac{s_1}{\chi_-}\right) + 2\ln\frac{s_1}{\chi_-}\ln\left(1+\frac{s_1}{\chi_-}\right),$$

$$G_+ = -\ln^2\frac{\chi_+}{s} + \frac{\pi^2}{3} - 2\mathrm{Li}_2\left(1-\frac{s_1}{s}\right) + 2\mathrm{Li}_2\left(-\frac{s_1}{\chi_+}\right) + 2\ln\frac{s_1}{\chi_+}\ln\left(1+\frac{s_1}{\chi_+}\right),$$

$$\tau_{22}(\chi_-,\chi_+) = \tau_{11}(\chi_+,\chi_-), a = -(\chi_+ + \chi_-), \quad b = s_1 + \chi_-, \quad c = s_1 + \chi_+.$$

The infrared singularity (the presence of the *photon mass* λ in ρ) is compensated by taking into account soft photon emission from the initial particles:

$$\begin{aligned} d\sigma^{\mathrm{soft}} &= d\sigma_0\frac{\alpha}{\pi}\Delta_{12},\\ \Delta_{12} = -\frac{1}{4\pi}\int\frac{d^3k}{\omega}\left(\frac{p_+}{p_+k}-\frac{p_-}{p_-k}\right)^2\Bigg|_{\omega\leqslant\Delta\varepsilon} &= 2(l_s-1)\ln\frac{m\Delta\varepsilon}{\lambda\varepsilon} + \frac{1}{2}l_s^2 - \frac{\pi^2}{3}. \end{aligned} \tag{31}$$

As a result, the quantity ρ in formula (24) will change to

$$\rho \to \rho_\Delta = \left(4\ln\frac{\Delta\varepsilon}{\varepsilon}+3\right)(l_s-1) - 3l_1 + \frac{2\pi^2}{3} - \frac{3}{2}. \tag{32}$$

Cross section of two-hard-photon emission for the case when one of them is emitted collinearly to the incoming electron or positron can be obtained by means of the quasi-real electron method [24]:

$$\begin{aligned}\frac{d\sigma^j_{\gamma\gamma,\,\mathrm{coll}}}{dx_-dc\,ds_1} &= dW_{p_-}(k_3)\frac{d\tilde{\sigma}^j_B(p_-(1-x_3),p_+;k_1,q_+,q_-)}{dx_-dc\,ds_1}+\\ &+ dW_{p_+}(k_3)\frac{d\tilde{\sigma}^j_B(p_-,p_+(1-x_3);k_1,q_+,q_-)}{dx_-dc\,ds_1}, \end{aligned} \tag{33}$$

with

$$dW_p(k_3)=\frac{\alpha}{\pi}\left[\left(1-x_3+\frac{x_3^2}{2}\right)\ln\frac{(\varepsilon\theta_0)^2}{m^2}-(1-x_3)\right]\frac{dx_3}{x_3},\quad x_3=\frac{\omega_3}{\varepsilon},\quad x_3>\frac{\Delta\varepsilon}{\varepsilon}. \tag{34}$$

Here we suppose that the polar angle θ_3 between the directions of the additional collinear photon and the beam axis does not exceed some small value $\theta_0 \ll 1$, $\varepsilon\theta_0 \gg m$.

The *boosted* differential cross section $d\tilde{\sigma}^j_B(p_-x,p_+y;k_1,q_+,q_-)$ with reduced momenta of the incoming particles reads (compare with Eq. (12))

$$\begin{aligned}\frac{d\tilde{\sigma}^\mu_B(p_+x_2,p_-x_1;k_1,q_+,q_-)}{dx_-dcds_1}&=\frac{\alpha^3(1+2\sigma-2\nu_-(1-\nu_-))(x_1^2(1-c)^2+x_2^2(1+c)^2)}{s_1sx_1^2x_2^2(1-c^2)(x_1+x_2+c(x_2-x_1))},\\ \frac{d\tilde{\sigma}^\pi_B(p_+x_2,p_-x_1;k_1,q_+,q_-)}{dx_-dcds_1}&=\frac{\alpha^3(\nu_-(1-\nu_-)-\sigma)(x_1^2(1-c)^2+x_2^2(1+c)^2)}{s_1sx_1^2x_2^2(1-c^2)(x_1+x_2+c(x_2-x_1))},\\ \nu_-=\frac{x_-}{y_2},\qquad y_2&=\frac{2x_1x_2}{x_1+x_2+c(x_2-x_1)}.\end{aligned} \tag{35}$$

