E17-2002-97

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THE EFFECTIVE INTERACTION BETWEEN ATOMS OF LIQUID ^{4}He

Submitted to «Physical Review Letters»

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1. Indroduction.

In 1941 Landay[1] have predicted, that the properties of ^{4}He can be described by language of excitations. In 1947 Bogoliubov[2] proposed the model of weakly non-ideal Bose-gas. Considering single particles motions of atoms he found the energy spectrum of liquid ^{4}He on language of excitations which are formed due to the presence of macroscopic number of condensate atoms. As early as in 1955 Bogoliubov and Zubarev[3] considering vibrations of density atoms in the liquid ^{4}He within the method of "collective variables" for a non-ideal Bose system. They'd obtained result for energetic spectrum in the liquid ^{4}He which is differ from result in the model of weakly non-ideal Bose-gas .

To solve a problem of liquid ^{4}He we will propose a new method of "mixed" representation according to the method of "mixed" representation for the Fermi system which was made by Zubarev [4].

According to prescription of Bohm and Pines[5], Zubarev had separated the motion of plasma electrons to "collective" and to "individual" components which correspond to vibrations of the plasma density of electrons and to single particle motions. He suggested that separating of motions of Fermi-particles in "collective" and "individual" components occurs simultaneously. It means that the every spatial coordinate of electron in the plasma can simultaneously correspond to "collective" and "individual" independent motions.

In analogy with the theory of Zubarev we will use the method of "mixed" representation for the Bose-system. Within this method we will show that the effective interaction exists between atoms of the liquid ^{4}He due to action vibrations of density atoms which represents as "collective" motion of atoms.

Henshaw and Woods[6] measured the dispersion branch for excitations in superfluid ^{4}He by the method of neutron inelastic scattering and showed that it coincides with Landau's predictions. Many experimental neutron studies of helium were aimed at correcting the position of the dispersion curve of elementary excitations in liquid ^{4}He[7-10]. The modern experimental facilities[11] permit to investigate the picture of dispersion laws with two branches of excitations with spectrum energies. The authors[11] of this experiment observed two types quasi-particles and separated them by the Bogoliubov-Zubarev's and Landau's quasi-particles. For theoretical explaining of their's results they take into account the Griffin-Glyde[12] model which predicts an existence of two types quasi-particles in the ^{4}He. In the Griffin-Glyde model is suggested that the physical nature of energy excitations can be connected with collective oscillations of the density and elementary phonon excitations by means of interaction through the condensate atoms.

2. The method of "mixed" representation for the Bose-system.

We study the model of the liquid ^{4}He as a system of N identical Bose spinless particles with the mass m and with "individual" $\vec{r}_1, ..., \vec{r}_N$ coordinates of particles confined within the volume V. These atoms interact between themselves through the S-wave pseudopotential as in the model of hard spheres[13]. By analogy of Zubarev's theory the every atom of liquid ^{4}He participates to "collective" and "individual" motions simultaneously which correspond to vibrations of density atoms and to single particle motions. In this case, we will introduce a spatial coordinates of N atoms $\vec{R}_1, ..., \vec{R}_N$ which coincide with a "individual" spatial coordinates and which will help to describe a vibrations of the density atoms.

The wave function of the liquid ^{4}He can be represented the following way:

$$\psi \;=\; \psi_s(\vec{r}_1, ..., \vec{r}_N; t)\psi_c(\vec{R}_1, ..., \vec{R}_N; t) \tag{1}$$

where $\psi_s(\vec{r}_1, ..., \vec{r}_N; t)$ is the wave function of "individual" component which describes the single particles motions; $\psi_c(\vec{R}_1, ..., \vec{R}_N; t)$ is the wave function of "collective" component which describes the state of Bose system at vibrations of density atoms.

