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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

z-SCALING IN HEAVY ION COLLISIONS AT THE RHIC

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Experimental data on transverse particle spectra obtained by the STAR, PHENIX, PHOBOS and BRAHMS collaborations at the RHIC are analyzed in the framework of the generalized concept of z-scaling. It was developed for analysis of inclusive particle production in proton–(anti)proton collisions at high p_T and high multiplicities. General scheme of the approach based on the physical principles of self-similarity, locality and fractality is reviewed. Independence of the scaling function $\psi(z)$ from energy, multiplicity and atomic weight for $h^{\pm}, \pi^{\pm,0}, K_S^0, \Lambda$ hadrons produced in Au–Au and Cu–Cu collisions at $\sqrt{s} = 130$ and 200 GeV is discussed. Based on z-scaling the multiplicity dependence of pion transverse spectra up to $p_T = 25$ GeV/c in Au–Au collisions at $\sqrt{s} = 200$ GeV for experiments at the RHIC is predicted.

Экспериментальные данные по поперечным спектрам частиц, полученные коллаборациями STAR, PHENIX, PHOBOS и BRAHMS на коллайдере RHIC, анализируются в рамках обобщенного z-скейлинга. Этот подход был развит для анализа инклюзивного рождения адронов в протон-(анти)протонных взаимодействиях при больших поперечных импульсах и больших множественностях. Описывается общая концепция подхода, основанная на принципах самоподобия, локальности и фрактальности. Обсуждается независимость скейлинговой функции $\psi(z)$ от энергии, плотности множественности и атомного номера для частиц $h^{\pm}, \pi^{\pm,0}, K_S^0, \Lambda$, рожденных в Au–Au- и Cu–Cu-столкновениях при $\sqrt{s} = 130$ и 200 ГэВ. Сделаны предсказания спектров π-мезонов при больших поперечных импульсах в Au–Au-взаимодействиях при $\sqrt{s} = 200$ ГэВ для экспериментов на RHIC.

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INTRODUCTION

Multiple particle production in nucleus–nucleus interactions at high energies has relevance to collective phenomena in produced nuclear matter. Therefore, search for a new state of matter (Quark–Gluon Plasma) and study of its properties are the main goals of heavy ion program at the Relativistic Heavy Ion Collider (RHIC) at BNL and at the Large Hadron Collider (LHC) at CERN.

Many phenomenological approaches to the description of particle production are used to search for regularities reflecting general principles in these systems at high energies [1–10]. The general principles of self-similarity, locality and fractality have heuristic strength. This allows one to formulate rules and laws beyond perturbation theory. These principles can be applied for description of both soft and hard regimes of particle production. The principle of locality states that interactions of hadrons and nuclei can be described in terms of

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interactions of their constituents. The principle of self-similarity states that physics phenomena can be described by nondimensional combinations of dimensional physical quantities. The existence of substructure of interacting constituents over a wide scale range is a specific feature connected with the fractal structure of the colliding objects, interaction of their constituents and particle formation.

In the paper we use the phenomenological approach (z-scaling) proposed in [11] for analysis of inclusive reactions in heavy ion collisions. They are characterized by high multiplicity density and production of particles with high transverse momentum p_T investigated at the RHIC. The approach is essentially based on the principles mentioned above. In the approach the z-presentation revealing simple symmetry properties instead of a usual presentation of experimental data (for example, $Ed^3\sigma/dp^3$ versus p_T) is suggested. The scaling function $\psi(z)$ and the variable z are constructed using the experimentally measured inclusive cross section $Ed^3\sigma/dp^3$ and the multiplicity density $dN/d\eta$.

In the original version of z-scaling [11], the construction was based on the assumption that gross features of the inclusive particle distribution for the inclusive reaction

$$M_1 + M_2 \rightarrow m_1 + X \tag{1}$$

at high energies can be described in terms of the corresponding exclusive subprocess

$$(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2). \tag{2}$$

Here M_1 and M_2 are the masses of the colliding hadrons (or nuclei) and m_1 is the mass of the inclusive particle. The mass parameter m_2 is introduced in connection with internal conservation laws (for isospin, baryon number, strangeness...). The symbols x_1 and x_2 stand for momentum fractions of the incoming four-momenta P_1 and P_2 of the colliding objects. The scaling variable z was constructed as a fractal measure

$$z = z_0 \Omega^{-1}(x_1, x_2), \quad \text{where} \quad \Omega(x_1, x_2) = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2} \tag{3}$$

with a characteristic power dependence on the nucleon fractal dimensions δ_1 and δ_2 in space of the momentum fractions $\{x_1, x_2\}$. The multiplicity density of charged particles produced in the central region of the interaction ($\eta = 0$) determines the scale of $z \sim [dN/d\eta|_0]^{-1}$. The independence of the scaling function $\psi(z)$ from the collision energy $\sqrt{s}$ and the angle θ of the produced inclusive particle were established for constant values of δ_1 and δ_2 [11].

The concept of the z-scaling was generalized [12, 13] for events of various multiplicities of charged particles produced in proton–(anti)proton collisions. The generalization was connected with introduction of a momentum fraction y in the final state of elementary subprocess written in the symbolic form

$$(x_1 M_1) + (x_2 M_2) \rightarrow \left(\frac{m_1}{y}\right) + \left(x_1 M_1 + x_2 M_2 + \frac{m_2}{y}\right). \tag{4}$$

It was shown that the generalized scaling represents a regularity in both soft and hard regimes in proton–(anti)proton collisions over a wide range of initial energies and multiplicities of the produced particles. In this paper we show that independence of the scaling function $\psi(z)$ from the collision energy $\sqrt{s}$ and multiplicity density $dN/d\eta$ can be restored for particle production in heavy ion collisions too. The results of analysis of experimental data obtained by the STAR, PHENIX, PHOBOS and BRAHMS collaborations at the RHIC confirmed the general properties of z-presentation established for proton–(anti)proton collisions.

1. GENERALIZED z-SCALING

We consider a collision of extended objects (hadrons, nuclei) at sufficiently high energies as an ensemble of individual interactions of their constituents. The constituents are partons in the parton model or quarks and gluons in the theory of Quantum Chromodynamics (QCD). A single interaction of the constituents is illustrated in Fig. 1. Structures of the colliding objects are characterized by the fractal dimensions δ_1 and δ_2 in the space of momentum fractions $\{x_1, x_2\}$. The interacting constituents carry the fractions x_1 and x_2 of the momenta P_1 and P_2 of the incoming objects. The subprocess is considered to be a binary collision with production of the scattered and recoil constituents, respectively. The inclusive particle carries the momentum fraction y_a of the scattered constituent with a fragmentation characterized by the fractal dimension ϵ_a. In the approximation [12], fragmentation of the recoil constituent is described by the same fractal dimension ϵ_b and the same momentum fraction y_b[1]. Multiple interactions are considered to be similar. This property is a manifestation of the self-similarity of the hadronic interaction at the constituent level. The interactions are governed by the local energy-momentum conservation law. The self-similarity at small scales reflects a fractality of the interacting objects and their constituents characterized by the corresponding fractal dimensions. The fractality concerns the parton content of the composite structures involved.

