Письма в ЭЧАЯ. 2005. Т. 2, № 5(128). С. 61–68

УДК 539.126+530.145

PSEUDOSCALAR MESONS WITH FINITE WIDTH IN DENSE MATTER

V. L. Yudichev[1]

Joint Institute for Nuclear Research, Dubna

The temperature dependence of the widths of pions in the hadronic phase and of open charm (D and D^*) mesons in the deconfined phase is investigated at zero chemical potential within the framework of a Nambu–Jona-Lasinio type chiral quark model.

Исследуется температурная зависимость ширин пионов в адронной фазе и отдельных очарованных (D и D^*) мезонов в фазе деконфайнмента при нулевом химическом потенциале в рамках киральной кварковой модели типа Намбу–Йона-Лазинио.

INTRODUCTION

The in-medium modification of hadron properties under extreme conditions (i.e., at high temperature and density) may significantly affect the observable yield of the particles produced in heavy-ion collision experiments and therefore should be taken into account when analyzing related experimental data. To study their effects one needs first to model the behavior of hadron properties (e.g., the mass and width) at finite temperature and chemical potential. An investigation of two particular aspects of this rather complicated problem is presented below: i) the temperature dependence of the pion width at zero baryonic chemical potential in a nonequilibrium pion gas within a simple effective model for the pion-to-pion interaction derived from the Nambu–Jona-Lasinio (NJL) model for the nonperturbative strong interaction of quarks; ii) the widths of the charmed D and D^* mesons are estimated above the critical temperature of the deconfinement phase transition. The results are used to explain an excess of dileptons in the invariant mass range $\sim 200 \div 400$ MeV in $\mathrm{Pb} + \mathrm{Au}$ (158 GeV/u) collisions [1] over simple on-shell pion annihilation on the one hand, and a mechanism for the anomalous J/ψ suppression in nucleus–nucleus collisions on the other.

1. PIONS IN HOT MATTER

In Ref. [2], the simultaneous mass reduction and broadening of the ρ-meson resonance has been found to be necessary for a realistic explanation of the mass and transverse momentum spectra of dileptons produced in $\mathrm{Pb} + \mathrm{Au}$ (158 GeV/u) collisions [1]. Moreover, it has also been shown in [2] that the introduction of the finite pion width improves the agreement of the fit with experimental data for the dilepton masses between ~ 200 and ~ 400 MeV. As one

[1]E-mail: yudichev@thsun1.jinr.ru

can see from Fig. 2 in [2], the finite pion width of the order 50÷100 MeV has a quite visible effect on the dilepton spectra. One can recall that the width of the pion in the vacuum is negligibly small, compared to these numbers, because in normal conditions the width of the pion is determined by its electroweak decays. On the contrary, in a very dense nonequilibrium pion gas, in-medium collisions of a pion with its comovers significantly shorten the average lifetime of a pion with certain quantum numbers and thus increase its width.

The description of the resonance's properties modification in hot and dense media has been systematically done by Kadanoff and Baym in a nonrelativistic approach [3], which, however, can be extended to the case of relativistic mesons. Following [3], the temperature dependence of the pion width at zero chemical potential has been investigated in [4], using a simple meson Lagrangian derived from the NJL (see, e.g., [5–8]) model for strong interaction of quarks in the mean-field approximation.

The pion broadening to a large extent depends on the mass of the resonance: the lighter the particle, the stronger effect. This is the reason why the pion is in focus. Moreover, due to the same cause, the pion dominates in a hot meson gas and pion annihilation is the dominant contribution to the yield of dileptons and photons.

Following the prescription given by Kadanoff and Baym [3], one needs an estimate for the contribution to the imaginary part of the hadron self-energy (i.e., the decay width) from collision integrals. Such contributions have a straightforward interpretation: they correspond to the inverse of the average lifetimes. To obtain them one needs to know cross sections for different collision processes averaged over the density of particles with the Bose amplification and Pauli suppression taken into account.

If the particle has a nonzero width, all the cross sections to which it contributes should be averaged using its spectral function, which can be chosen in the Breit–Wigner form

$$A(s) = \frac{M\Gamma}{(s - M^2)^2 + M^2\Gamma^2}, \tag{1}$$

where M is the mass of the resonance, and Γ is its width. In this approximation both quantities have a temperature and density dependence.

In general, the width is a function of energy and momentum, and one would finally come to a set of functional equations, difficult to solve. In Ref. [4] the pion width is approximated by a constant, thus leading to much more simple equation to be solved by iterations. Yet, the width is supposed to depend on the temperature and density.