As variables of wave function of "collective" component we take Fourier coefficients of the operator density of atoms $\varrho_{\vec{p}}$ with momentum $\vec{p}$:

$$\varrho_{\vec{p}} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \exp\left(\frac{i\vec{p}\vec{R}_i}{\hbar}\right), \vec{p} \neq 0. \tag{2}$$

Then, the wave function of "collective" component has the form:

$$\psi_c(\varrho_{\vec{p}_1} \cdots \varrho_{\vec{p}_i} \cdots; t)\,. \tag{3}$$

The Schrödinger equation for the liquid ^{4}He has the form:

$$i\hbar\frac{d\psi}{dt} = \hat{H}\psi \tag{4}$$

with the system Hamiltonian having the form

$$\hat{H} = \hat{H}_i + \hat{H}_c + \hat{H}_{i,c} \tag{5}$$

where $\hat{H}_i$ is the operator of Hamilton of "individual" component; $\hat{H}_c$ is the operator of Hamilton "collective" component; $\hat{H}_{i,c}$ is the operator of interaction between "individual" and "collective" components:

$$\hat{H}_i = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_{\vec{r}_i} + \frac{1}{2}\sum_{i\neq j} U(\vec{r}_i - \vec{r}_j) \tag{6}$$

$$\begin{aligned}\hat{H}_c &= \frac{1}{\sqrt{N}}\sum_{p_1\neq 0, p_2\neq 0}\frac{\vec{p_1}\vec{p_2}}{2m}\varrho_{\vec{p_1}+\vec{p_2}}\frac{d^2}{d\varrho_{\vec{p_1}}d\varrho_{\vec{p_2}}} + \sum_{p\neq 0}\frac{p^2}{2m}\varrho_{\vec{p}}\frac{d}{d\varrho_{\vec{p}}} + \\ &+ \frac{N}{2V}\sum_{p\neq 0} U_{\vec{p}}\varrho_{\vec{p}}\varrho_{-\vec{p}} + \frac{1}{2m\sqrt{N}}\sum_{i,\vec{p}\neq 0}\vec{p}\vec{p}_i \exp\left(-\frac{i\vec{p}\vec{R}_i}{\hbar}\right)\frac{d}{d\varrho_{\vec{p}}} - \\ &- \frac{N}{2V}\sum_{\vec{p}\neq 0} U_{\vec{p}} + \frac{U_0 N^2}{2V}\end{aligned} \tag{7}$$

where $\vec{p}_i$ is the momentum of i-th "individual" atom;

$$\hat{H}_{i,c} = \frac{1}{2}\sum_{i\neq j} U(\vec{r}_i - \vec{R}_j) \tag{8}$$

where $U_{\vec{p}}$ is the Fourier transform of S-wave pseudopotential as in the model of hard spheres:

$$U_{\vec{p}} = \frac{4\pi d\hbar^2}{m} \tag{9}$$

where d is the diameter of boson.

The operator of Hamilton in Eq.(7) is a non-Hermitian because the transition to variables $\vec{R}_i$, $\varrho_{\vec{p}}$ is not a canonical transformation. To make the operator of Hamilton in Eq.(7) to Hermitian, we use the random - phase approximation[5]

$$\varrho_{\vec{p_1}+\vec{p_2}} \approx \sqrt{N}\delta_{0,\vec{p_1}+\vec{p_2}}; \vec{p_1} \neq 0, \vec{p_2} \neq 0 \,.$$

Then, from Eq.(7) we can neglect a first and a third terms. At this case, we will introduce the wave function of Zubarev in the following form:

$$\Psi = \psi \exp\left(-\frac{1}{4}\sum_{\vec{p}} \varrho_{\vec{p}}\varrho_{-\vec{p}}\right).$$

Then, we obtain the operator of Hamilton for "collective" component $\hat{H}_c$ in the representation of Zubarev's wave function Ψ:

$$\hat{H}_c = \frac{1}{2}\sum_{\vec{p}\neq 0}\frac{p^2}{m}\Big(-\frac{d^2}{d\varrho_{\vec{p}}d\varrho_{-\vec{p}}}+\frac{1}{4}\varrho_{\vec{p}}\varrho_{-\vec{p}}-\frac{1}{2}\Big)+\frac{N}{2V}\sum_{\vec{p}\neq 0}U_{\vec{p}}\varrho_{\vec{p}}\varrho_{-\vec{p}} - - \frac{N}{2V}\sum_{\vec{p}\neq 0}U_{\vec{p}}+\frac{U_0N^2}{2V}\,. \qquad (10)$$