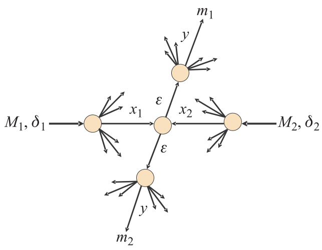


Fig. 1. Diagram of the constituent subprocess

1.1. Locality, Self-similarity and Fractality. The idea of the z-scaling is based on the assumption [6] that gross features of an inclusive particle distribution of reaction (1) can be described at high energies in terms of the kinematic characteristics of the corresponding constituent subprocesses. The subprocess is considered to be a binary collision (4) of the constituents (x_1M_1) and (x_2M_2) resulting in the scattered (m_1/y) and recoil $(x_1M_1 + x_2M_2 + m_2/y)$ objects in the final state. The inclusive particle with the mass m_1 and the 4-momentum p carries the fraction y of the 4-momentum of the scattered constituent. Its counterpart (m_2), moving in the opposite direction, carries the 4-momentum fraction y of the produced recoil. The binary subprocess satisfies the energy-momentum conservation law written in the form

$$\left(x_1P_1 + x_2P_2 - \frac{p}{y}\right)^2 = \left(x_1M_1 + x_2M_2 + \frac{m_2}{y}\right)^2. \tag{5}$$

The equation is an expression of the locality of the hadron interaction at a constituent level. It represents a kinematic constraint on the fractions x_1, x_2 and y.

We assume that self-similarity is a general property of hadron interactions at a constituent level. The self-similar solution is constructed in terms of the self-similarity parameters. Therefore, we search for a solution

$$\psi(z) = \frac{1}{N\sigma_{\text{in}}}\frac{d\sigma}{dz} \tag{6}$$

[1]More general case, $y_a \neq y_b$ and $\epsilon_a = \epsilon_b$, is considered in [14].

depending on a single self-similarity variable z. Here $\sigma_{\rm in}$ is an inelastic cross section of reaction (1) and N is an average particle multiplicity. The variable z is a specific dimensionless combination of quantities which characterize particle production in high-energy inclusive reactions. It depends on momenta and masses of the colliding and inclusive particles, structural parameters of the interacting objects and dynamical characteristics of the produced system. We define the self-similarity variable z as

$$z = z_0 \Omega^{-1}, \tag{7}$$

where

$$\Omega(x_1, x_2, y) = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}(1 - y)^{\epsilon}. \tag{8}$$

The variable z has a character of a fractal measure. A common property of fractal measures is their divergence with the increasing resolution

$$z(\Omega) \to \infty, \qquad \text{if} \;\; \Omega^{-1} \to \infty. \tag{9}$$

For an infinite resolution, all momentum fractions become unity ($x_1 = x_2 = y = 1$) and $\Omega = 0$. This kinematical limit corresponds to the fractal limit $z = \infty$. For a given reaction (1), its finite part z_0 is proportional to the transverse kinetic energy of the constituent subprocess consumed on the production of the inclusive particle (m_1) and its counterpart (m_2). The divergent factor Ω^{-1} describes a resolution at which the subprocess can be singled out of this reaction. The $\Omega(x_1, x_2, y)$ is a relative number of all parton configurations containing the incoming constituents which carry the fractions x_1 and x_2 of the momenta P_1 and P_2 and which fragment to the inclusive particle (m_1) and its counterpart (m_2) with the corresponding momentum fraction y. The factors δ_1 and δ_2 relate to the fractal structure of the colliding objects (hadrons or nuclei). The parameter ϵ characterizes the fractality of the fragmentation process in the final state. For inelastic proton–proton collisions we have $\delta_1 = \delta_2 \equiv \delta$. We also assume that the fragmentation of the scattered and recoil constituents is governed by the same fractal dimensions $\epsilon_a = \epsilon_b \equiv \epsilon/2$.

1.2. Principle of Minimal Resolution. The momentum fractions x_1, x_2 and y are determined from a principle of a minimal resolution of the fractal measure z. The principle states that the resolution Ω^{-1} should be minimal with respect to all binary subprocesses (4) in which the inclusive particle m_1 with the momentum p can be produced. This singles out the underlying interaction of the constituents. The momentum fractions x_1, x_2 and y are found from a minimization of $\Omega^{-1}(x_1, x_2, y)$,

$$\frac{\partial \Omega(x_1, x_2, y)}{\partial x_1} = 0, \qquad \frac{\partial \Omega(x_1, x_2, y)}{\partial x_2} = 0, \qquad \frac{\partial \Omega(x_1, x_2, y)}{\partial y} = 0 \tag{10}$$

taking into account the energy-momentum conservation law (5).

1.3. Scaling Variable z. Search for an adequate, physically meaningful but still sufficiently simple form of the self-similarity parameter z plays a crucial role in our approach. We define the scaling variable z in the form

$$z = \frac{s_\perp^{1/2}}{(dN/d\eta|_0)^c m} \Omega^{-1}. \tag{11}$$

Here m is a mass constant which we fix at the nucleon mass. The quantity $s_\perp^{1/2}$ is the transverse kinetic energy of the constituent subprocess consumed on the production of the inclusive particle (m_1) and its counterpart (m_2) (for more details see [12]). The quantity $dN/d\eta|_0$ is the average multiplicity density of charged particles produced in the central region of reaction (1) at pseudorapidity $\eta = 0$.

1.4. Scaling Variable z and Entropy S. The scaling variable (11) is equal to the ratio

$$z = \frac{s_\perp^{1/2}}{Wm} \tag{12}$$

of the transverse kinetic energy $s_\perp^{1/2}$ and the maximal number of the configurations W. The quantity

$$W = \left(\frac{dN}{d\eta}\bigg|_0 \right)^c \Omega \tag{13}$$

is proportional to all parton and hadron configurations of the colliding system which can contribute to the production of the inclusive particle with the momentum p. According to statistical physics, entropy of a system is given by a number of all statistical states W of the system as follows:

$$S = \ln W. \tag{14}$$

Using (13), we can write

$$S = c \ln \left[\frac{dN}{d\eta}\bigg|_0 \right] + \ln\left[(1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y)^{\epsilon}\right]. \tag{15}$$

Exploiting the analogy between entropy for ideal gas in thermodynamics and (15), the parameter c can be interpreted as a «heat capacity» of the produced medium. The multiplicity density $dN/d\eta|_0$ of particles in the central region characterizes a «temperature» of the colliding system. We would like to note that the principle of a minimal resolution (10) used to determine the momentum fraction x_1, x_2, y is equivalent to the principle of a maximal entropy S.