In a certain experimental situation, an estimate of the contribution of the additional hard photon emission outside the narrow cones around the beam axes is needed. It can be estimated by

$$\begin{aligned}\frac{d\sigma^j_{\gamma\gamma,\mathrm{noncoll}}}{dx_-dc\,ds_1}&=\frac{\alpha}{4\pi^2}\int\frac{d^3k_3}{\omega_3}\left[\frac{\varepsilon^2+(\varepsilon-\omega_3)^2}{\varepsilon\omega_3}\right]\left\{\frac{1}{k_3p_-}\frac{d\sigma^j_B(p_-(1-x_3),p_+;k_1,q_+,q_-)}{dx_-dc\,ds_1}+\right.\\ &\left.+\frac{1}{k_3p_+}\frac{d\sigma^j_B(p_-,p_+(1-x_3);k_1,q_+,q_-)}{dx_-dc\,ds_1}\right\}\Bigg|_{\theta_3\geqslant\theta_0,\quad \Delta\varepsilon<\omega_3<\omega_1},\quad x_3=\frac{\omega_3}{\varepsilon}.\end{aligned} \tag{36}$$

It is a simplified expression for the two-photon initial-state emission cross section. For the case of a large-angle emission, deviation of our estimate from the exact result is small. It does not depend on s and slightly depends on θ_0. For $\theta_0 \sim 10^{-2}$ we have

$$\frac{\pi}{\alpha}\left|\frac{\int(d\sigma^j_{\gamma\gamma,\,\mathrm{noncoll}}-d\sigma^j_{\gamma\gamma,\,\mathrm{noncoll\,exact}})}{\int d\sigma^j_B}\right|\lesssim 10^{-1}. \tag{37}$$

2.3. Master Formula. By summing up all contributions for the charge-even part, we can put the cross section of the radiative production in the form

$$\frac{d\sigma^j(p_+,p_-;k_1,q_+,q_-)}{dx_-dc\,ds_1}=\int\limits^1\int\limits^1\frac{dx_1dx_2}{|1-\Pi(sx_1x_2)|^2}\frac{d\tilde{\sigma}^j_B(p_+x_2,p_-x_1;k_1,q_+,q_-)}{dx_-dc\,ds_1}\times$$
$$\times D(x_1,l_s)D(x_2,l_s)\left(1+\frac{\alpha}{\pi}K^j\right)+\frac{\alpha}{2\pi}\int\limits_\Delta^1 dx\left[\frac{1+(1-x)^2}{x}\ln\frac{\theta_0^2}{4}+x\right]\times$$
$$\times\left[\frac{d\tilde{\sigma}^j_B(p_-(1-x),p_+;k_1,q_+,q_-)}{dx_-dc\,ds_1}+\frac{d\tilde{\sigma}^j_B(p_-,p_+(1-x);k_1,q_+,q_-)}{dx_-dc\,ds_1}\right]+\frac{d\sigma^j_{\gamma\gamma,\,\mathrm{noncoll}}}{dx_-dc\,ds_1},\tag{38}$$
$$D(x,l_s)=\delta(1-x)+\frac{\alpha}{2\pi}P^{(1)}(x)(l_s-1)+\ldots,$$
$$\Delta=\frac{\Delta\varepsilon}{\varepsilon},\ P^{(1)}(x)=\left(\frac{1+x^2}{1-x}\right)_+,\quad j=\mu,\pi.$$

The boosted cross section $d\tilde{\sigma}$ is defined above, in Eq. (35). The lower limits of the integrals over $x_{1,2}$ depend on the experimental conditions.

