Evolution of the Universe as collective motions of metrics in the light of the Supernova data and the local velocity field of galaxies

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Abstract

We consider the evolution of the Universe as collective motion of the volume. Equations of observational cosmology are derived averaging the local equations in general relativity over the spatial volume. The precise definition of the collective expansion of the volume allows as to define relative standards, as an alternative to the absolute ones of the Friedmann-Robertson-Walker Cosmology. On the basis of homogeneous anisotropic solutions of Hamiltonian equations we analyze Supernova data and the local velocity field of galaxies in the framework of conformal cosmology, in which the evolution of the Universe is an inertial motion of metrics. It is shown that conformal cosmology is able to describe Supernova data and the local velocity field of galaxies with different values of the Hubble parameter and to give contents of the Universe as $\Omega_{\text{dark energy}} = 0.72$, $\Omega_{\text{dark matter}} = 0.28$, where the dark energy is associated with the isotropic state of metric; while dark matter, with the anisotropic excitations.

Contents

1 Introduction

The concept of the "evolution of the Universe" in general relativity was formed in the context of the homogeneous approximation (Friedmann 1922, Friedmann 1924, Robertson 1933, Walker 1933, Walker 1935, Lemaitre 1927, Lemaitre 1931, Einstein, de-Sitter 1932). In particular, the homogeneous approximation is the basis of the modern inflation theory (Linde 1990).

However, general relativity admits also another interpretation of observational cosmology; namely considering the redshift - luminosity distance dependence as a consequence of the joint collective motion of fields of metrics and matter (Pervushin, Proskurin 2002). This point of view is more adequate to the field nature of matter and gravitation, especially in the absence of isotropy.

This field description supposes a separation of the collective (absolute) variables from the relative ones in accordance with the geometry of the field space. Its geometry was obtained by Borisov and Ogievetsky (1974) in terms of Cartan forms (Cartan 1946, Volkov 1973). The Cartan method (Cartan 1946, Volkov 1973) of construction of the nonlinear realization of the affine symmetry, in particular the operation of the group summation (Pervushin 1975, Pervushin 1976, Kazakov, Pervushin, Pushkin 1977, Isaev, Pervushin, Pushkin 1979), allows us to extend the concepts of "centre of mass" coordinates, "relative" coordinates, and "inertial motions" along the "geodesic" in the field space. In particular, the requirement that the canonical momentum of collective motion along geodesic line be constant unambiguously determines the integrals of the inertial motion of the Universe.

There is the Copernicus principle of relativity in the field space stating that an observer in the Universe measures quantities of relative motions, like when an Earth observer measures relative coordinates and velocities on Earth. The "relative" cosmology of the Universe is connected with the standard Friedmann cosmology by the conformal transformations of all measurable quantities with a cosmic scale factor. An observer in the "relative" Universe can measure only the conformal invariant quantities as so-called "conformal time", "coordinate distance", "conformal density", and conformal mass. All of the above quantities are connected to the corresponding ones in Standard Cosmology with the help of a cosmic scale factor. An observer within the relative standards of measurement lives in the Universe with a constant volume and constant temperature. It has been shown that the concept of the "relative" conformal cosmology with varying masses (Blaschke et al. 2001, Pervushin, Proskurin 2001, Behnke et al 2001) leads to the Hoyle - Narlikar type of cosmology (Narlikar 1989), and moreover, the "inertial motion" of the Universe along geodesic lines in the field space preserves all results of the Standard Cosmology concerning the abundance of primordial elements (Weinberg 1977). The "inertial motion" of the Universe is also compatible with the latest Supernova data on the redshift - luminosity-distance relation (Behnke et al. 2002).

Recent observation of the local velocity field of galaxies gives a three-dimensional ellipsoid with different values of the Hubble parameter, clearly showing its anisotropic character (Karachentsev 2001, Karachentsev, Makarov 2001). These new data require specific frames of reference for the description of anisotropic, homogeneous cosmological models.

In this paper we present a possible point of view that these frames should be regarded as a set of physical instruments allowing one to perform measurements. Such observational data connected with the dynamics of the fields of matter and metrics require a special type of embedding the 3–dimensional hypersurface into the 4–dimensional manifold (Dirac 1958, Dirac 1959, Arnowitt, Deser, Misner 1959, Arnowitt, Deser, Misner 1960, Arnowitt, Deser, Misner 1961, Zel'manov 1976, Vladimirov 1982).

We choose the reference frame of one time axis and a set of space-like hypersurfaces. In the present paper we consider the 3–dimensional hypersurface as a foliation of the 2–dimensional one, embedded in the 3–dimensional manifold. These 2–dimensional surfaces are congruent. After that we repeat the operation, considering the set of lines embedded in the respective 2–dimensional manifold. The procedure described above corresponds to the triangular tetrad representation of metrics

$$g_{\mu\nu} = e_{\alpha\mu}e_{\alpha\nu} = e_{0\mu}e_{0\nu} - e_{1\mu}e_{1\nu} - e_{2\mu}e_{2\nu} - e_{3\mu}e_{3\nu}, \tag{1}$$

where $e_{\alpha\mu} = 0$ for $\alpha > \mu$ and μ, ν=0, 1, 2, 3. Indices α and μ arise from the Minkowski and Riemannian spaces respectively. Our aim is to construct the Hamiltonian formalism for the description of the dynamics of the anisotropic and homogeneous models of the Universe. The analysis of the observational data will be based on the radial velocities of nearby galaxies, belonging to the Local Supercluster, as well as on the Supernova data. Our paper is organized in the following manner. In Section 2 the cosmic evolution is described as collective motion in superspace $A(4)/L$ in the framework of the 3+1 parametrization of metrics proposed by Zel'manov, Dirac, Arnowitt, Deser, Misner, and Vladimirov, hereafter refer to as ADM parametrization. In Section 3 we extend this approach to the $3 = 1+1+1$ spatial metrics adequate to the reference frame of observations, allowing us to determine the initial conditions for solving equations. In the next Sections 4 the Hamiltonian formalism for the homogeneous metrics is written down. We consider the simplest examples of the developed formalism in order to describe isotropic and anisotropic Universes, that is the Hubble diagram and the anisotropic velocity field . The paper ends with the conclusions.

2 Cosmic evolution as collective motion in the field space

2.1 Frame of reference as $3+1$ foliation of space–time

The problem of evolution of fields in General Relativity

$$S_{\rm GR} = \int d^4x \sqrt{-g}\frac{\varphi_0^2}{6}R(g), \qquad \left(\varphi_0 = M_{\rm Planck}\sqrt{\frac{3}{8\pi}}\right) \tag{2}$$

is considered in the kinemetric frame of reference chosen in the form of $3+1$ foliation of space–time (Dirac 1958, Dirac 1959, Arnowitt, Deser, Misner 1959, Arnowitt, Deser, Misner 1960, Arnowitt, Deser, Misner 1961, Zel'manov 1976, Vladimirov 1982)

$$(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu = (Ndx^0)^2 - {}^{(3)}g_{ij}\left(dx^i + N^i dx^0\right)\left(dx^j + N^j dx^0\right), \tag{3}$$

where ${}^{(3)}g_{ij}$ is a 3-dimensional metric, N^i is the shift–vector between two 3-dimensional hyper-surfaces, and N is a lapse-function. The Dirac-ADM parametrization characterizes a family of hypersurfaces $x^0 = \text{const}$ with the unit vector $\nu^\alpha = (1/N, -N^k/N)$ normal to a hypersurface. The next (external) form

$$\pi_{ij} \; = \; \partial_0 {}^{(3)}g_{ij} - N_{j|i} - N_{i|j} \equiv \left(\partial_0 - N^l\partial_l\right){}^{(3)}g_{ij} - {}^{(3)}g_{il}\partial^k N^l - {}^{(3)}g_{jl}\partial_i N^l \tag{4}$$

shows how this hypersurface is embedded into the four-dimensional space-time. Here $N_{i|j}$ is the covariant derivative with respect to the metric ${}^{(3)}g^{kj}$.

Then the action takes the form of:

$$S_{\rm GR} = \int d^3x dx^0 \sqrt{{}^{(3)}g}\frac{\varphi_0^2}{6}R = \int d^3x dx^0\,(\mathrm{K} - \mathrm{P} + \mathrm{S}), \tag{5}$$

where

$$\mathrm{K} \; = \; \frac{\varphi_0^2\sqrt{{}^{(3)}g}}{24N}\left[\pi_{ij}\pi^{ij} - \pi_i^i\pi_j^j\right] \tag{6}$$

is a kinetic term, while

$$\mathrm{P} = \frac{\varphi_0^2 N\sqrt{{}^{(3)}g}}{6}\,{}^{(3)}R \tag{7}$$

is a potential term, and

$$\mathrm{S} = \frac{\varphi_0^2}{6}(\partial_0 - \partial_k N^k)\frac{\sqrt{{}^{(3)}g}\pi_i^i}{N} - \frac{\varphi_0^2}{3}\partial_i({}^{(3)}g^{ij}\sqrt{{}^{(3)}g}\partial_j N) \tag{8}$$

and is a surface term.