1.5. Scaling Function $\psi(z)$. The scaling function $\psi(z)$ is expressed in terms of the experimentally measured inclusive invariant cross section $Ed^3\sigma/dp^3$, the multiplicity density $dN/d\eta$ and the total inelastic cross section σ_{in}. Exploiting the definition (6), one can obtain the expression

$$\psi(z) = -\frac{\pi s A_1 A_2}{(dN/d\eta)\sigma_{\text{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}, \tag{16}$$

where s is the square of the center-of-mass energy of the corresponding NN system; A_1 and A_2 are atomic weights and J is the corresponding Jacobian of transformation from the variables $\{p_z, p_T\}$ to $\{z, \eta\}$. The function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z)dz = 1. \tag{17}$$

The above relation allows us to interpret the function $\psi(z)$ as a probability density to produce an inclusive particle with the corresponding value of the variable z.

2. MULTIPLICITY DEPENDENCE OF z-SCALING IN HEAVY ION COLLISIONS

In this section we study the multiplicity dependence of p_T- and z-presentations of experimental data on inclusive cross sections of hadrons produced in heavy ion collisions. The data are obtained by the STAR [15, 16], PHOBOS [17], BRAHMS [18] and PHENIX [19, 20] collaborations at the RHIC.

The important ingredient of z-scaling is the multiplicity density $dN_{\rm ch}/d\eta(s,\eta)$ as a function of the collision energy $\sqrt{s}$ and pseudorapidity η. The scaling variable z is proportional to $[dN_{\rm ch}/d\eta(s,\eta)]^{-1}$ at $\eta = 0$, while the function $\psi(z)$ is expressed via multiplicity density depending on the energy $\sqrt{s}$ and pseudorapidity η. The multiplicity density in proton–(anti)proton collisions was measured up to the Tevatron energies (Fig. 2, a). Using the special selection of events the transverse hadron spectra were measured by the E735 collaboration at highest $dN_{\rm ch}/d\eta|_0 \simeq 26$. This value is much more larger than $dN_{\rm ch}/d\eta|_0/(0.5N_p)$ measured

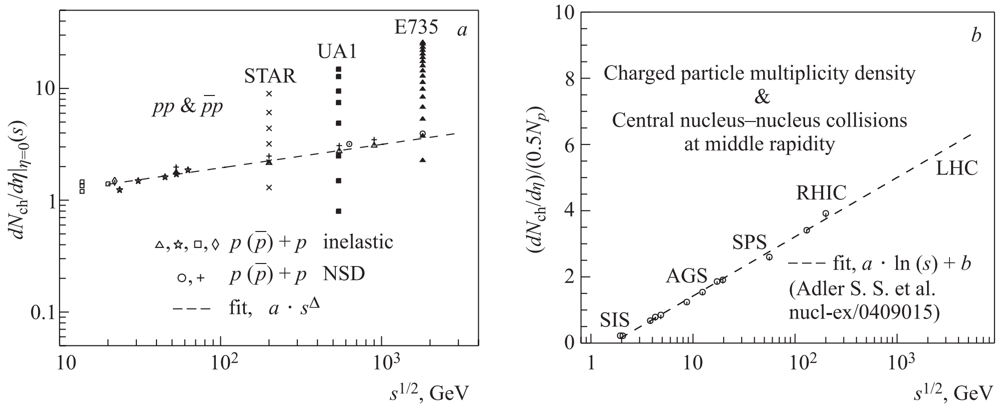
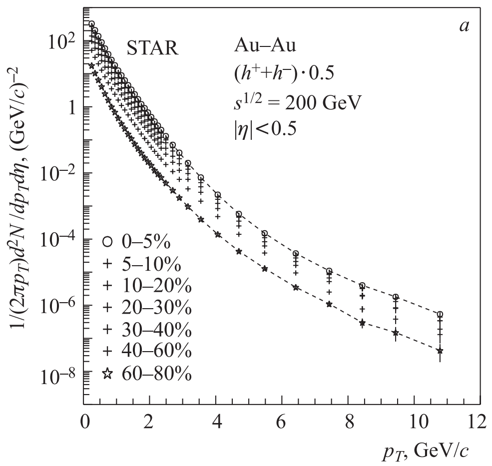


Fig. 2. The dependence of the multiplicity density in proton–(anti)proton (a) and central nucleus–nucleus (b) interactions on the collision energy $\sqrt{s}$

in central nucleus–nucleus collisions at the AGS, SPS and RHIC (Fig. 2, b). The multiplicity density in central Pb–Pb collisions at $\eta = 0$ is expected to be about 8000 particles per unit of pseudorapidity at the LHC energies. The regime of particle production at very high multiplicity density is believed to be more preferable for searching for clear signature of QGP formation.

2.1. Charged Hadrons. Multiplicity dependence of high-p_T spectra of charged and identified hadrons produced in proton–(anti)proton collisions at the RHIC, S$\bar{\rm p}$pS and Tevatron energies in p_T- and z-presentations was studied in [12, 13]. The strong sensitivity of spectra to $dN_{\rm ch}/d\eta$ was observed to increase with p_T. Independence of the corresponding scaling function $\psi(z)$ over a wide range of energy $\sqrt{s}$ and p_T was restored at $c = 0.25$, $\delta_N = 0.7$ and $\epsilon = 0.7$. We use this value to study the dependence of transverse spectra on multiplicity density $dN_{\rm ch}/d\eta$ in Au–Au and Cu–Cu collisions.

Figure 3 shows the dependence of the spectra of charged hadron production in Au–Au collisions on the transverse momentum p_T at the energy $\sqrt{s} = 200$ GeV and over a pseudorapidity range $|\eta| < 0.5$ for different centralities [15]. The data cover a wide



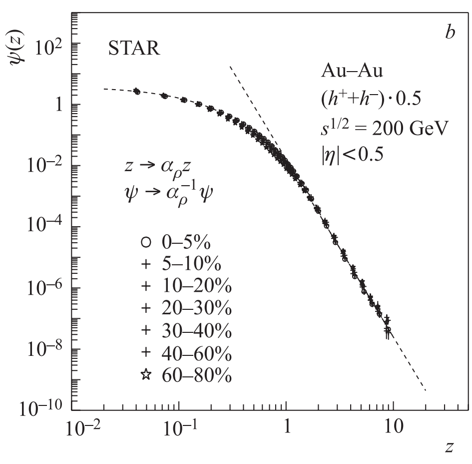
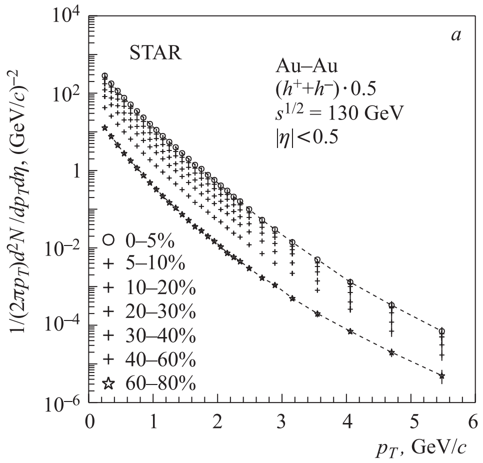


Fig. 3. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.5$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the data. Experimental data are taken from [15]

transverse momentum range, $p_T = 0.2-11$ GeV/c. We observe a strong sensitivity of high-p_T spectra to multiplicity density $dN_{\rm ch}/d\eta$. The scaling behavior of the data (Fig. 3, a) are restored at $c = 0.25$ and under the simultaneous transformation of $z \to \alpha_\rho z$ and $\psi \to \alpha_\rho^{-1}\psi$ ($\rho \equiv dN_{\rm ch}/d\eta$). The power behavior (the straight dashed line in Fig. 3, b) of the scaling function, $\psi(z) \sim z^{-\beta}$, for high z is observed. The soft regime of particle production demonstrates self-similarity in z-presentation for low z as well.