We will enter Zubarev's Bose-operators of "creation" $\hat{b}^+_{\vec{p}}$ and "annihilation" $\hat{b}_{\vec{p}}$ of the Bogoliubov-Zubarev's quasi-particles with momentum $\vec{p}$

$$\left.\begin{array}{l} \hat{b}_{\vec{p}} = \lambda_{\vec{p}}\frac{d}{d\varrho_{-\vec{p}}}+\frac{1}{2\lambda_{\vec{p}}}\varrho_{\vec{p}} \\ \hat{b}^+_{\vec{p}} = -\lambda_{\vec{p}}\frac{d}{d\varrho_{\vec{p}}}+\frac{1}{2\lambda_{\vec{p}}}\varrho_{-\vec{p}} \end{array}\right\} \qquad (11)$$

where $\lambda_{\vec{p}}$ is a real symmetrical function of momentum $\vec{p}$.

After a some calculations with take into consideration the Fourier transform of the potential interaction $U(\vec{R}_i-\vec{r}_j)$ and Eq.(2), Eq.(11), we get the following form for the Hamiltonian of system $\hat{H}$:

$$\hat{H} = \sum_{\vec{p}\neq 0}\varepsilon_{\vec{p}}\hat{b}^+_{\vec{p}}\hat{b}_{\vec{p}}+\frac{\sqrt{N}}{2V}\sum_{i,\vec{p}\neq 0}U_{\vec{p}}\lambda_{\vec{p}}\Big(\hat{b}^+_{-\vec{p}}+\hat{b}_{\vec{p}}\Big)\exp-\Big(\frac{i\vec{p}\vec{r}_i}{\hbar}\Big) - - \frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_{\vec{r}_i}+\frac{1}{2}\sum_{i\neq j}U(\vec{r}_i-\vec{r}_j)+\hat{H}_0 \qquad (12)$$

where $\varepsilon_{\vec{p}}$ is the energy of the Bogoliubov-Zubarev's quasi-particles with momentum $\vec{p}$:

$$\varepsilon_{\vec{p}} = \left[\Big(\frac{p^2}{2m}\Big)^2+\frac{p^2U_{\vec{p}}N}{mV}\right]^{1/2} \qquad (13)$$

or

$$\varepsilon_{\vec{p}} = \left[\Big(\frac{p^2}{2m}\Big)^2+p^2v^2\right]^{1/2}$$

where

$$v=\sqrt{\frac{4\pi d\hbar^2N}{m^2V}}$$

is the velocity of a first sound.

On the other hand, we have

$$\varepsilon_{\vec{p}} = \frac{p^2}{2m\lambda^2_{\vec{p}}} \tag{14}$$

$$\hat{H}_0 = \frac{1}{2}\sum_{\vec{p}\neq 0}\left(\varepsilon_{\vec{p}} - \frac{p^2}{2m} - \frac{NU_{\vec{p}}}{V}\right) + \frac{U_0 N^2}{2V}\ . \tag{15}$$

We will use a canonical transformation for the operator $\hat{H}$ in Eq.(12) which could expel a terms of interaction between the Bogoliubov-Zubarev's quasi-particles and "individual" particles. Then, taking into consideration that the operator $\hat{S}$ satisfies a condition:

$$\hat{S}^{+} = -\hat{S}$$

then, a new operator $\tilde{H}$ will be represented in the form:

$$\tilde{H} = \exp\left(\hat{S}^{+}\right)\hat{H}\exp\left(\hat{S}\right) = \hat{H} - [\hat{S},\hat{H}] + \frac{1}{2}[\hat{S},[\hat{S},\hat{H}]] - \cdots \tag{16}$$

here we can take into account

$$\hat{S} = \sum_{\vec{p}} \hat{S}_{\vec{p}}$$

and

$$\hat{S}_{\vec{p}} = \alpha^{+}_{\vec{p}}\hat{b}^{+}_{\vec{p}} - \alpha_{\vec{p}}\hat{b}_{\vec{p}}\ . \tag{17}$$