The Hilbert variational principle reproduces the classical Einstein equations

$$\frac{\varphi_0^2}{3}\sqrt{-g}\left[R^{\nu}_{\mu}(g)-\frac{1}{2}\delta^{\nu}_{\mu}R(g)\right]=\varepsilon^{\nu}_{\mu}\equiv\sqrt{-g}\,T^{\nu}_{\mu}\ , \tag{9}$$

where T^{ν}_{μ} is the matter energy - momentum tensor. The group of general coordinate transformations

$$x^{\mu}\to\tilde{x}^{\mu}=\tilde{x}^{\mu}(x^0,x^1,x^2,x^3)$$

reduces, in this case, to the so-called kinemetric ones

$$x^{i}\to\tilde{x}^{i}=\tilde{x}^{i}(x^0,x^1,x^2,x^3)$$

$$x^{0}\to\tilde{x}^{0}=\tilde{x}^{0}(x^0)$$

including reparametrizations of the coordinate parameter x^0.

The invariance of GR in the kinemetric frame with respect to reparametrizations of the coordinate parameter x^0 means that one of variables in the field space becomes the *dynamic evolution parameter* (Pawlowski, Pervushin 2001, Pervushin, Proskurin 2001, Barbashov, Pervushin 2001).

2.2 Separation of a collective motion of the spatial volume

In the light of the cosmological applications, it is conveniently to choose such *dynamic evolution parameter* as the functional of an spatial volume in the kinemetric frame (3)

$$\frac{1}{V_0}\int\limits_{V_0} d^3x\sqrt{{}^{(3)}g(x^0,x^i)}\equiv a^3[{}^{(3)}g]\ , \tag{10}$$

where V_0 is the finite constant volume of the coordinate space. In other words, in contrast to the conventional homogeneous approximation, we define the cosmic scale factor $a(x^0)$ as a collective variable $a^3[{}^{(3)}g]$ in the field space of the exact theory. This definition is compatible with the Dirac-ADM metric (3), as the scale functional $a^3[{}^{(3)}g]$ given by (10) is an invariant with respect to the kinemetric coordinate transformations.

It is worth to recall that in classical mechanics, a collective motion of composite many-particle systems $x_i,\ \ i=1,..,n$, is also described by the introduction of the "centre of mass" coordinate. The "centre of mass" coordinate X is separated from the relative coordinates z_i using the operation of adding coordinates along geodesic lines

$$x_i=z_i+\frac{X}{n} \tag{11}$$

and the constraint for relative coordinates

$$\sum_{i=1}^{n} z_i=0 \tag{12}$$

We can speak about the concept of a collective state if after the transformation (11) the total action is split into the sum of the action of the "centre of mass" coordinate and that of a "relative" motion

$$S(\{x\})=S_{\mathrm{CM}}(X)+S_{\mathrm{relative}}(\{z\})\ . \tag{13}$$

A set of all measurements can be separated into the measurements of the total motion X of the whole system and into relative motions. If the latter do not depend on the former, one can talk about the concept of an *inertial frame of reference.*

The cosmic evolution was introduced into GR by a similar collective motion of metrics in the "field space" (Pervushin, Proskurin 2002). A geometry of this ten-dimensional "field space" was described in paper (Borisov, Ogievetsky 1974) in terms of Cartan forms (Cartan 1946, Volkov 1973) as a geometry of the coset of the affine group $A(4)$ over the Lorentz one L

$$\frac{\text{group of affine transformations}}{\text{Lorentz group}}$$

with constant curvature.

The Cartan method of constructing the nonlinear realization of the affine symmetry (Cartan 1946, Volkov 1973) and the operation of the group summation, formulated in (Pervushin 1975, Pervushin 1976, Kazakov, Pervushin, Pushkin 1977, Isaev, Pervushin, Pushkin 1979), allows us to separate the "collective" and "relative" coordinates using the concepts of "geodesic lines" into the coset $A(4)/L$, in particular, to describe a class of "inertial motions" as motions with constant canonical momenta along the geodesic.

By analogy with formulae (11) and (12), one can introduce similar collective and relative variables and *inertial frames of reference* in the space of metric fields $g_{\mu\nu}$ using the geometry of geodesic lines in the space. The operation of addition along the geodesic line in terms of normal coordinates in the field "superspace" $g_{\mu\nu}(h) = [\exp(2h)]_{\mu\nu}$ is defined by (Pervushin 1976, Isaev, Pervushin, Pushkin 1979):

$$g_{\mu\nu}(h_{\text{coll.}}(+)h_{\text{rel.}}) = [\exp(h_{\text{coll.}})\exp(2h_{\text{rel.}})\exp(h_{\text{coll.}})]_{\mu\nu} \ . \tag{14}$$

In this case, the counterpart of formulae (11) that separate collective the motion of volume $a(x^0)$ from the relative metric $\bar{g}_{\mu\nu}(x^0, x^i)$ is the multiplication

$$g_{\mu\nu}(x^0, x^i) = \bar{g}_{\mu\nu}(x^0, x^i)a(x^0)^2 \ . \tag{15}$$

The normal coordinate in the field "space" along the geodesic line is the Misner exponential parametrization of the scale factor (Misner 1969)

$$a(x^0) = \exp X_0(x^0) \ . \tag{16}$$

The constant values of the canonical momentum of the Misner variable X_0 correspond to *the inertial motion* in the field "space" along the geodesic line.

Transformation (15) is a particular case of the Lichnerowicz conformal transformations of all field variables $\{{}^{(n)}f\}$

$${}^{(n)}f(x^0, x^i) = {}^{(n)}\bar{f}(x^0, x^i)a(x^0)^n \tag{17}$$

with the conformal weights n, including the metric as a tensor field with the conformal weight $n = 2$ (Lichnerowicz 1944, York 1971, Kuchar 1972). In line with the Lichnerowicz conformal transformations (17) each field contributes to the cosmic evolution of the Universe.

The analogue of constraint (12) is the condition of the constant spatial volume in the relative space $\bar{g}_{\mu\nu}$

$$\int\limits_{V_0} d^3x\sqrt{{}^{(3)}\bar{g}(x^0, x^i)} = V_0 \ . \tag{18}$$

To identify the *collective variable* $a^3[{}^{(3)}g]$ with the homogeneous cosmic scale factor in observational cosmology, we should verify that the exact Einstein equations averaged over the invariant three-dimensional volume in the theory coincide with the equations of homogeneous cosmic scale factor in the standard cosmology where the concept of the cosmic evolution of the universe is formulated.

2.3 Einstein equations for collective motions and relative variables

In order to find the Einstein action with collective motion and the corresponding Einstein equations, we use the well-known formula of conformal transformations (15) of four-dimensional curvature (Hawking, Ellis 1973)

$$-\sqrt{-g}\frac{\varphi_0^2}{6}R(g) = -\sqrt{-\bar{g}}\frac{\varphi^2}{6}R(\bar{g}) + \varphi\partial_\mu\left[\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\varphi\right] , \tag{19}$$

where $\varphi(x^0)$ is the dynamic Planck mass defined as the product of the Planck mass and the cosmic scale factor

$$\varphi(x^0) = a(x^0)\varphi_0 \qquad \left(\varphi_0 = M_{\text{Planck}}\sqrt{\frac{3}{8\pi}}\right) . \tag{20}$$

This formula leads to the Einstein action

$$S_{GR}[g|\varphi_0] = S_{GR}[\bar{g}|\varphi] + \int\limits_{x_1^0}^{x_2^0} dx^0 \int\limits_{V_0} d^3x\varphi\frac{d}{dx^0}\left(\sqrt{{}^{(3)}\bar{g}}\frac{d\varphi}{\bar{N}dx^0}\right) \tag{21}$$

$$+\int\limits_{x_1^0}^{x_2^0} dx^0\Lambda(x^0)\left[\int\limits_{V_0} d^3x\sqrt{{}^{(3)}\bar{g}(x^0,x^i)} - V_0\right] ,$$

where the Lagrangian factor $\Lambda(x^0)$ provides the conservation of the volume (18) of the local excitations;

$$S_{\text{GR}}[\bar{g}|\varphi] = -\int d^4x\sqrt{-\bar{g}}\frac{\varphi^2}{6}R(\bar{g}) . \tag{22}$$

is standard ADM action in GR in the relative metric $\bar{g}$ and with the running Planck mass (20) .