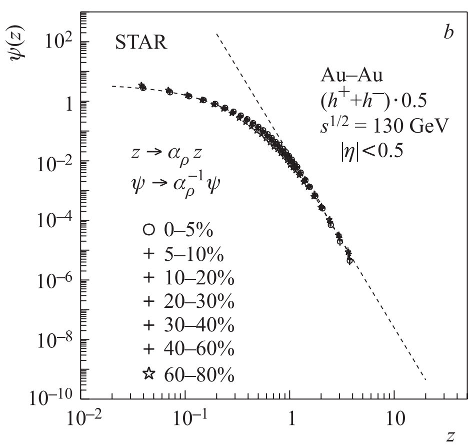


Fig. 4. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 130$ GeV and $|\eta| < 0.5$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the data at $\sqrt{s} = 200$ GeV. Experimental data are obtained by the STAR collaboration [16]

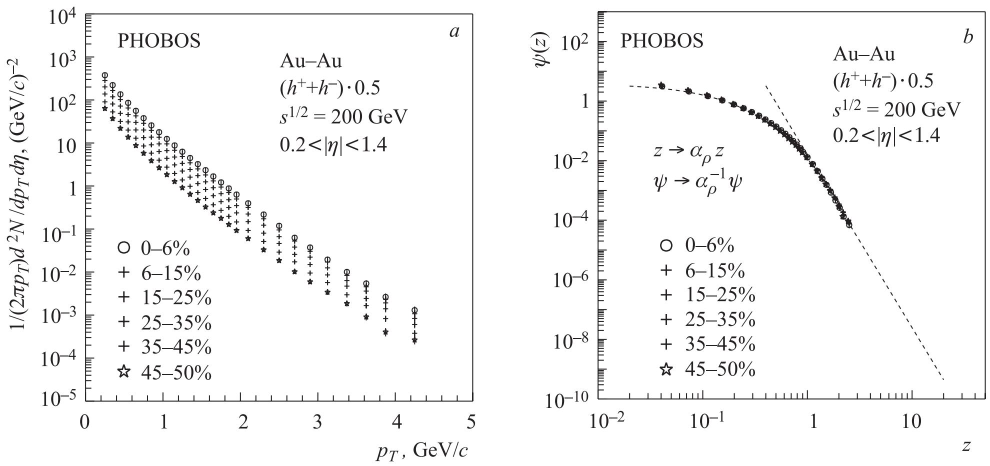


Fig. 5. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $0.2 < |\eta| < 1.4$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the STAR data at $\sqrt{s} = 200$ GeV. Experimental data are obtained by the PHOBOS collaboration [17]

Figure 4 demonstrates p_T (*a*) and z (*b*) presentations of the transverse of multiplicity density at the energy $\sqrt{s} = 130$ GeV. The dashed lines are obtained by fitting of $\psi(z)$ corresponding to the STAR data [15] at $\sqrt{s} = 200$ GeV. The independence of the scaling function from the collision energy is observed.

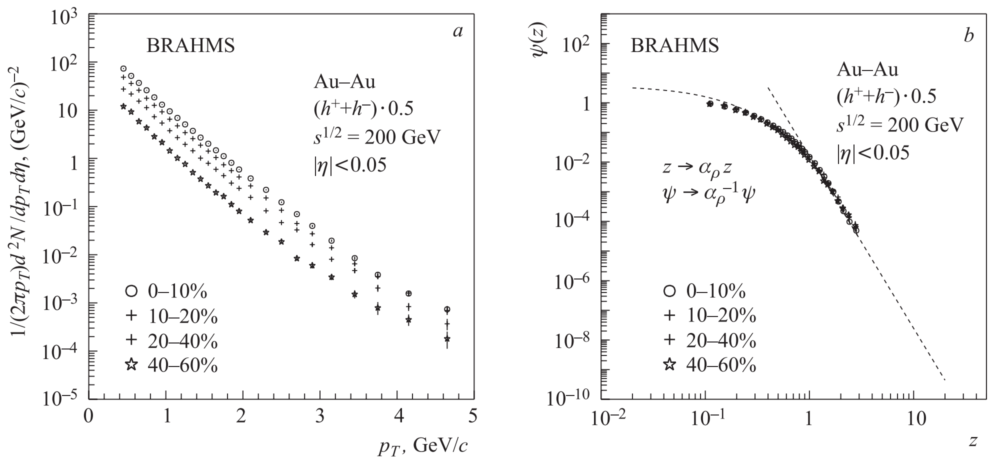


Fig. 6. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.05$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the STAR data at $\sqrt{s} = 200$ GeV. Experimental data are obtained by the BRAHMS collaboration [18]

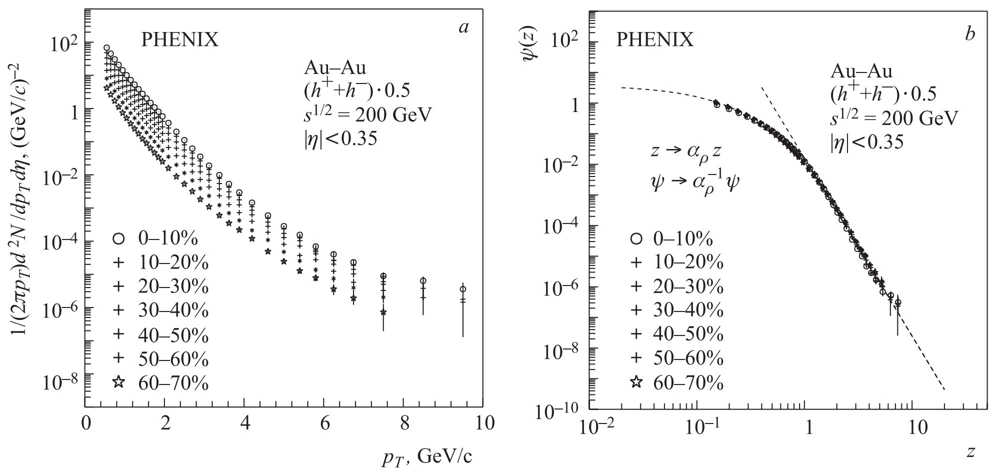


Fig. 7. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.35$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the STAR data at $\sqrt{s} = 200$ GeV. Experimental data are obtained by the PHENIX collaboration [19]

The PHOBOS collaboration measured the transverse spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $0.2 < |\eta| < 1.4$ for different centralities [17]. The data are shown in Fig. 5, a. The transverse momentum p_T reaches 4 GeV/c. The scaling presentation of the same data shown in Fig. 5, b demonstrates a good compatibility with the STAR data over a kinematical range measured by the PHOBOS collaboration.