We note that the operator $\alpha_{\vec{p}}$ is unknown. Then, using the form of the operator $\hat{H}$ from Eq.(12) in the formula Eq.(16) for a new operator $\tilde{H}$ with making a some calculations and then we will find

$$\begin{aligned}\tilde{H} &= \sum_{\vec{p}\neq 0}\varepsilon_{\vec{p}}\hat{b}^{+}_{\vec{p}}\hat{b}_{\vec{p}} - \sum_{\vec{p}\neq 0}\varepsilon_{\vec{p}}\hat{\alpha}^{+}_{\vec{p}}\hat{\alpha}_{\vec{p}} - \\ &- \frac{\hbar^2}{2m}\sum_{i=1}^{N}\Delta_{\vec{r}_i} + \frac{1}{2}\sum_{i\neq j}U(\vec{r}_i - \vec{r}_j) + \hat{H}_0\end{aligned} \tag{18}$$

where

$$\alpha_{\vec{p}} = \frac{\sqrt{N}}{V\varepsilon_{\vec{p}}}U_{\vec{p}}\lambda_{\vec{p}}\sum_{i=1}^{N}\exp\left(-\frac{i\vec{p}\vec{r}_i}{\hbar}\right)\ . \tag{19}$$

Then

$$\begin{aligned}\tilde{H} &= \sum_{\vec{p}\neq 0} \varepsilon_{\vec{p}} \hat{b}^{+}_{\vec{p}} \hat{b}_{\vec{p}} + \frac{1}{2}\sum_{i\neq j} \Phi(\vec{r}_i - \vec{r}_j) - \\ &- \frac{\hbar^2}{2m}\sum_{i=1}^{N} \Delta_{\vec{r}_i} + \frac{1}{2}\sum_{i\neq j} U(\vec{r}_i - \vec{r}_j) + \hat{H}_0 \,. \end{aligned} \tag{20}$$

In this case, the attractive potential between atoms has the form:

$$\Phi(\vec{r}_i - \vec{r}_j) = -2\frac{N}{V^2}\sum_{\vec{p}} \frac{U^2_{\vec{p}}\lambda^2_{\vec{p}} \exp\left(\frac{i\vec{p}\vec{r}}{\hbar}\right)}{\varepsilon_{\vec{p}}} = -\frac{a}{r}\exp(-br) \tag{21}$$

where $r = |\vec{r}_i - \vec{r}_j|$;

$$a = \frac{4mNU^2_{\vec{p}}}{V\hbar^2}$$

$$b^2 = \frac{16\pi mNU_{\vec{p}}}{V\hbar^2} \,.$$

Then, the Hamiltonian of the system has the form:

$$\tilde{H} = \sum_{\vec{p}\neq 0} \varepsilon_{\vec{p}} \hat{b}^{+}_{\vec{p}} \hat{b}_{\vec{p}} - \frac{\hbar^2}{2m}\sum_{i=1}^{N} \Delta_{\vec{r}_i} + \frac{1}{2}\sum_{i\neq j} V(\vec{r}_i - \vec{r}_j) + \hat{H}_0 \tag{22}$$

where $V(\vec{r}_i - \vec{r}_j)$ is the effective interaction between atoms of the liquid ^{4}He which equal

$$V(r) = U_{\vec{p}}\delta(r) - \frac{a}{r}\exp(-br) \,.$$

The Fourier transform of effective potential of interaction between atoms represents as:

$$V_{\vec{p}} = U_{\vec{p}} - \frac{4\pi\hbar^2 a}{p^2 + \hbar^2 b^2} \,. \tag{23}$$

This is a screening potential interaction between atoms of liquid ^{4}He which is a repulsive. At momentum $\vec{p} = 0$ we obtain $V_{\vec{p}} = 0$.