The action of collective motion allows us to define the global lapse function

$$\frac{1}{\bar{N}_0(x^0)} = \frac{1}{V_0}\int\limits_{V_0} d^3x\frac{\sqrt{{}^{(3)}\bar{g}}}{\bar{N}}. \tag{23}$$

and *the world geometrical time.* The global lapse function (23) determines a gauge-invariant *world geometrical time*

$$d\eta = \bar{N}_0(x^0)dx^0 = \tilde{\bar{N}}_0(\tilde{x}^0)d\tilde{x}^0 . \tag{24}$$

The GR action can be added by any action of matter fields

$$S_{\text{tot}} = S_{\text{GR}} + S_{\text{matter}} . \tag{25}$$

In terms of the *world geometric time*, the variation of the action (25) with respect to the metric components leads to the equations

$$\bar{N}\frac{\delta S_{\text{tot}}}{\delta\bar{N}} = 0 \implies \frac{\varphi'^2}{\mathcal{N}} = \mathcal{N}\bar{T}_0^0 , \tag{26}$$

$$\bar{g}^{ij}\frac{\delta S_{\text{tot}}}{\delta\bar{g}^{ij}} = 0 \implies \frac{2(\varphi^2)'' - 3\varphi'^2}{\mathcal{N}} + 3\Lambda = \mathcal{N}\bar{T}_k^k , \tag{27}$$

$$\frac{\delta S_{\text{tot}}}{\delta\bar{N}^k} = 0 \implies \bar{T}_k^0 = 0 , \tag{28}$$

$$\bar{g}^{ki}\frac{\delta S_{\text{tot}}}{\delta\bar{g}^{ij}} = 0 \implies \bar{T}_k^i = 0 \quad (i \neq k) , \tag{29}$$

where $f' = df/d\eta$, $\mathcal{N} = \bar{N}/\bar{N}_0$, and $\bar{T}^\nu_\mu = T^\nu_\mu - \varphi^2/3(R^\nu_\mu - 1/2\delta^\nu_\mu R)$ are the total components of the local energy-momentum tensor

$$\sqrt{{}^{(3)}\bar{g}}\mathcal{N}\bar{T}^0_0 = \mathrm{K}(\bar{g}|\varphi) + \mathrm{P}(\bar{g}|\varphi) + \varepsilon^0_0 \equiv \varepsilon^0_{0(\text{total})} \ , \tag{30}$$

$$\sqrt{{}^{(3)}\bar{g}}\mathcal{N}\bar{T}^k_k = 3\mathrm{K}(\bar{g}|\varphi) - \mathrm{P}(\bar{g}|\varphi) + 2\mathrm{S}(\bar{g}|\varphi) + \varepsilon^k_k \equiv \varepsilon^k_{k(\text{total})} . \tag{31}$$

$\bar{T}^\nu_\mu = 0$ is equal to zero, if the cosmic evolution is absent $\varphi(x^0) \equiv \varphi_0$. These equations contain the collective motion of the cosmic evolution which can be extracted by integrating of these equations over the spatial volume. As a result we get

$$\frac{1}{V_0}\int\limits_{V_0} d^3x \bar{N}\frac{\delta S_{\text{tot}}}{\delta \bar{N}} = 0 \ \Longrightarrow \ \varphi'^2 = \rho_{\text{total}}, \tag{32}$$

$$\frac{1}{V_0}\int\limits_{V_0} d^3x \bar{g}^{ij}\frac{\delta S_{\text{tot}}}{\delta \bar{g}^{ij}} = 0 \ \Longrightarrow \ (\varphi^2)'' - 3\varphi'^2 + 3\Lambda = -3p_{\text{total}} \ ; \tag{33}$$

here we designate

$$\rho_{\text{total}} = \frac{1}{V_0}\int d^3x \varepsilon^0_{0(\text{total})} \ , \qquad 3p_{\text{total}} = \frac{1}{V_0}\int d^3x \varepsilon^k_{k(\text{total})} .$$

These equations are accompanied by the equations of collective variables

$$\varphi\frac{\delta S_{\text{tot}}}{\delta \varphi} = 0 \ \Longrightarrow \ 2\varphi\varphi'' = \rho_{\text{total}} - 3p_{\text{total}}, \tag{34}$$

$$\frac{\delta S_{\text{tot}}}{\delta \Lambda} = 0 \ \Longrightarrow \ V[\bar{g}] - V_0 = 0. \tag{35}$$

The combination of eqs. (32), (33), and (34) leads to $\Lambda = 0$.

In this case, the exact equations (32) and (33) in *relative field space* for the collective variable fully coincide with the **conformal version** of the equations of the Friedmann-Robertson-Walker (FRW) cosmology in the homogeneous approximation (Friedmann 1922, Friedmann 1924, Robertson 1933, Walker 1933,

$$\boxed{\varphi_0^2 a'^2 = \rho_{\text{total}} \ ; \qquad \varphi_0^2\left[3a'^2 - (a^2)''\right] = 3p_{\text{total}} \ ,} \tag{36}$$

where ρ_{total} and p_{total} are *total* density and *total* pressure obtained by averaging the local ones over the coordinate space-volume.

This coincidence of the averaged equations in GR with their homogeneous approximation means that *homogeneous approximation of matter and metrics is not necessary.* The cosmic evolution can be interpreted as averaged value of the local energy density.

If the *total* density is equal to the *total* pressure $\rho_{\text{total}} = p_{\text{total}}$ the eqs. (36) convert into

$$(a^2)'' = 0 \ . \tag{37}$$

It is the rigid (i.e., most singular) state of the Universe evolution. The solution of eq. (37) with the present-day initial data $a(\eta_0) = 1$ and $a'(\eta_0) = H_0$ takes the form

$$a^2(\eta) = 1 + 2H_0(\eta - \eta_0) \ . \tag{38}$$

This evolution law follows from equation of motion obtained by variation of the relativistic action taking into account $d\eta = \bar{N}_0 dx^0$:

$$S_{\text{collective}} = -V_0 \int\limits_{x_1^0}^{x_2^0} dx^0 \left[\frac{(\partial_0 a(x^0))^2}{\bar{N}_0} + \bar{N}_0 \frac{\rho_{cr}}{a^2(x^0)} \right] , \tag{39}$$

where $\rho_{cr} = \varphi_0^2 H_0^2$ is the so-called critical density, if the collective motion of the universe is described rigid state, and $\bar{N}_0$ is the lapse function of the conformal time (24).

In terms of the variable (16) $a(x^0) = \exp X_0(x^0)$ along the geodesic in the coset A(4)/L, the action (39) takes form of the one of the inertial motion (with the constant momentum)

$$S_{\text{collective}} = -\gamma \int\limits_{x_1^0}^{x_2^0} dx^0 \left[\frac{(\partial_0 X_0(x^0))^2}{\bar{e}_0} + \bar{e}_0 H_0^2 \right] , \tag{40}$$

where ($\gamma = \varphi_0^2 V_0$) and $e_0 = e^{-2X_0} \bar{N}_0$ is the lapse function of the Misner time-interval

$$e_0 dx^0 = d\tau = a(\eta)^{-2} d\eta \, . \tag{41}$$

Thus, the rigid state corresponds to *the inertial motion* of the Universe along the geodesic line in the field "space" (Pervushin, Proskurin 2002).

The geometry of the coset shows us that there are two possibilities to choose the standard of measurement in the Universe.

If our measurements in the Universe corresponds to the absolute fields $F = (g, f)$, this means that we observe the expanding Universe with the z-history of temperature $T(z) = T_0(z+1) \equiv T_0/a(t)$ in terms of the absolute Friedmann time $dt = a(\eta)d\eta$. In this case, the square-root dependence $a(t) \sim \sqrt{t}$ describing the chemical evolution and primordial element abundance is the evidence of the radiation state of the Universe.

If we measure only the relative fields $\bar{F} = (\bar{g}, \bar{f})$ and coordinates including the conformal time (24), this means that we observe the steady Universe with the z-history of masses (including the Planck one) and constant temperature. In this case, the chemical evolution and primordial element abundance is the evidence of the square-root dependence (38)$a(\eta) \sim \sqrt{\eta}$. In the rigid state of the Universe moving inertially along geodesic in the field space (Behnke et al. 2002). To describe this free cosmic motion, one does not need the matter. Thus, the relative standard can explain us the most intriguing fact of the astroparticle physics that the visual matter almost does not take part in the cosmic evolution of the metrics.

It was shown that "inertial" motion of the universe along a geodesic line of the factor-space $A(4)/L$ does not contradict data of observational cosmology including the primordial element abundance, and the latest Supernova data on the redshift - luminosity-distance relation (Behnke et al. 2002, Pervushin, Proskurin 2002). There are the set of arguments in favor of that this "inertial" motion and the relative standards explains the origin of matter with the primordial temperature $T_I = (m_W^2 H_0)^{1/3} =\sim 2.7K$ as the intensive cosmological creation of vector W,Z-bosons (Blaschke et al. 2001, Pervushin, Proskurin 2002).