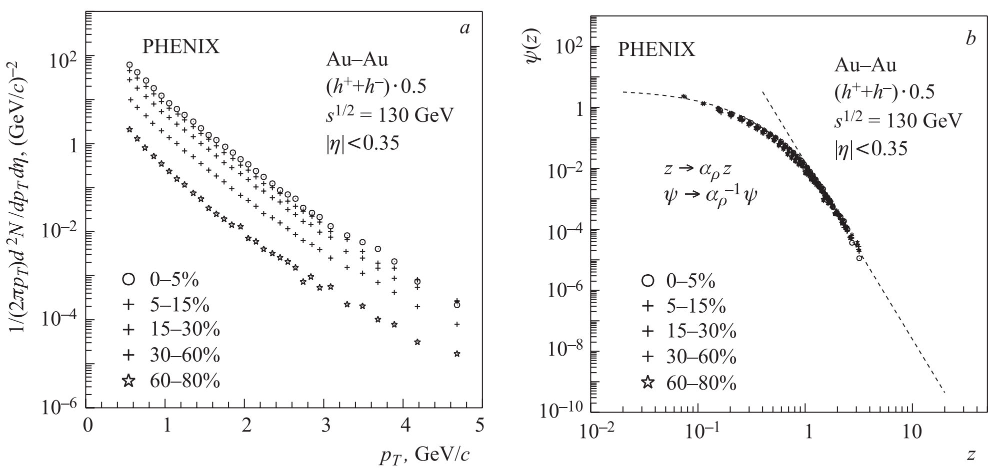


Fig. 8. *a*) The spectra of charged hadrons produced in Au–Au collisions at $\sqrt{s} = 130$ GeV and $|\eta| < 0.35$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$. Dashed lines are obtained by fitting the STAR data at $\sqrt{s} = 200$ GeV. Experimental data are obtained by the PHENIX collaboration [20]

Figure 6 shows p_T- and z-presentations of data on charged hadron inclusive spectra in the Au–Au interactions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.05$ obtained by the BRAHMS collaboration at the RHIC [18]. As seen from Fig. 6, the features of both presentations are similar to those observed for the STAR and PHOBOS data.

The PHENIX collaboration measured spectra of charged hadrons in $\mathrm{Au-Au}$ collisions at the energy $\sqrt{s} = 130$ and 200 GeV and central rapidity range $|\eta| < 0.35$ [19]. The data are presented in Figs. 7, a and 8, a. The transverse spectra cover the range $p_T = 0.5-10$ GeV/c. The z-presentation of data shown in Figs. 7, b and 8, b demonstrates power behavior (the dashed straight line) for high z and deviation from this law for low z.

Thus, based on the results of our analysis of the RHIC data, we conclude that the mechanism of charged hadron production in Au–Au interactions at high energies reveals self-similarity and fractality over a wide range of collision energy, transverse momentum and multiplicity density.

2.2. Strange Hadrons K_S^0, Λ. The multiplicity dependence of the scaling function $\psi(z)$ of K_S^0 mesons and Λ hyperons produced in pp collisions in the framework of z-presentation was studied in [13]. Here we analyze the data on inclusive spectra of K_S^0 and Λ production in Au–Au collisions obtained by the STAR collaboration at the RHIC [21, 22].

Figures 9 and 10 show p_T (a) and z (b) presentations of transverse momentum spectra of K_S^0 and Λ produced at $\sqrt{s} = 130, 200$ GeV and $|\eta| < 0.5$ and different centralities. The

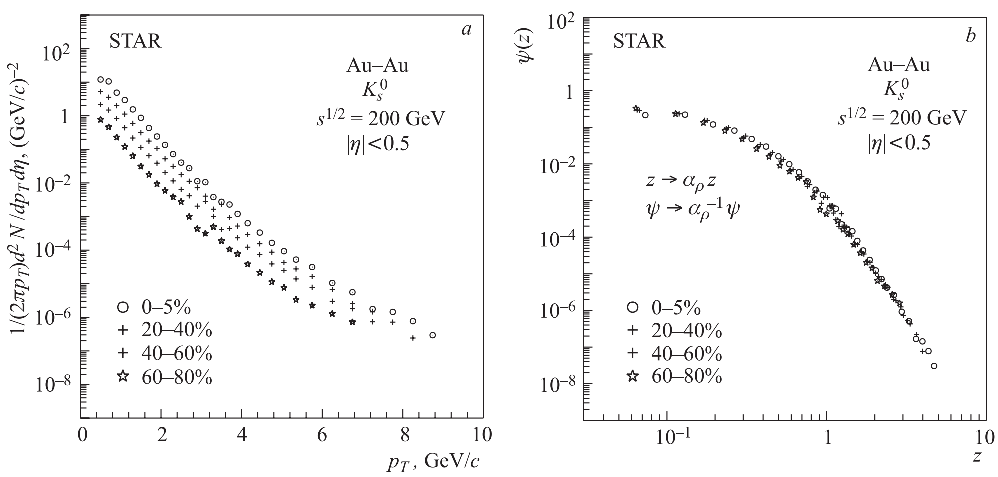


Fig. 9. a) The spectra of K_S^0 mesons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.5$ as a function of centrality and the transverse momentum p_T. b) The corresponding scaling function $\psi(z)$. Experimental data are obtained by the STAR collaboration [21]

values of the parameters $\delta_N = 0.7$, $\epsilon = 0.7$ and $c = 0.3$ for K_S^0 and 0.35 for Λ found in [13] allow one to restore the multiplicity independence of the scaling function $\psi(z)$ of strange hadron production in Au–Au collisions. The energy independence of $\psi(z)$ for Λ is demonstrated in Fig. 10, b. Indication of power behavior of the scaling function is seen both for K_S^0 mesons and Λ hyperons. To perform more detailed verification of flavor dependence