According to [3], the pion damping width is defined as follows:

$$\Gamma(p) = \Sigma^>(p) - \Sigma^<(p), \tag{2}$$

where

$$\Sigma^<(p) = \int\limits_{p_1}\int\limits_{p_3}\int\limits_{p_4}(2\pi)^4\delta_{p_1,p;\,p_3,p_4}|\mathcal{T}|^2 G^>(p_1)G^<(p_3)G^<(p_4), \tag{3}$$

$$\Sigma^>(p) = \int\limits_{p_1}\int\limits_{p_3}\int\limits_{p_4}(2\pi)^4\delta_{p_1,p;\,p_3,p_4}|\mathcal{T}|^2 G^<(p_1)G^>(p_3)G^>(p_4). \tag{4}$$

Here, $\mathcal{T}$ is the process amplitude; $G_i^>(p) = [1 + n_i(\mathbf{p}, s_i, T)]A_i(p^2)$; $G_i^<(p) =$

$n_i(\mathbf{p}, s, T)A_i(p^2)$, with n_i being the boson occupation numbers

$$n_i(\mathbf{p}, s_i, T) = \left[\exp\left(\sqrt{\mathbf{p}^2 + s_i}\Big/T\right) - 1\right]^{-1}, \tag{5}$$

and $A_i(p^2)$ — the spectral function of the ith state; here the notation $\int\limits_p = \int \frac{d^4p}{(2\pi)^4}$, where $\delta_{p_1,p_2;\,p_3,p_4} = \delta(p_1 + p_2 - p_3 - p_4)$, is used. The integration is performed in the four-dimensional momentum space over the momenta p_1, p_2, p_3, p_4, with the indices 1 and 2 corresponding to the initial states, and 3 and 4 to the final ones.

Then, for the pion width one obtains

$$\Gamma = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int ds_1 v_{\rm rel} A_\pi(s_1)[n_\pi(\mathbf{p}_1, s_1, T)\sigma^{\rm dir^*}(s; s_1, s_2) - \\ - (1 + n_\pi(\mathbf{p}_1, s_1, T))\sigma^{\rm inv^*}(s; s_1, s_2)], \tag{6}$$

where $v_{\rm rel}$ is the relative velocity of particles 1 and 2, and $\sigma^*(s; s_1, s_2)$ is the averaged cross section with the spectral functions of final states taken into account:

$$\sigma^*(s; s_1, s_2) = \int ds_3 \int ds_4 A_\pi(s_3)A_\pi(s_4)\sigma(s; s_1, s_2, s_3, s_4). \tag{7}$$

In Ref. [4], the processes $\pi^0\pi^0 \to \pi^0\pi^0$, $\pi^0\pi^0 \to \pi^+\pi^-$, $\pi^0\pi^\pm \to \pi^0\pi^\pm$ were considered as giving the main contribution to the pion damping width. Other possible processes, like $\pi\pi \to \sigma\sigma$, $\pi\sigma \to \pi\sigma$, $\pi\pi \to \sigma$, and $\pi\pi \to \bar{q}q$ were estimated to give a negligible contribution to the pion width and were omitted during calculations for simplicity. The lightest scalar isoscalar resonance, the σ meson, was allowed as an intermediate state ($\pi\pi \to \sigma \to \pi\pi$) because of its importance shown in various investigations of the pion–pion interaction [5, 9–11]. Using the $SU(2)_F \times SU_F(2)$ NJL model [5, 6], one gets the Lagrangian for pions and σ mesons in the form

$$\mathcal{L}_{\rm int} = 2mg_\sigma\sigma^3 + 2mg_\pi\sqrt{Z}\sigma(2\pi^+\pi^- + (\pi^0)^2) - g_\pi^2\sigma^2\pi^2 - \\ - \frac{g_\pi^2 Z}{2}(4\pi^+\pi^-\pi^+\pi^- + 4\pi^+\pi^-(\pi^0)^2 + (\pi^0)^4) - \frac{g_\sigma^2}{2}\sigma^4, \tag{8}$$

where m is the constituent quark mass ($m = g_\pi f_\pi$; $f_\pi \approx 93$ MeV is the pion weak decay constant in the vacuum). The constants g_π and g_σ describe the interaction of the pion and σ meson with quarks in the NJL model. In the vacuum one has: $m = 242$ MeV, $g_\pi = 2.61$, and $g_\sigma = 2.18$ (see [8]). Using their temperature dependence obtained in [8] for zero chemical potential, one can calculate the pion width at temperature from 0 to 180 MeV[1]. The resulting curves for a neutral pion rested in the heat-bath frame are shown in Fig. 1.