3 Supernova data in the context of conformal cosmology

The varying mass $\bar{M}(\eta) = Ma(\eta)$ in conformal cosmology means that the spectrum of atoms is described by the Schrödinger equation

$$\left[\frac{\hat{p}^2}{2m_0 a(\eta)} - \left(\frac{\alpha}{r} + E(\eta) \right) \right] \Psi_A = 0 \, . \tag{42}$$

It is easy to check that the exact solution of this equation is expressed throw the solution E_0 of a similar Schrödinger equation with constant masses m_0 at $a(\eta_0) = 1$

$$E(\eta) = a(\eta)E_0 \equiv \frac{E_0}{z(d)+1} , \qquad E_0 = -\frac{m_0\alpha^2}{n^2} , \tag{43}$$

where $z(d)$ is a redshift of the spectral lines of atoms at the coordinate distance $d/c = \eta_0 - \eta$, and η_0 is the present-day value of the geometric (conformal) time.

This type of conformal cosmology was developed by Hoyle and Narlikar (Narlikar 1989). A red photon emitted by an atom at a star two billion years (in terms of η) remembers the size of this atom, and after two billion years this photon is compared with a photon of the standard atom at the Earth that became blue due to the evolution of all masses. The redshift-coordinate distance relation is defined by the formula of the standard cosmology (43)

$$z(d) = \frac{a(\eta_0)}{a(\eta_0 - d/c)} - 1 , \qquad a(\eta_0) = 1 \tag{44}$$

(where d is the coordinate distance to an object) because the description of the conformal - invariant photons does not depend on the standard of measurements. As a light ray traces a null geodesic that satisfies the equation $dr/d\eta = 1$, the coordinate distance as a function of the redshift z in the Conformal Cosmology (CC)

$$H_0 r(z) = \int\limits_1^{1+z} \frac{dx}{\sqrt{\Omega_{\text{Rigid}}x^6 + \Omega_{\text{Radiation}}x^4 + \Omega_{\text{M}}x^3 + \Omega_\Lambda}} ,$$

where $\Omega_{\text{Rigid}} + \Omega_{\text{Radiation}} + \Omega_{\text{M}} + \Omega_\Lambda = 1$ coincides with the similar relation between coordinate distance and redshift in Standard Cosmology (SC).

The luminosity distance ℓ is defined so that the apparent luminosity of any object behaves as $1/\ell^2$. Therefore, in comparison with the stationary space in SC and stationary masses in CC, a part of photons is lost. To restore the full luminosity in both SC and CC, we should multiply the coordinate distance by the factor $(1+z)^2$. This factor comes from the evolution of the angular size of the light cone of emitted photons in SC and from the increase of the angular size of the light cone of absorbed photons in CC. This evolution sappears in the contrast to the stationary case as it is shown in Fig.1.

However, in SC, we have an additional factor (1+z) due to the expansion of the universe, since measurable distances in SC are related to measurable distances in CC (that coincide with the coordinate ones) by the relation

$$\ell = a\int \frac{dt}{a} = ar(z), \qquad a = \frac{\varphi}{\varphi_0} = \frac{1}{1+z} . \tag{45}$$

Thus we obtain the relations

$$\ell_{SC}(z) = (1+z)^2\ell = (1+z)r(z) ,$$
$$\ell_{CC}(z) = (1+z)^2 r(z) .$$

This means that the observational data are described by different regimes in SC and CC. In Fig. 2, we compare the results of SC and CC for the relation between the effective magnitude and redshift: m(z) = 5 log $[H_0\ell(z)]$ + M where M is a constant with the latest data for distant Supernovae (Perlmutter et al. 1999, Riess et al. 1998, Riess et al. 2001). In the region $0 \le z \le 2$, observational data, including the last SN 1997ff point $z = 1.7$ (Riess et al. 2001) cannot distinguish between Standard Cosmology

$$l_{SC}(z|\Omega_{\text{Rigid}} = 0, \Omega_M \ge 0.15, \Omega_\Lambda \le 0.85)$$

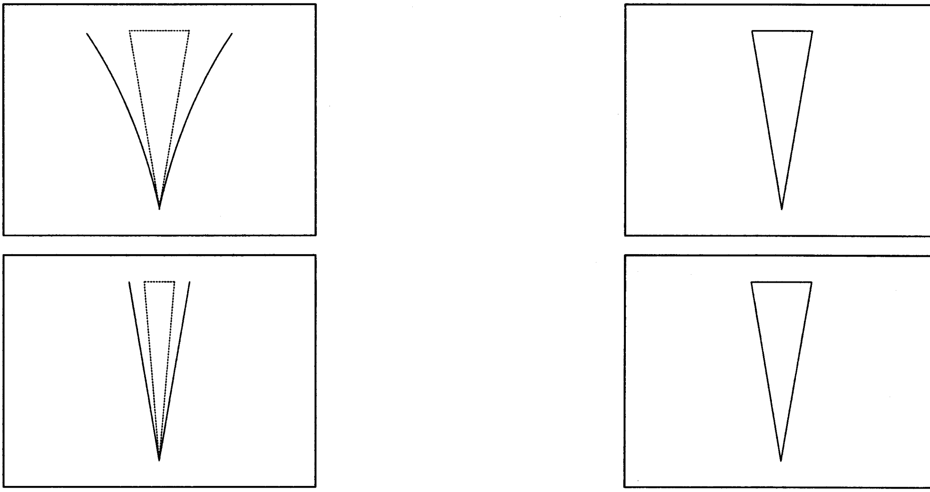

Figure 1: The comparison of standard FRW cosmology (SC) (left upper panel) with conformal cosmology (CC)(left lower panel) for a flat universe model (right panels).

and Conformal Cosmology

$$l_{CC}(z|\Omega_{\rm Rigid} \geq 0.7, \Omega_M \leq 0.3, \Omega_\Lambda = 0).$$

In the case of the inertial motion (38), this redshift - distance relation takes the form

$$z(d) = \frac{1}{(1 + 2H_0 d/c)^{1/2}} - 1 \,. \tag{46}$$

It results in the following simple relation

$$d(z) = \frac{c}{2H_0}\left[1 - \frac{1}{(1+z)^2}\right] \,. \tag{47}$$

The redshift - luminosity distance relation is determined by the formula

$$\ell_{\rm luminocity}(z) = (1+z)^2 d(z) \tag{48}$$

The factor $(1+z)^2$ comes from the evolution of the angular size of the light cone of absorbed photons (Behnke et al. 2002) like the same factor.

Since measurable distances in the conformal cosmology are the coordinate ones, we lose the factor $(1+z)^{-1}$ that was in the standard cosmology due to the expansion of the universe. Finally we obtain the redshift-luminosity distance relation

$$\ell_{\rm luminocity}(z) = (1+z)^2 d(z) = \frac{c}{H_0}\left[z + \frac{z^2}{2}\right] \tag{49}$$

as the consequence of the "inertial motion" of the universe along the geodesic line of the field *space* (i.e., the rigid state of dark energy with the most singular behaviour).

It has been shown (Behnke et al. 2002) that this relation does not contradict the latest Supernova data (Perlmutter et al. 1999, Riess et al. 1998, Riess et al. 2001). In Fig. 2 (Behnke et al. 2002) the predictions of the "absolute" standard cosmology (SC) and the "relative" conformal cosmology (CC) on the Hubble diagram are confronted with recent experimental data for distant supernovae (Perlmutter, et al. 1999, Riess et al. 1998, Riess et al. 2001). Among the CC models the pure rigid state of dark energy gives the best description and is equivalent to the SC fit, up to the distance of SN1997ff.

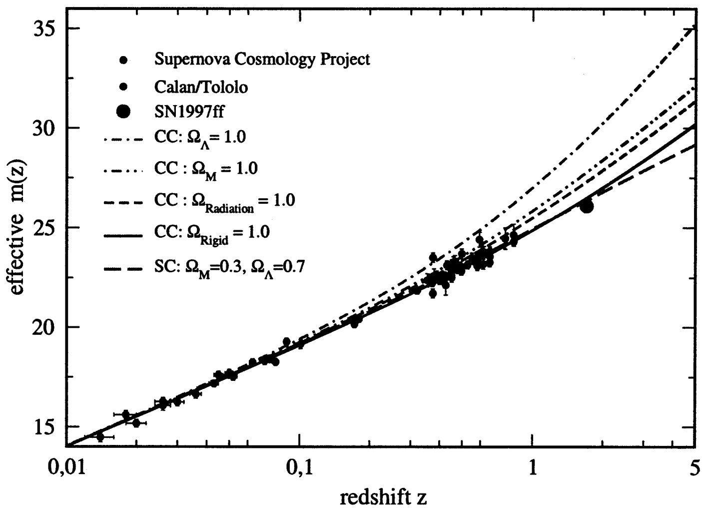

Figure 2: The Hubble diagram (Behnke et al. 2002) for a flat universe model in SC and CC. The points include 42 high-redshift Type Ia supernovae (Perlmutter et al. 1999, Riess et al. 1998) and the reported farthest supernova SN1997ff (Riess et al. 2001). The best fit to these data requires a cosmological constant $\Omega_\Lambda = 0.7$ in the case of SC, whereas in CC these data are consistent with the dominance of the rigid state.