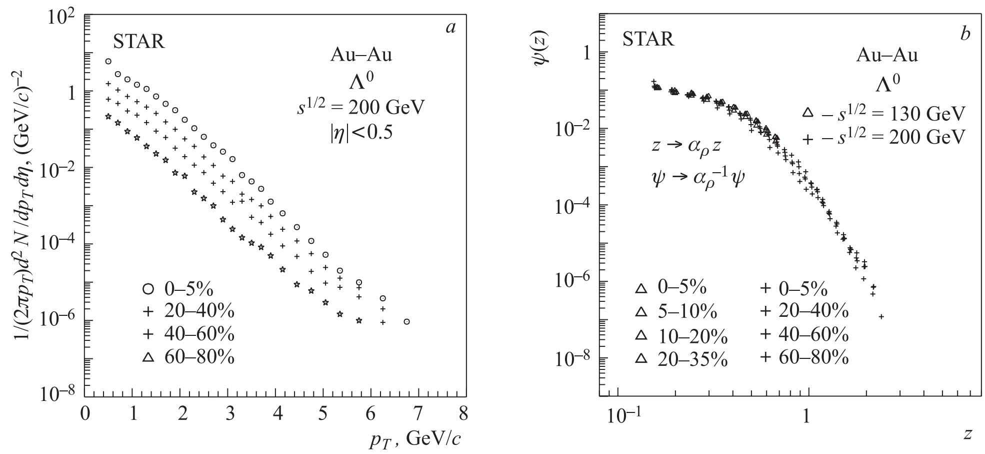


Fig. 10. *a*) The spectra of Λ hyperons produced in Au–Au collisions at energy $\sqrt{s} = 200$ GeV and $|\eta| < 0.5$ as a function of centrality and the transverse momentum p_T. *b*) The corresponding scaling function $\psi(z)$ at $\sqrt{s} = 130$ and 200 GeV. Experimental data are obtained by the STAR collaboration [21, 22]

of z-scaling, the analysis of strange particle spectra over a wider range of p_T (up to 20 GeV/c), multiplicity density ($dN_{\rm ch}/d\eta$) and atomic weights (A) is required.

2.3. π^+, π^-, π^0 **Mesons.** Here we analyze data on pion transverse spectra in Au–Au and Cu–Cu collisions at $\sqrt{s} = 200$ GeV obtained by the PHOBOS [23], PHENIX [24, 25] and STAR [26] collaborations and demonstrate a possibility to restore multiplicity and A independence of the scaling function $\psi(z)$.

The inclusive spectra of identified charged particles $\pi^\pm, K^\pm$ at very low transverse momenta ($p_T = 30-50$ MeV/c) in central Au–Au collisions at $\sqrt{s} = 200$ GeV are measured by the PHOBOS collaboration [23]. These data together with the STAR and PHENIX data for π^+ and π^0 mesons are used to construct the scaling function over a wide range of z. The results of the combined analysis are shown in Fig. 11, a. We would like to note that the PHOBOS data allow us to determine the asymptotic of $\psi(z)$ for low z. The power behavior of the scaling function of pions for high z is observed as well. The STAR and PHENIX data are crucial to determine the asymptotic of $\psi(z)$ for high z. A good agreement between the STAR and PHENIX data in overlapping range is observed.

Figure 11, b shows the independence of the scaling function $\psi(z)$ from the multiplicity density $dN_{\rm ch}/d\eta$ and the atomic weight A. The latter property means that scaling functions corresponding to different nuclei have the same shape. To compare them, the transformation of $z \to \alpha_A z$ and $\psi \to \alpha_A^{-1}\psi$ has been applied.

The obtained result means that nuclear medium produced in Au–Au and Cu–Cu collisions at different multiplicity densities modifies in a similar manner the mechanism of pion production. The modification is that self-similarity and fractality of the process take place over a wide range of p_T. The conclusion is confirmed by the results of our analysis of π^0-meson transverse spectra in Cu–Cu collisions at $\sqrt{s} = 200$ GeV [25] obtained by the PHENIX collaboration (see Fig. 12, a). The corresponding z-presentation is shown in Fig. 12, b.

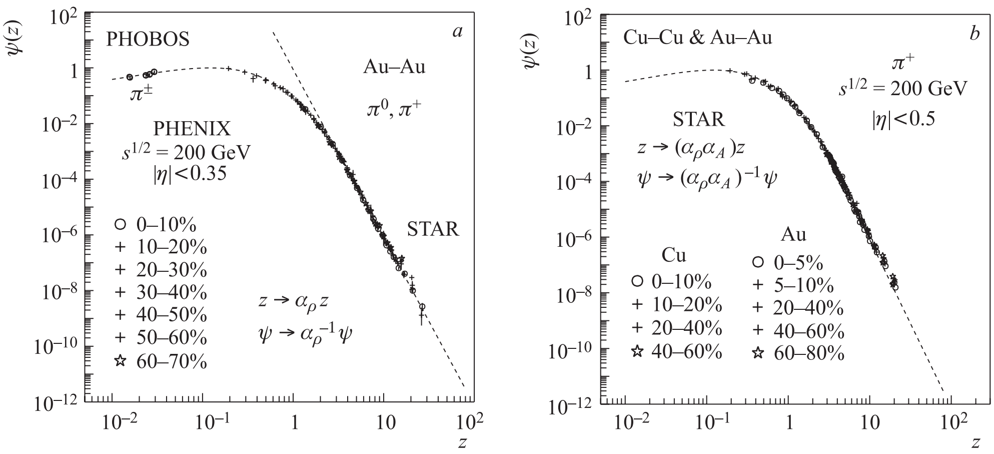


Fig. 11. *a*) z-presentation of inclusive transverse spectra of $\pi^{\pm}$, π^{0} mesons produced in Au–Au collisions at energy $\sqrt{s} = 200$ GeV as a function of centrality. Data are obtained by the PHOBOS [23], PHENIX [24] and STAR [26] collaborations. The combined fit of the RHIC data is shown by the dashed lines. *b*) z-presentation of inclusive transverse spectra of π^{+} mesons produced in Au–Au and Cu–Cu collisions at energy $\sqrt{s} = 200$ GeV as a function of centrality. Data are obtained by the STAR collaboration

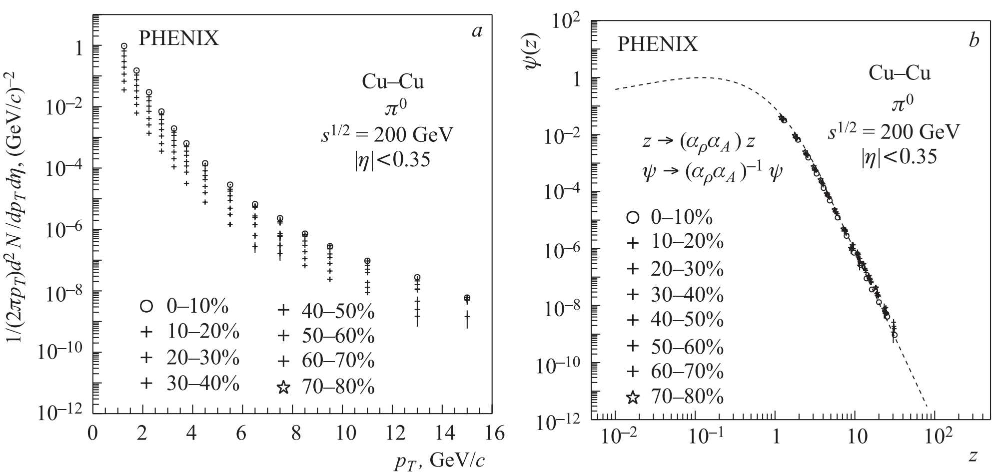


Fig. 12. The inclusive spectra of π^{0}-meson production in Cu–Cu collisions at energy $\sqrt{s} = 200$ GeV as a function of centrality in p_T (*a*) and z (*b*) presentations. Experimental data obtained by the PHENIX collaboration are taken from [25]. The dashed line is the combined fit of the RHIC data