The structure function D includes all dependence on the large logarithm l_s. The so-called K factor reads

$$K^j=\frac{1}{R^j}B^{\lambda\sigma}\left(i^{(vj)}_{\lambda\sigma}+i^{(sj)}_{\lambda\sigma}+i^{(hj)}_{\lambda\sigma}\right)+R^{(j)}_{\mathrm{compt}}.\tag{39}$$

Quantities $R^{(j)}_{\mathrm{compt}}$ include the «nonleading» contributions from the initial-state radiation. Generally, they are rather cumbersome expressions for the case $s_1\sim s$. For the case $s_1\sim M^2\ll s$ we obtain

$$R^{(\mu)}_{\mathrm{compt}}=R^{(\pi)}_{\mathrm{compt}}+\frac{c^2}{(1-2x_-x_++2\sigma)(1+c^2)},\tag{40}$$
$$R^{(\pi)}_{\mathrm{compt}}=\frac{1-c^2}{4(1+c^2)}\left\{\frac{5+2c+c^2}{1-c^2}\ln^2\left(\frac{2}{1+c}\right)-\frac{5-c}{1+c}\ln\left(\frac{2}{1+c}\right)+\right.$$
$$\left.+\frac{5-2c+c^2}{1-c^2}\ln^2\left(\frac{2}{1-c}\right)-\frac{5+c}{1-c}\ln\left(\frac{2}{1-c}\right)-4\frac{c^2}{1-c^2}\right\}+\frac{\pi^2}{3}.\tag{41}$$

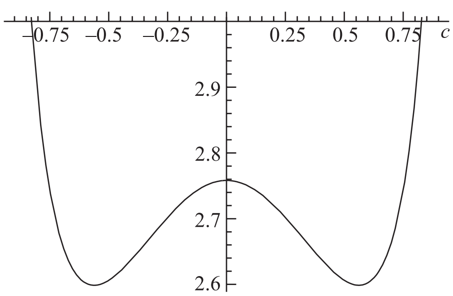

Fig. 2. $R^{(\mu)}_{\mathrm{compt}}$ for $\sigma=0.1$, $x_-=0.1$

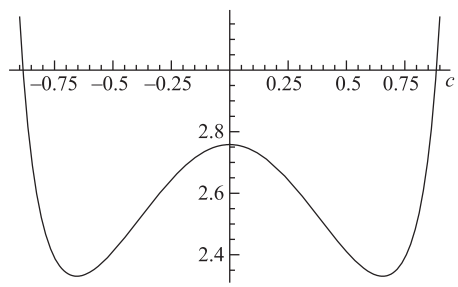

Fig. 3. $R^{(\pi)}_{\mathrm{compt}}$ for $\sigma=0.1$, $x_-=0.1$

Here we see the remarkable phenomena: the cancellation of terms containing $\ln\left(\frac{s}{s_1}\right)$. In such a way only one kind of large logarithm $\ln(s/m^2)$ enters in the final result. This fact is the consequence of the renormalization group invariance.

The values of $R^{(\mu)}_{\rm compt}$ and $R^{(\pi)}_{\rm compt}$ are depicted on Figs. 2 and 3.

CONCLUSIONS

We have considered radiative muon (pion) pair production in high-energy electron–positron annihilation for the charge-blind experimental set-up. Anyway, the charge-odd part of the cross section under consideration is suppressed by the factor $s_1/s \ll 1$ in the kinematics discussed here.

Using the heavy photon Compton tensor [22], we have calculated radiative corrections to this process. Although analogous calculations were performed earlier (see, for example, [13, 15–17]), our main result, equation (38), is new, we believe. This result shows that the cross section in our quasi $2 \to 2$ kinematics can be written in the form of the cross section of the Drell–Yan process. Thus the results of [25] for the RC to the one-photon e^+e^- annihilation into hadrons are generalized to the situation when a hard photon at a large angle is present in the final state.

This generalization is not a trivial fact because the two types of *large logarithms* are present in the problem.

Possible background from the peripheral process $e\bar{e} \to e\bar{e}\mu\bar{\mu}$ is negligible in our kinematics: it is suppressed by the factor $\frac{\alpha}{\pi}\frac{s_1}{s}$ and, besides, can be eliminated if the registration of the primary hard photon (see Eq. (1)) is required by the experimental cuts for event selection.