4 Anisotropic universe

4.1 $1+1+1$ foliation of space

Conformal cosmology allows us to describe the local velocity field of galaxies in the framework of a anisotropic model with the same rigid equation of state compatible with the Supernova data in conformal cosmology. In the case, we use $3 = 1+1+1$ foliation of space, where the tensor $\left({}^{(3)}g_{ij}\right)$ is defined as follows

$$ {}^{(3)}g_{ij}dx^i dx^j \;=\; e^{2\beta_1}(dx^1)^2 + \sum_{A,b=2,3} {}^{(2)}g_{AB}(dx^A + n^{(A)}dx^1)(dx^B + n^{(B)}dx^1), \tag{50} $$

where

$$ {}^{(2)}g_{AB}dx^A dx^B \;=\; e^{2\beta_2}dx^2 dx^2 + e^{2\beta_3}(dx^3 + \nu dx^2)^2. \tag{51} $$

and g_{AB} is given by

$$ {}^{(2)}g_{AB} = \begin{bmatrix} e^{2\beta_2} + \nu^2 e^{2\beta_3} & \nu e^{2\beta_3} \\ \nu e^{2\beta_3} & e^{2\beta_3} \end{bmatrix}, \qquad A,B = 2,3. \tag{52} $$

Then ${}^{(3)}g_{ij}$ is

$$ {}^{(3)}g_{ij} = \begin{bmatrix} e^{2\beta_1} + {}^{(2)}g_{AB}n^{(A)}n^{(B)} & {}^{(2)}g_{2B}n^{(B)} & {}^{(2)}g_{3B}n^{(B)} \\ {}^{(2)}g_{2B}n^{(B)} & e^{2\beta_2} + \nu^2 e^{2\beta_3} & \nu e^{2\beta_3} \\ {}^{(2)}g_{3B}n^{(B)} & \nu e^{2\beta_3} & e^{2\beta_3} \end{bmatrix} \tag{53} $$

or in explicit form

$$\begin{bmatrix} e^{2\beta_1}+e^{2\beta_2}(n^{(2)})^2+e^{2\beta_3}(n^{(3)}+\nu n^{(2)})^2 & e^{2\beta_2}n^{(2)}+\nu e^{2\beta_3}(n^{(3)}+\nu n^{(2)}) & e^{2\beta_3}(n^{(3)}+\nu n^{(2)}) \\ e^{2\beta_2}n^{(2)}+\nu e^{2\beta_3}(n^{(3)}+\nu n^{(2)}) & e^{2\beta_2}+\nu^2 e^{2\beta_3} & \nu e^{2\beta_3} \\ e^{2\beta_3}(n^{(3)}+\nu n^{(2)}) & \nu e^{2\beta_3} & e^{2\beta_3} \end{bmatrix}.$$

4.2 Homogeneous approximation of metric

Homogeneous approximation of metric (3) is the next ansatz

$$\begin{aligned} N &= N(x^0), \\ N^i &= 0, \\ {}^{(3)}g_{ij} &= {}^{(3)}g_{ij}(x^0). \end{aligned} \tag{54}$$

Next we choose the parametrization of β_i

$$\beta_1 = X_0 - 2X_1, \tag{55}$$

$$\beta_2 = X_0 + X_1 - \sqrt{3}X_2, \tag{56}$$

$$\beta_3 = X_0 + X_1 + \sqrt{3}X_2; \tag{57}$$

where X_0, X_1, and X_2 are functions of x^0. After substituting the ansatz (54) and (55)–(57) into equations (5)–(8) we get the following action

$$\begin{aligned} S_{\mathrm{GR}} = V_0\varphi_0^2 \int dx^0\, \frac{e^{3X_0}}{N} \times \Big\{ & - (\partial_0 X_0)^2 + (\partial_0 X_1)^2 + (\partial_0 X_2)^2 \\ & + \frac{1}{12}(\partial_0 n^{(3)} + \nu\partial_0 n^{(2)})^2 e^{2\triangle_{31}} + \frac{1}{12}(\partial_0 n^{(2)})^2 e^{2\triangle_{21}} \\ & + \frac{1}{12}(\partial_0\nu)^2 e^{2\triangle_{32}}\Big\}, \end{aligned} \tag{58}$$

where $V_0 = \int d^3x$, $\triangle_{ij} = \beta_i - \beta_j$.

We introduce dark energy in the form of a rigid state in action (58)

$$S_{\mathrm{DE}} = -\frac{1}{2}\int dx^0 N_{\mathrm{I}} C^2\,,$$

where density is equal to pressure ($\rho = p$). For convenience, we put

$$N_{\mathrm{I}} = \frac{N}{2V_0\varphi_0^2} e^{-3X_0}. \tag{59}$$

Now we have

$$\begin{aligned} S_{\mathrm{GR+DE}} = \frac{1}{2}\int dx^0 \Big\{\frac{1}{N_{\mathrm{I}}}\Big[& - (\partial_0 X_0)^2 + (\partial_0 X_1)^2 + (\partial_0 X_2)^2 \\ & + \frac{1}{12}(\partial_0 n_3 + \nu\partial_0 n_2)^2 e^{2\triangle_{31}} + \frac{1}{12}(\partial_0 n_2)^2 e^{2\triangle_{21}} \\ & + \frac{1}{12}(\partial_0\nu)^2 e^{2\triangle_{32}}\Big] - N_{\mathrm{I}} C^2\Big\}. \end{aligned} \tag{60}$$

Diagonalization of (60) is defined by equations:

$$\partial_0 n^{(2)} = \frac{\partial_0 u + \partial_0 v}{\sqrt{2a}}, \tag{61}$$

$$\partial_0 n^{(3)} = \frac{\partial_0 u - \partial_0 v}{\sqrt{2b}}, \tag{62}$$

where

$$a = \frac{1}{12}\Big(\nu^2 e^{2\Delta 31} + e^{2\Delta 21}\Big), \tag{63}$$
$$b = \frac{1}{12}e^{2\Delta 31}, \tag{64}$$
$$c = \frac{1}{6}\nu e^{2\Delta 31}. \tag{65}$$

The action (60) takes the form of:

$$S_{GR+DE} = \frac{1}{2}\int dx^0\Big\{\frac{1}{N_{\mathrm{I}}}\Big[- (\partial_0 X_0)^2 + (\partial_0 X_1)^2 + (\partial_0 X_2)^2 \tag{66}$$
$$+ \Delta_+(\partial_0 u)^2 + \Delta_-(\partial_0 v)^2 + \frac{1}{12}(\partial_0 \nu)^2 e^{4\sqrt{3}X_2}\Big] - N_{\mathrm{I}}C^2\Big\},$$

where

$$\Delta_\pm = 1 \pm \frac{c}{\sqrt{4ab}} = 1 \pm \frac{1}{\sqrt{1+\nu^{-2}\exp(-4\sqrt{3}X_2)}}. \tag{67}$$

The Hamiltonian form of this action is

$$S_{GR+DE} = \int dx^0\Big\{H_0\partial_0 X_0 + H_1\partial_0 X_1 + H_2\partial_0 X_2 + p_u\partial_0 u + p_v\partial_0 v + p_\nu\partial_0\nu \tag{68}$$
$$- \frac{N_{\mathrm{I}}}{2}\Big[-H_0^2 + H_1^2 + H_2^2 + \frac{p_u^2}{\Delta_+} + \frac{p_v^2}{\Delta_-} + 12e^{-4\sqrt{3}X_2}p_\nu^2 + C^2\Big]\Big\}.$$

Here H_i, $i = 0,1,2$ and p_u, p_v are generalized momenta of variables X_i, $i = 0,1,2$ and u, v, i.e.:

$$H_i = \frac{\partial L_{GR+DE}}{\partial(\partial_0 X_i)}, \quad i = 0,1,2; \tag{69}$$
$$p_u = \frac{\partial L_{GR+DE}}{\partial(\partial_0 u)}, \quad p_v = \frac{\partial L_{GR+DE}}{\partial(\partial_0 v)}, \quad p_\nu = \frac{\partial L_{GR+DE}}{\partial(\partial_0 \nu)}, \tag{70}$$

where L_{GR+DE} is the Lagrangian corresponding to the action S_{GR+DE}.