So now we will investigate the Hamiltonian of "individual" component within the model of Bogoliubov[2] for weakly non-ideal Bose gas. The Hamiltonian of Bogoliubov for "individual" component within the second quantization form represents as:

$$\tilde{H} = \sum_{\vec{p}\neq 0} \frac{p^2}{2m} \hat{a}^+_{\vec{p}} \hat{a}_{\vec{p}} + \frac{N}{2V} \sum_{\vec{p}\neq 0} V_{\vec{p}} \varrho_{\vec{p}} \varrho_{-\vec{p}} + \hat{H}_q \; . \tag{24}$$

The operator $\varrho_{\vec{p}}$ represents as

$$\varrho_{\vec{p}} = \frac{1}{\sqrt{N}} \sum_{\vec{p}_1} \hat{a}^+_{\vec{p}_1 - \vec{p}} \hat{a}_{\vec{p}_1} \tag{25}$$

where $\hat{a}^+_{\vec{p}}$ and $\hat{a}_{\vec{p}}$ are the "creation" and "annihilation" Bose-operators of a free atom with momentum $\vec{p}$; and

$$\hat{H}_q = \sum_{\vec{p}\neq 0} \varepsilon_{\vec{p}} \hat{b}^+_{\vec{p}} \hat{b}_{\vec{p}} + \hat{H}_0 \; . \tag{26}$$

Like to the Bogoliubov's approximation we will suggest that the number of atoms with the zero-momentum N_0 is a larger than 1 ($N_0 \gg 1$); then the operators can be written down as $\hat{a}^+_0 \simeq N_0^{\frac{1}{2}}$ and $\hat{a}_0 \simeq N_0^{\frac{1}{2}}$. It means that we can present the operators $\varrho_{\vec{p}}$ and $\varrho^+_{\vec{p}}$ in the following ways:

$$\varrho_{\vec{p}} \simeq \sqrt{\frac{N_0}{N}} \Big(\hat{a}^+_{\vec{p}} + \hat{a}_{-\vec{p}} \Big) \tag{27}$$

$$\varrho^+_{\vec{p}} \simeq \sqrt{\frac{N_0}{N}} \Big(\hat{a}_{\vec{p}} + \hat{a}^+_{-\vec{p}} \Big) . \tag{28}$$

Then, the Bogoliubov's Hamiltonian $\hat{H}_B$ has the form with taking into commiseration chemical potential μ :

$$\hat{H}_B = \sum_{\vec{p}\neq 0} \Big(\frac{p^2}{2m} + \frac{V_{\vec{p}} N_0}{V} - \mu \Big) \hat{a}^+_{\vec{p}} \hat{a}_{\vec{p}} + \frac{N_0}{2V} \sum_{\vec{p}\neq 0} V_{\vec{p}} \Big(\hat{a}^+_{-\vec{p}} \hat{a}^+_{\vec{p}} + \hat{a}_{-\vec{p}} \hat{a}_{\vec{p}} \Big) . \tag{29}$$

To carry the Hamiltonian $\hat{H}_B$ in the diagonal form we make use the Bogoliubov's transformation:

$$\hat{a}_{\vec{p}} = \frac{\hat{d}_{\vec{p}} + L_{\vec{p}} \hat{d}^+_{-\vec{p}}}{\sqrt{1 - L^2_{\vec{p}}}} \tag{30}$$

where $L_{\vec{p}}$ is a real symmetrical function of the momentum $\vec{p}$; $\hat{d}^+_{\vec{p}}$ and $\hat{d}_{\vec{p}}$ are, respectively, operators of the "creation" and "annihilation" of a quasi-particle with the momentum $\vec{p}$.

