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TWO COMMENTS TO UTILIZATION OF STRUCTURE FUNCTION APPROACH IN DEEP INELASTIC SCATTERING EXPERIMENTS

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Comment 1

It's now accepted the DIS cross section with taken into account the radiative corrections (RC) have form of a cross section of Drell-Yan process (see [1] and references therein):

$$\frac{d^2\sigma(p_1,p_2)}{dQ^2dy} = \int_{z_{1m}}^1 dz_1 \int_{z_{2m}}^1 dz_2 \frac{1}{z_2^2}\mathcal{D}(z_1,t)\mathcal{D}(z_2,t)\frac{d^2\tilde{\sigma}^{hard}(z_1p_1,p_2/z_2)}{d\tilde{Q}^2d\tilde{y}}\left(1+\frac{\alpha}{\pi}K\right), \quad (1)$$

$$t = \frac{\alpha}{\pi}L, \quad L = \ln(Q^2/m_e^2), \quad p_1^2 = p_2^2 = m_e^2, \quad Q^2 = -(p_1-p_2)^2 \gg m_e^2, \quad (2)$$

$$y = \frac{2p_1q}{2p_1P}, \quad q = p_1 - p_2,$$

with scaling parameters of the hard cross section:

$$\tilde{Q}^2 = \frac{z_1}{z_2}Q^2\,,\ \tilde{y} = 1 - \frac{1-y}{z_1z_2}\,,\ z_{1m} = \frac{1+z_{th}-y}{1-xy}\,,$$

$$z_{2m} = \frac{1-y+xyz_1}{z_1-z_{th}}\,,\ z_{th} = \frac{2m_\pi M}{2p_1P} \ll 1\,,\ P^2 = M^2.$$

Calculating the integral in (1) on parton's energy fraction z_1 it is necessary to keep in mind two enhancing tendencies. The main is related with the rapid decreasing of hard cross section with increasing of Q^2:

$$\frac{\mathrm{d}\tilde{\sigma}^{hard}}{\mathrm{d}Q^2\mathrm{d}y} = \left(\frac{1}{1-\Pi(Q^2)}\right)^2\frac{\mathrm{d}\sigma_B^{hard}}{\mathrm{d}Q^2\mathrm{d}y}, \quad (3)$$

$$\frac{d\sigma_B^{hard}}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{Q^4y}[(1-y-x^2y^2\frac{M^2}{Q^2})F_2(x,Q^2) + xy^2F_1(x,Q^2)], \quad (4)$$

with $\Pi(Q^2)$ is the polarization operator of virtual photon. The explicit value of K-factor can be found in paper of A. Afanasev et al. [1].

Another one is related with rather slow tendency of structure functions:

$$\mathcal{D}(z_1,L) \approx \varepsilon(1-z_1)^{-1+\varepsilon} \approx \delta(1-z_1), \quad \varepsilon = \frac{2\alpha}{\pi}L. \quad (5)$$

These tendencies are struggling. For the aim to extract the main contribution arising from the first mechanism (known in colliding beams experiments as a "returning to resonance" one).

Let put the right hand side (r.h.s.) of (1) in form:

$$\int_{z_{1m}}^1 \frac{dz_1}{z_1^2}\Psi(z_1) = \left(\frac{1}{z_{1m}}-1\right)\Psi(z_{1m}) + \int_{z_{1m}}^1 \frac{dz_1}{z_1}(1-z_1)\frac{d}{dz_1}\Psi(z_1)\,. \quad (6)$$

The second term in r.h.s. of (4) is much smaller than the first one at $z_{1m} \ll 1$. The enhancement tendencies in the second term integrand now become of the same rate: $\varepsilon z_1^{-1}(1-z_1)^{-1+\varepsilon}$.

When performing z_2 integration (suppressed here) we have only one enhancement tendency caused by structure function $\mathcal{D}(z_2,L) \sim \delta(1-z_2)$.We will not touch it here. The expression (4) provide the application of iteration procedure: using subsequently Ψ as an experimental data.

Comment 2

It's widely believed that the kinematical region $1-y \ll 1$ in DIS experiments cannot be described due to huge RC (exceeding 100%) of lowest order [3]. This fact was the reason why experimental data at $y > 0.8$ as a rule was excluded in analysis of data. We argue that all orders of perturbation theory (PT) are relevant here and must be taken into account.

The renormalization group approach is modified in such a way to include Sudakov-type suppression formfactor.

We will consider here the experimental set-up with no emission of hard photons along initial lepton.