[1]The upper limit corresponds to approximate condition for the deconfinement of quarks, above which the effective meson Lagrangian cannot be used.

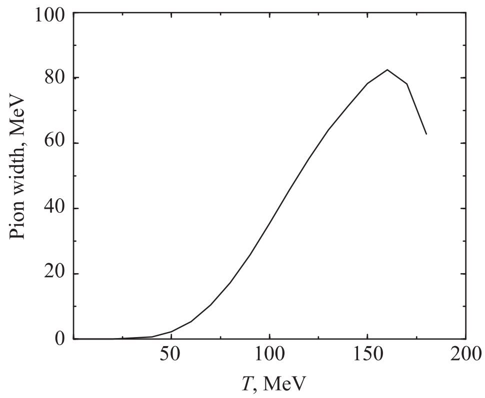

Fig. 1. Pion damping width as a function of temperature at zero chemical potential

As one can see from the figure, the pion state broadens noticeably already at $T \approx$ 60 MeV. At $T = 160$ MeV, the pion–pion scattering accounts for about 80 MeV in the total width; this is the maximum value. Then the curve in Fig. 1 turns down; this behavior of the damping width is caused by a noticeable decrease of the constant g_π after $T = 160$ MeV. The cross section is proportional to g_π^4 whose value reduces by half at $T = 180$ MeV, compared to $T = 160$ MeV. For the temperatures from 160 to 180 MeV, this weakening of the pion–pion interaction overcompensates the expected increase in collision integrals.

A self-consistent approach for the calculation of the pion width via a functional integral for the meson propagator has been suggested by van Hees and Knoll in [12]. However, the authors have chosen the pion damping width by hand to estimate the strength of the interaction rather than to predict it.

2. D AND D^* RESONANCES AND DECONFINEMENT

The modification of c-quark interaction in the quark–gluon plasma (QGP) suggested by Matsui and Satz [13] seems to be an explanation for the «abnormal» suppression of J/ψ production in Pb–Pb collisions found by the NA50 collaboration [14] at the CERN SPS. However, it is much likely that bound $\bar{c}c$ states disappear at temperatures higher than those reached at SPS. Therefore, one should examine other possible mechanisms contributing to the J/ψ suppression.

Recently, a new approach [15] based on the quark substructure of hadrons has been suggested which resulted in a characteristic energy dependence of the J/ψ breakup cross section by light hadron impact at the chiral/deconfinement phase transition. There were studied processes like $J/\psi + \pi \to D^* + \bar{D}$ and $J/\psi + \rho \to D^* + \bar{D}^*$, and it was found that the Mott effect for D mesons effectively reduces the threshold for a charmonium breakup leading to a strong enhancement in the J/ψ breakup rate. Therefore, it is of importance to know the in-medium behavior of the charmed mesons spectral functions which are input data for calculations of the J/ψ breakup rate.

Partially, D-meson properties (the D-meson mass) at finite temperature have already been studied within the standard NJL model (see, e.g., [16]). To study the spectral function in the mean field approximation, one has to allow a breakup of a D meson to free quark–antiquark pairs in the deconfined phase and forbid the free quark production at low temperatures, which is impossible to do within the usual NJL.

Recently, an approach has been suggested [8] where unphysical quark–antiquark thresholds in the domain of confinement were eliminated by means of a 3D infrared (IR) cutoff in quark-loop integrals over the relative quark momentum. The critical temperature, $T_c = 186$ MeV [8], was defined as corresponding to the Mott effect for the pion ($2m_u(T_c) = M_\pi(T_c)$). Below

T_c, spectral functions of all mesons are δ functions, and only above T_c, i.e., in the deconfined phase, meson spectral functions are smooth functions of energy and can be approximately described with the Breit–Wigner Ansatz.

Below, the temperature dependence of the D-meson spectral function in a matter with zero baryonic chemical potential from the NJL model with the IR cutoff is discussed. Only mesons with open charm are considered, and, for simplicity, terms like the 't Hooft term, which is necessary for the description of the singlet-octet mixing in the standard NJL, are omitted.