4.3 Hamiltonian equations

According to (68), the equations are

$$\partial_0 X_0 = -N_{\mathrm{I}}H_0, \tag{71}$$
$$\partial_0 H_0 = 0; \tag{72}$$
$$\partial_0 X_1 = N_{\mathrm{I}}H_1, \tag{73}$$
$$\partial_0 H_1 = 0; \tag{74}$$
$$\partial_0 X_2 = N_{\mathrm{I}}H_2, \tag{75}$$
$$\partial_0 H_2 = N_{\mathrm{I}}\Big\{2\sqrt{3}\left[\left(\frac{p_u}{\Delta_+}\right)^2 - \left(\frac{p_v}{\Delta_-}\right)^2\right]\frac{\nu^{-2}\exp(-4\sqrt{3}X_2)}{\left[1+\nu^{-2}\exp(-4\sqrt{3}X_2)\right]^{3/2}}$$
$$+24\sqrt{3}p_\nu^2\exp(-4\sqrt{3}X_2)\Big\}; \tag{76}$$
$$\partial_0 u = N_{\mathrm{I}}\frac{p_u}{\Delta_+}, \tag{77}$$
$$\partial_0 p_u = 0; \tag{78}$$

$$\begin{aligned}
\partial_0 v &= N_{\mathrm{I}} \frac{p_v}{\Delta_-}, &(79)\\
\partial_0 p_v &= 0; &(80)\\
\partial_0 \nu &= 12 N_{\mathrm{I}} p_\nu^2 \exp(-4\sqrt{3}X_2), &(81)\\
\partial_0 p_\nu &= N_{\mathrm{I}} \left[\left(\frac{p_u}{\Delta_+}\right)^2 - \left(\frac{p_v}{\Delta_-}\right)^2 \right] \frac{\nu^{-3}\exp(-4\sqrt{3}X_2)}{\left[1+\nu^{-2}\exp(-4\sqrt{3}X_2)\right]^{3/2}}; &(82)\\
H_0^2 &= H_1^2 + H_2^2 + \frac{p_u^2}{\Delta_+} + \frac{p_v^2}{\Delta_-} + 12 e^{-4\sqrt{3}X_2} p_\nu^2 + C^2. &(83)\\
+ & &\\
\partial_0 n^{(2)} &= \frac{\partial_0 u + \partial_0 v}{\sqrt{2a}}, &(84)\\
\partial_0 n^{(3)} &= \frac{\partial_0 u - \partial_0 v}{\sqrt{2b}}. &(85)
\end{aligned}$$

There are 13 initial conditions required for the solution. Twelve conditions come from the Hamiltonian, but due to equation (83) one condition is eliminated, and 2 further ones derive from inverse substitution.

The system of equations (71)–(85) is composed of 4 subsystems: (71)–(72), (73)–(74), (75)–(82) and (84)–(85). We start solving subsystem (71)–(72), and then subsystem (73)–(74) with a similar structure. Subsystem (84)–(85) can be integrated only after solving (73)–(74) and (75)–(82). In subsystem (75)–(82) equations (75) and (76) are the basic ones; they are independent, defining all solutions of the equations under consideration.

Each solution is characterized by the integrals of motion :

$$H_0,\ H_1,\ p_u,\ p_v \sim const. \tag{86}$$

4.4 Special case of the anisotropic Universe

In this subsection we consider a special case where metric ${}^{(3)}g_{ij}$ is given by equation (54) with $X_1 = X_2 = 0$, $n^{(3)} = \nu = 0$ and $n^{(2)} = n$:

$$\begin{aligned}
{}^{(3)}g_{ij} &= e^{2X_0} \sum_{a=1,2,3} (e_{ai} e_{aj}) = \\
&= e^{2X_0} \begin{bmatrix} 1+n^2 & n & 0 \\ n & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (87)
\end{aligned}$$

where

$$e_{ai} = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{88}$$

In this case, the action S_{GR+DE} (68) takes the form:

$$S_{GR+DE} = \int dx^0 \Big\{ H_0 \partial_0 X_0 + H_n \partial_0 n - \frac{N_{\mathrm{I}}}{2} \big[- H_0^2 + 12 H_n^2 + C^2 \big] \Big\} \tag{89}$$

where

$$H_n = \frac{\partial L_{GR+DE}}{\partial(\partial_0 n)}. \tag{90}$$

The Hamiltonian equations (A.4)–(A.16) are:

$$\begin{aligned} X_0' &= -H_0, && (91)\\ H_0' &= 0; && (92)\\ n' &= H_n, && (93)\\ H_n' &= 0; && (94)\\ H_0^2 &= 12H_n^2 + C^2 && (95) \end{aligned}$$

where $'$ denotes ∂_τ. The solution of the system is

$$\begin{aligned} X_0 &= -H_0(\tau - \tau_0), && (96)\\ H_0 &= \text{const}; && (97)\\ n &= H_n(\tau - \tau_0), && (98)\\ H_n &= \text{const}; && (99)\\ H_0^2 &= 12H_n^2 + C^2 && (100) \end{aligned}$$

where

$$\tau - \tau_0 = -\frac{1}{2H_0}\ln\left[1 + 2H_0\left(\eta_0 - \eta\right)\right] \;\; for \;\; H_0 \neq 0. \tag{101}$$

For this solution the matrix of Hubble parameters at point $\eta = \eta_0 - \frac{d}{c}$ (where η_0 is the present time, and $\frac{d}{c}$ is the distance to the object)

$$H_{ij} = \frac{1}{2}\frac{\partial_\eta {}^{(3)}g_{ij}}{{}^{(3)}g_{ij}}\Big|_{\eta=\eta_0-\frac{d}{c}} \tag{102}$$

is

$$H_{ij} = \begin{bmatrix} a_1 + 2b_1 & a_1 + a_1c_1 & 0 \\ a_1 + a_1c_1 & a_1 & 0 \\ 0 & 0 & a_1 \end{bmatrix}, \tag{103}$$

where

$$\begin{aligned} b_1 &= a_1c_1\frac{h^2}{h^2 + c_1^2}, && (104)\\ c_1 &= \frac{1}{\ln\left[1 + 2H_0\frac{d}{c}\right]}, && (105)\\ a_1 &= \frac{-2H_0}{1 + 2H_0\frac{d}{c}}, && (106)\\ h^2 &= \frac{H_n^2}{4H_0^2}. && (107) \end{aligned}$$

From (100) it follows that

$$h^2 < \frac{1}{4}. \tag{108}$$

The roots of the secular equation

$$\det\left[H_{ij} - \lambda\delta_{ij}\right] = \lambda^2 - \mathrm{tr}(H_{ij})\lambda + \det(H_{ij})$$

are

$$
\begin{aligned}
\lambda_0 &= a_1, && (109)\\
\lambda_\pm &= (a_1+b_1) \pm \sqrt{b_1^2 + a_1^2(1+c_1)^2}. && (110)
\end{aligned}
$$

From conditions:

$$
\begin{aligned}
\lambda_0 &> 0, && (111)\\
\lambda_\pm &> 0 && (112)
\end{aligned}
$$

one can obtain

$$(1+c_1)^2 < 1 - h^2, \qquad (113)$$

while the constraint

$$h^2 > 0 \qquad (114)$$

gives

$$b_1 < 0. \qquad (115)$$

If we know λ_0 and $\lambda_\pm$, then we can calculate a_1, b_1 and c_1. Their values are:

$$
\begin{aligned}
a_1 &= \lambda_0, && (116)\\
b_1 &= \frac{\lambda_+ + \lambda_-}{2} - \lambda_0, && (117)\\
c_1 &= -1 \pm \sqrt{\left(\frac{\lambda_+ - \lambda_-}{2\lambda_0}\right)^2 - \left(\frac{\lambda_+ + \lambda_-}{2\lambda_0} - 1\right)^2}. && (118)
\end{aligned}
$$

The $\pm$ sign in the last formula is connected with equation (110). The sign of c_1 is defined by the condition:

$$1 + c_1 > 0. \qquad (119)$$

The knowledge of a_1, b_1, c_1 allows us to calculate H_0, H_n, $\frac{d}{c}$ and dark matter C^2 from (104)–(107). During the calculations, values a, b and c taken from (116), (117) and (118) were used. In such a manner, we obtain:

$$
\begin{aligned}
H_0 &= -\frac{1}{2} a_1 e^{1/c_1}, && (120)\\
H_n^2 &= a_1^2 e^{2/c_1} \frac{b_1 c_1^2}{a_1 c_1 - b_1}, && (121)\\
C^2 &= \frac{1}{4} a_1^2 e^{2/c_1} \left(1 - 48 \frac{b_1 c_1^2}{a_1 c_1 - b_1}\right), && (122)\\
\frac{d}{c} &= \frac{e^{-1/c_1} - 1}{a_1}. && (123)
\end{aligned}
$$