For this aim let consider RC of two lowest order of PT. The emission of additional soft pions and soft pairs of the same order of energy as the one ε_2 of scattered lepton do not exceeding $\Delta\varepsilon \ll \varepsilon$ becomes relevant:

$$\Delta\varepsilon \sim \varepsilon_2 = \varepsilon(1-y) \ll \varepsilon_1 = \varepsilon. \tag{7}$$

The cross section with RC taken into account can be put in form :

$$\frac{d\sigma}{d\sigma_B} = 1+\delta,\ \delta = \frac{\alpha}{\pi}\Delta^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \Delta^{(2)} + ... \tag{8}$$

Lowest order RC are

$$\Delta^{(1)} = (l_t - 1)\left(\ln\frac{\Delta\varepsilon}{\varepsilon_1} + \ln\frac{\Delta\varepsilon}{\varepsilon_2}\right) + \frac{3}{2}l_t - \frac{1}{2}\ln^2(1-y) - \frac{\pi^2}{6} - 2 + Li_2\left(\frac{1+c}{2}\right), \tag{9}$$

with

$$-t = 2\varepsilon^2(1-y)(1-c) \gg m_e^2,\ l_t = \ln\left(\frac{-t}{m_e^2}\right),\ c = \cos\theta\ , \tag{10}$$

where $\theta = \widehat{\vec{p}_1\vec{p}_2}$ and ε_2 are the scattering angle and the energy of the scattered lepton in the laboratory frame. The reasons mentioned above allow us to put:

$$\ln\frac{\Delta\epsilon}{\epsilon_1} + \ln\frac{\Delta\epsilon}{\epsilon_2} = \ln(1-y). \tag{11}$$

We note that in this point we have some deviation from the known Δ-part of kernel of evolution equation

$$P^{(1)}(x) = \lim_{\Delta\to 0}\left\{\delta(1-x)P_\Delta^{(1)} + \theta(1-x-\Delta)\frac{1+x^2}{1-x}\right\} \tag{12}$$

Here in the our case θ-part do not work, $\Delta = 1-y$, (see the term containing l_t in (7)):

$$P_\Delta^{(1)} = 2\ln\Delta + \frac{3}{2} \to \left(2\ln(1-y) + \frac{3}{2}\right) - \ln(1-y). \tag{13}$$

In the second order of PT the emission of two soft photons and soft pair (with total energy not exceeding $\Delta\varepsilon$) as well as a single photon emission with 1-loop RC and, finally the 2-loop virtual corrections must be taken into account: $\Delta^{(2)} = \delta_{\gamma\gamma} + \delta_{sp}$. We will not consider here the contribution from emission of real and virtual pairs It can be taken into account by replacing the coupling constant by the moving one.

Contribution to RC from virtual and real photons emission have a form:

$$\delta_{\gamma\gamma} = \frac{1}{2}(\Delta^{(1)})^2 - \frac{\pi^2}{3}(l_t - 1)^2 + \frac{3}{2}l_t\left(2 + \frac{\pi^2}{6} - Li_2\left(\frac{1+c}{2}\right)\right) + O(1). \tag{14}$$

This result agree with predictions of renormalization groups (RG) [2] at $y = 0$ and contains, besides the term of type $\ln^2(1-y)$, $l_t \ln(1-y)$, which become relevant in the limit $y \to 1$.

Let discuss this points more closely. We suppose no hard photon emission by the initial lepton which can provide the "returning to resonance" mechanism. Really this mechanism for the case $\epsilon_2/\epsilon = 1 - y << 1$ will correspond to very small transversal momentum squared $Q_1^2 \sim \epsilon^2(1-y)^2 << Q^2$.

Let now average DIS cross section on small interval $\tilde{Q}^2 \sim Q^2$ introducing the additional integration in the right part of formula (1):

$$\int \mathrm{d}\tilde{Q}^2 \delta((z_1 x Q^2/z_2) - \tilde{Q}^2), x = 1 - (\Delta\epsilon/\epsilon). \tag{15}$$

Small variations of transfer momentum arises due to RC-emission of soft real and virtual partons (photons and leptons). Using the flatness of hard cross section in this region we obtain for the ratio of DIS cross sections with and without RC:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\sigma_B} = F(x,t) = \int\int D(z_1,t)D(z_2,t)\mathrm{d}z_1\mathrm{d}z_2\theta(xz_1 - z_2). \tag{16}$$

Using the differential evolution equations for nonsinglet structure functions $D(x,t)$

$$\frac{\partial D}{\partial t} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{\mathrm{d}y}{y}P(\frac{x}{y})D(y,t), D(y,0) = \delta(1-y).$$

One can obtain a differential equation for F

$$\frac{\partial F}{\partial t} = \frac{\alpha(t)}{\pi}\int_x^1 \mathrm{d}zP(\frac{x}{z})F(z,t), F(x,0) = 1. \tag{17}$$

This equation was solved in paper[2]:

$$F(x,t) = (\ln\frac{1}{x})^{2\chi}\frac{1}{\Gamma(1+2\chi)}e^{\chi(3/2-2C_E)}, \chi = -3\ ln(1 - \frac{1}{3}t). \tag{18}$$

Terms containing $\ln(1-y)$ are not taken into account in evolution procedure.We argue here that there is a reason to take them into account as a general factor which can be obtained from the known factor of Yienie,Frautchi and Suura [4],with replacement of logarithm of ratio of photon mass to lepton mass by $\ln\Delta, \Delta = \Delta\epsilon/\epsilon$ with accordance with Boch-Nordsick theorem.

