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SELF-SIMILARITY OF HIGH-p_T HADRON PRODUCTION IN $\pi^- p$ AND $\pi^- A$ COLLISIONS

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1 Introduction

Particle production with high transverse momenta is traditionally connected with local character of hadron interactions. Locality of the interaction is expressed in terms of the hadron constituents. Scaling features of high-p_T hadron spectra reflects self-similarity of constituent interactions. Therefore, search for scaling regularities in high energy collisions of hadrons and nuclei is of physical interest. Up to this date, the investigation of hadron properties in the high energy collisions has revealed widely known scaling laws. From the most popular and famous, let us mention the Feynman scaling [1] observed in inclusive particle production, the Bjorken scaling [2] in deep inelastic scattering (DIS), y-scaling [3] valid in DIS on nuclei, limiting fragmentation established for nuclei [4], scaling behaviour of the cumulative particle production [5, 6, 7], the Koba-Nielsen-Olesen (KNO) scaling [8], and others. Another expression of self-similarity in hadronic interactions are described by quark counting rules [9].

One of the methods to study the properties of particle structure, constituent interactions and particle formation is z-scaling [10]. The scaling was observed in proton-(anti)proton and proton-nucleus high energy collisions. In this paper we develope the concept of the scaling for $\pi^- p$ and $\pi^- A$ collisions.

2 Basic principles of z-scaling

The idea of z-scaling is based on the principles of locality, self-similarity and fractality of hadronic interactions. The properties are assumed to be revealed in high energy collisions of hadrons and nuclei. Their most pronounced manifestation is expected in the inclusive production of particles with large transverse momenta.

2.1 Locality principle

Following the ideas suggested in Ref. [6], we assume that gross features of the inclusive particle distributions for the reaction

$$M_1 + M_2 \to m_1 + X \tag{1}$$

can be described in terms of the constituent sub-process

$$(x_1M_1) + (x_2M_2) \to m_1 + (x_1M_1 + x_2M_2 + m_2) \tag{2}$$

satisfying to the condition

$$(x_1P_1 + x_2P_2 - p)^2 = (x_1M_1 + x_2M_2 + m_2)^2. \tag{3}$$

The assumption is expression of locality of the hadronic interactions at constituent level. The x_1 and x_2 are fractions of the incoming four-momenta P_1 and P_2 of the colliding objects with the masses M_1 and M_2. The p is four-momentum of the inclusive particle m_1. The parameter m_2 is introduced to satisfy the internal conservation laws (for isospin, baryon number, strangeness, and so on).

2.2 Self-similarity principle

Self-similarity is a scale-invariant property connected with dropping of certain dimensional quantities out of physical picture of the interaction. It means that for description of physical processes dimensionless quantities are used. One of the approaches to description of production cross sections complying with this requirement is the z-scaling. The scaling function $\psi(z)$ depends in a self-similar manner on a single variable z. The scaling function is expressed via the invariant differential cross section $Ed^3\sigma/dp^3$ as follows

$$\psi(z) = -\frac{\pi s}{\rho\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{4}$$

Here s is the collision center-of-mass energy squared, σ_{in} is the inelastic cross section, J is the corresponding Jacobian, and $\rho = dN/d\eta$ is the particle multiplicity density. The function $\psi(z)$ is normalized as

$$\int_{z_{min}}^{\infty} \psi(z)dz = 1. \tag{5}$$

The relation allows us to give the physical meaning of the scaling function $\psi(z)$ as a probability density to form a particle with the corresponding value of the variable z.

2.3 Fractality principle

Principle of fractality states that variables used in the description of the process diverge in terms of the resolution [11, 12]. This property is characteristic for the scaling variable

$$z = z_0\Omega^{-1}, \tag{6}$$

where

$$\Omega(x_1, x_2) = (1 - x_1)^{\delta_1}(1 - x_2)^{\delta_2}. \tag{7}$$

The variable z has character of a fractal measure. For a given production process (1), its finite part z_0 is the ratio of the transverse energy released in the binary collision of constituents (2) and the average multiplicity density $dN/d\eta|_{\eta=0}$. The divergent part Ω^{-1} describes the resolution at which the collision of the constituents can be singled out of this process. The $\Omega(x_1, x_2)$ represents relative number of all initial configurations containing the constituents which carry fractions x_1 and x_2 of the incoming momenta. The δ_1 and δ_2 are the anomalous fractal dimensions of the colliding objects (hadrons or nuclei). The momentum fractions x_1 and x_2 are determined in a way to minimize the resolution $\Omega^{-1}(x_1, x_2)$ of the fractal measure z with respect to all possible sub-processes (2) subjected to the condition (3). The variable z was interpreted [10] as a particle formation length.