The data for the local volume velocity field give the value of the Hubble parameter different for various directions. The three mutually perpendicular axes have the values (Karachentsev 2001, Karachentsev, Makarov , 2001)

$$(81 \pm 3)\ \mathrm{km\,s^{-1}Mpc^{-1}} : (62 \pm 3)\ \mathrm{km\,s^{-1}Mpc^{-1}} : (48 \pm 5)\ \mathrm{km\,s^{-1}Mpc^{-1}} \qquad (124)$$

respectively. The longest axis is directed toward the point with equatorial coordinates:

$$(\alpha = 13^h,\ \delta = -13.6^0) \qquad (125)$$

In the considered case, we assume the values of

$$81 \text{ km s}^{-1}\text{Mpc}^{-1} : 63 \text{ km s}^{-1}\text{Mpc}^{-1} : 44 \text{ km s}^{-1}\text{Mpc}^{-1}, \tag{126}$$

which are consistent with observation within the errors and satisfy the limit of $b < 0$. In such a way, our special case gives (Eqs. (116), (117), (118)) the following values of the parameters:

$$a_1 = 63, \tag{127}$$
$$b_1 = -0.5, \tag{128}$$
$$c_1 = -0.71. \tag{129}$$

This allows us to calculate the values of initial conditions

$$H_0 = -7.65, \tag{130}$$
$$H_n^2 = 1.33, \tag{131}$$
$$C^2 = 42.8. \tag{132}$$

In this way, we obtain the percentage of the dark energy contribution to total energy

$$\Omega_{\text{total}} = 1 = \Omega_{\text{C}} + \Omega_{\text{A}} \tag{133}$$

where

$$\Omega_{\text{C}} = \frac{\rho_C}{\rho_{\text{total}}} = \frac{C^2}{H_0^2}, \qquad \Omega_{\text{A}} = \frac{\rho_A}{\rho_{\text{total}}}. \tag{134}$$

We obtain

$$\Omega_{\text{C}} = 0.72, \tag{135}$$
$$\Omega_{\text{A}} = 0.28. \tag{136}$$

The direction of anisotropy is:

$$\alpha = 15^h, \tag{137}$$
$$\delta = 0^0. \tag{138}$$

5 Conclusions

We analyzed the status of homogeneity in cosmology. We consider the collective motion of the volume functional and we show that its equations averaged over the volume in exact GR completely coincide with analogical equations of the standard cosmology. In such a way we are able to show that the assumption of homogeneity it not a necessary condition for cosmology. We show also how observational data could be treated in order to expose the collective motion. The precise definition of the collective expansion of the volume allows us to define relative standards, as an alternative to the absolute standard of the standard cosmology. Considering the expansion of the volume means that all dimensions are changing too. So, the question arise what happen with our standard root? If it is expanding with the Universe, for the observation point of view it means that nothing expands. Instead of introduction of the Λ- term as possible explanation of the acceleration of the Universe we suggest to consider the relative standard of measurements. These standards expands exactly as the whole Universe. Such approach seems us more adequate considering the nature of the Λ-term.

Maxwell (1873) stated: "The most important aspect of any phenomenon from mathematical point of view is that of a measurable quantity". Accordingly, we have considered astrophysical data describing the anisotropic Universe, defining the frame of reference and the standards of observables. It is very important to emphasize the status and significance of conformal (relative) standard of observables. In the framework of the conformal cosmology, Supernova data and chemical abundance are described by the same equation of state when the density coincides with pressure ($p = \rho$) (Behnke et al. 2002), which corresponds to an inertial motion along the geodesic line in the coset of affine group $A_{(16)}(4)$ over the Lorentz one $L_{(6)}$ (Pervushin, Proskurin 2002). It is well known that anisotropic motion has also the same equation of state $p = \rho$, and it does not contradict the Supernova data. In the paper we formulated the problem in general terms. It has been shown that in the simplest example this approach is consistent with observations within their uncertainties. This means that conformal cosmology is able to describe the observables correctly. Nevertheless, it should be pointed out that the present-day accuracy of data, which moreover are restricted to the local volume ($d < 8Mpc$), does not allow us to use them for more complicated cases. The observables dealing with anisotropy have been used as a basis for the initial conditions of our problem. We determined the corresponding apex and the amount of dark energy considered now generally as Quintessence.

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Appendix A: Equations of motion

Let us discuss the complete system of equations obtained from the action (68):

$$\begin{aligned} S_{GR+DE} &= \int dx^0 \Big\{ H_0 \partial_0 X_0 + H_1 \partial_0 X_1 + H_2 \partial_0 X_2 + p_u \partial_0 u + p_v \partial_0 v + p_\nu \partial_0 \nu \\ &- \frac{N_{\rm I}}{2} \Big[-H_0^2 + H_1^2 + H_2^2 + \frac{p_u^2}{\Delta_+} + \frac{p_v^2}{\Delta_-} + 12 e^{-4\sqrt{3}X_2} p_\nu^2 + C^2 \Big] \Big\}. \end{aligned} \tag{A.1}$$

in the homogeneous approximation of metric (3)

$$(ds)^2 = g_{\mu\nu} dx^\mu dx^\nu = (N dx^0)^2 - {}^{(3)}g_{ij} \left(dx^i + N^i dx^0 \right) \left(dx^j + N^j dx^0 \right), \tag{A.2}$$

with ansatz (54)

$$\begin{aligned} N &= N(x^0), \\ N^i &= 0, \\ {}^{(3)}g_{ij} &= {}^{(3)}g_{ij}(x^0). \end{aligned} \tag{A.3}$$

We try to find the general solution of this set of equations:

$$X_0' = -H_0, \tag{A.4}$$
$$H_0' = 0; \tag{A.5}$$
$$X_1' = H_1, \tag{A.6}$$
$$H_1' = 0; \tag{A.7}$$
$$X_2' = H_2, \tag{A.8}$$
$$H_2' = 2\sqrt{3} \left[\left(\frac{p_u}{\Delta_+} \right)^2 - \left(\frac{p_v}{\Delta_-} \right)^2 \right] \frac{\nu^{-2} \exp(-4\sqrt{3}X_2)}{\left[1 + \nu^{-2} \exp(-4\sqrt{3}X_2)\right]^{3/2}} + 24\sqrt{3} p_\nu^2 \exp(-4\sqrt{3}X_2); \tag{A.9}$$
$$u' = \frac{p_u}{\Delta_+}, \tag{A.10}$$
$$p_u' = 0; \tag{A.11}$$
$$v' = \frac{p_v}{\Delta_-}, \tag{A.12}$$

$$
\begin{aligned}
p_v' &= 0; && \text{(A.13)}\\
\nu' &= 12p_\nu^2\exp(-4\sqrt{3}X_2), && \text{(A.14)}\\
p_\nu' &= \left[\left(\frac{p_u}{\Delta_+}\right)^2-\left(\frac{p_v}{\Delta_-}\right)^2\right]\frac{\nu^{-3}\exp(-4\sqrt{3}X_2)}{\left[1+\nu^{-2}\exp(-4\sqrt{3}X_2)\right]^{3/2}}; && \text{(A.15)}\\
H_0^2 &= H_1^2+H_2^2+\frac{p_u^2}{\Delta_+}+\frac{p_v^2}{\Delta_-}+12e^{-4\sqrt{3}X_2}p_\nu^2+C^2. && \text{(A.16)}\\
&+\\
n'^{(2)} &= \frac{u'+v'}{\sqrt{2a}}, && \text{(A.17)}\\
n'^{(3)} &= \frac{u'-v'}{\sqrt{2b}}; && \text{(A.18)}
\end{aligned}
$$

where $'$ denotes ∂_τ. Connection between η and τ is:

$$d\tau = N_{\rm I}d\eta. \tag{A.19}$$

For all solutions, we have that

$$H_0,\ H_1,\ p_u,\ p_v \tag{A.20}$$

are constant.

Integral (A.16) is a variation of action (68) with respect to $N_{\rm I}$. But if we take the derivative of (A.16) with respect to x^0/τ, then the result is equal to zero due to the equations of motion.

From (A.9), (A.14) and (A.15)

$$\left(H_2-2\sqrt{3}p_\nu\nu\right)'=0. \tag{A.21}$$

Thus

$$H_2=2\sqrt{3}p_\nu\nu+C_1, \tag{A.22}$$

where C_1 is an integration constant. After that, the three remaining independent equations are

$$
\left\{
\begin{aligned}
X_1' &= 2\sqrt{3}p_\nu\nu+C_1,\\
\nu' &= 12p_\nu\exp(-4\sqrt{3}X_2),\\
p_\nu' &= \left[\left(\frac{p_u}{\Delta_+}\right)^2-\left(\frac{p_v}{\Delta_-}\right)^2\right]\frac{\nu^{-3}\exp(-4\sqrt{3}X_2)}{\left[1+\nu^{-2}\exp(-4\sqrt{3}X_2)\right]^{3/2}}.
\end{aligned}
\right. \tag{A.23}
$$