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APPENDIX

The one-loop QED form factors of muon and pion are

$$\mathrm{Re}\,F_1^{(\mu)}(s_1) = 1 + \frac{\alpha}{\pi} f_1^{(\mu)}(s_1), \quad \mathrm{Re}\,F_2^{(\mu)}(s_1) = \frac{\alpha}{\pi} f_2^{(\mu)}(s_1),$$

$$f_1^{(\mu)}(s_1) = \left(\ln\frac{M}{\lambda} - 1\right)\left(1 - \frac{1+\beta^2}{2\beta} l_\beta\right) + \frac{1+\beta^2}{2\beta}\left(-\frac{1}{4} l_\beta^2 + l_\beta \ln\frac{1+\beta}{2\beta} + \frac{\pi^2}{3} + \right.$$
$$\left. + \, 2\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right) - \frac{1}{4\beta} l_\beta,$$

$$f_2^{(\mu)}(s_1) = -\frac{1-\beta^2}{8\beta} l_\beta, \tag{A.1}$$

$$\mathrm{Re}\,F_\pi^{\rm QED}(s_1) = 1 + \frac{\alpha}{\pi} f_\pi^{\rm QED}(s_1),$$

$$f_\pi^{\rm QED}(s_1) = \left(\ln\frac{M}{\lambda} - 1\right)\left(1 - \frac{1+\beta^2}{2\beta} l_\beta\right) + \frac{1+\beta^2}{2\beta}\left(-\frac{1}{4}l_\beta^2 + l_\beta \ln\frac{1+\beta}{2\beta} + \frac{\pi^2}{3} + \right.$$
$$\left. + \,\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right), \qquad \beta^2 = 1 - \frac{4M^2}{s_1}, \qquad l_\beta = \ln\frac{1+\beta}{1-\beta}.$$

The expressions for leptonic and hadronic [26] contributions to the vacuum polarization operator $\Pi(s)$ are

$$\Pi(s) = \Pi_l(s) + \Pi_h(s),$$
$$\Pi_l(s) = \frac{\alpha}{\pi}\Pi_1(s) + \left(\frac{\alpha}{\pi}\right)^2 \Pi_2(s) + \left(\frac{\alpha}{\pi}\right)^3 \Pi_3(s) + \ldots, \tag{A.2}$$
$$\Pi_h(s) = \frac{s}{4\pi^2\alpha}\left[\mathrm{PV}\int\limits_{4m_\pi^2}^{\infty} \frac{\sigma^{e^+e^-\to \rm hadrons}(s')}{s-s'} ds' - i\pi\sigma^{e^+e^-\to \rm hadrons}(s)\right].$$

The first-order leptonic contribution is well known [27]:

$$\Pi_1(s) = \frac{1}{3}\ln\frac{s}{m^2} - \frac{5}{9} + f(x_\mu) + f(x_\tau) -$$
$$- i\pi\left[\frac{1}{3} + \phi(x_\mu)\Theta(1-x_\mu) + \phi(x_\tau)\Theta(1-x_\tau)\right], \tag{A.3}$$

where

$$f(x) = \begin{cases} -\dfrac{5}{9} - \dfrac{x}{3} + \dfrac{1}{6}(2+x)\sqrt{1-x}\,\ln\left|\dfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right| & \text{for} \quad x \leqslant 1, \\ -\dfrac{5}{9} - \dfrac{x}{3} + \dfrac{1}{3}(2+x)\sqrt{x-1}\,\arctan\left(\dfrac{1}{\sqrt{x-1}}\right) & \text{for} \quad x > 1, \end{cases}$$
$$\phi(x) = \frac{1}{6}(2+x)\sqrt{1-x}, \qquad x_{\mu,\tau} = \frac{4m_{\mu,\tau}^2}{s}.$$

In the second order it is enough to take only the logarithmic term from the electron contribution [28]

$$\Pi_2(s) = \frac{1}{4}\left(\ln\frac{s}{m^2} - i\pi\right) + \zeta(3) - \frac{5}{24}. \tag{A.4}$$

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