3 Z-scaling in π^-p collisions

In this section we study the properties of data z-presentation for hadron production in π^-p collisions. One of the ingredients in the definition of the variable z is the particle multiplicity density $\rho(s) = dN/d\eta|_{\eta=0}$. We have simulated the multiplicity density [13] using the PYTHIA [14] in the energy range $\sqrt{s} = 10 - 200$ GeV. The generated values were parameterized in the form $\rho(s) = as^b$, where $a = 0.59 \pm 0.08$, $b = 0.126 \pm 0.017$. The energy dependence of $\rho(s)$ in π^-p collisions is found to be similar as for the experimental measured multiplicity density in the nucleon-nucleon interactions. Therefore we use the experimental values in our analysis.

The second ingredient in the variable z is the anomalous fractal dimension of the colliding objects. For the nucleon-nucleon collisions $\delta_1 = \delta_2 \equiv \delta_N$. The value of δ_N was determined in Ref. [10]. For pion-nucleon collisions, we deal with the asymmetric case $\delta_1 \equiv \delta_\pi \neq \delta_N$.

In our analysis we verify the hypothesis of the energy independence of data z-presentation for hadron production in $\pi^- p$ collisions to determine δ_π. We use the experimental data on cross sections for $\pi^{\pm,0}, K^{\pm}$, and $\bar{p}$ hadrons produced in $\pi^- p$ collisions obtained in Refs. [15, 16, 17, 18]. The measurements were made at pion momenta $p_{lab} = 40, 200$, and 300 GeV/c over the high transverse momentum range $0.8 < p_T < 10.$ GeV/c. Note that the data demonstrate strong energy dependence of cross sections as function of the transverse momentum. The difference between the hadron yields increases with p_T. The energy dependence of the cross sections is contrasted with z-presentation of the data obtained for $\delta_\pi = 0.1$. The result confirms the hypothesis of existence of the energy independence of the z-scaling in pion-nucleon interactions. The results are illustrated in Figs. 1-3.

Nevertheless, we would like to note that last points of the spectrum for the incident pion momentum $p_{lab} = 40$ GeV/c show some abundance of the hadron yields. From our point of view it means existence of considerable systematic errors of measurements at high transverse momentum in the $\pi^- p$ collisions at this energy [18]. We do not observe such anomalies in pp interactions (Fig. 4).

4 Z-scaling in $\pi^- A$ collisions

Using the concept of z-scaling, we extend our study to hadron production in $\pi^- A$ collisions. In this case we specify the scaling variable in analogy with its construction for the elementary pion-nucleon interaction. We determine the scale of the variable z in terms of the average multiplicity density of particles produced in the corresponding pion-nucleon interaction. The anomalous fractal dimensions of nuclei δ_A are expressed via the nucleon fractal dimension δ_N as follows [10]

$$\delta_A = A \cdot \delta_N. \tag{8}$$

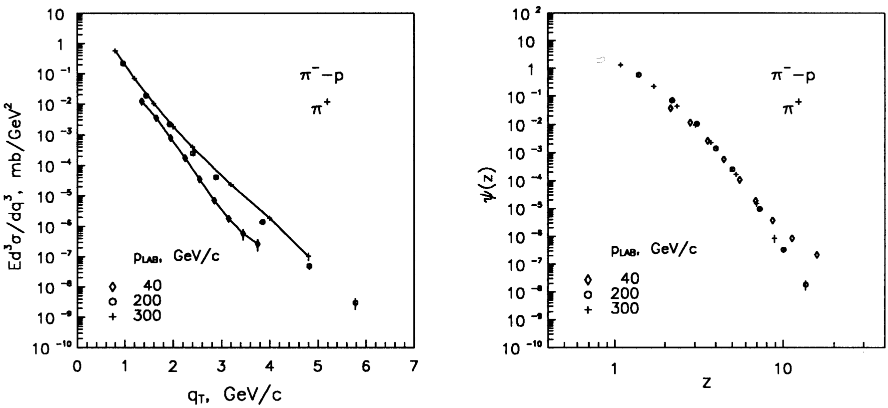
The value of pion δ_π is used the same as determined from the pion-nucleon interactions.

The experimental data on cross sections [15, 19, 20] have been analysed. The data are shown as function of p_T and z in Figs. 5-7. Z-presentation of the data manifests clear energy independence in the whole region of the transverse momentum considered. The scaling functions for different nuclei Be, Cu, and W are within the errors the same. The result is a confirmation of validity of the z-scaling in particle production in pion-nucleus collisions. The similar properties are observed for hadron production in proton-nucleus collisions, as well (Fig.8).