Or, in the new variable,

$$
\begin{aligned}
d\tau' &= \exp(-4\sqrt{3}X_2)d\tau, && \text{(A.24)}\\
f(\tau) &= \exp(-4\sqrt{3}X_2) && \text{(A.25)}
\end{aligned}
$$

the system (A.23) can be rewritten as

$$
\left\{
\begin{aligned}
\dot f &= -24p_\nu\nu-4\sqrt{3}C_1,\\
\dot\nu &= 12p_\nu,\\
\dot p_\nu &= \left[\left(\frac{p_u}{\Delta_+}\right)^2-\left(\frac{p_v}{\Delta_-}\right)^2\right]\frac{\nu^{-3}}{[1+\nu^{-2}f]^{3/2}},
\end{aligned}
\right. \tag{A.26}
$$

where dots denote the derivatives with respect to τ' and

$$\Delta\pm = 1\pm\frac{1}{\sqrt{1+f/\nu^2}}. \tag{A.27}$$

Appendix B: Diagonal metric

In this case, β_i is an explicit function of X_0, X_1 and X_2; $\nu = 0$, $n^{(2)} = n^{(3)} = 0$; i.e., metric (53) is

$$^{(3)}g_{ij} = diag\left(\; e^{2\beta_1}, \; e^{2\beta_2}, \; e^{2\beta_3}\right), \tag{B.1}$$

where β_i are defined in (55), (56) and (57). The Lagrangian is

$$L_{GR+DE} = \frac{1}{2N_{\rm I}}(-X_0'^2 + X_1'^2 + X_2'^2) - \frac{N_{\rm I}}{2}C^2; \tag{B.2}$$

$$H_{GR+DE} = \frac{N_{\rm I}}{2}(-H_0^2 + H_1^2 + H_2^2 + C^2). \tag{B.3}$$

Then X_0, X_1 and X_2 are linear functions of τ. The matrix of Hubble parameters is

$$\begin{aligned}(H_{ij}) &= \frac{1}{2}(\partial_t {}^{(3)}g_{ij}/{}^{(3)}g_{ij}) \\ &= diag\left(\; H_0 - 2H_1, \; H_0 + H_1 - \sqrt{3}H_2, \; H_0 + H_1 + \sqrt{3}H_2\right)\left(\frac{d\tau}{d\eta}\right) \\ &= diag\left(\; H_0 - 2H_1, \; H_0 + H_1 - \sqrt{3}H_2, \; H_0 + H_1 + \sqrt{3}H_2\right)\left(\frac{d\tau}{d\eta}\right).\end{aligned} \tag{B.4}$$

The eigenvalues of the matrix are the Hubble parameters, value of which are taken from observations. Namely:

$$(81 \pm 3) : (63 \pm 3) : (48 \pm 5), \tag{B.5}$$

so

$$h1 = 81 \pm 3, \quad h2 = 63 \pm 3, \quad h3 = 48 \pm 5.$$

From (B.4) and (B.5)

$$H_0 - 2H_1 = h1, \tag{B.6}$$

$$H_0 + H_1 - \sqrt{3}H_2 = h2, \tag{B.7}$$

$$H_0 + H_1 + \sqrt{3}H_2 = h3, \tag{B.8}$$

(B.9)

thus

$$h2_+ = \frac{h1 + h3}{2} \quad \& \quad h2_- = \frac{h1 - h3}{2\sqrt{3}}, \quad H_0 = \frac{h1 + h2 + h3}{3}. \tag{B.10}$$

Condition (B.5) satisfies the solution $h2_+$.

References

Arnowitt, R., Deser, S., Misner, C.W. 1959. *Phys. Rev.* **116** 1322

Arnowitt, R., Deser, S., Misner, C.W. 1960. *Phys. Rev.* **117** 1595

Arnowitt, R., Deser, S., Misner, C.W. 1961. *Phys. Rev.* **122** 997

Barbashov, B.M., Pervushin, V.N. 2001. Teor.Mat.Fiz. **127**, 91

Barbashov, B.M., Pervushin, V.N. 2001. Theor. Math. Phys. **127**, 483

Barbashov, B.M., Pervushin, V.N., Pawlowski, M. 2001. Phys. Particles and Nuclei **32**, 546

Behnke, D., Blaschke, D., Pervushin, V.N., Proskurin, D., 2002. Phys.Lett. B **530** 20-26, gr-qc/0102039

Blaschke, D., Pervushin, V., Proskurin, D., Vinitsky, S., Gusev, A. 2001. gr-qc/0103114

Borisov, A.B., Ogievetsky, V.I. 1974. Teor. Mat. Fiz., **21**, 329

Cartan E. 1946. *Lecons sur la geometric des espaces de Riemann* (Paris: Gauthier-Villars)

Dirac, P. A. M. 1958. *Proc.Roy.Soc.* **A 246** 333

Dirac, P. A. M. 1959. *Phys. Rev.* **114** 924

Einstein A., de-Sitter W. 1932. Proc. of Nat. Acad. of Scien., **18**, N3, 213

Friedmann A.A. 1922. Z. für Phys, **10**, 377

Friedmann A.A. 1924. Z. für Phys, **21**, 306

Hawking, S.W., Ellis, G.F.R. 1973. The large Scale Structure of Space–Time, Cambridge Univ. Press, Cambridge

Isaev G.V., Pervushin V.N., Pushkin S.V. 1979. J. Phys. A: Math. Gen., **12**, 1499

Karachentsev I.D., 2001. Usp.Fiz.Nauk 171, 860

Karachentsev I.D., Makarov D.I., 2001. Astrofizika 44, 5

Kazakov, D.I., Pervushin, V.N., Pushkin, S.V. 1977. Teor. Mat. Fiz., **31**, 169

Kuchar, K. 1972. *J. Math. Phys.* **13**, 768

Lemaitre G. 1931. Monthly Notice of Royal Soc, **91**, N5, 490

Lemaitre G. 1927. Ann. de la Soc. Scien. de Bruxelles, **A 47**, 49

Lichnerowicz, A. 1944. Journ. Math. Pures and Appl. B **37**, 23

Linde A.D. 1990. " Elementary Particle Physics and Inflation Cosmology" (in Russian), Nauka, Moscow.

Maxvell,J.C. 1873. "Treatise on Electrity and Magnetism" (Oxford)

Misner, C. 1969. *Phys. Rev.* **186** 1319

Narlikar, J.V. 1989. Space Sci. Rev. **50**, 523

Pawlowski, M., Pervushin, V.N. 2001. Int. J. Mod. Phys. **16**, 1715; [hep-th/0006116]

Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R.A, Nugent, P., Castro, P.G., Deustua, S., Fabbro, S., Goobar, A., Groom, D.E., Hook, I.M., Kim, A.G., Kim, M.Y., Lee, J.C., Nunes, N.J., Pain, R., Pennypacker, C.R., Quimby, R., Lidman, C., Ellis, R.S., Irwin, M., McMahon, R.G., Ruiz-Lapuente, P., Walton, N., Schaefer, B., Boyle, B.J., Filippenko, A.V., Matheson, T., Fruchter, A.S., Panagia, N., Newberg, H.J.M., Couch, W.J 1999. Astrophys. J. **517**, 565

Pervushin, V.N. 1975. Teor. Mat. Fiz., **22**, 201

Pervushin, V.N. 1976. Teor. Mat. Fiz., **27**, 16

Pervushin, V.N., Proskurin, D.V., 2001. Talk at the V International Conference on Cosmoparticle Physics (Cosmion-2001) dedicated to 80-th Anniversary of Andrei D. Sakharov (21-30 May 2001, Moscow-St.Peterburg, Russia), gr-qc/0106006.

Pervushin, V.N., Proskurin, D.V. 2001. Gravitation & Cosmology **7**, 89

Pervushin, V.N., Proskurin, D.V. 2002. [gr-qc/0205084]

Riess, A.G., Filippenko, A.V., Challis, P., Clocchiattia, A., Diercks, A., Garnavich, P.M., Gilliland, R.L., Hogan, C.J., Jha, S., Kirshner, R.P., Leibundgut, B., Phillips, M.M., Reiss, D., Schmidt, B.P., Schommer, R.A., Smith, R.Ch., Spyromilio, J., Stubbs, Ch., Suntzeff, N.B., Tonry, J. 1998. Astron. J. **116**, 1009

Riess, A.G., Nugent, P.E., Schmidt, B.P., Tonry, J., Dickinson, M., Gilliland, R.L., Thompson, R.I., Budavari, T., Casertano, S., Evans, A.S., Filippenko, A.V., Livio, M., Sanders, D.B., Shapley, A.E., Spinrad, H., Steidel, C.C., Stern, D., Surace, J., Veilleux, S. 2001. Astrophys.J. 560 49-71

Robertson H.P. 1933. Rev Mod Phys, **5**, 62

Walker A.G. 1933. Monthly Notice of Royal Soc, **94**, N2, 154

Walker A.G. 1935. Monthly Notice of Royal Soc, **95**, N3, 263

Weinberg, S. 1977. *First Three Minutes. A modern View of the Origin of the Universe* (Basic Books, Inc.,

Publishers, New-York)
Vladimirov, Yu.S. 1982. *Frame of references in theory of gravitation* (Moscow, Eneroizdat, in Russian)
Volkov, D.V. 1973. Particles and Nuclei,**4**, 3
York, J.W. (Jr.) 1971. *Phys. Rev. Lett.* **26**, 1658
Zel'manov, A.L. 1976. *Doklady AN SSSR* **227**, 78 (in Russian)