In this case, the operator $\hat{H}_B$ becomes the diagonal form

$$\hat{H}_B = \sum_{\vec{p}\neq 0} E_{\vec{p}} \hat{d}^+_{\vec{p}} \hat{d}_{\vec{p}} + \frac{1}{2}\sum_{\vec{p}\neq 0}\left(E_{\vec{p}} - \frac{p^2}{2m} - \frac{N_0 V_{\vec{p}}}{V}\right) \tag{31}$$

where $E_{\vec{p}}$ is the energy of Landau's quasi-particles with the momentum $\vec{p}$:

$$E_{\vec{p}} = \left[\left(\frac{p^2}{2m} - \mu\right)^2 + 2\left(\frac{p^2}{2m} - \mu\right)\frac{V_{\vec{p}} N_0}{V}\right]^{\frac{1}{2}}. \tag{32}$$

From experiment[6] has been known that at momentum $\vec{p} = 0$ we must obtain the energy of quasi-particles as $E_{\vec{p}} = 0$. Then, obviously from Eq.(32) we have $\mu = 0$. Then Eq.(32) gains the form:

$$E_{\vec{p}} = \left[\left(\frac{p^2}{2m}\right)^2 + \frac{p^2 N_0}{mV}\left(U_{\vec{p}} - \frac{4\pi\hbar^2 a}{p^2 + \hbar^2 b^2}\right)\right]^{\frac{1}{2}}. \tag{33}$$

This is a spectrum energy of Landau's quasi-particles which are excited due to single particle motions of atoms in the liquid helium. At the lambda transition $N_0 = 0$ the Landau's quasi-particles are a free because the energy of Landau's quasi-particles has the form $E_{\vec{p}} = \frac{p^2}{2m}$. Such result coincides with experimental data[11]. Obviously from the form of Eq.(33) it follows that the rotons part present in given expression.

3. Conclusion .

We proposed a new method of "mixed" representation for the Bose-system which let us to find an effective potential interaction between atoms of the ^{4}He according to the S-wave pseudopotential in the model of hard spheres. This approach is similar to finding of the screening Coulomb interaction in the electron gas. It means that the collective mode leads to the correction form of interaction between atoms of the liquid ^{4}He. This result is very important because the chemical potential of non-ideal Bose gas in the model of liquid ^{4}He must be equal zero ($\mu = 0$) which is connected with the necessary condition of thermal equilibrium. It means that requiring the free energy to be a minimum. Thus, due to a new method of "mixed" representation for Bose system we found two types bosons which can be excited in liquid ^{4}He. This result coincides with experimental data[11].

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Received on April 24, 2002.

Минасян В. Н., Турян К. В. E17-2002-97
Эффективное взаимодействие
между атомами жидкого гелия ^{4}He

Приводится новый метод «смешанного» представления для бозе-систем. На базе данного метода найден эффективный потенциал взаимодействия между атомами жидкого гелия, который формируется как результат взаимодействия колебаний плотности атомов и взаимодействия между двумя атомами, представленными в форме S-волны в модели твердых сфер. Показано, что два типа бозонов могут возбуждаться в жидком гелии. Данный результат совпадает с экспериментальными данными.

Работа выполнена в Научном центре прикладных исследований ОИЯИ и в Национальной лаборатории возобновляемых источников энергии (Голден, США).

Препринт Объединенного института ядерных исследований. Дубна, 2002

Minasyan V. N., Touryan K. J. E17-2002-97
The Effective Interaction between Atoms of Liquid ^{4}He

A new method of «mixed» representation for the Bose-system is presented. On the basis of this method we have found the effective potential interaction between atoms of ^{4}He, which is formed by the interaction of atoms density oscillations and the interaction between two atoms, represented in the form of S-wave pseudopotential in the model of hard spheres. It is also shown that two types of bosons can be axcited in liquid ^{4}He. This result coincides with the experimental data.

The investigation has been performed at the Scientific Center of Applied Research, JINR and at the National Renewable Energy Laboratory (Golden, USA).

Preprint of the Joint Institute for Nuclear Research. Dubna, 2002

Макет *Т. Е. Попеко*

ЛР № 020579 от 23.06.97.
Подписано в печать 14.05.2002.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 0,62. Уч.-изд. л. 1,0. Тираж 315 экз. Заказ № 53270.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.