As seen from Fig.6, the scaling function manifests a power behaviour in the region of large values of z

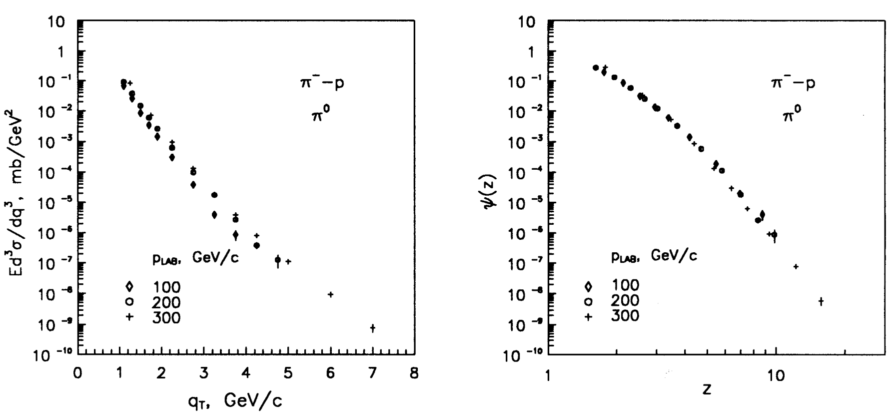
$$\psi(z) \sim z^{-\beta}. \tag{9}$$

From our point of view, such asymptotic regime reflects fractal character of particle formation. The fractality is connected with definition of the scaling variable z which has character of a fractal measure. The measure tends to infinity as the resolution increases and does not violate the power law for any value of z.



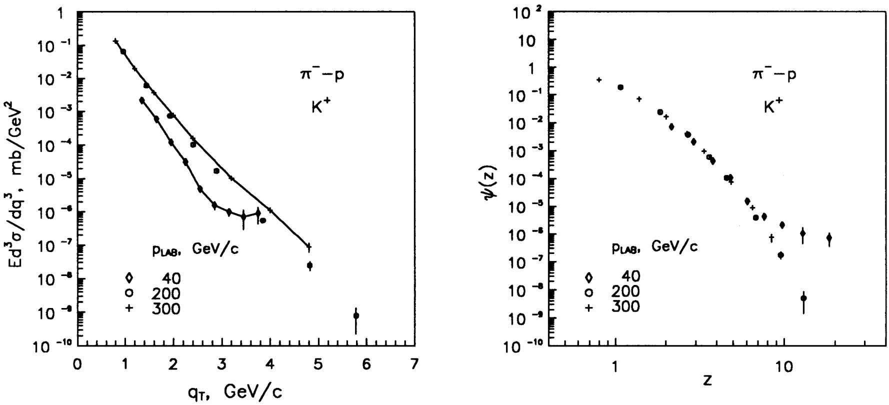
a) b)

Figure 1. (a) Dependence of the inclusive cross section of π^+-meson production on transverse momentum q_T at $p_{lab} = 40, 200$, and 300 GeV/c and $\theta_{cm}^{\pi p} \simeq 90^0$ in $\pi^- p$ collisions. Experimental data are taken from [15, 18]. (b) The corresponding scaling function $\psi(z)$.



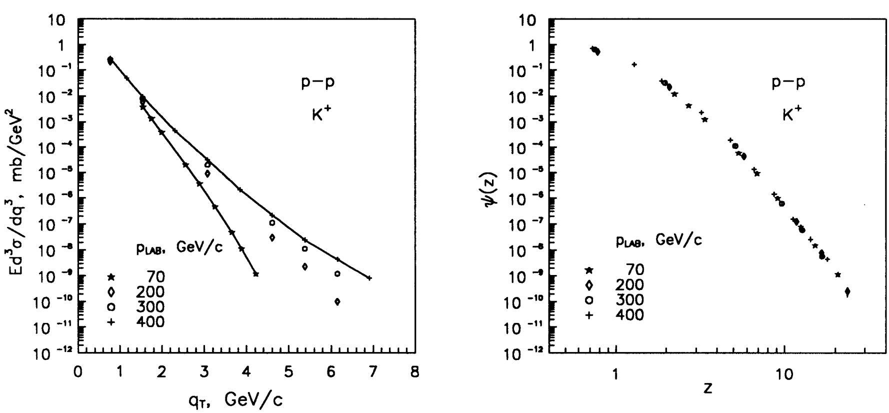
a) b)

Figure 2. (a) Dependence of the inclusive cross section of π^0-meson production on transverse momentum q_T at $p_{lab} = 100, 200$, and 300 GeV/c and $\theta_{cm}^{\pi p} \simeq 90^0$ in $\pi^- p$ collisions. Experimental data are taken from [16, 17]. (b) The corresponding scaling function $\psi(z)$.