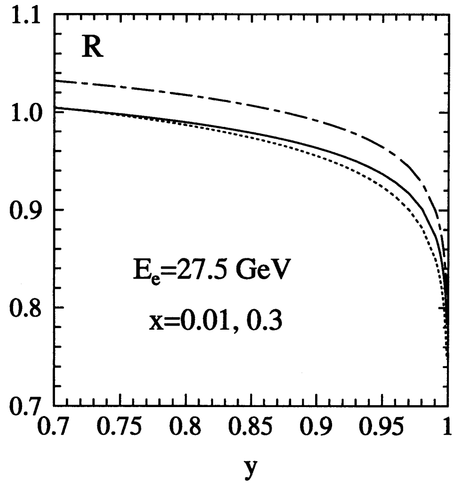

Figure 1: The ration $R = \mathrm{d}\sigma/\mathrm{d}\tilde{\sigma}_B$ for $x = 0.01$ (solid line) and $x = 0.3$ (dashed line) versus y for $0.7 \leq y \leq 0.999$. Dot-dashed line correspond to the ration $R = \mathrm{d}\sigma/\mathrm{d}\sigma_B$ with taken into account the polarization of vacuum for $x = 0.01$.

Replacing $\ln(1/x) = 1 - y$ we obtain for DIS cross section :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tilde{\sigma}_B}\bigg|_{y\to 1} = R(1 + \frac{\alpha}{\pi}K), \quad \mathrm{d}\tilde{\sigma}_B = \frac{\mathrm{d}\sigma_B}{(1 - \Pi(Q^2))^2}, \tag{19}$$

$$R = \frac{(1-y)^{2\chi}}{\Gamma(1+2\chi)} \mathrm{e}^{(3/2-2C_E)\chi} \mathrm{e}^{-\frac{\alpha}{2\pi}(\ln^2(1-y)+2l_t\ln(1-y))},$$

$$|K| \sim 1, \quad \chi = -3\ln\left(1 - \frac{\alpha}{3\pi}l_t\right) = \frac{\alpha}{\pi}l_t + \frac{\alpha^2}{6\pi^2}l_t^2 + \ldots.$$

with $C_E = 0.577$ is Euler constant and $\mathrm{d}\sigma_B$ is the DIS cross-section in Born approximation. One can be convinced with the agreement (18) with the results of lowest order of calculation (8,13) up to non-leading terms, which are parameterized in form of K-factor. Formula (18) provides for $|K| \sim 1$ the accuracy on the level of 1%. The behaviour of quantity $R(y, x)$ for different values of Bjorken parameter x is illustrated in fig.1.

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Два комментария к использованию метода структурных функций в экспериментах по глубоконеупругому рассеянию

Механизм «возвращения на резонанс» может быть использован для расчета радиационных поправок к сечениям глубоконеупругого рассеяния в рамках модели Дрелла–Яна. Предложена процедура итераций.

Кинематическая область $y \to 1$ может быть описана в рамках представления сечений в форме Дрелла–Яна. Большая величина радиационных поправок низшего порядка отражает эффект формфактора Судакова. Этот эффект может быть учтен в высших порядках теории возмущений. На основе расчетов первых двух порядков теории возмущений сконструировано сечение, удовлетворяющее уравнениям ренормгруппы и содержащее формфактор Судакова.

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Two Comments to Utilization of Structure Function Approach in Deep Inelastic Scattering Experiments

The «returning to resonance» mechanism can be used to obtain the simple procedure of taking radiative corrections (RC) to deep inelastic scattering (DIS) cross sections into account in the framework of Drell–Yan picture. Iteration procedure is proposed.

Kinematical region $y \to 1$ can be described in the framework of Drell–Yan picture using the structure function approach. The large RC in the lowest order reflect the Sudakov form factor suppression, which can be taken into account in all orders of perturbation theory. Based on explicit calculation in two lowest orders of perturbation theory we construct the cross section in $y \to 1$ region obeying renormalization group equations and including the Sudakov-like form factor suppression.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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