3. α_ρ AND α_A TRANSFORMATION OF z AND $\psi(z)$

The method for construction of the scaling function used in the present analysis shows that shapes of the scaling functions for different multiplicity densities and atomic weights coincide, whereas their absolute values are different. To compare them the transformation of $z \to \alpha_i z$ and $\psi \to \alpha_i^{-1}\psi$, where $i = dN_{\rm ch}/d\eta, A$, was used. Note that the function α_i depends on $dN_{\rm ch}/d\eta$ or on A. It is not an explicit function of the kinematical variables $\sqrt{s}, p_T$ and η.

The dependences of α_ρ on $\rho \equiv dN_{\rm ch}/d\eta$ and of α_A on A are shown by dashed lines in Fig. 13, a and b, respectively. These functions are convenient to parameterize by power

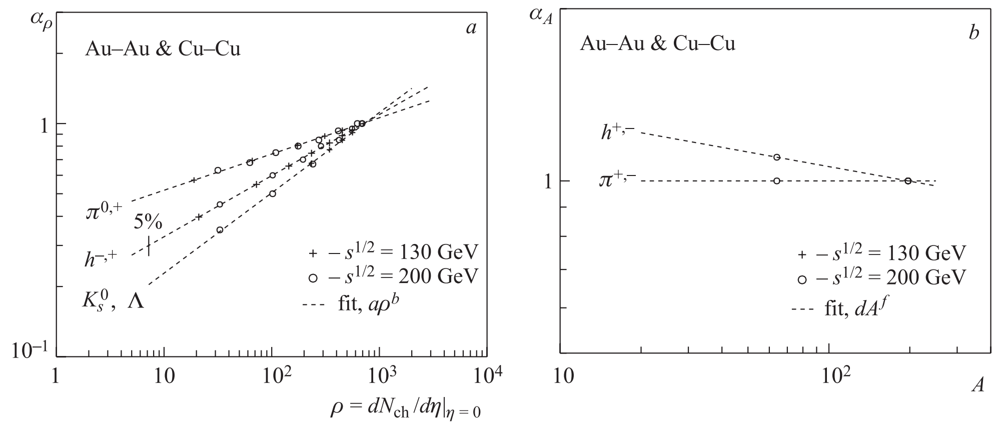


Fig. 13. The dependence of the transformation functions α_ρ (a) and α_A (b) on the multiplicity density $dN_{\rm ch}/d\eta$ and atomic weight A, respectively

dependence on ρ and A. The slope parameter b of the function $\alpha_A = a\rho^b$ is found to be independent of energy ($+ -$ 130 GeV, $\circ -$ 200 GeV) and dependent on type of the particles ($\pi^{\pm,0}, h^\pm, K_S^0, \Lambda$). A value of the slope parameter b increases with the mass of produced particle. We found that the function $\alpha_A = dA^f$ is independent of A for pions. It reveals the growth for unidentified charged hadrons as atomic weight decreases.

4. PREDICTIONS OF THE SPECTRA FOR RHIC EXPERIMENTS

The properties of z-presentation allow us to construct the scaling function $\psi(z)$ for hadron production in heavy ion collisions and then calculate particle spectra versus collision energy, multiplicity density, and atomic weight over a wide region of p_T. Figure 14 demonstrates transverse distributions of π^0 and π^+ mesons produced in Au–Au collisions in the central rapidity range at $\sqrt{s} = 200$ GeV as a function of multiplicity density. The points $(+, \circ, \star)$ shown in Fig. 14 are the experimental data taken from [24, 27]. The theoretical results are plotted by the dashed lines. We would like to emphasize that asymptotic behavior of the predicted spectra is determined by the power dependence of the scaling function $\psi(z)$ for high z. Therefore, the experimental verification of the predictions and measurement of spectra for high-p_T and multiplicity density $dN_{\rm ch}/d\eta$ is of interest both to test z-scaling in heavy ion collisions and to search for boundary of its validity.

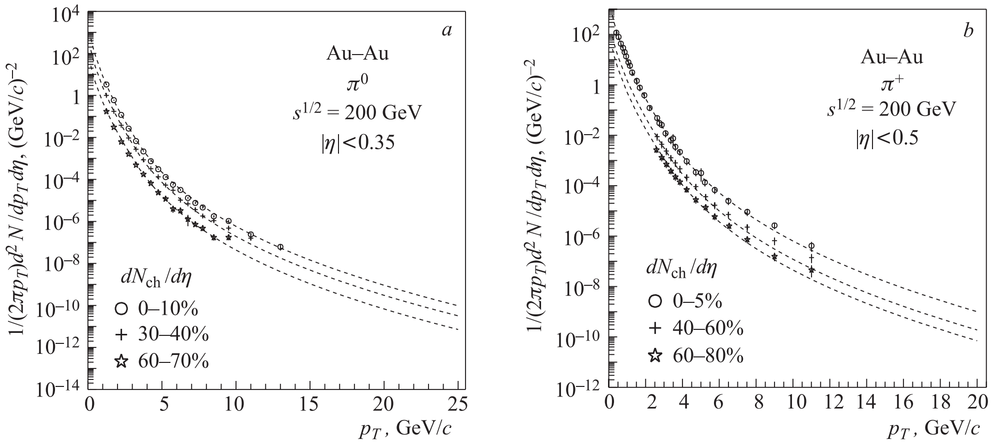


Fig. 14. The spectra of π^0 (a) and π^+ (b) mesons produced in Au–Au collisions at $\sqrt{s} = 200$ GeV and in the central pseudorapidity region as a function of the transverse momentum p_T and centrality. Symbols are experimental data taken from [24, 27]. Dashed lines are predictive calculations based on z-scaling

CONCLUSIONS

Results of analysis of the data on inclusive transverse spectra of hadrons produced in Au–Au and Cu–Cu collisions obtained by the STAR, PHENIX, PHOBOS and BRAHMS collaborations at the RHIC in the framework of generalized z-scaling are presented. The scaling function of particle production in nucleus–nucleus interaction was constructed. The dependence of z-presentation on collision energy, multiplicity density and atomic weight was studied. It was shown that taking into account the transformation of $z \to \alpha_i z$ and $\psi \to \alpha_i^{-1}\psi$, where $i = dN_{\rm ch}/d\eta$ and A, the general properties of z-scaling can be restored. The dependences of α_ρ on $dN_{\rm ch}/d\eta$ and of α_A on A were established. The values of the fractal dimensions $\delta_A = A\delta_N$, ϵ and the parameter c are found to be constant over the range $\sqrt{s} = 130-200$ GeV, $A =$ Cu–Au, $dN_{\rm ch}/d\eta = 20-700$, $|\eta| < 1.4$, and p_T up to 10 GeV/c. The power dependence of the scaling function $\psi(z)$ of hadron production in heavy ion collisions for high z was confirmed.