One might argue that an inclusion of the charm quantum number would require an $SU_F(4)$ extension of the NJL model. But, following Ref. [16] and projecting on a subgroup, which is $SU_F(3)$, only u, d, and c are kept instead of constructing a full $SU_F(4)$ version of NJL. The treatment of open-charmed mesons thus is the same as of the strange mesons. In the pseudoscalar channel, the $SU_F(3)$ symmetry is assumed and possible effects of the explicit $SU_F(4)$ breaking in QCD for the effective quark interaction are disregarded, while in the vector channel a modification of the quark interaction constant is admitted to reproduce the experimental D^*-meson mass. The Lagrangian is

$$\mathcal{L}_q = \bar{q}(i\hat{\partial} - \hat{m}^0)q + \frac{G_1}{2}\sum_{a=1}^{7}\left[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\right] - \\ - \frac{G_2}{2}\sum_{a=1}^{3}\left[(\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda^a q)^2\right] - \frac{G_2^*}{2}\sum_{a=4}^{7}\left[(\bar{q}\gamma_\mu\lambda^a q)^2 + (\bar{q}\gamma_5\gamma_\mu\lambda^a q)^2\right], \quad (9)$$

where q and $\bar{q}$ are u, d, and c quarks, respectively; the constant G_1 describes the quark interaction in the scalar and pseudoscalar channels, while G_2 and G_2^* do the same in the vector and axial-vector channels[1], and λ^a are the Gell-Mann lambda matrices acting in flavor space. The chiral symmetry is explicitly broken in (9) by current quark masses entering into the diagonal matrix $\hat{m}^0 = \mathrm{diag}\,(m_u^0, m_d^0, m_c^0)$, where the isotopic symmetry is assumed ($m_u^0 = m_d^0$).

At zero temperature and chemical potential, chiral symmetry is spontaneously broken and quarks acquire constituent masses determined by gap equations of the type[2]

$$m_q^0 = m_q[1 - 8G_1 I_1^\Lambda(m_q)] \quad (q = u, d, c). \quad (10)$$

Here, $I_1^\Lambda(m_q)$ is the tadpole integral regularized by a 3D ultraviolet (UV) cutoff Λ:

$$I_1^\Lambda(m_q) = -i\frac{N_c}{(2\pi)^4}\int \frac{\theta(\Lambda^2 - |\mathbf{k}|^2)d^4k}{m_q^2 - k^2 - i\varepsilon}. \quad (11)$$

Here, $N_c = 3$ is the number of colors; the 3D momentum $\mathbf{k}$ refers to the heat-bath frame.

[1] In the case of strange mesons one just put $G_2 = G_2^*$, while for the charmed mesons one should choose G_2^* different from G_2 in order to fit the model D^*-meson mass to its experimental value.

[2] In general, one has a system of gap equations. Only if the 't Hooft term is excluded, the equations decouple and can be solved separately.

In the NJL model the masses of pseudoscalar mesons are determined by the equation

$$1 - G_1 \Pi_{\rm ps}(P^2) = 0, \tag{12}$$

where $\Pi_{\rm ps}(P^2)$ is the pseudoscalar polarization operator depending on meson's 4-momentum squared, P^2. Further, all calculations are done for the mesons resting in the heat-bath frame. For Π_D one obtains

$$\Pi_D(P^2) = 4I_1^\Lambda(m_u) + 4I_1^\Lambda(m_c) + 4\left(P^2 - (m_u - m_c)^2\right) I_2^{(\lambda_P,\Lambda)}(P^2, m_u, m_c), \tag{13}$$

where

$$I_2^{(\lambda_P,\Lambda)}(P, m_1, m_2) = -i\frac{N_c}{(2\pi)^4}\int \frac{\theta(\Lambda^2 - |\mathbf{k}|^2)\theta(|\mathbf{k}|^2 - \lambda_P^2)d^4k}{(m_1^2 - k^2 - i\varepsilon)(m_2^2 - (k-P)^2 - i\varepsilon)}, \tag{14}$$

with $P = (M, 0, 0, 0)$ for a meson at rest.