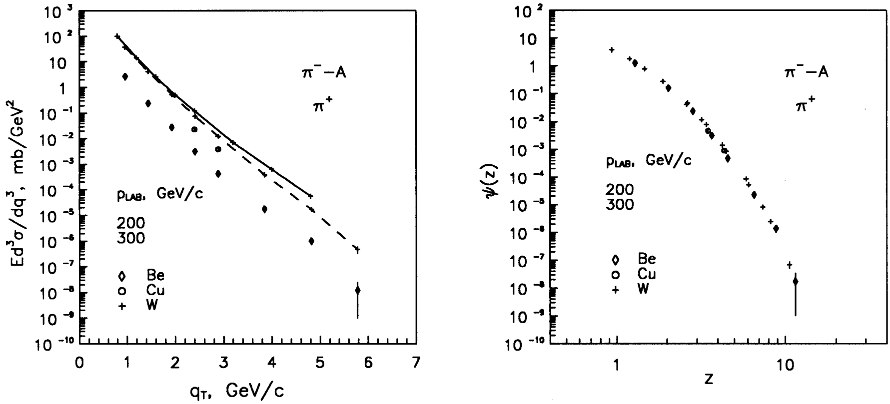
a) b)

Figure 3. (a) Dependence of the inclusive cross section of K^+-meson production on transverse momentum q_T at $p_{lab} = 40, 200$, and 300 GeV/c and $\theta_{cm}^{\pi p} \simeq 90^0$ in $\pi^- p$ collisions. Experimental data are taken from [15, 18]. (b) The corresponding scaling function $\psi(z)$.



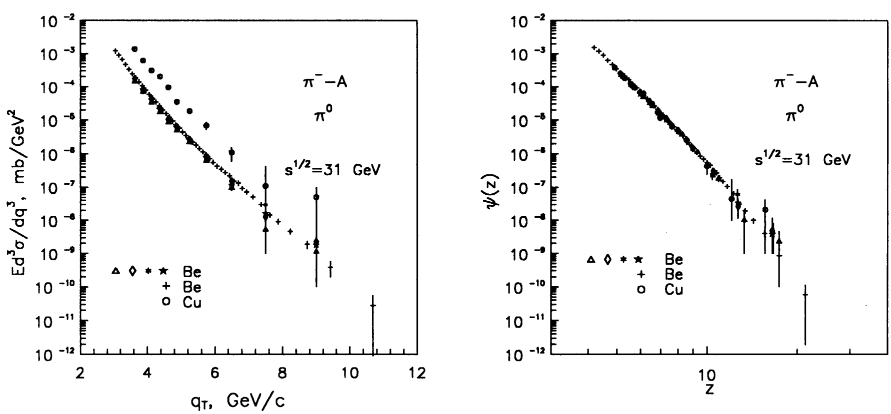
a) b)

Figure 4. (a) Dependence of the inclusive cross section of K^+-meson production on transverse momentum q_T at $p_{lab} = 70, 200, 300$, and 400 GeV/c and $\theta_{cm}^{pp} \simeq 90^0$ in pp collisions. Experimental data are taken from [21, 22]. The solid lines are obtained by fitting of the data at $p_{lab} = 70$ and 400 GeV/c. (b) The corresponding scaling function $\psi(z)$.



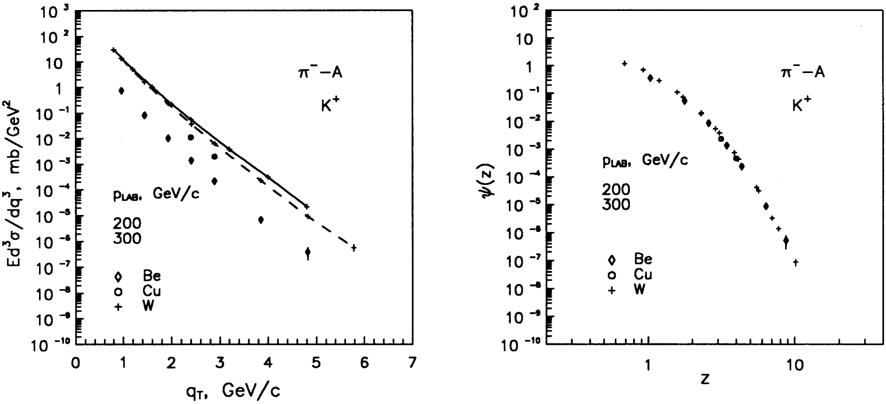
a) b)

Figure 5. (a) Dependence of the inclusive cross section of π^+-meson production on transverse momentum q_T in $\pi^- A$ collisions at $p_{lab} = 200$ and 300 GeV/c. Experimental data are taken from [15]. (b) The corresponding scaling function $\psi(z)$.