The presented results are considered as confirmation of the fact that medium produced in heavy ion collisions at high energy and high multiplicity density modifies the mechanism of particle formation in a self-similar manner. The property is valid over a wide range of transverse momentum. It means that the process possesses fractal properties as well. Therefore, if we choose the relevant kinematic conditions of particle production and special selection of events, we can hope to reach the transition regime from hadron to quark-gluon degrees of freedom. We consider that the developed method of data analysis can be useful for searching for new physics phenomena of particle production in collisions of hadrons and nuclei at high transverse momenta and high multiplicities at the U70, Tevatron, RHIC and LHC.

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REFERENCES

1. *Feynman R. P.* // Phys. Rev. Lett. 1969. V. 23. P. 1415.
2. *Bjorken J. D.* // Phys. Rev. 1969. V. 179. P. 1547;
   Bjorken J. D., Paschos E. A. // Ibid. V. 185. P. 1975.
3. *Bosted P. et al.* // Phys. Rev. Lett. 1972. V. 49. P. 1380.
4. *Benecke J. et al.* // Phys. Rev. 1969. V. 188. P. 2159.
5. *Baldin A. M.* // Sov. J. Part. Nucl. 1977. V. 8. P. 429.
6. *Stavinsky V. S.* // Sov. J. Part. Nucl. 1979. V. 10. P. 949.
7. *Leksin G. A.* Report No. ITEF-147. 1976;
   Leksin G. A. // Proc. of the XVIII Intern. Conf. on High Energy Physics, Tbilisi, 1976 / Eds. N. N. Bogolubov et al. Dubna, 1977.
8. *Koba Z., Nielsen H. B., Olesen P.* // Nucl. Phys. B. 1972. V. 40. P. 317.
9. *Matveev V. A., Muradyan R. M., Tavkhelidze A. N.* // Part. Nucl. 1971. V. 2. P. 7; Lett. Nuovo Cim. 1972. V. 5. P. 907; 1973. V. 7. P. 719.
10. *Brodsky S., Farrar G.* // Phys. Rev. Lett. 1973. V. 31. P. 1153; Phys. Rev. D. 1975. V. 11. P. 1309.
11. *Zborovský I. et al.* // Phys. Rev. D. 1996. V. 54. P. 5548;
    Zborovský I. et al. // Phys. Rev. C. 1999. V. 59. P. 2227;
    Tokarev M. V., Dedovich T. G. // Intern. J. Mod. Phys. A. 2000. V. 15. P. 3495;
    Tokarev M. V., Rogachevski O. V., Dedovich T. G. // J. Phys. G: Nucl. Part. Phys. 2000. V. 26. P. 1671;
    Tokarev M. V., Rogachevski O. V., Dedovich T. G. JINR Commun. E2-2000-90. Dubna, 2000.
    Tokarev M. et al. // Intern. J. Mod. Phys. A. 2001. V. 16. P. 1281;
    Tokarev M. hep-ph/0111202;
    Tokarev M., Toivonen D. hep-ph/0209069;
    Skoro G. P. et al. hep-ph/0209071;
    Tokarev M. V., Efimov G. L., Toivonen D. E. // Phys. At. Nucl. 2004. V. 67. P. 564;
    Tokarev M. V. // Acta Phys. Slovaca. 2004. V. 54. P. 321;
    Tokarev M. V., Dedovich T. G. // Phys. At. Nucl. 2005. V. 68. P. 404.
12. *Zborovský I., Tokarev M. V.* // Part. Nucl., Lett. 2006. V. 3, No. 5(134). P. 68; hep-ph/0506003.
13. *Tokarev M. V., Zborovský I.* // Phys. At. Nucl. (in press).
14. *Zborovský I., Tokarev M. V.* JINR Commun. E2-2006-34. Dubna, 2006.
15. *Adams J. et al.* // Phys. Rev. Lett. 2003. V. 91. P. 172302.
16. *Adams J. et al.* // Phys. Rev. C. 2004. V. 70. P. 044901; nucl-ex/0404020.
17. *Back B. B. et al.* // Phys. Lett. B. 2004. V. 578. P. 297.
    Back B. B. et al. // Phys. Rev. C. 2002. V. 65. P. 061901(R).
18. *Arsene I. et al.* // Phys. Rev. Lett. 2003. V. 91. P. 072305.

19. *Adler S. S. et al.* // Phys. Rev. C. 2004. V. 69. P. 034910; nucl-ex/0307022; nucl-ex/0308006.

20. *Adcox K. et al.* // Phys. Rev. Lett. 2001. V. 86. P. 3500; nucl-ex/0012008; nucl-ex/0207009.

21. *Lamont M. A. C.* // Proc. of the Intern. Conf. on Strangeness in Quark Matter «Strange Quark Matter 2006», Los Angeles, USA, March 25–31, 2006; nucl-ex/0608017; http://www.physics.ucla.edu/SQM2006/

22. *Adler C. et al.* // Phys. Rev. Lett. 2002. V. 89. P. 092301; nucl-ex/0203016.

23. *Trzupek A.* // Proc. of the 18th Intern. Conf. on Ultra-Relativistic Nucleus–Nucleus Collisions «Quark Matter 2005», Budapest, Aug. 4–9, 2005; Nucl. Phys. A. 2006. V. 774. P. 469; nucl-ex/0510039; http://qm2005.kfki.hu/

24. *Adler S. S. et al.* // Phys. Rev. Lett. 2003. V. 91. P. 072301; nucl-ex/0304022.

25. *Shimomura M.* // Proc. of the 18th Intern. Conf. on Ultra-Relativistic Nucleus–Nucleus Collisions «Quark Matter 2005», Budapest, Aug. 4–9, 2005; Nucl. Phys. A. 2006. V. 774. P. 457; nucl-ex/0510023; http://qm2005.kfki.hu/
Isobe T. // Proc. of Particles and Nuclei Intern. Conf. «PANIC 2005», Santa Fe, New Mexico, USA, Oct. 24–28, 2005; nucl-ex/0604016; http://www.panic05.lanl.gov/

26. *Ruan L.* // Proc. of Particles and Nuclei Intern. Conf. «PANIC 2005», Santa Fe, New Mexico, USA, Oct. 24–28, 2005; nucl-ex/0606003; http://www.panic05.lanl.gov/

27. *Barannikova O.* // Proc. of the 18th Intern. Conf. on Ultra-Relativistic Nucleus–Nucleus Collisions «Quark Matter 2005», Budapest, Aug. 4–9, 2005; http://qm2005.kfki.hu/

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