In the integrals $I_2^{(\lambda_P,\Lambda)}(P^2, m_1, m_2)$ a 3D infrared (IR) cutoff λ_P is implemented such that unphysical imaginary parts are removed from the integrals. One can check that an IR cutoff in the form

$$\lambda_P = \left[\theta(m_{q1} + m_{q2} - m_{q1}^{\rm cr} - m_{q2}^{\rm cr}) + \frac{m_{q1} + m_{q2}}{m_{q1}^{\rm cr} + m_{q2}^{\rm cr}}\theta(m_{q1}^{\rm cr} + m_{q2}^{\rm cr} - m_{q1} - m_{q2})\right] \times \\ \times \sqrt{\frac{(P^2 - (m_{q1}^{\rm cr} + m_{q2}^{\rm cr})^2)(P^2 - (m_{q1}^{\rm cr} - m_{q2}^{\rm cr})^2)}{4P^2}} \tag{15}$$

ensures the absence of unphysical imaginary parts in the integrals I_2 with different quark masses[1]. Critical values for the quark masses $m^{\rm cr}$ are defined as $m_q^{\rm cr} = m_q(T_c)$.

In the vector-meson sector, the polarization is

$$\Pi_{\mu\nu}(P^2) = \left(\frac{P_\mu P_\nu}{P^2} - g_{\mu\nu}\right)\Pi_{\rm vec}^T(P^2) + \frac{P_\mu P_\nu}{P^2}\Pi_{\rm vec}^L(P^2), \tag{16}$$

with $\Pi_{\rm vec}^T$ and $\Pi_{\rm vec}^L$ being the transversal and longitudinal parts, respectively. For the D^* meson one has

$$\Pi_{D^*}^T(P^2) = \frac{8}{3}\left(P^2 - \frac{3}{2}(m_u - m_c)^2\right) I_2^{(\lambda_P,\Lambda)}(P^2, m_u, m_c), \tag{17}$$

$$\Pi_{D^*}^L(P^2) = -4(m_u - m_c)^2 I_2^{(\lambda_P,\Lambda)}(P^2, m_u, m_c). \tag{18}$$

Masses can be found by solving equations of the type

$$1 - G_2 \Pi_{\rm vec}^T(P^2) = 0. \tag{19}$$

---

[1] In Ref. [8] the quark mass was unique and here the definition of the IR cutoff has been adapted for a bound state of quarks with different masses.

For each pseudoscalar meson, the quark-meson and the weak-decay coupling constants can be calculated using the formulas

$$g_D = \left(\Pi'_D(M_D^2)\right)^{-1/2}, \qquad F_D = \frac{m_u + m_c}{2g_D}. \tag{20}$$

Using the parameter fixing procedure described in [8], one obtains: $\Lambda = 1.094$ GeV, $G_1 = 2.978$ GeV^{-2}, $G_2 = 11.842$ GeV^{-2}, $G_2^* = 22.334$ GeV^{-2}, $m_u^0 = 2.1$ MeV, $m_c^0 = 1017$ MeV, $m_u = 242$ MeV, $m_c = 1728$ MeV. The model predicts the quark-meson coupling and weak-decay constants: $g_D = 3.58$ and $F_D = 275$ MeV. In the NJL model without IR cutoff one should choose the following model parameters: $\Lambda = 1.034$ GeV, $G_1 = 3.443$ GeV^{-2}, $G_2 = 15.9$ GeV^{-2}, $G_2^* = 20.6$ GeV^{-2}, $m_u^0 = 2.1$ MeV, $m_s = 50.3$ MeV, $m_c^0 = 780$ MeV, and obtain the constituent quark masses: $m_u = 281$ MeV, $m_c = 1459$ MeV, the constants: $g_D = 5.17$ and $F_D = 168$ MeV. Note that $m_u + m_c < M_D$ in the model without the IR cutoff, and the D meson is therefore unstable in the vacuum[1].

The extension to the case of finite temperature consists in introducing the temperature dependence into the loop integrals I_1 and I_2, using, e.g., the Matsubara formalism (see [8] for details). Solving the gap and mass equations at arbitrary temperature, one then obtains the temperature dependence of the masses. The results are shown in Figs. 2 and 3. In Ref. [17] strange quarks and kaon were considered for comparison, their masses are also plotted in Figs. 2 and 3.