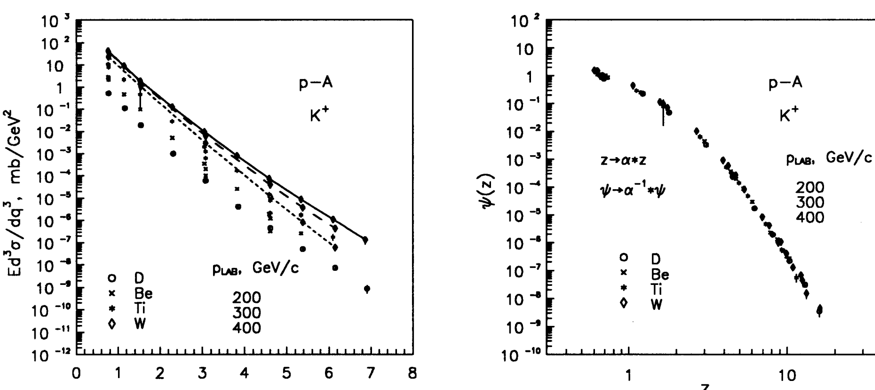
a) b)

Figure 6. (a) Dependence of the inclusive cross section of π^0-meson production on transverse momentum q_T in $\pi^- A$ collisions at $\sqrt{s} = 31$ GeV. Experimental data are taken from [19, 20]. (b) The corresponding scaling function $\psi(z)$.



a) b)

Figure 7. (a) Dependence of the inclusive cross section of K^+-meson production on transverse momentum q_T in $\pi^- A$ collisions at p_{lab} = 200 and 300 GeV/c. Experimental data are taken from [15]. (b) The corresponding scaling function $\psi(z)$.



a) b)

Figure 8. (a) Dependence of the inclusive cross section of K^+-meson production on transverse momentum q_T in pA collisions at p_{lab} = 200, 300 and 400 GeV/c. Experimental data are taken from [21, 23]. (b) The corresponding scaling function $\psi(z)$.

5 Conclusions

The z-presentation of experimental data on cross sections of hadron production in $\pi^- p$ and $\pi^- A$ collisions was constructed. The anomalous fractal dimension of pion, $\delta_\pi = 0.1$, allows to reproduce the general properties of the z-scaling established in pp and pA interactions. The scaling function $\psi(z)$ demonstrates energy independence in a wide region of the colliding energy and the transverse momentum. Universal character of $\psi(z)$ for different nuclei was found. For sufficient large values of z, the scaling function is governed by a power law.

The z-scaling confirmed in $\pi^- p$ and $\pi^- A$ collisions reflects general properties of particle production such as self-similarity, locality, and fractality. The scaling function $\psi(z)$ is interpreted as the probability density to form a particle with the formation length z. The scaling can serve as an effective tool in searching for new physical phenomena in pion induced reactions at small scales.

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Самоподобие рождения адронов с большими поперечными импульсами в $\pi^- p$- и $\pi^- A$-взаимодействиях

Изучаются свойства самоподобия рождения адронов с большими поперечными импульсами в $\pi^- p$- и $\pi^- A$-взаимодействиях. Проведен анализ экспериментальных данных в рамках концепции z-скейлинга. Определена аномальная фрактальная размерность для налетающего пиона $\delta_\pi \approx 0{,}1$. Установлена энергетическая независимость z-представления экспериментальных данных. Анализ A-зависимости z-скейлинга показал, что формирование адронов в $\pi^- A$-взаимодействиях обладает свойством самоподобия.

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Self-Similarity of High-p_T Hadron Production in $\pi^- p$ and $\pi^- A$ Collisions

Self-similar properties of hadron production in $\pi^- p$ and $\pi^- A$ collisions over a high-p_T region are studied. The analysis of experimental data is performed in the framework of z-scaling. The scaling variable depends on the anomalous fractal dimension of the incoming pion. Its value is found to be $\delta_\pi \approx 0.1$. Independence of the scaling function $\psi(z)$ on the collision energy is shown. A-dependence of data z-presentation confirms self-similarity of particle formation in $\pi^- A$ collisions.

The investigation has been performed at the Veksler–Baldin Laboratory of High Energies, JINR.

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