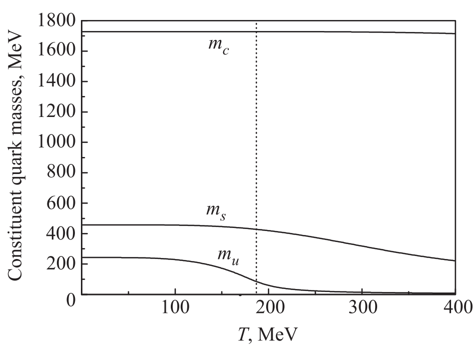

Fig. 2. The $u(d)$, s, and c quark masses as functions of temperature

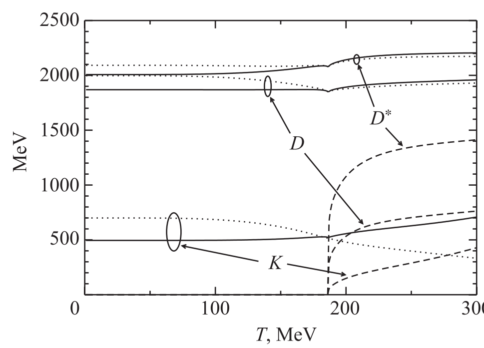

Fig. 3. The D, D^*, and kaon masses (solid curves) and widths (dashed curves) at various temperatures

At the critical temperature $T_c \approx 186$ MeV, the Mott effect for the pion takes place and all mesons are allowed to decay into free quarks if they exceed breakup thresholds. Equating the meson propagator to the Breit–Wigner Ansatz near the pole

$$M^2 - P^2 - iM\Gamma = (\mathrm{Re}\,\Pi'(M^2))^{-1}\left(\frac{1}{G} - \mathrm{Re}\,\Pi(P^2) - i\,\mathrm{Im}\,\Pi(P^2)\right), \tag{21}$$

[1]The D meson is stable in the vacuum for the parametrization used in [7].

one obtains the width

$$\Gamma_{\bar{q}q} = \frac{\operatorname{Im}\Pi(M^2)}{M \operatorname{Re}\Pi'(M^2)}. \tag{22}$$

Using Eq. (22) at temperatures above T_c, one can draw a curve on the Γ–T plane, also depicted in Fig. 3. Using this correspondence between the temperature and the width, one can, in particular, estimate the contribution to the «abnormal» J/ψ suppression due to an in-medium impact by a comover ($J/\psi + \pi \to D + \bar{D}^*$) (see [17]). It has been shown in [17] that due to the Mott effect for D mesons at the QGP phase transition a reduction of the threshold for charmonium breakup occurs, which leads to a steplike behavior of the J/ψ breakup rate corresponding to a drop in the J/ψ lifetime.

Acknowledgements. The author thanks M. K. Volkov and D. Blaschke for useful discussions. The work has been supported by RFBR grant No. 02-02-16194, Heisenberg–Landau Program and «Dynasty» Foundation.

REFERENCES

1. *Agakichiev G. et al. (CERES Collab.)* // Phys. Lett. B. 1998. V. 422. P. 405;
Lenkeit B. et al. (CERES Collab.) // Nucl. Phys. A. 1999. V. 661. P. 23.
2. *Schulze H.-J., Blaschke D.* // Part. Nucl., Lett. 2004. V. 1. No. 2(119). P. 27.
3. *Kadanoff L. P., Baym G.* Quantum Statistical Mechanics. N. Y., 1962.
4. *Blaschke D., Volkov M. K., Yudichev V. L.* // Phys. At. Nucl. 2003. V. 66. P. 2233.
5. *Volkov M. K.* // Sov. J. Part. Nucl. 1986. V. 17. P. 186.
6. *Ebert D., Reinhardt H., Volkov M. K.* // Prog. Part. Nucl. Phys. 1994. V. 35. P. 1.
7. *Klevansky S. P.* // Rev. Mod. Phys. 1992. V. 64. P. 649.
8. *Blaschke D. B. et al.* // Eur. Phys. J. A. 2001. V. 11. P. 319.
9. *Hatsuda T., Kunihiro T.* // Phys. Rep. 1994. V. 247. P. 221.
10. *Volkov M. K., Radzhabov A. E., Russakovich N. L.* hep-ph/0203170; Phys. At. Nucl. 2003. V. 66. P. 997.
11. *Quack E. et al.* // Phys. Lett. B. 1995. V. 348. P. 1.
12. *van Hees H., Knoll J.* // Nucl. Phys. A. 2000. V. 683. P. 369.
13. *Matsui T., Satz H.* // Phys. Lett. B. 1986. V. 178. P. 416.
14. *Abreu M. C. et al. (NA50 Collab.)* // Phys. Lett. B. 2000. V. 477. P. 28.
15. *Blaschke D. B. et al.* nucl-th/0211058.
16. *Gottfried F. O., Klevansky S. P.* // Phys. Lett. B. 1992. V. 286. P. 221.
17. *Blaschke D. B. et al.* // Prog. Theor. Phys. Suppl. 2003. V. 149. P. 182.